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Partially Conserved Currents and Weak Form Factors

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A modification of the Goldberger-Treiman relations (for G_A and for the induced pseudoscalar G_P) is obtained by assuming, in addition to the hypothesis of partially conserved axial-vector currents, that the matrix element of the axial-vector current between two nucleon states is dominated by the $A_1(1080)$ and π. This yields a more detailed structure of the weak form factors. The result is discussed. Extending this line of thought to the case of strangeness-changing currents, which we assume to be "partially conserved," we obtain corresponding expressions for the strangeness-changing induced scalar and pseudoscalar form factors. A relation is obtained relating the induced scalar coupling to the K_{l3} form-factor ratio $\xi=f_-(0)/f_+(0)$. These results are useful in a detailed study of the Cabibbo theory. A value for the K_{l3} parameter ξ is obtained by combining our results with the soft-pion current-algebra result of Callen and Treiman. It ranges from -0.15 to -0.10, depending on the experimental value of f_K/f_π.

I. INTRODUCTION

The concept of the partially conserved axial-vector current (PCAC)[1,2] has proved to be extremely useful in correlating parameters characterizing various physical processes. Notable among the successes are the derivation[1,2] of the Goldberger-Treiman relations[3,4]:

$$G_A(0)=-\sqrt{2}f_\pi g_{\pi NN}/M_\pi, \quad (g_{\pi NN}\simeq 1.0) \quad (1)$$

$$G_P(q^2)/G_A(0)=2M_N(q^2+M_\pi^2)^{-1}, \quad (2)$$

and the recent Adler-Weisberger[5] type calculations. While (1) is satisfied experimentally to within $\sim 13\%$, there has been no firm verification of (2). However, the results of almost all experiments on μ capture are consistent[6] with (2).

In the derivation of (1) and (2), both in the original treatment of Goldberger and Treiman[3,4] and in the PCAC version,[1,2] the spinless pion plays the central role. On the other hand, it is the spin-1 ρ meson that dominates isovector-vector transitions. One would naturally ask: What is the dynamical role of the 1^+ $A_1(1080)$ meson, which is the $SU(2)\times SU(2)$ partner[7] of the ρ meson, in axial-vector transitions? We shall adopt the point of view that the A_1 meson does play the primary role in axial-vector transitions and that the A_1 and π parameters are related in such a way that the PCAC condition is satisfied. What is gained from such a consideration is more detailed information about the structure of the weak form factors, a modification of the original Goldberger-Treiman relations. The implications of the results will be discussed.

We shall generalize the idea of PCAC to other currents, namely the strangeness-changing vector and axial-vector currents, and obtain results concerning the corresponding weak form factors. These results are useful in processes like $\nu+p\to\Lambda+\mu^+$ and in a detailed study of the Cabibbo theory.[8]

II. PCAC AND MODIFIED GOLDBERGER-TREIMAN RELATIONS

One can formulate[9] the hypothesis of PCAC in a number of ways, depending upon the degree of sophistication with which one is willing to use the language. We shall regard PCAC as meaning that *the matrix elements of the divergence of the axial-vector current are dominated by the single-particle pion state and that they vanish in the limit* $M_\pi\to 0$. To bring the 1^+ $A_1(1080)$ meson into the picture, we shall further assume that the matrix elements of the axial-vector current itself are dominated by the single-particle states A_1 and π. Consider the matrix element of the strangeness-conserving weak axial-vector current A_μ between two nucleon states[10]:

$$\langle p'|A_\mu(0)|p\rangle=\bar{u}(p')[i\gamma_\mu\gamma_5 G_A(q^2)+\gamma_5 q_\mu G_P(q^2)]u(p), \quad q=p'-p. \quad (3)$$

The dominance of this matrix element by the single-particle intermediate states A_1 and π gives

$$\langle p'|A_\mu(0)|p\rangle=\bar{u}(p')\left[f_{A_1}(q^2)\sqrt{2}g_{A_1NN}(q^2)\frac{g_{\mu\nu}+q_\mu q_\nu/M_{A_1}^2}{q^2+M_{A_1}^2}+f_\pi(q^2)\sqrt{2}\frac{g_{\pi NN}(q^2)}{M_\pi}\frac{q_\mu q_\nu}{q^2+M_\pi^2}\right]i\gamma^\nu\gamma_5 u(p), \quad (4)$$

