dominant role in determining the spectrum of τ and τ' decay, but rather that it *can* be much more important than one would have guessed. It is always possible to have direct $K\rightarrow\rho+\pi$ weak coupling,[6,7] but we have shown here that even without any direct coupling, final-state multiple scattering can generate enough p wave to dominate the K-decay spectrum.[8]

[6] G. Barton and C. Kacser, Phys. Rev. Letters 8, 226, 353(E) (1962).

[7] M. A. B. Bég and P. C. DeCelles, Phys. Rev. Letters 8, 46 (1962).

[8] The large role played by final-state interactions in this calculation is not necessarily inconsistent with the recent successful current-commutator calculations of $K\rightarrow 3\pi$ decay [e.g., H. D. I. Abarbanel, Phys. Rev. **153**, 1547 (1967)] since the resulting matrix element has a phase near 0° or 180° and is linear in the pion energies. These two features rather than the total lack of final-state interactions are sufficient to allow the extrapolation required by current-commutator calculators.

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New Formalism for the Quantization of a Spin-$\frac{3}{2}$ Field

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The general equation satisfied by a vector-spinor field is considered and it is found that in addition to the spin-$\frac{3}{2}$ solution there are two spin-$\frac{1}{2}$ solutions of arbitrary masses. The conditions for these masses to be infinite are identical to the irreducibility conditions of the Rarita-Schwinger formalism. It is shown that a consistent quantization can be achieved, and some of the usual difficulties avoided, if the limit of infinite masses is taken after the quantization. This is similar to what happens in Lee and Yang's ξ-limiting formalism for vector bosons. It is also found that the spin-$\frac{1}{2}$ part acts as a regulator for the propagator of the field.

I. INTRODUCTION

PROBABLY the main difficulty in a relativistic field theory for high-spin particles is the one related to the quantization of the field. Since in the usual representations of such fields[1–5] there are too many components, some of these have to be eliminated as field variables. This is achieved through the imposition of supplementary conditions which permit one to express these components in terms of a smaller set of field variables. In order to obtain a consistent theory, the supplementary conditions are required to be a consequence of the Euler-Lagrange equations of motion. Subsequently, the canonical commutation relations are imposed on the set of independent field variables.[6] This is a procedure that can be applied without trouble as long as there are no interaction terms in the Lagrangian. When an interaction is introduced, however, there appear inconsistencies, mainly related to Lorentz invariance.[7,8] Serious difficulties also appear in field theories in which there are no redundant components.[9,10]

Some time ago, Lee and Yang[11] introduced the so-called ξ-limiting formalism for a massive vector-boson field. In that formalism the original equations of motion for a pure spin-1 field are modified in order to display the simultaneous presence of a scalar field. The mass of the particles associated with the scalar field goes to infinity when the equations are made to go back to the original ones. The procedure followed by Lee and Yang is, then, to quantize the fields and to calculate physical processes before taking the limit. With this prescription some of the difficulties mentioned above do not arise because all the field variables are independent. Moreover, they obtain for the field a Feynman propagator which for high values of the momentum does not have the divergent behavior of the pure spin-1 propagator. This allows them to expect finite results from a theory that would, otherwise, be unrenormalizable.[12]

We give here an analogous limiting formalism for noninteracting spin-$\frac{3}{2}$ fields. We show that in this case all the essential features of the ξ-limiting formalism are present although the theory is more involved because of the greater complexity of the spin-$\frac{3}{2}$ representations. We shall use the Rarita-Schwinger[4,5] formalism for spin-$\frac{3}{2}$ fields. In this formalism, when there are no supplementary conditions, the field represents the superposition of a spin-$\frac{3}{2}$ plus two spin-$\frac{1}{2}$ fields. We show that the mass of each one of these fields depends on the values of the parameters in the equations of motion. We see then that when the parameters attain the values that

[1] P. A. M. Dirac, Proc. Roy. Soc. (London) **A155**, 447 (1936).

[2] M. Fierz, Helv. Phys. Acta **12**, 3 (1939).

[3] M. Fierz and W. Pauli, Proc. Roy. Soc. (London) **A173**, 211 (1939).

