Nucleon-Nucleon and Nucleon-Antinucleon Charge-Exchange Scattering and Conspiracy of Singularities in the Complex Angular Momentum Plane*

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We analyze the experimental data on the reactions $pn\rightarrow np$ and $p\bar{p}\rightarrow n\bar{n}$ near the forward direction for evidence of "conspiracy". From a comparison between the forward differential cross section of $pn\rightarrow np$ and the total cross sections for pp and pn, we conclude that conspiracy exists, and proceed to investigate the nature of the conspiracy. Using experimental data, we can rule out all Regge-pole conspiracy schemes involving the pion and at most one other conspiring family, under the assumption of constant residue functions. An extremely simple semiphenomenological formula that agrees with experiments is then proposed. The formula is consistent with a model which includes π exchange and conspiracies of branch cuts in the angular momentum plane. We suggest that the relevant branch cuts are those arising from ρ-P and A_2-P exchange, where P is the vacuum trajectory.

I. INTRODUCTION AND SUMMARY OF RESULTS

WHEN two nucleons scatter in the forward direction the component of total angular momentum along the direction of relative motion is conserved. Therefore, the helicity amplitudes for which the total helicity is different in the initial and final states must vanish in the forward direction. By crossing symmetry, each helicity amplitude is a linear combination of helicity amplitudes in the crossed channel. Therefore, certain linear combinations of crossed-channel helicity amplitudes must vanish at zero total energy of the crossed channel. To satisfy this requirement, the relevant helicity amplitudes may vanish individually, or they may be finite individually but cancel each other. These alternatives are referred to as "evasion" and "conspiracy," respectively. The linear relation among helicity amplitudes referred to above will be called a "conspiracy condition."

It is always possible (though not necessarily fruitful) to describe the scattering process in terms of the singularities of partial-wave amplitudes in the complex angular momentum plane. In such a language, the conspiracy condition becomes a condition on these singularities. If we consider, for example, exchange of Regge poles, then for "evasion" the residues of the Regge poles in the relevant amplitudes must vanish at zero crossed-channel energy. For "conspiracy," they remain finite there but must fulfill specific conditions, which relate residues and trajectories of Regge poles of different quantum numbers. The latter raises an intriguing theoretical possibility. The purpose of this paper is to investigate whether nature makes use of it, and if so in what form.

A promising reaction to analyze is forward proton-neutron charge-exchange scattering, hereafter designated by $pn\rightarrow np$, which is synonymous with backward proton-neutron elastic scattering. The data[1] show an unusually sharp forward peak, whose width in the squared 4-momentum transfer $-t$ is of the order of a squared pion mass, or about five times smaller than normal diffraction peak widths. The magnitude of the forward peak is

$$(d\sigma/d\Omega)_{t=0}=1.01\pm0.09\text{ mb/sr}\qquad(\text{at }p_{\text{lab}}=8\text{ BeV}/c),\quad(1)$$

and it falls off with energy roughly like s^{-1}, where s is the squared total center-of-mass (c.m.) energy. These facts suggest that π exchange is important.[2] To exhibit this we define a reduced differential cross section

$$I(y)\equiv\left[\frac{1}{4m^2}\left(\frac{g^2}{4\pi}\right)^2x^{-1+2\alpha_\pi(y)}\right]^{-1}\frac{d\sigma}{d\Omega},\qquad(2)$$
$$x=s/m^2,$$
$$y=-t/\mu^2,$$

where m and μ are, respectively, the nucleon and pion mass, and

$$g^2/4\pi=15,$$
$$(4m^2)^{-1}(g^2/4\pi)^2=25.6\text{ mb/sr},\qquad(3)$$
$$\alpha_\pi(y)=0.02(1+y).$$

The quantity $\alpha_\pi(y)$ is the pion Regge trajectory, taken to be linear with slope $1(\text{BeV}/c)^2$. At $y=0$, the quantity enclosed in brackets in Eq. (2) is $\frac{1}{4}$ of what we would obtain for the forward differential cross section from a conspiring π trajectory (without the contribution of its

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[1] 8 BeV/c: G. Manning, A. G. Parham, J. D. Jafar, H. B. van der Roay, D. H. Reading, D. G. Ryan, B. D. Jones, J. Malos, and N. H. Lipman, Nuovo Cimento **41**, 167 (1966); 2.83 and 3.67 BeV/c: H. Palevsky, J. A. Moore, R. L. Stearns, H. R. Muether, R. J. Sutter, R. E. Chrien, A. P. Jain, and K. Otnes, Phys. Rev. Letters **9**, 509 (1962).

[2] For phenomenological models using π exchange, see G. A. Ringland and R. J. N. Phillips, Phys. Letters **12**, 62 (1964); E. M. Henley and I. J. Muzinich, Phys. Rev. **136**, B1783 (1964); L. Durand III and Y. T. Chiu, *ibid.* **137**, B1530 (1964); N. Byers, *ibid.* **156**, 1703 (1967).

co-conspirators). The energy dependence of $I(0)$ is shown in Fig. 1. The data for $pn \to np$ are consistent with $I(0)=1$. In Fig. 2 we show $I(y)$ for $0<y<7$. Although existing data are not sufficient to establish definitely the energy dependence of $I(y)$, they suggest that the dependence is weak, if it exists.

A reaction closely related to $pn \to np$ is the annihilation of proton-antiproton into neutron-antineutron, designated by $p\bar{p} \to n\bar{n}$. The crossed-channel reactions for these processes are, respectively, $p\bar{n} \to \bar{n}p$ and $\bar{n}p \to \bar{n}p$, which differ only in their initial states, which are images of each other under G conjugation. An exchanged object gives the same contribution to both processes if it has odd G parity. It gives contributions equal in magnitude but opposite in sign if it has even G parity. In Figs. 1 and 2, data[3] for $p\bar{p} \to n\bar{n}$ are shown on the same plot as for $pn \to np$. As we can see, the energy dependence of $I(y)$ is again weak, if present at all. The sum total of the data in Fig. 2 provides a stringent test for specific theories.

The data for $pn \to np$ indicate that we have a case of conspiracy, rather than evasion. As we detail later, the contribution of those crossed-channel amplitudes not involved in the conspiracy condition can be estimated by relating it to the difference of total cross sections for pp and pn reactions, through use of the optical theorem. We find that it amounts only to 20% of Eq. (1), and therefore conclude that conspiracy, and not evasion, occurs in $pn \to np$.

A natural question is: "Who are the conspirators?" The pion is an obvious suspect. But, as we show in more detail later, π conspiracy alone fails, because it leads to

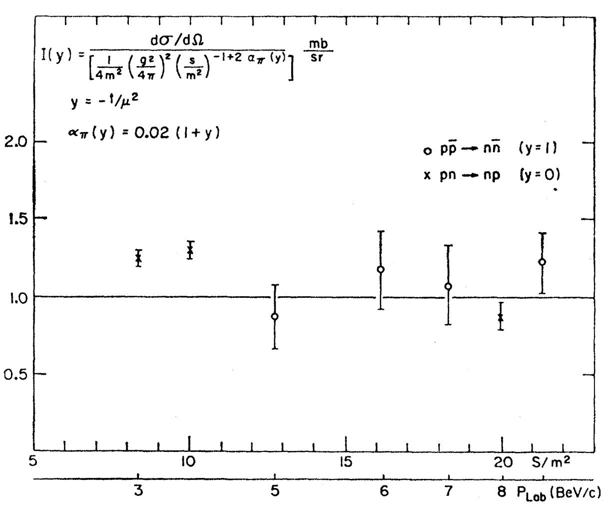

FIG. 1. Energy dependence of the reduced differential cross section in the forward direction. The $pn \to np$ data are from Ref. 1, and the $pp \to nn$ data are from Ref. 3. For $p\bar{p} \to n\bar{n}$, no data are available at $y=0$. Extrapolation to $y=0$ is dangerous because of possible rapid variations there.