[1] Y. Nambu, Phys. Rev. Letters 4, 380 (1960); Chou Kuang-Chao, Zh. Eksperim. i Teor. Fiz. 39, 703 (1960) [English transl.: Soviet Phys.—JETP 12, 492 (1961)].
[2] M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).
[3] M. L. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958).
[4] M. L. Goldberger and S. B. Treiman, Phys. Rev. 111, 354 (1958).
[5] W. I. Weisberger, Phys. Rev. Letters 14, 1047 (1965); S. L. Adler, Phys. Rev. Letters 14, 1051 (1965).
[6] T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. 15, 381 (1965).
[7] S. Weinberg, Phys. Rev. Letters 18, 507 (1967).
[8] N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).
[9] For a review and critique of PCAC, see, for example, Y. Nambu, Lectures given at the Istanbul Summer School (unpublished).
[10] G-parity conservation is assumed so that the $\gamma_5\sigma_{\mu\nu}q^\nu$ term is absent from the matrix element. We note, however, that even in the presence of this term, (7) and consequently (8) remain unchanged. In (3), the two nucleon states are the proton and the neutron.

where $f_{A_1}(q^2)$, $g_{A_1NN}(q^2)$, $f_\pi(q^2)$, and $g_{\pi NN}(q^2)$ are the A_1-lepton, A_1-nucleon, π-lepton, and π-nucleon form factors, respectively, the corresponding decay or coupling constants being defined to be $f_{A_1}(-M_{A_1})$, $g_{A_1NN}(-M_{A_1}^2)$, $f_\pi(-M_\pi^2)$, and $g_{\pi NN}(-M_\pi^2)$, respectively. According to (4), we have the following expressions for the weak form factors[11]:

$$G_A(q^2)=f_{A_1}(q^2)\sqrt{2}g_{A_1NN}(q^2)/(q^2+M_{A_1}^2)\,, \tag{5}$$

$$G_P(q^2)=-2M_N\left[\frac{G_A(q^2)}{M_{A_1}^2}+f_\pi(q^2)\sqrt{2}\frac{g_{\pi NN}(q^2)}{M_\pi}\frac{1}{q^2+M_\pi^2}\right]. \tag{6}$$

From (4), one obtains the matrix element of the divergence of the axial-vector current

$$\langle p'|\partial^\mu A_\mu(0)|p\rangle=2M_N\bar{u}(p')\left[\frac{f_{A_1}(q^2)\sqrt{2}g_{A_1NN}(q^2)}{M_{A_1}^2}+f_\pi(q^2)\sqrt{2}\frac{g_{\pi NN}(q^2)}{M_\pi}\frac{q^2}{q^2+M_\pi^2}\right]\gamma_5 u(p)\,. \tag{7}$$

The PCAC condition that $\langle p'|\partial^\mu A_\mu(0)|p\rangle$ vanishes in the limit $M_\pi\to 0$ requires that[12]

$$f_{A_1}(q^2)\sqrt{2}g_{A_1NN}(q^2)/M_{A_1}^2+f_\pi(q^2)\sqrt{2}g_{\pi NN}(q^2)/M_\pi=0. \tag{8}$$

Making use of (8), we obtain from (5) and (6) that

$$G_A(q^2)=-\frac{f_\pi(q^2)\sqrt{2}g_{\pi NN}(q^2)/M_\pi}{1+q^2/M_{A_1}^2}\,, \tag{9}$$

$$G_P(q^2)=G_A(q^2)2M_N\left(\frac{1-M_\pi^2/M_{A_1}^2}{q^2+M_\pi^2}\right). \tag{10}$$

When the assumption is made that $f_\pi(q^2)$ and $g_{\pi NN}(q^2)$ are slowly varying functions of q^2 so that

$$f_\pi(0)\simeq f_\pi(-M_\pi^2)\equiv f_\pi\,,$$
$$g_{\pi NN}(0)\simeq g_{\pi NN}(-M_\pi^2)\equiv g_{\pi NN}\,,$$

