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p-Wave Interactions in τ Decay*

R. L. SCHULT
Department of Physics, University of Illinois, Urbana, Illinois

AND

I. M. BARBOUR†
Physics Department, University of Sussex, Brighton, England
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We have calculated the effect of final-state s- and p-wave interactions on the spectra in $K \to 3\pi$ decay using the Faddeev formalism derived in an earlier paper. In our model, p waves are not produced directly by the weak interaction but only by the recoupling of pion pairs in the multiple scattering series. For large s-wave scattering lengths ($\sim 1.0 m_\pi^{-1}$), the p state has very little influence on the spectrum, and a fit to the experimental spectra requires $a_2^2 > a_0^2$. However, for small s-wave scattering lengths ($\sim 0.2 m_\pi^{-1}$) the p wave can become dominant, and fits to the data can be achieved with $a_2^2 < a_0^2$. These p-wave-dominant solutions are cutoff-sensitive, so that there is no unique fit. The final-state interaction phase is also calculated.

We have calculated the final-state s- and p-wave interactions in $K \to 3\pi$ decay using the Faddeev formalism previously derived.[1] The model has a weak-interaction matrix element which would populate the Dalitz plot uniformly if there were no final-state interactions. This means that the weak interaction itself produces no p waves between any pair of particles. However, an s-wave rescattering between one pair of pions does produce a small amount of p state in a different pairing of the pions. This small p wave may then be enhanced by whatever forces produce the ρ meson even though the ρ itself is very far from the physical region. Our numerical results show that it is possible to explain the observed density of events on the Dalitz plot for K decay with this model if two very interesting conditions are met by the s-wave interactions:

(1) The s-wave scattering lengths must be small ($\sim 0.2 m_\pi^{-1}$) so as not to dominate the p wave. The s-wave interaction cannot be removed completely, of course, because it must produce the p state by providing the initial rescattering. The small size of the scattering length required is interesting in view of Weinberg's current-commutator calculation[2] which gives $a_0 = 0.2$, $a_2 = -0.06$.[3]

(2) The second feature of interest is that, while the s-wave interactions alone produce a slope on the Dalitz plot which has the wrong sign if $a_0^2 > a_2^2$, the p-wave interaction can produce the correct sign only if driven by an $I=0$ potential which is stronger than the $I=2$ interaction.

The equations we have used in calculating the final-state interactions for $K \to 3\pi$ are those of Barbour and Schult (BS).[1] These equations assume that the two-body pion-pion scattering can be parametrized by a single separable potential for each two-pion isospin state. For the $I=1$ state this is a p-wave potential which has been fitted to a resonance at 755 MeV with a width of 96 MeV.[4] (Nonrelativistic kinematics have been used so that an exact fit to the ρ meson is not meaningful.[5]) For the $I=0$ and $I=2$ states the potential is purely s wave and can be characterized by its scattering length and effective range. In BS the weak interaction which initiates the decay was assumed to have no more than linear dependence on pion energy and in the present calculations is assumed to be constant (i.e., in the notation of BS the parameter A is zero).

The equations can be thought of as describing a multiple rescattering of the pions from each other where the weak interaction H_w produces three pions which can propagate in space (factor G_0) until a pair (j,k) of them scatter from each other (scattering amplitude t_i), after which there may be further pairwise scatterings,

$$U = H_w + \sum_{i=1}^{3} t_i G_0 H_w + \sum_{i \neq j=1}^{3} t_i G_0 t_j G_0 H_w + \cdots. \quad (1.1)$$

The series is summed by defining

$$U = H_w + \sum_{i=1}^{3} U_i, \quad (1.2)$$

where

$$U_i = t_i G_0 (H_w + \sum_{j \neq i} U_j). \quad (1.3)$$

Using separable potentials and the identity of the pions, Eq. (1.3) can be reduced to a set of three coupled

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† Present address: Department of Natural Philosophy, University of Glasgow, Glasgow, Scotland.

[1] I. M. Barbour and R. L. Schult, Phys. Rev. **155**, 1712 (1967).