[4] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).

[5] For general reference see H. Umezawa, *Quantum Field Theory* (North-Holland Publishing Company, Amsterdam, 1956).

[6] J. Schwinger, Phys. Rev. **82**, 914 (1951).

[7] K. Johnson and E. C. G. Sudarshan, Ann. Phys. (N.Y.) **13**, 126 (1961).

[8] J. Schwinger, Phys. Rev. **130**, 800 (1963).

[9] W. K. Tung, Phys. Rev. Letters **16**, 763 (1966).

[10] S. Chang, Phys. Rev. Letters **17**, 1024 (1966).

[11] T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962).

[12] T. D. Lee, Phys. Rev. **128**, 899 (1962).

imply the supplementary conditions, the masses of the spin-$\frac{1}{2}$ particles tend to infinity. The quantization can be carried out consistently before taking that limit, although it is necessary to introduce a negative metric in the Hilbert space for at least one of the spin-$\frac{1}{2}$ fields. This is similar to what happens in the ξ-limiting formalism of Yang and Lee, and does not present any physical difficulties as long as the spin-$\frac{1}{2}$ masses are sufficiently large.[11] We also find that the canonical commutation relations do not allow, in this formalism, a representation which might contain both a spin-$\frac{3}{2}$ and a spin-$\frac{1}{2}$ finite-mass field. There is no restriction on the way the masses tend to infinity. This freedom in the theory might be a useful feature when an interaction is introduced.

We study also the Feynman propagator, and we show that its high-momentum dependence is similar to that of a spin-$\frac{1}{2}$ field, quite independently of the value of the spin-$\frac{1}{2}$ masses as long as they are finite. It means that, as in Lee and Yang's formalism, the propagators of the redundant fields act as regulators for the spin-$\frac{3}{2}$ propagator, which would otherwise be highly divergent.

We discuss also the interaction with the electromagnetic field and we show that the diffidulties pointed out by Johnson and Sudarshan[7] do not seem to appear within this formalism.

II. KINEMATICS

In the Rarita-Schwinger formalism,[4] a spin-$\frac{3}{2}$ free field satisfies the equation[13]

$$(p-m)\psi_\mu=0\,, \tag{1}$$

and the supplementary conditions

$$\begin{aligned}\gamma_\mu\psi^\mu&=0\,,\\ p_\mu\psi^\mu&=0\,.\end{aligned} \tag{2}$$

These conditions eliminate the solutions of Eq. (1) which would represent two spin-$\frac{1}{2}$ fields.

In a Lagrangian formulation, (1) and (2) are required to be a consequence of the Euler-Lagrange equations. So we consider the most general first-order Hermitian Lagrangian density

$$\begin{aligned}\mathcal{L}(x)&\equiv-\bar\psi^\mu L_{\mu\nu}\psi^\nu\\ &=-\bar\psi^\mu(p^\lambda A_{\lambda,\mu\nu}+mC\gamma_\mu\gamma_\nu-mg_{\mu\nu})\psi^\nu\,,\end{aligned} \tag{3}$$

where

$$A_{\lambda,\mu\nu}=\gamma_\lambda g_{\mu\nu}+A(\gamma_\mu g_{\lambda\nu}+\gamma_\nu g_{\mu\lambda})+B\gamma_\mu\gamma_\lambda\gamma_\nu\,. \tag{4}$$

A, B, and C are real arbitrary constants. By variation of $\mathcal{L}$, we obtain the equation

$$L_{\mu\nu}\psi^\nu=0\,. \tag{5}$$

[13] We use throughout this paper the metric $g_{00}=-g_{11}=-g_{22}=-g_{33}=1$. p^μ is the operator $i\partial/\partial x_\mu$. The Dirac matrices satisfy $\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2g_{\mu\nu}$, and $\gamma_\mu{}^\dagger=\gamma_0\gamma_\mu\gamma_0$. We suppress the spinor indices, and $\bar\psi_\mu\equiv\psi_\mu{}^\dagger(x)\gamma_0$.