[3] P. Astbury, G. Brautti, G. Finocchiaro, A. Michelini, D. Websdale, C. H. West, E. Polgar, W. Beusch, W. E. Fischer, B. Gobbi, and M. Pepin, Phys. Letters 23, 160 (1966).

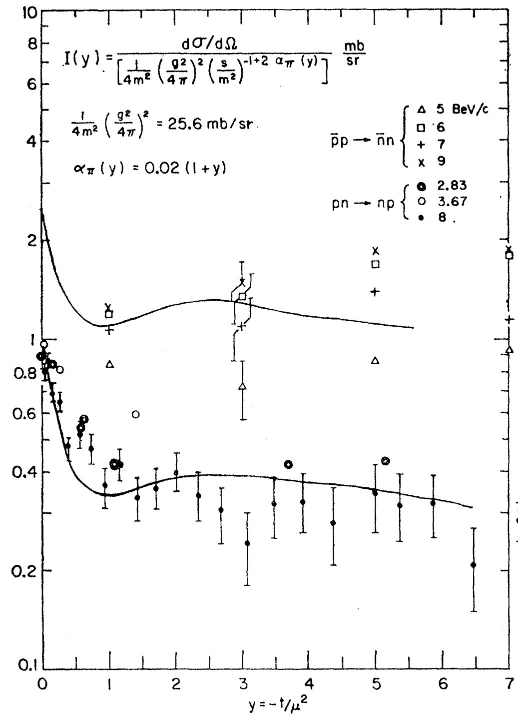

FIG. 2. Reduced differential cross section. For significance of $I(y)$ see introduction. Sources of data are the same as Fig. 1. The data for $pn \to np$ at 2.83 and 3.67 BeV/c have been reduced by an assumed systematic error of 30% relative to the 8-BeV/c data. The solid curves are from the semiphenomenological fit, Eq. (40), at 8 BeV/c, which can be interpreted in terms of π exchange and conspiracies involving branch cuts.

the prediction $I(0)=2$ for both $pn \to np$ and $p\bar{p} \to n\bar{n}$. This is definitely ruled out by experiments. Thus we have to find other conspirators coupled to the same amplitude as the pion.

We consider then a general class of conspiracy schemes involving only Regge poles. The class includes all schemes involving the pion and at most one other conspiring family of trajectories. Under the assumption that the residue functions of all conspiring trajectories can be taken as constants in a neighborhood of $t=0$, the size of a few squared pion masses, we show that all these schemes are in disagreement with experiments, no matter how we vary the available free parameters. The main reason for the failure lies in the stringent requirements imposed on Regge poles by the energy independence of $I(0)$, and the definite connection between $pn \to np$ and $p\bar{p} \to n\bar{n}$.

With the failure mentioned above, there remain many possibilities. For example, the residue functions of the conspirators may vary rapidly. This has recently been investigated and found not entirely satisfactory.[4] The possibilities also remain that the conspiracy may involve a larger number of Regge poles or other types of singularities, such as branch cuts. Because of the same

[4] R. J. N. Phillips (to be published); F. Arbab and J. W. Dash, Phys. Rev. 163, 1603 (1966).

difficulties we mentioned, the possibility that more Regge poles might turn the trick is unlikely, unless they are very numerous with closely spaced trajectories. In the latter case they are indistinguishable from a branch cut.

We proceed by adopting a more phenomenological approach. The lessons learned in the earlier failure enable us to write down a successful and extremely simple semiphenomenological formula. The interpretation of the formula is not unique, but we suggest a simple model for it. The model consists of a π trajectory, which either evades or conspires, and two self-conspiring branch cuts. It is consistent with present knowledge to identify them as the cuts arising from ρ-P and A_2-P exchange, where P denotes the vacuum trajectory.

Since at present little is known about the properties of cuts, it is not possible to make the model more definite. The contributions of ρ and A_2 to the helicity-flip amplitude are not important for $y<3$, as we shall show explicitly.

Our conclusions, then, are as follows:

(1) Nature makes use of "conspiracy."

(2) In $pn\rightarrow np$ and $p\bar{p}\rightarrow n\bar{n}$, simple conspiracy schemes involving only Regge poles are ruled out by experiments.

(3) The experiments can be explained by a model involving π exchange and conspiracies of branch cuts in the J plane, but the imperfect state of the theory of cuts prevents us from making unequivocal claims.

II. KINEMATICS

The reactions $pn\rightarrow np$ and $p\bar{p}\rightarrow n\bar{n}$ are kinematically identical when the neutron-proton mass difference is neglected. The developments of this section then apply to both reactions. We define, as usual,

$$s=4(k^2+m^2)\,,$$
$$t=-2k^2(1-z_s)\,,$$

where k is the c.m. momentum, $z_s=\cos\theta_s$ (θ_s being the c.m. scattering angle), and m is the nucleon mass. The helicity amplitudes in the direct channel (the s channel) are denoted by $f_{cd,ab}{}^s(s,t)$, where c,d are final helicities, and a,b are initial helicities, which may take on the values $+,-$. The partial-wave decomposition[5]

$$f_{cd,ab}{}^s(s,t)=\sum_J (2J+1)\langle cd|F^J(s)|ab\rangle d_{\lambda\mu}{}^J(z_s)\,,\qquad \lambda=a-b\,,\quad \mu=c-d\,,\tag{4}$$

defines the partial-wave amplitudes $\langle cd|F^J(s)|ab\rangle$, where J is the total angular momentum. The sum in (4) extends over all integer values $J\geq\max(|\lambda|,|\mu|)$. The differential cross section, averaged over initial helicities and summed over final helicities, is

$$d\sigma/d\Omega=(16\pi^2s)^{-1}\sum_{a,b,c,d}|f_{cd,ab}{}^s(s,t)|^2\,.\tag{5}$$

[5] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) 1, 404 (1959).

In the crossed channel (the t channel), the helicity amplitudes are denoted by $f_{cA,Db}{}^t(s,t)$, where a capital subscript denotes the helicity of a crossed particle. It admits an analogous partial-wave expansion

$$f_{cA,Db}{}^t(s,t)=\sum_J (2J+1)\langle cA|G^J(t)|Db\rangle d_{\lambda\mu}{}^J(z_t)\,,\qquad \lambda=D-b\,,\quad \mu=c-A\,,\qquad z_t=1-[2s/(4m^2-t)]\,,\tag{6}$$

and it is related to $f_{cd,ab}{}^s$ by the crossing relation[6]

$$f_{cd,ab}{}^s(s,t)=\sum_{c',A',D',b'} d_{A'a}{}^{1/2}(\chi)d_{b'b}{}^{1/2}(\pi-\chi)\times d_{c'c}{}^{1/2}(\pi-\chi)d_{D'd}{}^{1/2}(\chi)f_{c'A',D'b'}{}^t(s,t)\,,\tag{7}$$

where

$$\cos\chi=[st/(s-4m^2)(t-4m^2)]^{1/2}\,,\tag{8}$$

$$d_{\lambda\mu}{}^{1/2}(\chi)=\begin{bmatrix}\cos(\chi/2) & -\sin(\chi/2)\\ \sin(\chi/2) & \cos(\chi/2)\end{bmatrix}.\tag{9}$$

Because of the orthogonality of $d_{\lambda\mu}{}^{1/2}$, the differential cross section (5) can also be written as

$$d\sigma/d\Omega=(16\pi^2s)^{-1}\sum_{c,A,D,b}|f_{cA,Db}{}^t(s,t)|^2\,.\tag{10}$$

There are 16 helicity amplitudes, of which only 5 are independent, by invariance under reflection and time reversal, and conservation of total spin.[7] We choose them to be

$$f_{++,++},\ f_{++,--},\ f_{+-,+-},\ f_{+-,-+},\ f_{++,+-},\tag{11}$$

either in the s or t channel. All others are equal to one of the above, and (10) can be reduced to

$$d\sigma/d\Omega=(8\pi^2s)^{-1}(|f_{++,++}{}^t|^2+|f_{++,--}{}^t|^2+|f_{+-,+-}{}^t|^2+|f_{+-,-+}{}^t|^2+4|f_{++,+-}{}^t|^2)\,.\tag{12}$$

This equation, of course, holds also if all superscripts t are replaced by s.