[2] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966); N. Khuri, Phys. Rev. **153**, 1477 (1967); J. Sucher and C. H. Woo, Phys. Rev. Letters **18**, 723 (1967).

[3] We use the sign convention $k \cot\delta = +1/a$. This is a change from the notation of BS in order to conform to standard usage in particle physics.

[4] The parameters for the p-wave potential were given incorrectly in BS. A suitable choice is $\lambda_1 = -1.001 \times 10^{10}$ with $\beta_1 = 31.3 + 25.6i$.

[5] Since we never need the p-wave interaction in the region of the ρ-meson pole itself, the criticisms by J. L. Basdevant and R. L. Omnes [Phys. Rev. Letters **17**, 775 (1966)] apply to our use of a separable potential. We do not intend to imply that our p-wave parametrization is in any way unique, but only that it is a possible one.

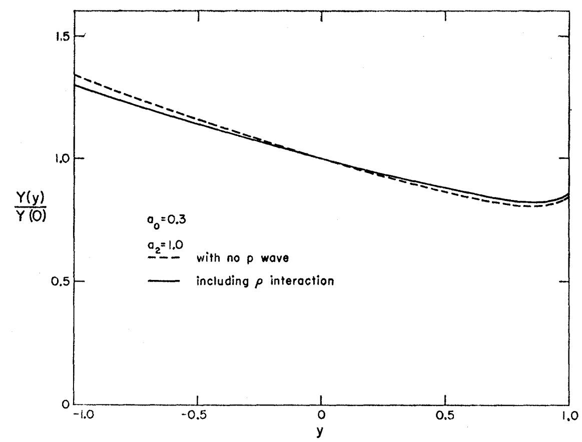

FIG. 1. The normalized odd-pion energy spectrum of τ' decay for $a_0=0.3$ and $a_2=1.0$ with and without inclusion of a resonant p-wave interaction. The τ decay spectrum is not shown since by the $\Delta I=\frac{1}{2}$ rule its slope is only half as large as for τ' decay and thus it is not as easy to display differences in τ decay slopes.

single-variable integral equations—one for each isospin state [Eq. (3.1) of BS].

If H_w is taken to be a constant $H_w(0)$ in momentum space, it produces no p waves between any pair of pions, since a p wave implies a $\cos\theta$ factor which is, of course, not constant. (In position space such an H_w is a delta function at zero separation between all three pions. The centrifugal barrier keeps the p waves from being generated by this H_w.) We therefore see that in any term of Eq. (1.1) the t_i which is nearest to the H_w can only be an s-wave interaction. Once the s-wave interaction has produced a nonconstant amplitude, however, subsequent rescattering can occur in a p state, since the energy of one pion pair is a function of the energy and angle of another pair. Specifically, if the relative momentum $\mathbf{k}_i=\frac{1}{2}(\mathbf{P}_k-\mathbf{P}_j)$ and center-of-mass momentum $\mathbf{q}_i=\mathbf{P}_j+\mathbf{P}_k=-\mathbf{P}_i$ of the pair (j,k) are used, then

$$\begin{aligned}\mathbf{k}_i^2&=\tfrac{1}{4}(\mathbf{P}_k-\mathbf{P}_j)^2\\&=\tfrac{1}{4}(\tfrac{1}{2}(\mathbf{P}_k-\mathbf{P}_i)+\tfrac{1}{2}(\mathbf{P}_k+\mathbf{P}_i)-\mathbf{P}_j)^2\\&=\tfrac{1}{4}(-\mathbf{k}_j+\tfrac{3}{2}\mathbf{q}_j)^2=\tfrac{1}{4}k_j^2+\tfrac{9}{16}q_j^2-\tfrac{3}{4}\mathbf{q}_j\cdot\mathbf{k}_j\,,\end{aligned}$$

and the $\cos\theta_j$ of the last term produces higher angular momenta for the pair (i,k).

Without an s-wave interaction the equations have only the trivial solution $U=H_w(0)$. Any effect the p-state interaction has on the Dalitz plot comes purely from multiple rescattering.

