Event Horizons in Static Vacuum Space-Times

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The following theorem is established. Among all static, asymptotically flat vacuum space-times with closed simply connected equipotential surfaces g_{00}=constant, the Schwarzschild solution is the only one which has a nonsingular infinite-red-shift surface $g_{00}=0$. Thus there exists no static asymmetric perturbation of the Schwarzschild manifold due to internal sources (e.g., a quadrupole moment) which will preserve a regular event horizon. Possible implications of this result for asymmetric gravitational collapse are briefly discussed.

1. INTRODUCTION

THE peculiar properties of the infinite-red-shift surface $g_{00}=0$ $(r=2m)$ in Schwarzschild's spherically vacuum field, and the question of whether analogous surfaces exist in asymmetric space-times[1–4] have become a focus of attention in connection with recent interest in gravitational collapse.

For static fields (to which we confine ourselves in this paper) the history of an infinite-red-shift surface can be defined as a 3-space S on which the Killing vector becomes null. Then S itself is null, and acts as a stationary unidirectional membrane for causal influence.[2]

In the special case of axial symmetry, the effect on S of static perturbations of the Schwarzschild metric can be worked out explicitly.[1,3,5] A fundamental difference emerges according to whether the source of the perturbation is external or internal. If the perturbation is due solely to the presence of exterior bodies, and if it is not too strong (e.g., if the spherically symmetric particle is encircled by a ring of mass some distance away), the effect is merely to distort S while preserving its essential qualitative features as a nonsingular event horizon.[3] On the other hand, superimposing a quadrupole moment q, no matter how small, causes S to become singular.[1] (The square of the four-dimensional Riemann tensor diverges according to

$$R_{ABCD}R^{ABCD}\sim q^2/g_{00} \quad \text{as} \quad g_{00}\to 0). \tag{1}$$

A study of small (linearized) static perturbations of the Schwarzschild manifold[4] points to similar conclusions.

Partial results of this type suggest strongly that Schwarzschild's solution is uniquely distinguished among all static, asymptotically flat, vacuum fields by the fact that it alone possesses a nonsingular event horizon. It is the aim of this paper to give a precise formulation (see Sec. 4) and proof of this conjecture.

2. IMBEDDING FORMULAS

We begin by collecting some general formulas for the immersion of hypersurfaces in an $(n+1)$-dimensional Riemannian space.[6]

Let the equations

$$x^\alpha = x^\alpha(\theta^1,\cdots,\theta^n,V), \quad V=\text{const} \tag{2}$$

represent an orientable hypersurface Σ with unit normal $\mathbf{n}$;

$$\mathbf{n}\cdot\mathbf{e}_{(i)}=0, \quad \mathbf{n}\cdot\mathbf{n}=\epsilon(\mathbf{n})=\begin{cases}+1 & (\text{spacelike } \mathbf{n})\\ -1 & (\text{timelike } \mathbf{n})\end{cases}. \tag{3}$$

The n holonomic base vectors $\mathbf{e}_{(i)}$ tangent to Σ, with components

$$e_{(i)}{}^\alpha = \partial x^\alpha/\partial\theta^i, \tag{4}$$

are such that an infinitesimal displacement in Σ has the form $\mathbf{e}_{(i)}d\theta^i$.

The Gauss-Weingarten relations

$$\delta e_{(a)}{}^\mu/\delta\theta^b = -\epsilon(\mathbf{n})K_{ab}n^\mu + \Gamma_{ab}{}^c e_{(c)}{}^\mu \tag{5}$$

decompose the absolute derivative $\delta/\delta\theta^b$ [referred to the $(n+1)$-dimensional metric] of the vector $\mathbf{e}_{(a)}$ with respect to the $(n+1)$-dimensional basis $\{\mathbf{e}_{(i)},\mathbf{n}\}$. They may be regarded as defining the extrinsic curvature tensor K_{ab} and the intrinsic affine connection $\Gamma_{ab}{}^c$ of Σ. From (3) and (5),

$$\delta n^\mu/\delta\theta^b = K_b{}^a e_{(a)}{}^\mu. \tag{6}$$

The Ricci commutation relations

$$\left(\frac{\delta}{\delta\theta^c}\frac{\delta}{\delta\theta^b}-\frac{\delta}{\delta\theta^b}\frac{\delta}{\delta\theta^c}\right)e_{(a)}{}^\mu = -R^\mu{}_{\alpha\beta\gamma}e_{(a)}{}^\alpha\frac{\partial x^\beta}{\partial\theta^b}\frac{\partial x^\gamma}{\partial\theta^c} \tag{7}$$