To discuss the contribution of Regge poles to the partial-wave amplitudes $\langle cA|G^J(t)|Db\rangle$, it is convenient to decompose each of them into amplitudes for transitions between states of definite parity. A nucleon-antinucleon helicity state of total angular momentum J, denoted by $|J,c,A\rangle$, transforms under spatial reflection according to $P|J,c,A\rangle=(-)^J|J,-c,-A\rangle$. We define parity eigenstates by

$$|J,c,A\rangle_\pm=2^{-1/2}(|J,c,A\rangle\pm|J,-c,-A\rangle)\,,\tag{13}$$

with

$$P|J,c,A\rangle_\pm=\pm(-)^J|J,c,A\rangle\,.\tag{14}$$

For each J, there are four such states, which have definite total spin S, G parity, and, of course, parity. They

[6] T. L. Trueman and G. C. Wick, Ann. Phys. (N. Y.) 26, 322 (1964). We use the phase conventions of L. C. Wang, Phys. Rev. 142, 1187 (1966).

[7] M. L. Goldberger, M. J. Grisaru, S. W. McDowell, and D. Y. Wong, Phys. Rev. 120, 2250 (1960); I. J. Muzinich, *ibid.* 130, 1571 (1963).

TABLE I. Parity-helicity states of $N\bar{N}$ system.

	S	P	PG	G	$I=1$ trajectories
$\lvert ++\rangle_- \equiv \lvert 0-\rangle$	0	$-(-)^J$	$+$	$+$	$B(1^+)$
				$-$	π
$\lvert +-\rangle_- \equiv \lvert 1-\rangle$	1	$-(-)^J$	$-$	$+$	$B(2^-)$
				$-$	A_1
$\lvert ++\rangle_+ \equiv \lvert 0+\rangle$	1	$(-)^J$	$-$	$+$	ρ
				$-$	A_2
$\lvert +-\rangle_+ \equiv \lvert 1+\rangle$	1	$(-)^J$	$-$	$+$	ρ
				$-$	A_2

are listed in Table I together with the possible $I=1$ Regge trajectories coupled to them. In the second column of Table I they are relabeled by the difference of helicities (the total angular momentum along the direction of relative motion). The only possible transition occurs between $\lvert 0+\rangle$ and $\lvert 1+\rangle$, all other transitions being forbidden either by conservation of P or S. Thus we define five new partial-wave amplitudes[8] $G_{00}{}^{J-}$, $G_{11}{}^{J-}$, $G_{00}{}^{J+}$, $G_{11}{}^{J+}$, $G_{10}{}^{J+}$, where $G_{\lambda\mu}{}^{J\pm}$ denotes the amplitude leading from $\lvert\mu\pm\rangle$ to $\lvert\lambda\pm\rangle$. Using (13), we easily work out the relation between $G_{\lambda\mu}{}^{J\pm}$ and $\langle cA\lvert G^J\rvert Db\rangle$:

$$
\begin{aligned}
\langle ++\lvert G^J\rvert ++\rangle &= \tfrac{1}{2}(G_{00}{}^{J+}+G_{00}{}^{J-}),\\
\langle ++\lvert G^J\rvert --\rangle &= \tfrac{1}{2}(G_{00}{}^{J+}-G_{00}{}^{J-}),\\
\langle +-\lvert G^J\rvert +-\rangle &= \tfrac{1}{2}(G_{11}{}^{J+}+G_{11}{}^{J-}),\\
\langle +-\lvert G^J\rvert -+\rangle &= \tfrac{1}{2}(G_{11}{}^{J+}-G_{11}{}^{J-}),\\
\langle ++\lvert G^J\rvert +-\rangle &= \tfrac{1}{2}G_{10}{}^{J+}.
\end{aligned}
\tag{15}
$$

Since $G_{\lambda\mu}{}^{J\pm}$ are the amplitudes to which Regge poles are coupled, it is convenient to define the following combination of helicity amplitudes:

$$
\begin{aligned}
M_{00}{}^{\pm}(t,z_t) &= \sum_J (2J+1)G_{00}{}^{J\pm}(t)d_{00}{}^J(z_t),\\
M_{11}{}^{\pm}(t,z_t) &= \sum_J (2J+1)G_{11}{}^{J\pm}(t)d_{11}{}^J(z_t),\\
M_{-1,1}{}^{\pm}(t,z_t) &= \sum_J (2J+1)G_{11}{}^{J\pm}(t)d_{-1,1}{}^J(z_t),\\
M_{10}{}^{\pm}(t,z_t) &= \sum_J (2J+1)G_{10}{}^{J\pm}(t)d_{10}{}^J(z_t).
\end{aligned}
\tag{16}
$$

Note that $M_{-1,1}{}^{\pm}$ are not independent, but are determined by $M_{11}{}^{\pm}$, respectively. By (15) and (16), the cross section can be expressed directly in terms of these amplitudes as

$$
\begin{aligned}
d\sigma/d\Omega = (16\pi^2 s)^{-1}[&\lvert M_{00}{}^+\rvert^2+\lvert M_{00}{}^-\rvert^2\\
&+\tfrac{1}{2}\lvert M_{11}{}^+ + M_{11}{}^-\rvert^2+\tfrac{1}{2}\lvert M_{-1,1}{}^+\\
&-M_{-1,1}{}^-\rvert^2+\lvert M_{10}\rvert^2],
\end{aligned}
\tag{17}
$$

where the arguments of $M_{\lambda\mu}{}^{\pm}$ are t, z_t.

The contribution of a single Regge pole to $M_{\lambda\mu}{}^{\pm}$ may be obtained by carrying out the Watson-Sommerfeld transform on (16) in a standard manner,[9] assuming the Mandelstam symmetry $G_{\lambda\mu}{}^{J\pm}=G_{\lambda\mu}{}^{(-J-1)\pm}$ for $J=$half-integer, which enables us to push the "background integral" far enough to the left for it to be negligible. The contribution to $M_{\lambda\mu}{}^{\pm}$ from a Regge pole of trajectory $\alpha(t)$, residue function $\beta(t)$, and signature η, is given below,[10] where it should be understood that the set $\{\alpha(t),\beta(t),\eta\}$ is not meant to be the same for all amplitudes:

$$
\begin{aligned}
M_{00}{}^{\pm} &= -[\pi\beta(\alpha+\tfrac{1}{2})/\sin\pi\alpha][E_{00}{}^{\alpha}(-z_t)+\eta E_{00}{}^{\alpha}(z_t)],\\
M_{11}{}^{\pm} &= [\pi\beta(\alpha+\tfrac{1}{2})/\sin\pi\alpha](1-z_t)\\
&\qquad\times[E_{1,-1}{}^{\alpha}(-z_t)-\eta E_{11}{}^{\alpha}(z_t)],\\
M_{10}{}^{\pm} &= [\pi\beta(\alpha+\tfrac{1}{2})/\sin\pi\alpha](1-z_t{}^2)^{1/2}\\
&\qquad\times[E_{10}{}^{\alpha}(-z_t)+\eta E_{10}{}^{\alpha}(z_t)].
\end{aligned}
\tag{18}
$$

For the contribution of each Regge pole, we have

$$
M_{-1,1}{}^{\pm}(t,z_t) = -\eta M_{11}{}^{\pm}(t,-z_t). \tag{19}
$$

The functions $E_{\lambda\mu}{}^{\alpha}$ are defined and discussed in Ref. 9. Their explicit forms are given in the Appendix.

When $s\to\infty$, $-2z_t\to s/m^2$. We then have the asymptotic forms

$$
\begin{aligned}
M_{00}{}^{\pm} &\approx -[\beta K(\alpha)/\sin\pi\alpha](1+\eta e^{-i\pi\alpha})(s/m^2)^{\alpha},\\
M_{11}{}^{\pm} &\approx -[\alpha/(1+\alpha)][\beta K(\alpha)/\sin\pi\alpha]\\
&\qquad\times(1+\eta e^{-i\pi\alpha})(s/m^2)^{\alpha},
\end{aligned}
\tag{20}
$$

where

$$
K(\alpha)=\pi^{1/2}\Gamma(\alpha+\tfrac{3}{2})/\Gamma(\alpha+1). \tag{21}
$$

Again we emphasize that (20) merely illustrates the contribution of a single Regge pole in each amplitude $M_{\lambda\mu}{}^{\pm}$. The set $\{\alpha(t),\beta(t),\eta\}$ is not meant to be the same for all amplitudes.