Using the method described in BS, the integral equations (1.3) [Eq. (3.1) of BS] were solved numerically for various s-wave scattering lengths both with and without inclusion of the p-wave interaction. Without the p wave we have shown in BS that a fit to the data requires the scattering lengths a_0 and a_2 to be fairly large (of order m_π^{-1}) and to satisfy the inequality $a_2^2>a_0^2$. For these large scattering lengths inclusion of the p-wave interaction has very little effect on the Dalitz plot but tends to reduce the slope slightly. (See Fig. 1.)

However, as the s-wave scattering lengths are made smaller, thereby reducing the slope on the Dalitz plot in the absence of p-wave interaction, the p-wave makes more of a difference and can even change the sign of the slope. (See Fig. 2.) This provides us with interesting alternative fits to the experimental data with small scattering lengths satisfying $a_2^2<a_0^2$. (See, for example, Fig. 3.)

The curves of Fig. 3 are not unique to the particular parameters $a_0=+0.2$, $a_2=+0.08$, but we find that in

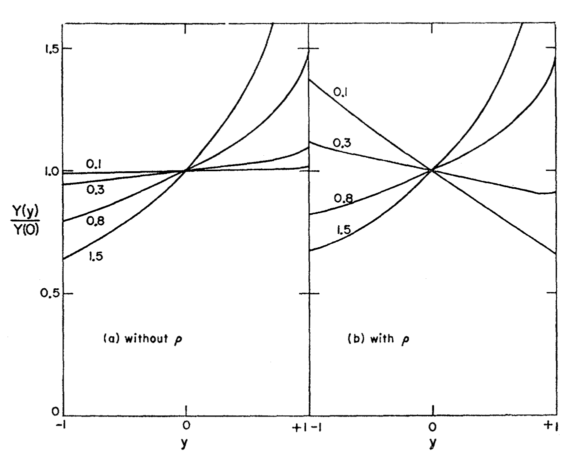

FIG. 2. The normalized odd-pion energy spectrum of τ' decay for several values of a_0 with no $I=2$ interaction included. The p-wave interaction (ρ meson) is omitted in (a) and included in (b).

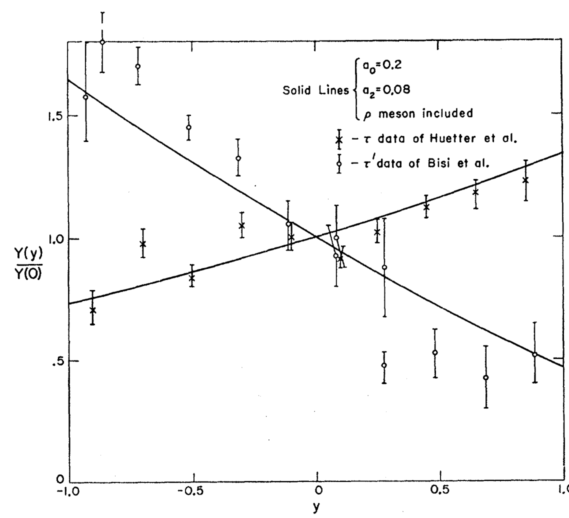

FIG. 3. The normalized odd-pion energy spectra of τ and τ' decay for resonant p-wave interaction, $a_0=0.2m_\pi^{-1}$, and $a_2=0.08m_\pi^{-1}$ are compared with experimental data. The data for τ are those of T. Huetter *et al.* [Phys. Rev. **140**, B655 (1965)] and for τ' those of V. Bisi *et al.* [Nuovo Cimento **35**, 768 (1965)].

TABLE I. The slope[a] of the odd-pion spectrum, the absolute decay rate in an arbitrary system of units, and the phase of the decay amplitude at the center of the Dalitz plot are given for τ' decay for a variety of pion-pion interactions. The s-wave parameters in the notation of BS and the corresponding effective ranges ($k\cot\delta=1/a+\frac{1}{2}r_0k^2+\cdots$) are given at the bottom of the table. The primes on the values of the scattering lengths are used to distinguish between potentials which give the same scattering length but have different effective ranges.