Eq. (9) reduces to the Goldberger-Treiman relation Eq. (1). It is interesting to note that if we forget the q^2 dependence of $f_{A_1}(q^2)$ and $g_{A_1NN}(q^2)$ in (5), or that of $f_\pi(q^2)$ and $g_{\pi NN}(q^2)$ in (9), the q^2 dependence of $G_A(q^2)/G_A(0)$ is given by

$$G_A(q^2)/G_A(0)=(1+q^2/M_{A_1}^2)^{-1}\,, \tag{11}$$

which, incidentally, is in good agreement with Adler's[13] (model-dependent) analysis of the preliminary data of the CERN high-energy neutrino experiment.[14] If this result is eventually confirmed, that would suggest that all the form factors $f_\pi(q^2)$, $g_{\pi NN}(q^2)$, $f_{A_1}(q^2)$, and $g_{A_1NN}\times(q^2)$ are quite insensitive to q^2 for a considerable range of q^2 values, a range corresponding to that of the CERN experiment. It is interesting to note that (11) is also in numerical agreement with the current-algebra result of Furlan *et al.*[15]

Equation (10) is our result for the induced pseudoscalar form factor; it is a modification of (2). Since $M_{A_1}^2\gg M_\pi^2$, the modification is not numerically significant. For the process of muon capture by hydrogen, the modification amounts to about just 2%.

For a general strangeness-conserving baryon leptonic decay $B\to B'+l+\nu$, the generalization of (9) and (10) is immediate. One only needs to replace $\sqrt{2}g_{\pi NN}(q^2)/M_\pi$ by the corresponding $\pi^\pm$-baryon form factor in (9), and $2M_N$ by $(M_B+M_{B'})$ in (10).

III. OTHER PARTIALLY CONSERVED CURRENTS

We shall assume that the strangeness-changing vector[16] and axial-vector currents are also partially conserved, in the sense of the preceding section; the matrix elements of the divergence of these currents vanish in the limit of $M_\kappa\to 0$ and $M_K\to 0$, respectively. In the case of the strangeness-changing axial-vector current, there has been some evidence[17] indicating that PCAC is a reasonably good approximation. For the strangeness-changing vector current, however, there has been no evidence at all; we are not even sure that the scalar meson $\kappa(725)$ exists.[18] Nonetheless, we shall pursue the idea of partially conserved vector current in this case as well. For definiteness, we *formally* introduce a $\kappa(M_\kappa)$ "meson" as an effective way of taking into account the effects of the strangeness-changing scalar excitations, which may or may not appear as a particle or a resonance. In addition to the assumption of partially conserved current, stated above, we shall further assume that the matrix elements of the strangeness-changing axial-vector current are dominated by $K_A(1320)$[19] and

[11] In reducing (4) to the form of (3), the Dirac equation for the nucleon spinor has been used.

[12] The relation (8) is probably known to a few experts in this field [D. Majumda (private communication)]. But the effect of the spin-1 state on the induced pseudoscalar form factor does not seem to be generally realized. After the submission of the original manuscript for publication, the work of P. Dennery and H. Primakoff [Phys. Rev. Letters **8**, 350 (1962)] was brought to our attention. These authors have a treatment similar to ours. However, they did not include the contribution of the 1^+ state to the induced pseudoscalar form factor.

[13] S. Adler, in Proceedings of the Argonne International Conference on Weak Interactions, 1965, Argonne National Laboratory Report No. ANL-7130 (unpublished).

[14] CERN NPA/Int. 65-11 April 1965 (unpublished).

[15] G. Furlan, R. Jengo, and E. Remiddi, Phys. Letters **20**, 679 (1966).

[16] It seems to be Nambu and Sakurai who first speculated on this possibility. See Y. Nambu and J. J. Sakurai, Phys. Rev. Letters **11**, 43 (1963).

[17] See, e.g., W. I. Weisberger, Phys. Rev. **143**, 1302 (1966).