The canonical momenta conjugate to the ψ^μ are

$$\Pi_\mu\equiv\partial\mathcal{L}/\partial\frac{\partial\psi^\mu}{\partial t}=-i\bar\psi^\nu A_{0,\nu\mu}\,. \tag{6}$$

We can find also the divergenceless current.

$$J_\mu(x)\equiv-\bar\psi^\nu A_{\mu,\nu\lambda}\psi^\lambda\,. \tag{7}$$

This current suggests the scalar product

$$(\psi(x),\psi(x))=-\int\bar\psi^\nu(x)A_{0,\nu\mu}\psi^\mu(x)d^3x\,. \tag{8}$$

Multiplying (5) by γ^μ and p^μ, respectively, we obtain two coupled equations for $\gamma_\lambda\psi^\lambda$ and $p_\lambda\psi^\lambda$. Simple algebra shows that, with the restriction $A\neq-\frac{1}{2}$, if

$$B=\tfrac{1}{2}(3A^2+2A+1)\,,$$

and

$$C=A+2B\,, \tag{9}$$

then the conditions (2) follow. We shall discuss the case $A=-\frac{1}{2}$ later on. The usual treatments of spin-$\frac{3}{2}$ fields use the relations (9) and some arbitrary value of A. Hence, Eqs. (1) and (2) have, for each given momentum, a set of four spacelike solutions which correspond to the four different projections of the spin. Quantization of the field is carried out in the conventional way without major troubles as long as there are no interaction terms in the Lagrangian. When these terms are present, however, there appear difficulties[7,8] due to the nonindependence of the canonical momenta. This nonindependence follows from the fact that the matrix $A_{0,\mu\nu}$ is singular when Eqs. (9) are satisfied.

III. SOLUTIONS

We proceed now to study the solutions of Eq. (5) in the general case in which the relations (9) do not hold.

As before, we have the four spin-$\frac{3}{2}$ solutions satisfying (1) and (2). In addition, we find two spin-$\frac{1}{2}$ solutions with different masses M_i, ($i=1$, 2), which have the form

$$\psi_\mu(i)=(p_\mu+\alpha_i\gamma_\mu)\psi(i)\,, \tag{10}$$

where

$$(p-M_i)\psi(i)=0\,, \tag{11}$$

and

$$\alpha_i=[m-M_i(1+A)]/(2+4A)\,. \tag{12}$$

By substitution in (5), we find that the M_i are the roots of the equation

$$\begin{aligned}M^2(3A^2+2A+1-2B)+Mm(2A+4B-2C)\\ +m^2(4C-1)=0\,.\end{aligned} \tag{13}$$

For $A=\frac{1}{2}$, Eq. (13) does not apply. In such a case one of the solutions will be $M=2m$, independent of B and C. We will see below that the corresponding field would have to be quantized with a negative metric. Hence, we disregard this value of A. It is an immediate consequence of the equations of motion that any two solutions of different mass are orthogonal with respect to the scalar product (8). We find it convenient to have as parameters of the theory the constants A, M_1, and M_2 instead of A, B, and C.[14] We therefore get from (13)

$$C=\tfrac{1}{4}+(M_1M_2/4m^2)(3A^2+2A+1-2B), \quad (14)$$

$$B=[\tfrac{1}{2}m-2mA+(M_1M_2/2m-M_1-M_2)(3A^2+2A+1)\times[M_1M_2/m-2M_1-2M_2+4m]^{-1}. \quad (15)$$

We note that we can choose a particular value of A without losing any generality. This is a consequence of the existence of the point transformation[7]

$$\psi_\mu'=(g_{\mu\nu}+\tfrac{1}{4}\delta\gamma_\mu\gamma_\nu)\psi^\nu\equiv\Delta_{\mu\nu}\psi^\nu, \quad (16)$$

with $\delta\neq-1$. This transformation is nonsingular. It does not alter the spin-$\frac{3}{2}$ solutions nor the masses M_i. It will, however, change the values of A, B, and C, since Eq. (5) will now be

$$(\Delta^{-1})_{\mu\sigma}L^{\sigma\lambda}(\Delta^{-1})_{\sigma\nu}\psi^\nu=0.$$