III. "CONSPIRACY"

The functions $d_{\lambda\mu}{}^J(z)$ appearing in the partial-wave expansion (4) are polynomials of z, multiplied by $(1+z)^{\lvert\lambda+\mu\rvert/2}(1-z)^{\lvert\lambda-\mu\rvert/2}$. Thus it follows from (4) that

$$
\begin{aligned}
f_{+-,-+}{}^{s}(s,0) &= 0,\\
f_{++,+-}{}^{s}(s,0) &= 0,
\end{aligned}
\tag{22}
$$

which merely express angular momentum conservation. By (7), these requirements are converted into requirements on $f_{cA,Db}{}^{t}(s,0)$. The crossing angle χ in (7) is equal to $\pi/2$ at $t=0$. It is a simple matter to verify that in the t channel the second requirement of (22) is identically satisfied, by virtue of conservation of parity and total spin, and the first requirement is translated into

$$
f_{++,++}{}^{t}+f_{+-,-+}{}^{t}-f_{++,--}{}^{t}-f_{+-,+-}{}^{t}=0, \quad (\text{at } t=0). \tag{23}
$$

In terms of the amplitudes defined in (16), it takes the form

$$
M_{00}{}^{-}+\tfrac{1}{2}(M_{-1,1}{}^{+}-M_{11}{}^{+})-\tfrac{1}{2}(M_{-1,1}{}^{-}+M_{1,1}{}^{-})=0, \quad (\text{at } t=0). \tag{24}
$$

[8] These are related to the amplitudes defined in Ref. 7 by $G_{00}{}^{J-}=f_0{}^J$, $G_{11}{}^{J-}=f_1{}^J$, $G_{00}{}^{J+}=f_{11}{}^J$, $G_{11}{}^{J+}=f_{22}{}^J$, $G_{10}{}^{J+}=f_{12}{}^J$.

[9] M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B145 (1964), Appendices.

[10] $\alpha(t)$ and $\beta(t)$ are defined by
$\lim_{J\to\alpha(t)}\{[J-\alpha(t)]G_{\lambda\mu}{}^{\pm}(t)\}=\beta(t)$.

TABLE II. Volkov-Gribov conspiring triplets.

Amplitude	Trajectory	P	G	Signature ($\eta=\pm1$)	Solution 1 $\alpha(0)$	Solution 1 $\beta(0)$	Solution 2 $\alpha(0)$	Solution 2 $\beta(0)$
M_{00}^-	1	$-\eta$	$-\eta$	η	α	β	α	β
M_{11}^+	2	η	$-\eta$	η	α	$[(1+\alpha)/\alpha]\beta$	α	$[\alpha/(\alpha+1)]\beta$
M_{11}^-	3	η	$-\eta$	$-\eta$	$\alpha-1$	$-\dfrac{(2\alpha+1)(\alpha-1)}{(2\alpha-1)\alpha}\beta$	$\alpha+1$	$\dfrac{(2\alpha+1)(\alpha+2)}{(2\alpha+3)(\alpha+1)}\beta$

Expanding $M_{\lambda\mu}^{\pm}$ according to (16), we obtain, after certain manipulations using the properties of $d_{\lambda\mu}{}^J$, the condition

$$\sum_{J=1}^{\infty}\left[G_{00}{}^{(J-1)-}-G_{00}{}^{(J+1)-}-\frac{2J+1}{J(J+1)}G_{11}{}^{J-}-\frac{J-1}{J}G_{11}{}^{(J-1)+}+\frac{J+2}{J+1}G_{11}{}^{(J+1)+}\right]P_J{}^1(z_0)=0\,, \tag{25}$$

where $z_0=1-(s/2m^2)$, and the argument of the G's is $t=0$. For $|z_0|<1$, which corresponds to the unphysical region $|s|<2m^2$, the partial-wave expansion converges, and the functions $P_J{}^1(z_0)$ are linearly independent. Thus the summand of (25) must vanish. This condition was first noted in Ref. 7. An alternative form of (25) valid beyond the region of convergence of the partial-wave series may be obtained by applying the Watson-Sommerfeld tranform to (25). This is done in the Appendix.

Without further assumptions, however, (25) is not very restrictive and therefore not very interesting. Volkov and Gribov[11] first noted that when Regge poles are assumed, (25) requires that the residues of relevant Regge poles either vanish or cancel each other in a specific way, at $t=0$. In the first instance we have a case of "evasion," and in the second instance a case of "conspiracy."

Volkov and Gribov gave two particularly simple solutions to (25), with one Regge pole each in the amplitudes M_{00}^-, M_{11}^+, and M_{11}^-. The ones in M_{00}^- and M_{11}^+ have degenerate trajectories at $t=0$, and the third trajectory lies either one unit above or below. These solutions are displayed in Table II.

Actually the Volkov-Gribov solutions are special cases. As shown in the Appendix, the most general solution involves infinite families of trajectories.[12] A family characterized by given quantum numbers consists of a leading parent trajectory, with daughter trajectories spaced successively two units below. The residues of trajectories in the same family as well as in different families are related by recursion relations. The Volkov-Gribov solutions correspond to cases in which the recursion relation terminates without daughters. In solution 1 of Table II, the relation between residues of trajectories 1 and 2 is the same whether or not there are daughters. Solution 2, however, will be changed if there are daughters.

In our later work, it turns out that solution 2 of Volkov and Gribov can be ruled out fairly easily. When we turn to solution 1, the distinction between finite and infinite families becomes lost at high energies, because daughter trajectories do not contribute to the cross section, and the residues of the parents are independent of the daughters. An exception occurs when the parents have trajectory near $\alpha=0$, in which case the residues of the daughters diverge like α^{-1}. This will be taken into consideration later.

If other singularities in the complex J plane are important, such as branch cuts, they could of course conspire, but only with singularities of the same type. For branch cuts the conspiracy condition imposes relations among their discontinuities in the various amplitudes, similar to the case of poles. The essential difference lies in the fact that the same branch can contribute to all amplitudes, so that a branch cut can "conspire with itself," so to speak. Our analysis later suggests that cuts may be important.

It should be noted that, should the Mandelstam symmetry turn out to be invalid for relativistic scattering, then the "background integral" could become important, and dominate the poles with $\alpha<-\frac{1}{2}$. In that event, the conspiracy condition does not lead to definite restrictions, unless we know something about the detailed nature of the "background integral."

We now give experimental evidence for the case of conspiracy, rather than evasion, in $pn\to np$.

In the cross section (17), two amplitudes are not involved in the conspiracy condition (24), namely M_{00}^+ and M_{10}^+. Of these $M_{10}^+=f_{++,+-}{}^t$ vanishes at $t=0$ by virtue of a kinematic factor $t^{1/2}$.[7] The amplitude M_{00}^+ is coupled to the ρ and A_2 trajectories. Taking the intercepts of these trajectories at $t=0$ to be $\frac{1}{2}$, we find that their separate contributions have equal real and imaginary parts. The imaginary part of M_{00}^+ can be found through the optical theorem:

$$\begin{aligned}\sigma(s)&=k^{-1}s^{-1/2}\,\mathrm{Im}\,[f_{++,++}{}^s(s,0)+f_{+-,+-}{}^s(s,0)]\\&=k^{-1}s^{-1/2}\,\mathrm{Im}\,M_{00}{}^+(t=0)\,,\end{aligned} \tag{26}$$

where the last step is obtained through use of the crossing relation (7) and the definition (16). For pn charge-

[11] D. V. Volkov and V. N. Gribov, Zh. Eksperim. i Teor. Fiz 44, 1068 (1963) [English transl.: Soviet Phys.—JETP, 17 720 (1963)]; Interest in this subject has been revived by M. Gell-Mann and E. Leader, and W. Frazer and R. Phillips, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, California, 1967).