[18] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

[19] The $K_A(1320)$ is not definitely known to be a 1^+ resonance, but this is a plausible interpretation of the data. This interpretation makes possible a reasonable calculation of the ratio f_K/f_π, whose deviation from 1 measures the $SU(3)$ symmetry-breaking effect on the phenomenological axial-vector Cabibbo angle. See H. T. Nieh, Phys. Rev. Letters **19**, 43 (1967); S. L. Glashow, H. J. Schnitzer, and S. Weinberg, *ibid.* **19**, 139 (1967).

K, and those of the strangeness-changing vector current by $K^*(890)$ and $\kappa(M_\kappa)$.

For strangeness-changing baryon leptonic decays $B \to B'+l+\nu$, one can readily write down relations corresponding to (9) and (10):

$$G_A(q^2)\Big|_{B\to B'+l+\nu}^{|\Delta S|=1} = -\frac{f_K(q^2)g_{K^+BB}(q^2)/M_K}{1+q^2/M_{K_A}{}^2}, \qquad (12)$$

$$\frac{G_P(q^2)}{G_A(q^2)}\Big|_{B\to B'+l+\nu}^{|\Delta S|=1} = (M_B+M_{B'})\frac{1-M_K{}^2/M_{K_A}{}^2}{q^2+M_K{}^2}. \qquad (13)$$

The effect of the $K_A(1320)$ in (13) is about 15%. If the q^2 dependence of $f_K(q^2)g_{K^+BB'}(q^2)$ is insignificant, then

$$G_A(q^2)/G_A(0)\,|^{|\Delta S|=1} = (1+q^2/M_{K_A}{}^2)^{-1}.$$

Experimentally, there is no information available concerning the q^2 dependence of the strangeness-changing axial-vector form factor.

In the case of strangeness-changing vector current, PCVC together with the dominance assumptions relate the induced scalar form factor to the vector form factor:

$$\frac{G_S(q^2)}{G_V(q^2)}\Big|_{B\to B'+l+\nu}^{|\Delta S|=1} = (M_B-M_{B'})\frac{1-M_\kappa{}^2/M_{K^*}{}^2}{q^2+M_\kappa{}^2}, \qquad (14)$$

where $G_S(q^2)$ is defined in a way similar to $G_P(q^2)$ is defined. The "induced" scalar form factor is of the second class, according to the classification of Weinberg,[20] and should vanish in the limit of $SU(3)$ symmetry.[21] Our result (14) is consistent with this general theorem, since in the $SU(3)$ limit $M_B-M_{B'}=0$. According to (14), $G_S(q^2)$ is a first-order symmetry-breaking effect. The q^2 dependence of the strangeness-changing $G_V(q^2)$ is given by $(1+q^2/M_{K^*}{}^2)^{-1}$, if the q^2 dependence of $f_{K^*}(q^2)g_{K^*BB'}(q^2)$ is slight.

When the PCVC hypothesis is applied to the K_{l3} decay, which is a vector transition, we obtain

$$\frac{f_-(q^2)}{f_+(q^2)} = (M_K{}^2-M_\pi{}^2)\frac{1-M_\kappa{}^2/M_{K^*}{}^2}{q^2+M_\kappa{}^2}, \qquad (15)$$

where the form factors $f_\pm(q^2)$ are defined by

$$\langle\pi|V_\mu{}^{\Delta S=1}(0)|K\rangle = f_+(q^2)(p_K+p_\pi) + f_-(q^2)(p_K-p_\pi), \quad q=p_K-p_\pi. \qquad (16)$$

The parameter ξ is given by[22]

$$\xi \equiv f_-(0)/f_+(0) = (M_K{}^2-M_\pi{}^2)(M_\kappa{}^{-2}-M_{K^*}{}^{-2}). \qquad (17)$$

The experimental situation concerning ξ is not yet definite; its value varies according to the kind of experiment performed.[23] If we take κ to be the $\kappa(725)$ (however, we are noncommital), we get $\xi=+0.14$. If the K^* form factors $f_{K^*}(q^2)$ and $g_{K^*K\pi}(q^2)$ are reasonably slowly varying functions of q^2, the q^2 dependence of $f_+(q^2)$ is approximately given by

$$f_+(q^2)/f_+(0) \simeq (1+q^2/M_{K^*}{}^2)^{-1}. \qquad (18)$$

The recent fit of Kalmus[24] seems to confirm this. In terms of the usual linear parametrization:

$$f_\pm(q^2)/f_\pm(0) \equiv (1+\lambda_\pm q^2/M_\pi{}^2), \qquad (19)$$

the results (15), (17), and (18) correspond to

$$\lambda_+ \simeq -M_\pi{}^2/M_{K^*}{}^2 \qquad (20)$$

$$\lambda_- \simeq -(M_\pi{}^2/M_{K^*}{}^2+M_\pi{}^2/M_\kappa{}^2) = \lambda_+(2+3.45\xi). \qquad (21)$$