Under transformation (16) we have

$$A'=(2A-\delta)/2(1+\delta),$$

which shows that A can attain the forbidden value $-\frac{1}{2}$ only if $\delta=\pm\infty$, or if the original A is already $-\frac{1}{2}$. Hence, from now on, we shall use the particular value $A=-1$. The corresponding singular values of B and C are both 1. From Eqs. (14) and (15) we learn that when $|M_1|$ and $|M_2|$ tend to infinity in any order, B and C tend to the singular values (9). This is similar to what happens in the ξ-limiting formalism for vector bosons, and suggests that the correct procedure to follow with Eq. (5) is to quantize the fields before taking the limit $|M_i|\to\infty$. The alternative procedure spoils the completeness of the solutions of the equation and is the source of some of the difficulties commonly encountered.

IV. QUANTIZATION

Since the canonical momenta (6) are independent, we can impose the anticommutation relations

$$\{\psi_\mu(x),\pi_\nu(y)\}_{x_0=y_0}=ig_{\mu\nu}\delta^3(\mathbf{x}-\mathbf{y}), \quad (17)$$

$$\{\psi_\mu(x),\psi_\nu(y)\}_{x_0=y_0}=0. \quad (18)$$

Using (6), we can write (17) in the form

$$\{\psi_\mu(x),\bar{\psi}^\lambda(y)A_{0,\lambda\nu}\}_{x_0=y_0}=-g_{\mu\nu}\delta^3(\mathbf{x}-\mathbf{y}). \quad (19)$$

The particular sign of the right side insures the correct anticommutation relations for the spin-$\frac{3}{2}$ part of the field, but the indefiniteness of $g_{\mu\nu}$ suggests that there will be difficulties with the metric of the spin-$\frac{1}{2}$ fields. To see the nature of the difficulty, let us consider (19) in greater detail. We express the field as a superposition of the spin-$\frac{3}{2}$ and the two spin-$\frac{1}{2}$ fields as follows:

$$\psi_\mu(x)=\psi_\mu^{3/2}(x)+\psi_\mu(x,1)+\psi_\mu(x,2), \quad (20)$$

where the three fields on the right side anticommute with each other. For $\psi_\mu^{3/2}(x)$ we assume, as usual,[15] the anticommutation relations

$$\{\psi_\mu^{3/2}(x),\bar{\psi}_\nu^{3/2}(y)\}=i\Lambda_{\mu\nu}(p)\Delta(x-y;m), \quad (21)$$

with

$$\Lambda_{\mu\nu}(p)=-(p+m)[g_{\mu\nu}-2p_\mu p_\nu/3m^2+(p_\mu\gamma_\nu-p_\nu\gamma_\mu)/3m-\tfrac{1}{3}\gamma_\mu\gamma_\nu], \quad (22)$$

where the derivatives are acting on x, and $\Delta(x-y;m)$ is the solution of the Klein-Gordon equation which satisfies the initial conditions

$$\Delta(\mathbf{x},0;m)=0, \quad (23)$$
$$p_0\Delta(x;m)|_{x_0=0}=-i\delta^3(\mathbf{x}).$$

The spin-$\frac{1}{2}$ fields are expressed in the form

$$\psi_\mu(x,i)=N_i(p_\mu-\tfrac{1}{2}m\gamma_\mu)\psi(x,i). \quad (24)$$

The N_i are normalization factors which will be determined later on, and $\psi(x,i)$ obeys the standard anticommutation relations for a spin-$\frac{1}{2}$ field;

$$\{\psi(x,i),\bar{\psi}(y,i)\}=i(p+M_i)\Delta(x-y;M_i). \quad (25)$$

From (25) we obtain the representation in terms of creation and annihilation operators

$$\psi(x,i)=(2\pi)^{-3/2}\sum_{\pm s}\int d^3p(|M_i/E_p|)^{1/2}\times[a_i(p,s)u_i(p,s)e^{-ip\cdot x}+b_i^\dagger(p,s)v_i(p,s)e^{ip\cdot x}], \quad (26)$$

with

$$\{b_i(p,s),b_j^\dagger(p',s')\}=\{a_i(p,s),a_j^\dagger(p',s')\}=\delta^3(\mathbf{p}-\mathbf{p}')\delta_{ss'}\delta_{ij}, \quad (27)$$

$$(p-M_i)u_i(p,s)=0, \quad (28)$$
$$(p+M_i)v_i(p,s)=0,$$

and the normalization

$$u_i^\dagger(p,s)u_i(p,s)=v_i^\dagger(p,s)v_i(p,s)=|p_0/M_i|, \quad (29)$$
$$\bar{u}_i(p,s)u_i(p,s)=-\bar{v}_i(p,s)v_i(p,s)=M_i/|M_i|.$$