[12] See also D. Z. Freedman and J. M. Wang, Phys. Rev. Letters 18, 863 (1967); Phys. Rev. 160, 1560 (1967).

exchange scattering the relevant total cross section in (26) is, by isotopic spin conservation, $\sigma=\sigma_{pp}-\sigma_{np}$. At $p_{\rm lab}=8$ BeV/c, experiments indicate $\sigma\approx1.5$ mb,[13] which gives Im $M_{00}^+\approx60$. This corresponds to a contribution to the forward differential cross section of 0.12 mb/sr. Assuming that ReM_{00}^+ gives a comparable contribution,[14] we obtain the estimate

$$(16\pi^2 s)^{-1}|M_{00}^+|^2\approx0.2\text{ mb/sr (at }p_{\rm lab}=8\text{ BeV}/c), \quad (27)$$

which should hold in a neighborhood of $t=0$, the size of a few squared pion masses. The same estimate holds for $p\bar{p}\to n\bar{n}$, for this process is obtainable from $pn\to np$ by G-conjugating the initial state in the t channel. Since experimentally $(d\sigma/d\Omega)_{t=0}=1$ mb/sr for $pn\to np$ at 8 BeV/c, we see that at $t=0$ either $M_{00}^-\neq0$, or $M_{11}^+\neq0$, or both. Therefore, we have a case of conspiracy rather than evasion.

The only question now is whether the conspiracy involves solely Regge poles, or other types of singularities. In the next section we show that all Regge-pole conspiracy schemes involving the pion and at most one other conspiracy family are in disagreement with experiments. We prove the contention under the assumption that residue functions of conspiring Regge poles are constant in a neighborhood of $t=0$, the size of a few squared pion masses.

IV. FAILURE OF SIMPLE REGGE-POLE CONSPIRACIES

It is instructive to calculate first the Feynman diagram for one-pion exchange. The amplitude is real and gives the same contribution to $pn\to np$, and $p\bar{p}\to n\bar{n}$:

$$(d\sigma/d\Omega)_{\pi\text{ exch}}=(1/s)(g^2/4\pi)^2[t/(t-\mu^2)]^2, \quad (28)$$

where μ is the pion mass, and $g^2/4\pi=15$. We note that the cross section vanishes at $t=0$, but rises very sharply with half-width the order of μ^2, to a plateau of $s^{-1}(g^2/4\pi)^2$. The last expression happens to be 5 mb/sr at $p_{\rm lab}=8$ BeV/c, or about five times the experimental value of $(d\sigma/d\Omega)_{t=0}$ for $pn\to np$. This indicates that the magnitude of the observed cross section might be characterized by the pion-nucleon coupling constant. It suggests that a forward peak of the right magnitude and sharpness can result from π conspiracy, or interference between π and a conspiring object, or both.

We confine our attention to the region $0\leqslant y<3$, or

$$0\leqslant -t<0.06\ (\text{BeV}/c)^2, \quad (29)$$

and neglect the amplitudes M_{10}^+ and M_{00}^+, which do not enter into the conspiracy condition. The former is neglected because it vanishes at $t=0$, as pointed out earlier. The latter is neglected because, as estimated earlier, it contributes only about 20% to the cross section, an amount that lies within the systematic errors in the experimental absolute cross sections.

The basic assumptions in this section are as follows:

(1) The only singularities in the J plane are Regge poles.

(2) Residue functions of conspiring Regge poles can be taken as roughly constant in the region (29).

(3) The residue function of the π trajectory, conspiring or not, can be completely determined by extrapolation to the physical pion pole at $t=\mu^2$.

We investigate three general cases: (a) π conspiracy only; (b) both π conspiracy and B conspiracy; (c) B conspiracy only. Here B denotes a trajectory (other than π) that couples to M_{00}^-, and hence interferes with π. It need not be a known trajectory. Both signatures will be considered for B, whose G parity is then determined according to Table II. The Volkov-Gribov conspiracy schemes will be considered first. Later we consider the possibility of daughter trajectories, and show that they do not alter our conclusions. The possible conspiring triplets will be given the following names:

$$M_{00}^-;\ \pi,B,$$
$$M_{11}^+;\ \pi',B',$$
$$M_{11}^-;\ \pi'',B''.$$

The general procedure is as follows:

(1) Calculate $I(0)$ for $pn\to np$, and require $I(0)=1$, independent of energy, as Fig. 1 indicates.

(2) Require that $I(y)$ for $pn\to np$ has a forward peak with half-width at $y=\frac{1}{2}$, as Fig. 2 indicates.

(3) Go over to $p\bar{p}\to n\bar{n}$ by G conjugation in the t channel, and require agreement with the available data in Fig. 2.

A. π Conspiracy Only

Solution 2 of Table II, in which π'' lies one unit above π, can be immediately ruled out, for it would predict that $I(0)$ grows linearly with s. Thus only solution 1 of Table II need be considered, in which π'' lies one unit below π, and may be ignored in the cross section.

The relevant trajectories and residues are as follows:

$$\pi;\quad \alpha=-a(1+y),\quad \beta=ag^2, \quad (30)$$

$$\pi';\quad \alpha=-\alpha,\qquad \beta=-g^2, \quad (31)$$

where $y=-t/\mu^2$, $g^2/4\pi=15$, and $a=0.02$. The π residue is taken to be that at $t=\mu^2$, obtained through a straightforward calculation. The π' residue is taken from Table II, using the approximation $a\ll1$. The slope of the π' trajectory is neglected, because it must be small in order not to give rise to a physical scalar boson almost degenerate with the pion. From (20), we obtain, after some reductions using the approximation $a\ll1$,

$$M_{00}^-=g^2(1+y)^{-1}e^{i\pi a(1+y)/2}x^{-a(1+y)}, \quad (32)$$

$$M_{11}^+=g^2e^{i\pi a/2}x^{-a}, \quad (33)$$

[13] D. V. Bugg, D. C. Salter, G. H. Stafford, R. F. George, K. F. Riley, and R. J. Tapper, Phys. Rev. 146, 980 (1966).

[14] This assumption is consistent with ρ, A_2 couplings in other processes.

where $x=s/m^2$. The reduced differential cross section defined in (2) is, both for $pn\to np$ and $p\bar{p}\to n\bar{n}$,

$$I(y)=4[1+(1+y)^{-2}], \tag{34}$$

in which the approximation $x^{\alpha y}\approx 1$ has been made. This predicts $I(0)=8$, in disagreement with experiments. Therefore, we rule out this case.

B. Both π Conspiracy and B Conspiracy

As we have seen, π conspiracy alone yields too high a forward cross section in $pn\to np$. We may hope to remedy this by introducing a conspiring B to cancel part of the π contribution. The contribution of B to M_{00}^- is essentially a t-independent complex number, depending on its trajectory, residue, and signature. For $pn\to np$, we must arrange the real part of this number so as to reduce the π contribution by $2^{-3/2}$, independent of energy. This is possible only if B is degenerate with π, and has even signature. But then the B triplet transforms under G conjugation like the π triplet, and we would predict that $d\sigma/d\Omega$ is identical for $pn\to np$ and $\bar{p}p\to\bar{n}n$, in disagreement with the data in Fig. 2. Therefore this case is ruled out.

C. B Conspiracy Only

For the same reason as given in the previous case, the signature of B must be odd, for otherwise we would predict identical $d\sigma/d\Omega$ for $pn\to np$ and $p\bar{p}\to n\bar{n}$. Thus B has even G parity, and its residue is equal in magnitude but opposite in sign for $pn\to np$ and $p\bar{p}\to n\bar{n}$.

Since in the present case π does not conspire, its residue must vanish at $t=0$. However, a sharp forward peak can result from interference between π and B. If we arrange this to happen in $pn\to np$, as we must, then we predict a sharp forward dip in $p\bar{p}\to n\bar{n}$. Such a dip is not ruled out by experiments. Let us consider separately the two solutions of Table II.