lead, with the aid of (5) and (6), to the equations of

[1] A. G. Doroshkevich, Ya. B. Zel'dovich, and I. D. Novikov, Zh. Eksperim. i Teor. Fiz. **49**, 170 (1965) [English transl.: Soviet Phys.—JETP **22**, 122 (1966)].
[2] C. V. Vishveshwara, University of Maryland Report, 1966 (unpublished).
[3] L. A. Mysak and G. Szekeres, Can. J. Phys. **44**, 617 (1966); W. Israel and K. A. Khan, Nuovo Cimento **33**, 331 (1964).
[4] T. Regge and J. A. Wheeler, Phys. Rev. **108**, 1063 (1957).
[5] G. Erez and N. Rosen, Bull. Res. Council Israel **F8**, 47 (1959).

[6] Greek indices run from 1 to $n+1$. Italic indices distinguish quantities defined on the imbedded manifold (e.g., R_{abcd} is the intrinsic curvature tensor of Σ) and have the range $1-n$. Covariant differentiation with respect to the $(n+1)$-dimensional or n-dimensional metric is denoted by a stroke or a semicolon, respectively.

Gauss and Codazzi,

$$R_{\alpha\beta\gamma\delta}e_{(a)}{}^{\alpha}e_{(b)}{}^{\beta}e_{(c)}{}^{\gamma}e_{(d)}{}^{\delta} = R_{abcd}+\epsilon(\mathbf{n})(K_{ad}K_{bc}-K_{ac}K_{bd}), \quad (8)$$

$$R_{\alpha\beta\gamma\delta}n^{\alpha}e_{(b)}{}^{\beta}e_{(c)}{}^{\gamma}e_{(d)}{}^{\delta}=K_{bc;d}-K_{bd;c}. \quad (9)$$

We consider next a regular family of hypersurfaces (2), parametrized so that θ^i are constant along the orthogonal trajectories: $n^{\alpha}\partial\theta^i/\partial x^{\alpha}=0$. Then

$$\partial\theta^i/\partial x^{\alpha}=g^{ij}e_{(j)\alpha}\equiv e^{(i)}{}_{\alpha}, \quad (10)$$

$$\partial x^{\alpha}(\theta^i,V)/\partial V=\rho n^{\alpha}, \quad (11)$$

with ρ defined by[7]

$$n_{\alpha}=\epsilon(\mathbf{n})\rho\partial_{\alpha}V, \quad (12)$$

i.e.,

$$\rho=[\epsilon(\mathbf{n})g^{\alpha\beta}(\partial_{\alpha}V)(\partial_{\beta}V)]^{-1/2}. \quad (13)$$

It follows that

$$\rho^2V_{|\mu\nu}n^{\nu}=-\partial_{\mu}\rho. \quad (14)$$

From (4), (11), and (6),

$$\delta e_{(i)}{}^{\mu}/\delta V=\delta(\rho n^{\mu})/\delta\theta^i=(\partial_i\rho)n^{\mu}+\rho K_i{}^je_{(j)}{}^{\mu}, \quad (15)$$

and hence

$$\partial g_{ab}/\partial V=(\delta/\delta V)(\mathbf{e}_{(a)}\cdot\mathbf{e}_{(b)})=2\rho K_{ab}, \quad (16)$$

$$\delta n_{\alpha}/\delta V=-\epsilon(\mathbf{n})(\partial_{\alpha}\rho)e_{\alpha}{}^{(a)}, \quad (17)$$

with the aid of (3). Again from (6),

$$K_{ab}=e_{(a)}{}^{\mu}\delta n_{\mu}/\delta\theta^b=e_{(a)}{}^{\mu}n_{\mu|\nu}e_{(b)}{}^{\nu}=\rho e_{(a)}{}^{\mu}e_{(b)}{}^{\nu}V_{|\mu\nu}. \quad (18)$$

By virtue of the completeness relation

$$g^{\mu\nu}=g^{ab}e_{(a)}{}^{\mu}e_{(b)}{}^{\nu}+\epsilon(\mathbf{n})n^{\mu}n^{\nu}, \quad (19)$$