From (20), (21), (24), and (25) it follows that

$$\{\psi_\mu(x),\bar{\psi}_\nu(y)\}=i\Lambda_{\mu\nu}(p)\Delta(x-y;m)+i\sum_{i=1,2}|N^i|^2(p_\mu-\tfrac{1}{2}m\gamma_\mu)(p+M_i)\times(p_\nu-\tfrac{1}{2}m\gamma_\nu)\Delta(x-y;M_i). \quad (30)$$

[14] Equation (13) can have as solution a pair of complex conjugate masses without altering the Hermiticity of the Lagrangian. We shall consider here, however, only real masses M_i.

[15] R. E. Behrends and C. Fronsdal, Phys. Rev. **106**, 345 (1957).

Multiplying (30) from the right by $A_{0,\nu\lambda}$, we obtain

$$\{\psi_\mu(x),\bar{\psi}^\nu(y)A_{0,\nu\lambda}\}=i[\Lambda_{\mu\lambda}(p)\gamma_0-\Lambda_{\mu0}(p)\gamma_\lambda]\Delta(x-y;\,m)+i\sum_{i=1,2}|N_i|^2(p_\mu-\tfrac{1}{2}m\gamma_\mu)(p+M_i)\times\{p_\lambda\gamma_0-p_0\gamma_\lambda-(M_i-m)g_{0\lambda}+[m+B(M_i-2m)]\gamma_0\gamma_\lambda\}\Delta(x-y;\,M_i). \quad (31)$$

We have to check now whether (31) is consistent with (19). We consider the (00) component of (31) at $x_0=y_0$, and get

$$\{\psi_0(x),\bar{\psi}^\nu(y)A_{0,\nu0}\}_{x_0=y_0}=\sum_{i=1,2}|N_i|^2\{(M_i-i\boldsymbol{\gamma}\cdot\nabla)(M_i-2m)(B-1)-\tfrac{1}{2}m(M_i-2m)(B-1)\}\delta^3(\mathbf{x}-\mathbf{y}). \quad (32)$$

If the right side has to be equal to $-\delta^3(\mathbf{x}-\mathbf{y})$, then we must have

$$\sum_{i=1,2}|N_i|^2(M_i-2m)=0, \quad (33)$$

and

$$(B-1)\sum_{i=1,2}|N_i|^2M_i(M_i-2m)=-1. \quad (34)$$

Substituting B from (15), we obtain

$$\sum_{i=1,2}|N_i|^2M_i(M_i-2m)=\frac{2}{3m^2}(M_1-2m)(M_2-2m). \quad (35)$$

From (33) and (35) we get

$$|N_1|^2=\frac{2}{3m^2}\frac{M_2-2m}{M_1-M_2},\qquad |N_2|^2=\frac{2}{3m^2}\frac{M_1-2m}{M_2-M_1}. \quad (36)$$

We see that the right-hand terms in (36) can not be positive simultaneously for any choice of the masses M_i. This shows that at least one of the fields $\psi(x,i)$ has to be quantized with a negative metric in the Hilbert space. We can see, too, that if one of the masses goes to infinity, the other one remaining finite, then the finite mass field has to be quantized with a negative metric. To introduce the appropriate metric we substitute $\bar{\psi}_\mu$ by $\tilde{\psi}_\mu$, where

$$\tilde{\psi}_\mu=\eta\bar{\psi}_\mu\eta^{-1}, \quad (37)$$

with

$$\eta\bar{\psi}_\mu{}^{3/2}\eta^{-1}=\bar{\psi}_\mu{}^{3/2}, \quad (38)$$

and

$$\eta\bar{\psi}_\mu(i)\eta^{-1}=\sigma_i\bar{\psi}_\mu(i). \quad (39)$$

The σ_i can be ±1 for each index, but not $+1$ for both. They are to be obtained from the modified Eq. (36), namely,

$$|N_i|^2\sigma_i=\frac{2}{3m^2}\frac{M_j-2m}{M_i-M_j},\quad (j\neq i). \quad (40)$$