Solution 2 of Table II, with B'' one unit above B, can be ruled out as follows. For $I(0)$ to be energy-independent, B'' cannot lie higher than π. Thus, B lies at least one unit below π, but must have a sufficient large coupling to be able to interfere with π to produce the forward peak in $pn\to np$. Consequently, B must contribute appreciably to the energy dependence of $I(0)$. The total contributions of B, B', B'' always lead to a wrong energy dependence, whatever the choice of the B'' trajectory. The argument is independent of whether or not there are daughters. Thus we rule out this case.

For solution 1 of Table II, B'' lies one unit below B, and may be ignored. For $I(0)$ to be energy-independent, B should be approximately degenerate with π. The interference term between π and B then vanishes, owing to the odd signature of B. Thus there would be no forward peak in $pn\to np$, and this case is also ruled out.

D. Daughter Trajectories

It is shown in the Appendix that there are solutions to the conspiracy condition in which the Gribov-Volkov leading triplet are supplemented by infinite families of daughter trajectories. They lie at least two units below the leading member of the family, and are normally expected to be negligible in the cross section. To examine the effect of the daughters more closely, it suffices to consider the special case in which there are no conspiring trajectories in M_{11}^- (i.e., B'' and its possible daughters are all absent). In this case, B' must be followed by an infinite family of daughters, with trajectories spaced successively two units below each other. Their residues are determined by that of B', which is in turn determined by that of B. The residue of the daughters may be arbitrarily large compared to that of B' if the trajectory of B' is arbitrarily close to 0. Denoting the residue of B' and that of the nth daughter of B' by β_0 and β_{2n}, respectively, and denoting by α the trajectory of B', we have from (A28)

$$\beta_{2n}\underset{\alpha\to 0}{\longrightarrow}\frac{1}{\alpha}\left(\frac{2n}{2n-1}\right)\left[\frac{1\times3\times5\times\cdots\times(2n-1)}{2\times4\times6\times\cdots\times2n}\right]^2\beta_0,\quad (n=1,2,3,\cdots). \tag{35}$$

The daughters may in fact make an appreciable contribution to the cross section at a given energy. But since their contributions are proportional to $x^{\alpha-2n}$, they vary rapidly with energy, and in all cases produce wrong energy dependences of $I(0)$.

The inclusion of daughters, therefore, does not alter the conclusion reached in this section, namely, under the basic assumptions stated earlier, all Regge-pole conspiracy schemes involving the pion and at most one other conspiring family are in disagreement with experiments.

V. MODEL

In the last section we have seen that simple Regge-pole conspiracies do not lead to a simultaneous understanding of both $pn\to np$ and $p\bar{p}\to n\bar{n}$. The main difficulty was that Regge poles generally give strong energy dependences (i.e., s^α), and that their contributions to $pn\to np$ and $p\bar{p}\to n\bar{n}$ are strictly correlated. For these reasons it is unlikely that adding a few more Regge poles will lead to a good fit. If a large number of Regge poles with closely spaced trajectories are introduced, their contribution can hardly be distinguished from that of a branch cut. Accordingly, we try a semiphenomenological approach.

In the last section we have completely ignored the contributions of the ρ and A_2 trajectories, which may interfere with the conspirators in M_{11}^+. This was justifiable because they evade instead of conspire, hence in the neighborhood of $t=0$ they are unimportant. At larger $-t$, however, they could be important, for it is known that ρ and A_2 have very large helicity-flip couplings, from simultaneous analysis of $\pi^-p\to\pi^0n$ and $\pi^-p\to\eta n$, and πp and Kp elastic scattering.[15] We shall

[15] R. J. N. Phillips and W. Rarita, Phys. Rev. 139, B1336 (1965).

include their effects in our phenomenological formula, mainly for the purpose of ascertaining that they do not have much influence in the interesting region $0 \leqslant y < 3$. We shall continue to neglect the contributions of ρ and A_2 to M_{10}^+, because in this amplitude they cannot interfere with any conspirator, and because their residues there are linear instead of quadratic in the helicity-flip coupling constant, and hence small.

To write down a semiphenomenological formula that works, all we have to do is to add to (32) and (33) an energy-independent interference term that may be different for $pn \to np$ and $p\bar{p} \to n\bar{n}$. With the contributions of ρ and A_2 included, we write

$$M_{00}^- = g^2 \exp(-\tfrac{1}{2}i\pi\alpha_\pi)x^{\alpha_\pi}e^{-cy}[-(y/(1+y))+\lambda], \quad (36)$$

$$\begin{aligned} M_{11}^+ = g^2 \exp(-\tfrac{1}{2}i\pi\alpha_\pi)x^{\alpha_\pi}e^{-cy}\{&\lambda - iy(\mu/m)^2\gamma_\rho \\ &\times \exp[-\tfrac{1}{2}i\pi(\alpha_\rho-\alpha_\pi)]x^{\alpha_\rho-\alpha_\pi} - y(\mu/m)^2\gamma_{A_2} \\ &\times \exp[-\tfrac{1}{2}i\pi(\alpha_{A_2}-\alpha_\pi)]x^{\alpha_{A_2}-\alpha_\pi}\}, \quad (37) \end{aligned}$$

where c and λ are constants, and α_ρ and α_{A_2} are, respectively, the ρ and A_2 trajectory, and γ_ρ and γ_{A_2} are constants characterizing their couplings. We may look upon (36) and (37) as results of the exchange of an evading π, plus some other conspiracy that gives the constant λ. We show later that λ is consistent with the contribution from self-conspiring cuts.

The factor e^{-cy} represents the t dependence of residue functions and cut discontinuities, taken here to be the same for all poles and cuts. It corresponds to a choice of the energy scale s_0 in the factor $(s/s_0)^{\alpha_\pi}$.

We now take up the ρ and A_2 contributions. Choosing their trajectories to be degenerate, with

$$\alpha_\rho = \alpha_{A_2} = \tfrac{1}{2} - ay, \quad (38)$$

where $a = 0.02$, we obtain

$$M_{11}^+ = g^2 \exp(-\tfrac{1}{2}i\pi\alpha_\pi)x^{\alpha_\pi}e^{-cy} \times[\lambda - y(\mu/m)^2(x/2)^{1/2}(\gamma - i\gamma')], \quad (39)$$

where $\gamma = \gamma_\rho + \gamma_{A_2}$, $\gamma' = \gamma_\rho - \gamma_{A_2}$. The term $i\gamma'$ will be neglected because it is small and does not interfere with the conspirators. The final formula for the reduced cross section defined in (2) is extremely simple:

$$I(y) = 4e^{-2cy}\{[\lambda - y/(1+y)]^2 + [\lambda - y(\mu/m)^2(x/2)^{1/2}\gamma]\}. \quad (40)$$

The constant c should be the same for $pn \to np$ and $p\bar{p} \to n\bar{n}$, but λ and γ may be different for the two processes. An illustrative fit to the data consists of taking

$$c = 0.1, \quad (41)$$

$$\begin{aligned} \lambda &= 0.35, \quad (pn \to np) \\ &= 0.55, \quad (p\bar{p} \to n\bar{n}) \end{aligned} \quad (42)$$

$$\begin{aligned} \gamma &= 1, \quad (pn \to np) \\ &= -0.75, \quad (p\bar{p} \to n\bar{n}). \end{aligned} \quad (43)$$

The fits for $p_{\text{lab}} = 8$ BeV/c are shown as the solid curves in Fig. 2. The values of γ appear reasonable when compared with the ρ and A_2 helicity-flip couplings in other processes.[15] However, the fit here is very insensitive to γ, and our values should be regarded as illustrative rather than definitive.

It is clear that λ cannot be the contribution of only a few Regge poles, for that would give λ a strong energy dependence. Furthermore, it cannot be the contribution of a single cut, for that would require λ to have equal magnitude for $pn \to np$ and $p\bar{p} \to n\bar{n}$.

