# Test of a Superconvergent Relation for Meson-Baryon Scattering*

R. H. GRAHAM AND M. HUQ†
*Physics Department, Syracuse University, Syracuse, New York*
(Received 16 August 1967)

The assumptions that (1) meson-baryon scattering amplitudes are dominated at high energies by Regge-pole exchange and (2) the residues characterizing the couplings of the Regge poles to the pseudoscalar mesons are $SU(3)$-symmetric lead to a superconvergent relation for the following combination of forward scattering amplitudes:

$$\{[T_{\pi^-p}-T_{\pi^+p}]+[T_{K^-n}-T_{K^+n}]-[T_{K^-p}-T_{K^+p}]\}.$$

Using experimental data on total cross sections and scattering lengths, we have tested this relation and found it to be well satisfied for cutoffs on the integrals at 3, 5, and 20 BeV/$c$.

RECENTLY there has been a great deal of interest in superconvergence relations.[1] Attempts to saturate these by low-lying single-particle states have met with moderate success.[2] In this paper we will make use of experimental data on cross sections and scattering lengths to test a superconvergent relation involving a particular combination of meson-baryon forward scattering amplitudes.

We will assume here that Regge theory accounts for the high-energy behavior of the meson-baryon scattering amplitudes. Then for elastic scattering, which will be our only concern here, the forward scattering amplitude $T(s,t=0)\sim\text{const}\times s^{\alpha(t=0)}$ as $s\to\infty$ where $\alpha(0)$ is the intercept of the leading Regge trajectory. In particular the combinations[3]

$$\Delta(\pi p)\equiv[T_{\pi^-p}-T_{\pi^+p}] \quad (1)$$

and

$$\Delta(KN)\equiv\{[T_{K^-p}-T_{K^+p}]-[T_{K^-n}-T_{K^+n}]\} \quad (2)$$

receive contributions only from the $\rho$ trajectory and hence behave asymptotically as $s^{\alpha_\rho(0)}$.

In the $SU(3)$-symmetric limit, assuming vector meson exchange and degenerate masses, we have

$$\Delta(\pi p)=\Delta(KN). \quad (3)$$

This relation was first obtained by Barger and Rubin[4] and was found to be satisfied at high energies.[5] We will assume the factorization property[6] of Regge residues and further that the factored residues are related by $SU(3)$.[7,8] Then

$$\Delta(\pi p)-\Delta(KN)\underset{s\to\infty}{\sim}\text{const}\times s^{\alpha_\rho(0)-2}. \quad (4)$$

Since $\alpha_\rho(0)<1$, we can write the following (nontrivial) superconvergent relation:

$$\int_{-\infty}^{+\infty}d\omega\,\{[\text{Im}T_{\pi^-p}(\omega)-\text{Im}T_{\pi^+p}(\omega)]+[\text{Im}T_{K^-n}(\omega)-\text{Im}T_{K^+n}(\omega)]-[\text{Im}T_{K^-p}(\omega)-\text{Im}T_{K^+p}(\omega)]\}=0, \quad (5)$$

where $\omega$ is the energy of the meson in the lab system. The optical theorem

$$\text{Im}T(\omega)=2m_Nk_{\text{lab}}\sigma_{\text{tot}}(\omega) \quad (6)$$

relates the imaginary part of the forward scattering amplitude above the physical threshold to the total cross section. The relation to be tested is then

$$-\frac{2m_\pi^2}{4m_N^2}\pi g_{\pi NN}^2+\frac{1}{2m_N}\left\{(m_N-m_\Sigma)+\frac{m_\Sigma^2-m_N^2-m_K^2}{2m_N}\right\}\pi g_{KN\Sigma}^2-\frac{1}{2m_N}\left\{(m_N-m_\Lambda)+\frac{m_\Lambda^2-m_N^2-m_K^2}{2m_N}\right\}\pi g_{KN\Lambda}^2$$
$$+\int_{m_\pi}^{\infty}d\omega\,2m_N(\omega^2-m_\pi^2)^{1/2}\{\sigma_{\pi^-p}(\omega)-\sigma_{\pi^+p}(\omega)\}+\int_{m_K}^{\infty}d\omega\,2m_N(\omega^2-m_K^2)^{1/2}\{[\sigma_{K^-n}(\omega)-\sigma_{K^+n}(\omega)]$$
$$-[\sigma_{K^-p}(\omega)-\sigma_{K^+p}(\omega)]\}+\int_{\omega(\Lambda\pi)}^{m_K}d\omega\,\{\text{Im}T_{K^-n}(\omega)-\text{Im}T_{K^-p}(\omega)\}=0. \quad (7)$$