The operator η is given by

$$\eta=\eta_1\eta_2,$$
$$\eta_i=\exp\{-\tfrac{1}{2}i\pi(1-\sigma_i)\sum_{\pm s}[a_i^\dagger(p,s)a_i(p,s)-b_i^\dagger(p,s)b_i(p,s)]\}. \quad (41)$$

As a consequence of the new metric, in all previous equations $\bar{\psi}_\mu$ and $|N_i|^2$ have to be substituted by $\tilde{\psi}_\mu$ and $|N_i|^2\sigma_i$, respectively. It is a simple task, now, to check that (19) and (31) are consistent. The particular values of the N_i are related to the normalization of the spin-$\frac{1}{2}$ solutions under the scalar product (8). Thus, if

$$\psi_\mu(x,i)=u_\mu(p,i)e^{-ipx},$$

then from (8) and (24), (29), and (40) we have

$$-\bar{u}^\mu(p,i)A_{0,\mu\nu}u^\nu(p,j)=\sigma_i|p_0/M_i|\delta_{ij}. \quad (42)$$

V. HAMILTONIAN

We can evaluate the Hamiltonian which because of the new metric turns out to be positive definite. We have

$$H=\int d^3x\,\pi_\mu(x)\psi^\mu(x)=-\int d^3x\,\tilde{\psi}^\mu(x)A_{0,\mu\nu}p_0\psi^\nu(x). \quad (43)$$

Using (42), we get for the normal-ordered Hamiltonian

$$H=\int d^3p[\sum_{r=1}^{4}(a^\dagger(p,r)a(p,r)+b^\dagger(p,r)b(p,r))\times(\mathbf{p}^2+m^2)^{1/2}+\sum_{i=1,2}\sum_{\pm s}(a_i^\dagger(p,s)a_i(p,s)+b_i^\dagger(p,s)b_i(p,s))(\mathbf{p}^2+M_i^2)^{1/2}, \quad (44)$$

where the $a(p,r)$ and $b(p,r)$ are the particle and antiparticle annihilation operators for the four possible states of spin-$\frac{3}{2}$.

VI. PROPAGATOR

The propagator for this field is given by the time-ordered product

$$P_{\mu\nu}(x)\equiv-i\langle0|\psi_\mu(x)\tilde{\psi}_\nu(0)|0\rangle\theta(x_0)+i\langle0|\tilde{\psi}_\nu(0)\psi_\mu(x)|0\rangle\theta(-x_0). \quad (45)$$

As a consequence of the equations of motion we have

$$L_\lambda{}^\mu(p)P_{\mu\nu}(x)=\langle0|\{A_{0,\lambda\mu}\psi^\mu(x),\tilde{\psi}_\nu(0)\}|0\rangle\delta(x_0). \quad (46)$$

From the commutation relations (19) and the fact that A_0 has an inverse, we get

$$L_\lambda{}^\mu(p)P_{\mu\nu}(x)=-g_{\lambda\nu}\delta^4(x). \quad (47)$$

We see that the propagator defined by the time-ordered product coincides with Feynman's Green's function of Eq. (5). $P_{\mu\nu}(x)$ has the Fourier representation

$$P_{\mu\nu}(x)=\frac{1}{(2\pi)^4}\int d^4p\, P_{\mu\nu}(p)e^{-ip\cdot x}, \qquad (48)$$

where

$$P_{\mu\nu}(p)=\frac{\Lambda_{\mu\nu}(p)}{p^2-m^2+i\epsilon}+\sum_{i=1,2}\left(\frac{2}{3m^2}\frac{M_j-2m}{M_i-M_j}\right)\times\left(p_\mu-\frac{m}{2}\gamma_\mu\right)\left(\frac{p+M_i}{p^2-M_i^2+i\epsilon}\right)(p_\nu-\tfrac{1}{2}m\gamma_\nu). \qquad (49)$$

Here $\Lambda_{\mu\nu}(p)$ is given by (22) and $j\neq i$. The first term is the Feynman propagator for a pure spin-$\frac{3}{2}$ field. As is well known,[5] it differs from its time-ordered product, which contains noncovariant equal-time terms. However, in (49) they do not appear. They cancel with identical terms that come from the spin-$\frac{1}{2}$ part.