The simplest model consistent with (40) consists of taking λ to be the contribution from two branch cuts, namely, those arising from $\rho - P$ and $A_2 - P$ exchange, where P is the vacuum trajectory.[16] These cuts conspire separately with themselves, each contributing equally to M_{00}^- and M_{11}^+. Since these cuts have even and odd G parity, respectively, a relatively small contribution from each, constructively interfering in $pn \to np$, and destructively interfering in $p\bar{p} \to n\bar{n}$, would reproduce the λ required by (42). The magnitude of the cut contributions are in rough agreement with cut contributions in other processes.[17]

The trajectories of the two branch cuts are the same by virtue of (38). They are given by

$$\begin{aligned} \alpha_c(t) &= \max[\alpha_P(t_1) + \alpha_\rho(t_2) - 1], \\ \sqrt{t_1} &+ \sqrt{t_2} = \sqrt{t}. \end{aligned} \quad (44)$$

In the region we are interested in we can take

$$\alpha_c(t) \approx \alpha_\rho(0) = \tfrac{1}{2}. \quad (45)$$

We do not know the phase and energy dependence of the cuts too well. If we assume an energy dependence of the form $\lambda \propto x^{\alpha_c}/\ln x$, then (45) leads to $\lambda \propto x^{1/2}/\ln x$, which is very slowly varying, changing only by 10% in the interval $10 < x < 20$ ($3 < p_{\text{lab}} < 8$ BeV/c). The data are not sufficiently accurate for a comparison.

The above is merely a model for (40), and by no means a unique interpretation. In fact, (40) by itself does not even imply π evasion. To see this, note that the π contribution in (36) may be rewritten in the form $(1+y)^{-1} - 1 = -y/(1+y)$. The constant -1 can be incorporated into λ, and the remaining term $(1+y)^{-1}$ is precisely the contribution of a conspiring π. Thus the case of π conspiracy cannot be distinguished from that of π evasion, except by the numerical value of λ, on which we have no independent information.

We conclude with a few remarks:

(1) Regardless of the model that leads to the semiphenomenological fit, the basic feature remains the

[16] One expects that Regge trajectories coupled to $p\bar{p}$ and $\pi\pi$, such as P, ρ, and $A2$, evade rather than conspire, by virtue of the factorization property of residue functions and the finiteness of the total $\pi\pi$ cross section. The cuts arising from $\rho - P$ and $A_2 - P$ exchange, however, may conspire. A model example of a conspiring cut arising from an iteration of the vacuum trajectory is given by D. Branson, S. Nussinov, S. B. Treiman, and W. I. Weissberger, Phys. Letters **25B**, 141 (1967).

[17] K. Huang, C. E. Jones, and V. L. Teplitz, Phys. Rev. Letters **18**, 146 (1967); M. de Lany, D. J. Gross, I. J. Muzinich, and V. L. Teplitz, *ibid.* **18**, 148 (1967).

same, namely, that the sharp forward peak in $pn \to np$ is governed by the pion mass and the pion-nucleon coupling constant.

(2) Our choice of parameters, which is subject to some uncertainty, predicts a sharp forward peak in $p\bar{p} \to n\bar{n}$, as shown in Fig. 2, with the same shape but twice the magnitude as that in $pn \to np$. It would be helpful to have experimental data in this region for comparison.

(3) The data shown in Fig. 2 seem to show some weak energy dependence. For both $pn \to np$ and $p\bar{p} \to n\bar{n}$, it seems that the diffraction peaks first expand and then shrink, as $p_{\rm lab}$ increases from 3–8 BeV/c. If this should turn out to be a real effect, it might be accommodated in our model by a suitable choice of the energy scale in the cut contribution, making it $x^{1/2}/(\ln x + C)$ instead of $x^{1/2}/\ln x$.

(4) We can make a rough projection of the features of $d\sigma/d\Omega$ for $pn \to np$ at very high energies, say $p_{\rm lab} = 200$ BeV/c ($x \approx 400$). In the forward direction, the conspiring π contribution, which falls like s^{-1}, would become negligible, and the main contributions comes from the amplitude $M_{00}{}^{+}$. The dominant contribution to this amplitude is the ρ trajectory, and should be roughly energy-independent. It should yield $d\sigma/d\Omega \approx 0.2$ mb/sr, as estimated in (27). As t increases there should be an extremely sharp rise to a maximum and then a falloff, with a shape similar to that observed in $\pi^- p \to \pi^0 n$,[18] and for the same reason, i.e., the large ρ helicity-flip coupling. At these extremely high energies, the effect of π exchange is almost completely masked.

(5) Conspiracy can be shown to occur also in other processes, such as $\gamma p \to \pi^+ n$. Analysis of this reaction in terms of π and cut conspiracies also seem to fit the data, while pure Regge-pole conspiracies seem to fail.[19]

APPENDIX: GENERAL SOLUTION OF THE CONSPIRACY CONDITION FOR REGGE POLES

Let

$$\begin{aligned} A &= M_{00}{}^{-}, \\ B &= \tfrac{1}{2}(M_{-1,\,1}{}^{+} - M_{11}{}^{+}), \\ C &= \tfrac{1}{2}(M_{-1,\,1}{}^{-} + M_{11}{}^{+}), \end{aligned} \tag{A1}$$

all evaluated at fixed s and $t=0$. The conspiracy condition then reads

$$A + B - C = 0. \tag{A2}$$

Assume that $M_{00}{}^{-}$, $M_{11}{}^{+}$, and $M_{11}{}^{-}$ are each superpositions of an arbitrary number of Regge poles, with individual contributions given by (18). Since the contribution of a Regge pole of trajectory α is asymptotically proportional to s^α, two trajectories can conspire with each other only if their trajectories differ at most by an integer. Otherwise, they must separately found their own conspiring families. Hence it suffices to consider trajectories in all three amplitudes differing from one another by integer units. Assume that among all these trajectories there is a leading trajectory, denoted by α. We need consider only trajectories of the form $\alpha - n$ ($n = 0, 1, 2, \cdots$). There is no loss in generality to assume that A,B,C each contains one and only one trajectory $\alpha - n$, for, if the trajectory were absent, we would set its residue equal to zero, and if there were more than one trajectories at $\alpha - n$, we would regard β as the sum of their residues. It suffices, therefore, to consider the set of trajectories listed in Table III.

TABLE III. Assignment of conspiring trajectories.

Amplitude	Trajectories	Reduced residue $\bar{\beta}$	Signature of $n=0$ member
A	$\alpha - n$ ($n=0, 1, 2, \cdots$)	$-A_n$	η
B	$\alpha - n$ ($n=0, 1, 2, \cdots$)	$(\alpha-n)(\alpha-n+1)B_n$	$\eta\eta'$
C	$\alpha - n$ ($n=0, 1, 2, \cdots$)	$(\alpha-n)(\alpha-n+1)C_n$	$\eta\eta''$

We define a reduced residue $\bar{\beta}$ for the trajectory α to be

$$\bar{\beta} = \pi\beta(2\alpha+1)/\sin\pi\alpha, \tag{A3}$$

which is further redefined in Table III. To put (18) in more explicit form, we need the following formulas, which may be obtained from Ref. 9:

$$E_{00}{}^{\alpha}(z) = \mathcal{P}_\alpha(z), \tag{A4}$$

$$E_{11}{}^{\alpha}(z) = [\alpha(\alpha+1)]^{-1}[\mathcal{P}_\alpha{}'(z) + (z-1)\mathcal{P}_\alpha{}''(z)], \tag{A5}$$

$$E_{-1,\,1}{}^{\alpha}(z) = [\alpha(\alpha+1)]^{-1}[\mathcal{P}_\alpha{}'(z) + (z+1)\mathcal{P}_\alpha{}''(z)], \tag{A6}$$

where

$$\begin{aligned} \mathcal{P}_\alpha(z) &= -(\tan\pi\alpha/\pi)Q_{-\alpha-1}(z) \\ &= [(2z)^\alpha \Gamma(\alpha+\tfrac{1}{2})/\pi^{1/2}\Gamma(\alpha+1)] \\ &\qquad \times F(-\tfrac{1}{2}\alpha, \tfrac{1}{2}(1-\alpha), \tfrac{1}{2}-\alpha; 1/z^2), \end{aligned} \tag{A7}$$

$$\mathcal{P}_\alpha(z) = e^{-i\pi\alpha}\mathcal{P}_\alpha(-z). \tag{A8}$$

Some useful relations are

$$(1-z^2)\mathcal{P}_\alpha{}'' - 2z\mathcal{P}_\alpha{}' + \alpha(\alpha+1)\mathcal{P}_\alpha = 0, \tag{A9}$$

$$(z^2-1)\mathcal{P}_\alpha{}' = \alpha(z\mathcal{P}_\alpha - \mathcal{P}_{\alpha-1}), \tag{A10}$$

$$\mathcal{P}_\alpha = (2\alpha+1)^{-1}(\mathcal{P}_{\alpha+1}{}' - \mathcal{P}_{\alpha-1}{}'). \tag{A11}$$