* Work supported in part by the U. S. Atomic Energy Commission.
† On leave of absence from Pakistan Atomic Energy Commission.
[1] V. de Alfaro, S. Fubini, G. Rossetti, and G. Furlan, Phys. Letters **21**, 576 (1966); L. D. Soloviev, Dubna report, 1965 (unpublished); A. P. Balachandran, P. G. O. Freund, and C. R. Schumacher, Phys. Rev. Letters **12**, 209 (1964).
[2] There have been many such attempts; among the earliest were: G. Altarelli, F. Buccella, and R. Gatto, Phys. Letters **24B**, 57 (1967); P. Babu, F. J. Gilman, and M. Suzuki, *ibid.* **24B**, 65 (1967); B. Sakita and K. C. Wali; Phys. Rev. Letters **18**, 29 (1967).
[3] $T_{PB}$ represents the forward elastic scattering amplitude for PB scattering.
[4] V. Barger and M. W. Rubin, Phys. Rev. **140**, B1366 (1965).
[5] V. Barger and M. Olsson, Phys. Rev. Letters **15**, 930 (1965).
[6] M. Gell-Mann, Phys. Rev. Letters **8**, 263 (1962); V. M. Gribov and I. Ya. Pomeranchuk, *ibid.* **8**, 343 (1962).
[7] G. Costa and A. H. Zimerman, Nuovo Cimento **46A**, 198 (1966).
[8] Notice that we need to assume that only the pseudoscalar meson coupling to the Regge trajectory is $SU(3)$ symmetric,

The experimental data on total cross sections for the processes appearing in Eq. (7) are, in general, good.[9] The total cross sections for $\pi N$ and $KN$ scattering are well known up to 20 BeV/$c$. For $\bar{K}N$ the total cross section is reliably given between 600 MeV/$c$ and 20 BeV/$c$. Below 600 MeV/$c$ the total cross sections have not been accurately determined.

At low energies the $\bar{K}N$ amplitudes can be reasonably estimated by scattering-length and effective-range approximations.[10] The effective range and scattering lengths are chosen to reproduce the $Y_0^*$ at 1405 MeV and the $Y_1^*$ at 1385 MeV which lie below the physical threshold of the $\bar{K}N$ system. We have used Kim's solution for the $S$-wave scattering lengths.[11] For the $P$ wave we have used an effective-range approximation due to Watson.[12]

The coupling constants appearing as residues of the baryon pole terms are assumed to be given by $SU(3)$ where we have taken a $D/F$ ratio of $\frac{3}{2}$ and we take $g_{\pi NN}^2/4\pi=15$.

In Table I we show the various contributions to the sum rule. We have naturally had to cut the integrals off owing to the finite range of experimental data. We have chosen three cutoffs: at 3, at 5, and at 20 BeV. Our original set of assumptions implies that the Barger-Rubin relation should be increasingly well satisfied with increasing energy. But the relation seems to hold within experimental errors down to energies in the neighborhood of 5 BeV. Each of the three cases considered represents the assumption that the relation holds identically beyond the particular cutoff chosen.

TABLE I. The contributions of the six amplitudes are shown with cutoffs at 3, 5, and 20 BeV. The uncertainties in the contributions for $T_{K^-n}$ and $T_{K^-p}$ are due to the uncertainties in the scattering-length parameters (see footnotes 11 and 12). The last but one row shows the total contributions to the left-hand side of Eq. (7). In the last row the percentage deviations from the sum rule with respect to the average magnitude of the six contributions are displayed.

| Amplitude | Pole terms | Pole terms+continuum Cutoff 3 BeV | Cutoff 5 BeV | Cutoff 20 BeV |
|---|---|---|---|---|
| $T_{\pi^-p}$ | −6.4 | 5 536 | 14 083 | 190 075 |
| $T_{\pi^+p}$ | ··· | 4 945 | 13 039 | 175 492 |
| $T_{K^-n}$ | −2.4 | 3 724±92 | 9 859±92 | 146 569 |
| $T_{K^+n}$ | ··· | 2 642 | 7 560 | 124 254 |
| $T_{K^-p}$ | −40 | 4 711±50 | 11 742±50 | 157 236 |
| $T_{K^+p}$ | ··· | 2 466 | 7 250 | 120 808 |
| Left-hand side of Eq. (7) | ··· | −572±142 | −1 149±142 | 470 |
| % deviation | ··· | (14.3±3.6)% | (10.8±1.3)% | 0.3% |