(14) and (18) yield

$$V_{|\mu\nu}=\rho^{-1}K_{ab}e^{(a)}{}_{\mu}e^{(b)}{}_{\nu}-\epsilon(\mathbf{n})\rho^{-2}(\partial_a\rho)\times(e^{(a)}{}_{\mu}n_{\nu}+e^{(a)}{}_{\nu}n_{\mu})-\epsilon(\mathbf{n})\rho^{-3}(\partial\rho/\partial V)n_{\mu}n_{\nu}. \quad (20)$$

For the mean curvature $K\equiv g^{ab}K_{ab}$, we thus obtain

$$K=\rho g^{\mu\nu}V_{|\mu\nu}+\epsilon(\mathbf{n})\rho^{-2}\partial\rho(V,\theta^i)/\partial V. \quad (21)$$

We form $[(\delta/\delta V)(\delta/\delta\theta^i)-(\delta/\delta\theta^i)(\delta/\delta V)]n^{\mu}$, and take into account (5), (6), (15), (17), and the Ricci commutation relations. The result is

$$\rho R_{\alpha\mu\nu\beta}e_{(a)}{}^{\alpha}n^{\mu}n^{\nu}e_{(b)}{}^{\beta}=\epsilon(\mathbf{n})\rho_{;ab}+g_{ap}\partial K_b{}^p/\partial V+\rho K_{ap}K_b{}^p. \quad (22)$$

Contraction of (8), (9), and (22) with use of (19) yields the decomposition of the Ricci tensor $R_{\alpha\beta}\equiv R^{\mu}{}_{\alpha\beta\mu}$ and the associated Einstein tensor $G_{\alpha\beta}$ with respect to the basis $\{\mathbf{e}_{(i)},\mathbf{n}\}$;

$$2G_{\alpha\beta}n^{\alpha}n^{\beta}=-\epsilon(\mathbf{n})g^{ab}R_{ab}+K^{ab}K_{ab}-K^2, \quad (23)$$

$$R_{\alpha\beta}n^{\alpha}e_{(b)}{}^{\beta}=\partial_bK-K_b{}^c{}_{;c}, \quad (24)$$

$$R_{\alpha\beta}e_{(a)}{}^{\alpha}e_{(b)}{}^{\beta}=R_{ab}+\rho^{-1}\rho_{;ab}+\epsilon(\mathbf{n})KK_{ab}+\epsilon(\mathbf{n})\rho^{-1}g_{ap}\partial K_b{}^p/\partial V. \quad (25)$$

[7] It is assumed that the right side of (13) vanishes nowhere in the region of interest; see Sec. 4.

3. STATIC VACUUM FIELDS

A space-time manifold is called static if it admits a hypersurface-orthogonal Killing vector field $\boldsymbol{\xi}$ which is timelike over some domain. From this definition it follows that the four-dimensional curl of $V^{-2}\boldsymbol{\xi}$ vanishes, if $V\neq 0$, where

$$V^2=|\boldsymbol{\xi}\cdot\boldsymbol{\xi}|. \quad (26)$$

Hence, in a simply connected region which has $\boldsymbol{\xi}\cdot\boldsymbol{\xi}<0$ throughout, there will exist a scalar field t such that $V^{-2}\boldsymbol{\xi}=\nabla t$. If we introduce special coordinates $x^0=t$, x^1, x^2, x^3 chosen so that $(\boldsymbol{\xi}\cdot\nabla)x^{\alpha}=0$, the line element reduces locally to

$$ds^2=g_{\alpha\beta}(x^1,x^2,x^3)dx^{\alpha}dx^{\beta}-V^2dt^2,\quad V=V(x^1,x^2,x^3). \quad (27)$$

(Greek indices run from 1 to 3; lower case and upper case italic indices are reserved for the ranges 1 to 2 and 0 to 3, respectively.)