For fixed masses M_i, we find from (22) and (49) that for high values of the momentum $P_{\mu\nu}(p)\sim O(1/p)$. The contribution to the propagator from the spin-$\frac{1}{2}$ part cancels the divergence of the spin-$\frac{3}{2}$ propagator, which would otherwise behave like $O(p)$.

VII. INTERACTION WITH THE ELECTROMAGNETIC FIELD

As discussed by Johnson and Sudarshan,[7] there appear inconsistencies when one introduces the interaction with the electromagnetic field in the conventional theory for a spin-$\frac{3}{2}$ field. These inconsistencies are related to the fact that the matrix $A_{0,\mu\nu}$ is singular and consequently some of the equations are constraint conditions rather than equations of motion.

This does not seem to be the case in the formalism developed here when the parameters B and C are different from their singular values because all the equations are true equations of motion. We introduce the interaction with the electromagnetic field $\mathcal{A}_\mu$ through the "minimal" substitution

$$p_\mu\rightarrow p_\mu-e\mathcal{A}_\mu, \qquad (50)$$

and we get an interaction Lagrangian

$$\mathcal{L}_I=e\bar{\psi}^\mu A_{\lambda,\mu\nu}\psi^\nu\mathcal{A}^\lambda. \qquad (51)$$

With this interaction the canonical momenta still have the form given by Eq. (6) and are still independent. The action principle[6] allows us to write for the interacting vector-spinor field the same equal-time anticommutation relations (17) and (18) we had for the free field, namely,

$$\{\psi_\mu(x),\bar{\psi}^\lambda(y)A_{0,\lambda\nu}\}_{x_0=y_0}=-g_{\mu\nu}\delta^3(\mathbf{x}-\mathbf{y}), \qquad (52)$$

$$\{\psi_\mu(x),\psi_\nu(y)\}_{x_0=y_0}=0. \qquad (53)$$

Since in (52) and (53) the fields are interacting fields, a meaning has to be given to $\bar{\psi}^\lambda(y)$. This can be done by invoking translational invariance of the theory. That is, if P_0 is the generator of time translations, we have

$$\bar{\psi}_\mu(\mathbf{y},y_0)=e^{iP_0y_0}\bar{\psi}^\lambda(\mathbf{y},0)e^{-iP_0y_0}, \qquad (54)$$

and at time $y_0=0$ we assign to $\bar{\psi}^\lambda(\mathbf{y},0)$ the same formal structure given to the free field. This will ensure the validity of (52) and (53) at all times.

The inconsistencies pointed out by Johnson and Sudarshan do not seem to appear in any obvious way within this approach. We do not know whether these will appear in a more careful examination. At least, these are reflected in the fact that the S matrix is not unitary. This is a difficulty that can be handled as in the ζ-limiting formalism by taking the masses $|M_i|$ larger than the total energy of the initial state in the calculation of any particular process.

VIII. CONCLUSION

From the preceding discussion it becomes clear that some of the usual difficulties met with in the quantization of a vector-spinor field derive from the fact that one deals with a singular equation. This equation has two spin-$\frac{1}{2}$ solutions of infinite mass, and if they are not taken into account the completeness of the field variables is lost. A correct procedure seems, therefore, to be the limiting formalism developed above. Moreover, the subsidiary conditions (2) appear as necessary for a consistent quantization, since if either of these is not satisfied one gets a spin-$\frac{1}{2}$ solution of finite mass which would have to be quantized with a negative metric. These features and the similar ones for vector bosons strongly suggest that a limiting procedure should be applicable also to fields of any spin.

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