a_0	a_2	ρ included	Slope	Rate	Phase $-\pi<\varphi<+\pi$	Comment
···	···	no	0.0	0.105	0°	No final-state interaction.
0.8	1.5	no	−0.32	1.2×10^4	−104°	
0.8	1.5	yes	−0.32	2.3×10^4	−127°	
0.3	1.0	no	−0.33	1.8×10^5	−132°	
0.3	1.0	yes	−0.30	6.3×10^5	+150°	
0.3	···	yes	−0.12	1.2×10^2	−168°	
0.3′	···	yes	0.0	41	7°	
0.1	···	yes	−0.37	1.7×10^3	−172°	
0.1′	···	yes	−0.37	83	−179°	
0.1″	···	yes	−0.08	2.4	− 21°	
0.1′	0.1″	yes	+0.55	26	+ 11°	
0.1″	0.1′	yes	−0.69	17	+ 9°	Any of these are good fits to the data.
0.056	···	yes	−0.70	25	+170°	
0.2	0.08	yes	−0.64	44	−166°	
0.056	0.1″	yes	+0.46	33	+ 10°	

a	r_0	λ	β
1.5	0.00	-3.772×10^7	$4.742+6.708i$
0.8	0.00	-3.073×10^9	$8.89+12.57i$
0.3	0.00	-2.944×10^{12}	$23.7+33.52i$
0.3′	−0.02	-2.650×10^9	$8.89+12.57i$
0.2	0.00	-5.032×10^{13}	$35.55+50.28i$
0.1	0.00	-6.446×10^{15}	$71.11+100.6i$
0.1′	−0.004	-4.532×10^{13}	$35.55+50.28i$
0.1″	−0.10	-1.830×10^9	$8.89+12.57i$
0.08	−0.12	-1.630×10^9	$8.89+12.57i$
0.056	−0.009	-4.032×10^{13}	$35.55+50.28i$

[a] Note: Interchanging the $I=0$ and $I=2$ potentials changes the sign of the slope.

general a large slope of the proper sign is much easier to produce if the s-wave interaction is stronger for $I=0$ than for $I=2$. A few cases are summarized in Table I, where we also show the large fluctuations in the absolute rate which were already noted in BS. Also included in Table I is the phase of the decay amplitude at the center of the Dalitz plot. This phase would be zero in the absence of final-state interactions, and since the p-wave contribution vanishes at this point, the phase comes from the s-wave parts alone.

We see from Table I that the precise size of the slope is model-dependent. In particular, when the constraint of exactly zero range is relaxed to allow the effective-range parameter to be as large as $0.1m_\pi^{-1}$ (for small scattering lengths this has very little effect on the low-energy phase shift), we get results which depend on more than just the scattering length. (Compare the three $a_0=0.1$ cases in Table I.) The amount of p wave present appears to depend on the far-off-the-energy-shell structure of the amplitudes.

Another way to see this is to consider the relative sign of the s- and p-wave terms. If only the energies near the physical region were important, then we would expect that any slope generated by the s-wave interaction would be enhanced by the attractive p-wave interaction and not reduced or changed in sign. If, on the other hand, the p wave is generated by the variation of the s-wave term at large (unphysical) energies, then the enhanced p-wave amplitude has its sign determined by this distant variation rather than by the slope of the s-wave amplitude in the physical region. Our numerical solutions conform to the second alternative and several simple cutoff models can be made which illustrate this behavior.

This cutoff sensitivity is in distinct contrast to the situation with s waves only where the solution was found to be insensitive to the potential shape (see Ref. 1). There are two reasons why the p wave is more cutoff-sensitive than the s waves, both related to the fact that there is no $l=1$ state present unless some asymmetry is first produced by the s-wave interactions. Firstly, the p-wave amplitude cannot be subtracted since there is no point at which its value is known. Secondly, it must depend on an asymmetry in the s-wave interaction, since constant s-wave amplitudes also generate no p waves. This is further complicated by the fact that if the $I=0$ and $I=2$ π-π interactions were energy-dependent but identical, no pion would be singled out to distinguish $\cos\theta_j=+1$ from $\cos\theta_j=-1$, and again no p wave would be generated. [This can be seen by studying Eq. (1.27) of BS and noting that, if the $I=0$ and $I=2$ potentials are identical, a solution exists with $W(q,0)=(5/4)^{\frac{1}{2}}W(q,2)$ and $W(q,1)=0$.]