In terms of the three-dimensional quantities characterizing a hypersurface $t=$ const, the vanishing of the four-dimensional Ricci tensor for a static *vacuum* manifold is expressible as follows[8]:

$$g^{\alpha\beta}R_{\alpha\beta}=0,\quad R_{\alpha\beta}+V^{-1}V_{|\alpha\beta}=0 \quad (V\neq 0). \quad (28)$$

Thus V is harmonic;

$$g^{\alpha\beta}V_{|\alpha\beta}=0 \quad (V\neq 0). \quad (29)$$

These equations are again recast by projecting onto the two-dimensional subspaces $V=\text{const}\neq 0$ of the 3-spaces $t=$ const. We apply the formulas of Sec. 2 with $n=2$, $\epsilon(\mathbf{n})=+1$. From (21) and (29),

$$\partial\rho/\partial V=\rho^2K. \quad (30)$$

On the other hand, we have from (16),

$$\partial\ln(\sqrt{g})/\partial V=\rho K, \quad (31)$$

where g is the 2×2 determinant of g_{ab}. Hence

$$(\partial/\partial V)[(\sqrt{g})/\rho]=0. \quad (32)$$

Equations (23) and (24) [in combination with (28),

[8] This is immediately verifiable from the general formulas (23), (25) of the previous section (with appropriate changes of notation), upon noting from (16) that the extrinsic curvature $\frac{1}{2}V^{-1}\partial g_{\alpha\beta}/\partial t$ of the 3-spaces $t=$const vanishes; see A. Lichnerowicz, *Théories Relativistes de la Gravitation et de L'électromagnétisme* (Masson, Paris, 1955), Chap. VII.

(14), and (30)] yield, respectively,

$$^{(2)}R \equiv g^{ab}R_{ab} = K_{ab}K^{ab} - K^2 - 2K/\rho V\,, \qquad (33)$$

$$\partial_a \rho = \rho^2 V(\partial_a K - K_a{}^b{}_{;b})\,, \qquad (34)$$

From (25) we obtain

$$(\partial/\partial V + V^{-1})K_a{}^b = -\rho_{;a}{}^{;b} - \tfrac{1}{2}\rho\,{}^{(2)}R\delta_a{}^b - \rho K K_a{}^b\,, \qquad (35)$$

with the aid of (28), (18), and (16).

Equations (16), (30), and (35) provide a complete system for determining the variables g_{ab}, ρ, K_{ab} as functions of V. They respect the constraint equations (33) and (34); i.e., if (33) and (34) are satisfied for one value of V they will hold identically.[9]

Finally, we record an expression for the invariant square of the four-dimensional Riemann tensor $^{(4)}R_{ABCD}$. From the general formulas (8), (9), and (22) (with appropriate changes of notation) we readily find

$$^{(4)}R_{\alpha\beta}{}^{\gamma\delta} = {}^{(3)}R_{\alpha\beta}{}^{\gamma\delta} = \epsilon_{\alpha\beta\mu}\epsilon^{\gamma\delta\nu}\,{}^{(3)}G_\nu{}^\mu\,, \qquad (36)$$

$$^{(4)}R_{0\alpha\beta\gamma} = 0\,, \quad {}^{(4)}R_{\alpha 00\beta} = -VV_{|\alpha\beta}\,,$$

where $\epsilon_{\alpha\beta\gamma}$ is the Levi-Civita permutation symbol. Hence

$$\tfrac{1}{4}\,{}^{(4)}R_{ABCD}\,{}^{(4)}R^{ABCD} = {}^{(3)}G_{\mu\nu}\,{}^{(3)}G^{\mu\nu} + V^{-2}V_{|\mu\nu}V^{|\mu\nu} \qquad (37)$$

for an arbitrary static field. *In vacuo*, this reduces, by virtue of (28) and with the aid of (20), to

$$\tfrac{1}{8}\,{}^{(4)}R_{ABCD}\,{}^{(4)}R^{ABCD} = V^{-2}V_{|\mu\nu}V^{|\mu\nu} \qquad (38)$$

$$= (V\rho)^{-2}[K_{ab}K^{ab} + 2\rho^{-2}\rho_{;a}\rho^{;a} + \rho^{-4}(\partial\rho/\partial V)^2]\,. \qquad (39)$$

4. THE THEOREM

In a static space-time, let Σ be any spatial hypersurface $t=$ const, maximally extended consistent with $\xi\cdot\xi<0$. We consider the class of static fields such that the following conditions are satisfied on Σ:

(a) Σ is regular, empty, noncompact, and "asymptotically Euclidean." More precisely, the last term means that the metric (27) (in suitable coordinates) has the asymptotic form

$$g_{\alpha\beta} = \delta_{\alpha\beta} + O(r^{-1})\,, \quad \partial_\gamma g_{\alpha\beta} = O(r^{-2})\,,$$
$$V = (-g_{00})^{1/2} = 1 - m/r + \eta\,, \quad m = \text{const}\,, \qquad (40)$$
$$\eta = O(r^{-2})\,, \quad \partial_\alpha \eta = O(r^{-3})\,, \quad \partial_\alpha\partial_\beta\eta = O(r^{-4})\,,$$

when $r \equiv (\delta_{\alpha\beta}x^\alpha x^\beta)^{1/2} \to \infty$.