It would be very interesting to see whether the relation (21) will agree with experiment. As of now, there is no experimental information about λ_-.

It is interesting to combine (15) and (18) with the soft-pion current-algebra result of Callen and Treiman,[25] which is

$$[f_+(-M_K{}^2)+f_-(-M_K{}^2)]/f_+(0) = f_K/f_\pi. \qquad (22)$$

This will yield information about the effective-mass value M_κ and, in turn, a value for the parameter ξ. From (15), (18), and (22), we obtain

$$\frac{1}{1-M_K{}^2/M_{K^*}{}^2}\left[1+\left(\frac{M_K{}^2-M_\pi{}^2}{M_{K^*}{}^2}\right)\times\frac{1-M_\kappa{}^2/M_{K^*}{}^2}{M_\kappa{}^2/M_{K^*}{}^2-M_K{}^2/M_{K^*}{}^2}\right] = \frac{f_K}{f_\pi}, \qquad (23)$$

or

$$\frac{M_\kappa{}^2}{M_{K^*}{}^2} = \frac{0.74 f_K/f_\pi - 0.07}{2.38 f_K/f_\pi - 2.45}. \qquad (24)$$

This yields

$$\begin{aligned} M_\chi{}^2/M_{K^*}{}^2 &= 2.0 \quad \text{for} \quad f_K/f_\pi = 1.20 \text{ (see Ref. 26)}; \\ &= 1.5 \quad \text{for} \quad f_K/f_\pi = 1.28 \text{ (see Ref. 27)}. \end{aligned} \qquad (25)$$

We note that for $M_\kappa = 725$ MeV, Eq. (24) can be satisfied only with the unreasonably large value $f_K/f_\pi=1.85$. This seems to discredit the $\chi(725)$.[28] Substituting (24)

[20] S. Weinberg, Phys. Rev. **112**, 1375 (1958).

[21] See, e.g., L. Wolfenstein, Phys. Rev. **135**, B1436 (1964).

[22] Professor B. W. Lee kindly pointed out that this relation is contained in Primakoff's 1962 Bergen lectures. See H. Primakoff, in *Weak Interactions and Topics in Dispersion Physics*, edited by C. Fronsdal (W. A. Benjamin, Inc., New York, 1963).

[23] For a view, see, for example, N. Cabibbo, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, California, 1967).

[24] George E. Kalmus and A. Kernan, Phys. Rev. **159**, 1187 (1967).

[25] C. G. Callen and S. B. Treiman, Phys. Rev. Letters **16**, 153 (1966).

[26] L. B. Auberbach, J. M. Dobbs, A. K. Mann, W. K. McFarlane, D. H. White, R. Coester, P. T. Eschstruth, G. K. O'Neill, and D. Yount, Phys. Rev. **155**, 1505 (1967).

[27] Reference quoted in Ref. 23.

[28] In Sec. IV, we shall point out another possible difficulty with the $\kappa(725)$.

into (17), we obtain a formula for the parameter ξ:

$$\xi=0.29\left[\frac{2.38f_K/f_\pi-2.45}{0.74f_K/f_\pi-0.07}-1\right], \qquad (26)$$

which yields

$$\begin{aligned}\xi&=-0.15 \quad \text{for} \quad f_K/f_\pi=1.20\\ &=-0.10 \quad \text{for} \quad f_K/f_\pi=1.28.\end{aligned} \qquad (27)$$

Since the experimental results concerning this parameter are still conflicting, we will not take seriously the comparison of (27) with experiment. It is hopeful that a general clarification of the experimental situation is in view, presumably by including the q^2 dependence of the form factors in the analysis of the data, which will enable a meaningful comparison of the result (27) with experiment.