From Table I we see that, for the 20-BeV cutoff the sum rule holds to within a very small percentage (compared with the average magnitude of the individual contributions). This is not surprising as it reflects the validity of the Barger-Rubin relation (which holds over much of the high-energy region) which dominates the individual integrals. The superconvergent sum rule of Eq. (7) goes beyond the Barger-Rubin relation by constraining the meson-baryon scattering amplitudes in an energy region below that at which the Barger-Rubin sum rule holds. We can see from the table that, for the cutoff of 5 BeV the sum rule is still well satisfied and reasonably well satisfied even for the cutoff at 3 BeV.[13]

In conclusion, we have seen that a superconvergence sum rule holds for a particular combination of meson-baryon forward scattering amplitudes provided (1) these amplitudes are dominated at high energy by Regge-pole exchange and (2) the (factored) Regge residues describing the coupling of the pseudoscalar mesons to the vector meson trajectory satisfy $SU(3)$. The sum rule was tested using experimental cross-section data and found to be well satisfied for three different choices of cutoff for the integrals.


## ACKNOWLEDGMENT

The authors are grateful to Professor Ramesh Chand for helpful discussions on low-energy $\bar{K}N$ scattering.


[9] V. S. Barashenkov and V. M. Maltsev, Fortschr. Physik **9**, 549 (1961); S. J. Lindenbaum *et al.*, Phys. Rev. Letters **7**, 352 (1961); V. Cook *et al.*, in *Proceedings of the International Conference on High-Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN Scientific Information Service, Geneva, 1962), p. 364; P. L. Bastien *et al.*, in *Proceedings of the International Conference on High-Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN Scientific Information Service, Geneva, 1962), p. 373; A. N. Diddens *et al.*, Phys. Rev. Letters **10**, 262 (1963); W. Galbraith *et al.*, in *Proceedings of the Twelfth Annual Conference on High-Energy Physics, Dubna* (Atomizdat, Moscow, 1965). p. 109; J. L. Brown *et al.*, in *Proceedings of the Twelfth Annual Conference on High-Energy Physics, Dubna* (Atomizdat, Moscow, 1965), p. 715; Robert Good and Nguyen-hun Xuong, Phys. Rev. Letters **14**, 191 (1965); R. L. Cool *et al.*, *ibid.* **17**, 102 (1966); M. Ferro-Luzzi, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, California, 1967), p. 183.

[10] R. H. Dalitz and S. F. Tuan, Ann. Phys. (N. Y.) **8**, 100 (1959); Ramesh Chand, Nuovo Cimento **30**, 1445 (1963); **31**, 827 (1964).

[11] J. K. Kim, Phys. Rev. Letters **14**, 29 (1965). We take solution I of this paper, which is

$$A_{I=0}=(-1.674\pm0.038)+i(0.722\pm0.040);$$
$$A_{I=1}=(-0.003\pm0.058)+i(0.688\pm0.033).$$

The scattering lengths are in fermis.

[12] M. Watson, University of California Radiation Laboratory Report No. UCRL-10175, 1962 (unpublished). The complex scattering length at momentum $k$ of the $K$ meson is given by

$$1/A(k)=1/A(0)+\tfrac{1}{2}Rk^2,\quad I=1.$$

The complex parameters $A(0)$ and $R$ are obtained by requiring that at the position of the $Y_1^*$ resonance $A=1.357-i(0.768)$ and at 400 MeV/$c$

$$A=(-0.016\pm0.012)+i(0.0004\pm0.0004).$$

The unit of scattering length is (F)$^3$. The $I=0$ $P$-wave amplitude is neglected.

[13] As shown in Table I, the contributions of the pole terms to Eq. (7) are very small. Thus our results are not sensitive to whether or not the meson-baryon coupling constants are $SU(3)$ symmetric. Notice from Table I that the sum rule would be exactly satisfied for some energy between 5 and 20 BeV. This should not be taken too seriously since there is some uncertainty in the experimental data for this region.