(b) The equipotential surfaces $V=$ const >0, $t=$ const are regular, simply connected closed 2-spaces.

(c) The invariant $R_{ABCD}R^{ABCD}$ formed from the four-dimensional Riemann tensor is bounded on Σ.

(d) If V has a vanishing lower bound on Σ, the intrinsic geometry (characterized by $^{(2)}R$) of the 2-spaces $V=c$ approaches a limit as $c\to 0+$, corresponding to a closed regular 2-space of finite area.

Theorem. The only static space-time satisfying (a), (b), (c), and (d) is Schwarzschild's spherically symmetric vacuum solution.

To prove this statement, we note the fact (proved in the Appendix) that, since V is harmonic on Σ [see (29)], a point where V has zero gradient would be a point of bifurcation of the equipotential surfaces. This possibility is ruled out by condition (b). It follows that two distinct equipotential surfaces are associated with different values of V, and that ρ, as defined by (13), is finite for $V>0$.

Let us dispose first of the trivial case where V has a positive lower bound on Σ. In this case, Σ is complete and simply connected and the harmonic function V is regular because of (c) [see (38)] and tends to unity uniformly at infinity. Hence $V=1$ everywhere, space-time is flat, and the theorem holds trivially.[10]

If the greatest lower bound of V on Σ is zero, condition (d) in combination with (32) implies that ρ approaches a regular nonzero limit as $V\to 0+$. From (c), we have

$$K_{ab}(0+,\theta^1,\theta^2) = 0\,, \qquad (41)$$

$$\rho(0+,\theta^1,\theta^2) \equiv \rho_0 = \text{const}\,, \qquad (42)$$

$$\lim_{V\to 0+}(K/V) = -\tfrac{1}{2}\rho_0\,{}^{(2)}R(0+,\theta^1,\theta^2)\,, \qquad (43)$$

where use has been made of (39) and (33).

By integrating (32) over Σ, and using (42) and the asymptotic conditions (40), we find

$$S_0/\rho_0 = 4\pi m\,, \qquad (44)$$

where

$$S_0 = \iint g^{1/2}(0+,\theta^1,\theta^2)\,d\theta^1 d\theta^2$$

is the area of the 2-space $V=0+$. This implies that the constant m in (40) is necessarily positive.

The identities

$$\frac{\partial}{\partial V}\left(\frac{g^{1/2}}{\rho^{1/2}}\frac{K}{V}\right) = -\frac{2g^{1/2}}{V}[\nabla^2(\rho^{1/2}) + \rho^{-3/2}(\tfrac{1}{2}\rho_{;a}\rho^{;a} + \psi_{ab}\psi^{ab})]\,, \qquad (45)$$

$$\frac{\partial}{\partial V}\left[\frac{g^{1/2}}{\rho}\left(KV + \frac{4}{\rho}\right)\right] = -g^{1/2}V[\nabla^2(\ln\rho) + \rho^{-2}(\rho_{;a}\rho^{;a} + 2\psi_{ab}\psi^{ab}) - {}^{(2)}R] \qquad (46)$$

are obtainable in a straightforward way from (30), (32), (33), and (35). Here, $\nabla^2(\cdots) = g^{ab}(\cdots)_{;ab}$ repre-

[9] A similar (though different) way of formulating the static field equations has been noted by R. K. Sachs, *Perspectives in Geometry and Relativity*, edited by B. Hoffmann (Indiana University Press, Bloomington, 1966), p. 340.