Thus, we *cannot* conclude that the ρ *must* play a dominant role in determining the spectrum of τ and τ' decay, but rather that it *can* be much more important than one would have guessed. It is always possible to have direct $K \rightarrow \rho+\pi$ weak coupling,[6,7] but we have shown here that even without any direct coupling, final-state multiple scattering can generate enough p wave to dominate the K-decay spectrum.[8]

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[7] M. A. B. Bég and P. C. DeCelles, Phys. Rev. Letters **8**, 46 (1962).

[8] The large role played by final-state interactions in this calculation is not necessarily inconsistent with the recent successful current-commutator calculations of $K \rightarrow 3\pi$ decay [e.g., H. D. I. Abarbanel, Phys. Rev. **153**, 1547 (1967)] since the resulting matrix element has a phase near 0° or 180° and is linear in the pion energies. These two features rather than the total lack of final-state interactions are sufficient to allow the extrapolation required by current-commutator calculators.

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New Formalism for the Quantization of a Spin-$\frac{3}{2}$ Field

HERMAN MUNCZEK
Department of Physics, Northwestern University, Evanston, Illinois

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The general equation satisfied by a vector-spinor field is considered and it is found that in addition to the spin-$\frac{3}{2}$ solution there are two spin-$\frac{1}{2}$ solutions of arbitrary masses. The conditions for these masses to be infinite are identical to the irreducibility conditions of the Rarita–Schwinger formalism. It is shown that a consistent quantization can be achieved, and some of the usual difficulties avoided, if the limit of infinite masses is taken after the quantization. This is similar to what happens in Lee and Yang's ξ-limiting formalism for vector bosons. It is also found that the spin-$\frac{1}{2}$ part acts as a regulator for the propagator of the field.

I. INTRODUCTION

PROBABLY the main difficulty in a relativistic field theory for high-spin particles is the one related to the quantization of the field. Since in the usual representations of such fields[1–5] there are too many components, some of these have to be eliminated as field variables. This is achieved through the imposition of supplementary conditions which permit one to express these components in terms of a smaller set of field variables. In order to obtain a consistent theory, the supplementary conditions are required to be a consequence of the Euler-Lagrange equations of motion. Subsequently, the canonical commutation relations are imposed on the set of independent field variables.[6] This is a procedure that can be applied without trouble as long as there are no interaction terms in the Lagrangian. When an interaction is introduced, however, there appear inconsistencies, mainly related to Lorentz invariance.[7,8] Serious difficulties also appear in field theories in which there are no redundant components.[9,10]

Some time ago, Lee and Yang[11] introduced the so-called ξ-limiting formalism for a massive vector-boson field. In that formalism the original equations of motion for a pure spin-1 field are modified in order to display the simultaneous presence of a scalar field. The mass of the particles associated with the scalar field goes to infinity when the equations are made to go back to the original ones. The procedure followed by Lee and Yang is, then, to quantize the fields and to calculate physical processes before taking the limit. With this prescription some of the difficulties mentioned above do not arise because all the field variables are independent. Moreover, they obtain for the field a Feynman propagator which for high values of the momentum does not have the divergent behavior of the pure spin-1 propagator. This allows them to expect finite results from a theory that would, otherwise, be unrenormalizable.[12]

We give here an analogous limiting formalism for noninteracting spin-$\frac{3}{2}$ fields. We show that in this case all the essential features of the ξ-limiting formalism are present although the theory is more involved because of the greater complexity of the spin-$\frac{3}{2}$ representations. We shall use the Rarita-Schwinger[4,5] formalism for spin-$\frac{3}{2}$ fields. In this formalism, when there are no supplementary conditions, the field represents the superposition of a spin-$\frac{3}{2}$ plus two spin-$\frac{1}{2}$ fields. We show that the mass of each one of these fields depends on the values of the parameters in the equations of motion. We see then that when the parameters attain the values that

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[5] For general reference see H. Umezawa, *Quantum Field Theory* (North-Holland Publishing Company, Amsterdam, 1956).

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