Before concluding this section, we derive one more useful relation by combining (14) and (17):

$$\left.\frac{G_S(0)}{G_V(0)}\right|^{|\Delta S|=1}=\frac{M_B-M_{B'}}{M_K{}^2-M_\pi{}^2}\xi, \qquad (28)$$

which correlates the strangeness-changing induced scalar coupling constant to the K_{l3} parameter ξ. It is our opinion that this relation may be a relatively reliable result. Since the induced scalar coupling is still experimentally unknown, we are not able to compare it with experiment. In the absence of any other theoretical estimate of the strangeness-changing scalar coupling constant, the relation (28) is useful, at least as a rough guide, for processes like $\nu+p\rightarrow\Lambda+\mu^+$ and in a detailed analysis of the baryon-leptonic-decay data.

IV. CONCLUDING REMARKS

We have pursued the idea of a "partially conserved current" in the context that the matrix elements of the partially conserved current are dominated by one spin-1 and one spin-0 state. By bringing the spin-1 particles into the picture, we have obtained somewhat more detailed results concerning the various weak form factors. They are the modified version of the Goldberger-Treiman type relations. While the effect of the spin-1 state on the induced pseudoscalar coupling is negligibly small in the strangeness-conserving case, it amounts to about 15% in the strangeness-changing case. We have derived a formula relating the strangeness-changing induced scalar coupling constant to the K_{l3} decay parameter ξ. These results are useful in a detailed study of the Cabibbo theory.[8]

A value for the K_{l3} parameter ξ is predicted by combining our results with the soft-pion current-algebra result of Callen and Treiman.[25] It ranges from -0.15 to -0.10 depending upon the experimental value of f_K/f_π. A meaningful comparison with experiment awaits a general clarification of the experimental situation.

It is interesting to point out a general pattern of the various strong form factors. If the final analysis of the CERN high-energy neutrino experiment confirms Adler's fit[13] of the strangeness-conserving axial-vector form factor,

$$G_A(q^2)/G_A(0)=(1+q^2/M^2)^{-1}, \quad M\simeq1200 \text{ MeV},$$

the pion and A_1 form factors, $f_\pi(q^2)$, $g_{\pi NN}(q^2)$, and $g_{A_1NN}(q^2)$, would seem to be slowly varying functions of q^2 for a considerable range of the q^2 values. We note that this seems to be the case for the strong K^* form factors as is indicated by the recent fit[24] of the K_{l3} form factor $f_+(q^2)$:

$$f_+(q^2)/f_+(0)=(1+q^2/M^2)^{-1}, \quad M=810_{-140}{}^{+320} \text{ MeV}.$$

We also recall that the fits of the nucleon electromagnetic form factors suggest that the strong ρ form factors also approximately exhibit similar properties.

Finally, we should like to comment on the relation (8). Without further information on either f_{A_1} or g_{A_1NN}, one is not able to say any thing more than (8). But we know beyond reasonable doubt what f_{A_1} is. According to the results of Weinberg,[7]

$$f_{A_1}{}^2\simeq f_\rho{}^2\simeq 2M_\rho{}^2f_\pi{}^2\simeq M_{A_1}{}^2f_\pi{}^2.$$

Combining this result with (8) yields[29]

$$g_{A_1NN}\simeq(M_{A_1}/M_\pi)g_{\pi NN}\simeq 9.4,$$

where the notation is such that $\sqrt{2}g_{A_1NN}$ is the $A_1{}^+\bar{p}n$ coupling constant in the form of axial-vector coupling. This result may be of interest to the Regge-pole theorists. It would be interesting to see if the inclusion of the A_1 meson can provide a better understanding of the strong scattering processes, such as $N+N\rightarrow N+N$, $\pi+N\rightarrow\rho+N$, etc. But this is outside the scope of the present note.