[10] Cf. Lichnerowicz, Ref. 8.

sents the two-dimensional Laplacian, and

$$\psi_{ab}=\rho(K_{ab}-\tfrac{1}{2}g_{ab}K). \tag{47}$$

These equations are now integrated over Σ. We observe that

$$\begin{aligned}&\iint(\nabla^2 f)g^{1/2}d\theta^1 d\theta^2=0,\\&\iint {}^{(2)}Rg^{1/2}d\theta^1 d\theta^2=-8\pi\end{aligned} \tag{48}$$

(Gauss-Bonnet theorem), for a simply connected, closed regular 2-space and any regular function f, and we employ (40), (42), and (43) to evaluate the surface integrals. We find

$$\rho_0\geq 4m, \tag{49}$$

$$S_0\geq\pi\rho_0{}^2 \tag{50}$$

from (45) and (46), respectively, with equality if and only if

$$\partial_a\rho=0=\psi_{ab} \tag{51}$$

everywhere on Σ.

Comparison of (44), (49), and (50) shows that equality must hold. The spherical symmetry of the field then follows immediately from (51), and establishes the theorem. To verify that the Schwarzschild solution (for any $m\geq 0$) indeed satisfies conditions (a)–(d) is, of course, trivial.

5. DISCUSSION

The search for a space-time possessing a regular event horizon can be regarded as a nonlinear eigenvalue problem. For the class of static, asymptotically flat vacuum fields, it has here been formulated as the problem of selecting well-behaved solutions of the system of differential Eqs. (16), (30), and (35), which have $V=0$ as a singular boundary point. It has been shown that the eigensolutions are the Schwarzschild fields, characterized by a continuous spectrum of nonnegative eigenvalues m. Extensions of this result[11] would be of great interest. In particular, it is natural to ask whether the 2-parameter family of Kerr solutions[12] embraces all eigenfields in the stationary case.

The result of this paper would have important astrophysical consequences if it were permissible to consider the limiting external field of a gravitationally collapsing asymmetric (nonrotating) body as static. In that case, only two alternatives would be open—either the body has to divest itself of all quadrupole and higher moments by some mechanism (perhaps gravitational radiation), or else an event horizon ceases to exist.[13]

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APPENDIX

In connection with the argument of Sec. 4, it will be shown here that the level surfaces of a regular harmonic function V (defined on a three-dimensional Riemannian space) are many-sheeted in the neighborhood of a point P_0, where V has vanishing gradient.

Let $V_{|\alpha_1\cdots\alpha_n}(n\geq 2)$ be the covariant derivative of lowest order which does not vanish at P_0. In terms of Riemannian normal coordinates with origin at P_0, we have

$$\begin{gathered}g^{\alpha\beta}|_{P_o}=\delta^{\alpha\beta},\quad \Gamma_{\alpha\beta}{}^{\mu}|_{P_o}=0,\\ \partial_{\alpha_1}\cdots\partial_{\alpha_m}V|_{P_o}=V_{|\alpha_1\cdots\alpha_m}|_{P_o}=0,\quad(m<n)\\ \partial_{\alpha_1}\cdots\partial_{\alpha_n}V|_{P_o}=V_{|\alpha_1\cdots\alpha_n}|_{P_o}=n!c_{\alpha_1\cdots\alpha_n}\neq 0.\end{gathered}$$

The harmonic condition $g^{\alpha_1\alpha_2}V_{|\alpha_1\alpha_2\cdots\alpha_n}=0$ requires that $c_{\alpha_1\cdots\alpha_n}$ be traceless. Thus the leading term in the power series expansion

$$V-V_0=c_{\alpha_1\cdots\alpha_n}x^{\alpha_1}\cdots x^{\alpha_n}+\cdots\quad[V_0\equiv V(P_0)]$$

in a solid spherical harmonic $Y_n(x^1,x^2,x^3)$ of degree n. Hence $V-V_0$ vanishes on n distinct curves of a (sufficiently small) geodesic sphere with center P_0, and the surface $V=V_0$ has more than one sheet.

[11] It has been shown recently [W. Israel (to be published)] that the Reissner-Nordström solutions with $m\geq|e|$ comprise all eigenfields in the class of static, asymptotically flat electrovac space-times.

[12] R. P. Kerr, Phys. Rev. Letters **11**, 237 (1963); R. H. Boyer and R. W. Lindquist, J. Math Phys. **8**, 265 (1967).

[13] In this connection, it is perhaps significant that one can construct a sequence of static vacuum fields with axial symmetry, which are nonsingular to well within the gravitational radius—they display only a "pointlike" multipole singularity at the origin of Weyl's coordinates—which are free of event horizons, and which deviate *arbitrarily little* from spherical symmetry for $r>(1+\sqrt{2})m$. See W. Israel, Nature **216**, 148, 312 (1967).