Using (19) and (A4)–(A6), we find

$$A = \tfrac{1}{2}\sum_{n=0}^{\infty}[1 + \eta(-)^n e^{-i\pi\alpha}]A_n \mathcal{P}_{\alpha-n}(-z), \tag{A12}$$

$$B = \tfrac{1}{2}\sum_{n=0}^{\infty}[1 + \eta\eta'(-)^n e^{-i\pi\alpha}]B_n \mathcal{R}_{\alpha-n}(-z), \tag{A13}$$

$$C = \tfrac{1}{2}\sum_{n=0}^{\infty}[1 + \eta\eta''(-)^n e^{-i\pi\alpha}]C_n \mathcal{P}_{\alpha-n}{}'(-z), \tag{A14}$$

[18] A. V. Sterling, P. Sonderegger, J. Kirz, P. Falk-Variant, O. Guisan, C. Bruneton, P. Borgeand, M. Yvert, J. P. Guillard, C. Caberzasio, and B. Amblard, Phys. Rev. Letters 14, 763 (1965).

[19] D. Gordon and J. Frøyland (private communication).

where $z=1-(s/2m^2)$, and

$$\mathcal{R}_\alpha(z)=\alpha(\alpha+1)\mathcal{P}_\alpha(z)-z\mathcal{P}_\alpha'(z) \quad \text{(A15)}$$
$$=(2\alpha+1)^{-1}[\alpha^2\mathcal{P}_{\alpha+1}'(z)-(1+\alpha)^2\mathcal{P}_{\alpha-1}'(z)].$$

By using (A11) and (A15), we find that the conspiracy condition can be expressed in the form

$$\sum_{n=0}^{\infty}[f_n\mathcal{P}_{\alpha-n+1}'(z)-g_n\mathcal{P}_{\alpha-n-1}'(z)-h_n\mathcal{P}_{\alpha-n}'(z)]=0, \quad \text{(A16)}$$

where

$$f_n=\tfrac{1}{2}[1+\eta(-)^ne^{-i\pi\alpha}][A_n/(2\alpha-2n+1)]$$
$$+\tfrac{1}{2}[1+\eta\eta'(-)^ne^{-i\pi\alpha}]$$
$$\times[(\alpha-n)^2B_n/(2\alpha-2n+1)], \quad \text{(A17)}$$

$$g_n=\tfrac{1}{2}[1+\eta(-)^ne^{-i\pi\alpha}][A_n/(2\alpha-2n+1)]$$
$$+\tfrac{1}{2}[1+\eta\eta'(-)^ne^{-i\pi\alpha}]$$
$$\times[(1+\alpha-n)^2B_n/(2\alpha-2n+1)], \quad \text{(A18)}$$

$$h_n=\tfrac{1}{2}[1+\eta\eta''(-)^ne^{-i\pi\alpha}]C_n. \quad \text{(A19)}$$

The coefficients of $\mathcal{P}_\alpha'(-z)$ have different asymptotic behavior for different α, i.e., $\mathcal{P}_\alpha'(-z)\to(-2z)^{\alpha-1}$. Therefore, the conspiracy conditions are

$$f_0=0,$$
$$f_1=h_0, \quad \text{(A20)}$$
$$f_{n+1}-g_{n-1}-h_n=0, \quad (n=1,2,3,\cdots).$$

The general solution is

$$A_0+\alpha^2B_0=0,$$
$$A_1+(1-\alpha)^2B_1=(2\alpha-1)C_0,$$
$$(2\alpha-2n-1)^{-1}[A_{n+1}+(\alpha-n-1)^2B_{n+1}] \quad \text{(A21)}$$
$$=(2\alpha-2n+3)^{-1}$$
$$\times[A_{n-1}+(\alpha+2-n)^2B_{n-1}]+C_n,$$
$$(n=1,2,3,\cdots),$$

with additional requirements depending on the relative signatures η' and η'', as specified below:

η'	η''	Additional requirement	
1	−1	None	
−1	1	$A_n=0$ $(n=0,1,2,\cdots)$	(A22)
−1	−1	$B_n=0$ $(n=0,1,2,\cdots)$	
1	1	$C_n=0$ $(n=0,1,2,\cdots)$.	

The two Volkov-Gribov solutions are special cases. Solution 1 consists of choosing $\eta'=1$, $\eta''=-1$, and all A_n, B_n, C_n zero except A_0, B_0, C_1. Thus,

$$B_0=-\alpha^{-2}A_0, \quad \text{(A23)}$$
$$C_1=\alpha^{-2}A_0.$$

Solution 2 consists of choosing $\eta'=1$, $\eta''=-1$, and all A_n, B_n, C_n zero except A_1, B_1, C_0. Thus,

$$B_1=-\alpha^{-2}A_1, \quad \text{(A24)}$$
$$C_0=\alpha^{-2}A_1.$$

When A_n, B_n, C_n are translated into residue functions by Table III, we obtain the results in Table II.

An interesting special case is obtained by taking $\eta'=1$, $\eta''=-1$, and setting $A_n=0$ (all $n>0$), $C_n=0$ (all $n\geq 0$). Then there must be an infinite number of daughters in the amplitude B, with all residues determined by A_0:

$$B_0=-(1/\alpha^2)A_0,$$
$$B_{n+1}=\frac{2\alpha-2n-1}{2\alpha-2n+3}\left(\frac{\alpha+2-n}{\alpha-n-1}\right)^2B_{n-1}, \quad \text{(A25)}$$
$$(n=1,3,5,\cdots).$$

Let the residue corresponding to B_n be denoted by β_n. Then from Table III we can translate (A25) into

$$\beta_{n+1}=\frac{(\alpha-n)(\alpha-n+2)}{(\alpha-n-1)(\alpha-n+1)}\beta_{n-1}, \quad (n=1,3,5,\cdots), \quad \text{(A26)}$$

or

$$\beta_2=\left(\frac{\alpha+1}{\alpha}\right)\left(\frac{\alpha-1}{\alpha-2}\right)\beta_0,$$
$$\beta_4=\left(\frac{\alpha+1}{\alpha}\right)\left(\frac{\alpha-3}{\alpha-4}\right)\left(\frac{\alpha-1}{\alpha-2}\right)^2\beta_0, \quad \text{(A27)}$$
$$\beta_6=\left(\frac{\alpha+1}{\alpha}\right)\left(\frac{\alpha-5}{\alpha-6}\right)\left[\frac{(\alpha-3)(\alpha-1)}{(\alpha-4)(\alpha-2)}\right]^2\beta_0,\cdots.$$

Thus, when $\alpha\to 0$, $|\beta_{2n}/\beta_0|$ diverges like α^{-1} for all $n>0$:

$$\beta_{2n}\underset{\alpha\to 0}{\longrightarrow}\frac{1}{\alpha}\left(\frac{2n}{2n-1}\right)\left[\frac{1\times 3\times 5\times\cdots\times(2n-1)}{2\times 4\times 6\times\cdots\times 2n}\right]^2\beta_0, \quad \text{(A28)}$$
$$(n=1,2,3,\cdots).$$