One could similarly derive information about the coupling constants $g_{K_ABB'}$, $g_{\kappa BB'}$, and $g_{\kappa K\pi}$. This would require knowledge in f_{K_A} and f_κ. It is provided by a generalization of Weinberg's results. According to this generalization[30]

$$f_{K_A}{}^2\simeq f_{A_1}{}^2\simeq f_\rho{}^2,$$
$$f_\kappa\simeq 0.75f_\pi.$$

Using this, the coupling constant $g_{\chi K\pi}$ can be expressed in terms of f_π and $f_+(0)$. If we identify κ to be the $\kappa(725)$, the calculated decay width will be

$$\Gamma(\kappa(725)\rightarrow K+\pi)\simeq 90 \text{ MeV},$$

which disagrees with the reported $\kappa(725)$ width of $\lesssim 15$

[29] The same numerical result for g_{A_1NN} is obtained by assuming the dominance of the vector and axial-vector transitions by ρ and A_1, respectively, and making use of $f_{A_1}\simeq f_\rho$ and the conserved-vector-current theory.

[30] S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 139 (1967).

MeV. This seems to be another indication against the $\kappa(725)$.

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Are Quarks Really Heavy?*

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Superconvergence relations for quark-antiquark elastic scattering and quark-antiquark scattering to 0^- mesons are derived and saturated with S-wave quark-antiquark states. It is found that the masses of the vector mesons are the sum of the masses of the constituent quarks. A high symmetry of quark-antiquark couplings to 0^- and 1^- mesons is also found.

I. INTRODUCTION

In this paper we consider the consequences of the following assumptions:

(i) Quarks exist as physical particles (and not only as mathematical entities).

(ii) They obey dynamics that are conventional enough to satisfy a Regge representation.

(iii) Trajectories with nonintegral baryon number either do not exist or are far to the right, so that their intercept at the origin is negative.

Assumption (i) is quite controversial and far from settled, and this is true *a fortiori* for assumption (ii). Assumption (iii) for the one-quark trajectory is equivalent to assuming that the quarks are elementary particles. For multiquark trajectories, it either means that multiquark states with nonintegral baryon number do not exist[1] or that the quark mass is very large (similar assumptions have been previously made for $I=2$ trajectories).[2] Our results support the former view.

Assumptions (ii) and (iii) allow us to derive superconvergence relations for quark-antiquark elastic scattering and for quark-antiquark scattering to pseudoscalar mesons. We saturate these sum rules with S-wave quark-antiquark states (pseudoscalar and vector mesons), and derive relations among masses and coupling constants.

In Sec. II we examine the relations following from quark-antiquark scattering to 0^- mesons; the elastic quark-antiquark scattering is examined in Sec. III. We discuss our results in the final section.

II. QUARK-ANTIQUARK SCATTERING TO 0^- MESONS

The general form for the T matrix is

$$\bar{v}(p')[-A+i\gamma\cdot QB]u(p)$$
$$Q=\tfrac{1}{2}(q+q'), \tag{1}$$

where p, p' (q,q') are the momenta of the quark and antiquark (two pseudoscalar mesons), respectively. $u(p)$ and $\bar{v}(p)$ are the quark and antiquark spinors.

Regge trajectories in the t channel have nonintegral baryon number; assumption (iii) along with the results of Trueman and Wick[3] ensure that the following combination of A and B will superconverge for fixed momentum transfer:

$$F\equiv\tfrac{1}{2}(t+M^2-\mu^2)A+\mathfrak{F}(s,t)B, \tag{2}$$

where

$$\mathfrak{F}(s,t)\equiv\frac{(2t+s-2M^2-2\mu^2)}{4}\times\left\{1-\frac{1}{t-4M^2}[2t+s-2M^2-2\mu^2+2(2t(M^2+\mu^2)-t^2-st-(M^2-\mu^2)^2)^{1/2}]\right\}, \tag{3}$$

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[1] Strictly speaking, the absence of multiquark resonances with nonintegral baryon number imply the vanishing of the corresponding trajectory only if it extends to infinity as $s\rightarrow\infty$.

[2] See, for example, V. de Alfaro, S. Fubini, G. Rossetti, and G. Furlan, Phys. Letters **21**, 576 (1966).

[3] T. L. Trueman and G. C. Wick, Ann. of Phys. (N. Y.) **26**, 322 (1964). I am indebted to Dr. A. M. Gleeson for a crucial discussion on this point.