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# Ionization Loss at Relativistic Velocities

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The variation of grain density in the tracks of 5-, 8-, 12-, and 24-GeV/$c$ protons incident at angles of 75°, 60°, 50°, and 40°, respectively, and of 5-GeV/$c$ pions incident at an angle of 75° to the edge of the plate has been investigated as a function of velocity in the same Ilford G-5 pellicle. About 80 000 blobs were counted for each beam. The ratio of grain densities at 5-GeV/$c$ pion to proton is $\sim 1.114 \pm 0.01$. The results are in agreement with the Sternheimer theory using an ionization potential for AgBr of 434 eV and a cutoff energy $T_0$ of 2–5 keV, which is in contradiction with the result due to Jongejans (who uses the cutoff energy $T_0 = 100$ keV), but in agreement with Shapiro's result ($T_0$ between 2 and 5 keV) and with Barkas's result (with 0–10% correction due to secondary ionization). The data are also consistent with the slow rate of rise in the $g^*$ value as a function of velocity which is predicted by the theory.

## 1. INTRODUCTION

When a fast charged particle passes through matter it loses energy by inelastic collision with atomic electrons. We are concerned here with the ionization loss of a heavy singly charged particle going through a condensed medium such as a nuclear emulsion.

Bohr[1] initiated the theory by predicting an indefinitely continued relativistic increase in ionization with velocity. Following a suggestion by Swann,[2] Fermi[3] improved the theory by taking into account the polarization of the medium. Fermi, however, assigned a single frequency to all the electrons in the medium while Wick,[4] Halpern and Hall,[5] and Sternheimer[6,7] showed that it is essential to construct a multifrequency theory. In order to apply their results to energy loss by ionization in emulsion, it is necessary to calculate the restricted rate of ionization loss arising only from the energy transfer to electrons up to a few keV which are absorbed along the track. The theoretical expression for the restricted energy loss (energy losses smaller than $T_0$ per incidental collision) of a singly charged heavy particle (not an electron) due to distant collisions is given as follows[8]:

$$\left(\frac{dE}{dx}\right)_{\mathrm{coll}\ll T_0} = \frac{A}{\beta^2}\left[\ln\frac{2m_e c^2\beta^2\gamma^2 T_0}{I^2(Z)} - \beta^2 - 2C\right], \qquad (1)$$

where $A = 0.06705$ MeV cm$^2$/g for AgBr. Here $dE/dx$ is the energy loss per unit path length; $E$ is the energy of the ionizing particle and $v = \beta c$ is its velocity; $m_e$ is the electron rest mass; $\gamma = (1-\beta^2)^{-1/2}$; $T_0$ is the upper limit of $\delta$-ray energy corresponding to the maximum energy deposited in a single grain (assumed $\ll E$); $I(Z)$ is the mean ionization potential of atoms in the medium (AgBr), and was calculated according to the formula given by Sternheimer[9] (434 eV); and $C$ is the "density-effect correction" which is a function of the particle velocity and has been tabulated by Barkas.[8]

In Eq. (1) the constant $A$ is unimportant for relative ionization. According to this theory, energy loss per unit path length varies as $1/v^2$ at lower velocities while for a relativistic particle the energy loss increases logarithmically with the energy of the particle. The logarithmic term in Eq. (1) which plays an important part has two not precisely known quantities, i.e., the mean ionization potential $I$ which is a measure of the smallest amount of energy which can be transferred on the average to a bound electron and the cutoff energy $T_0$. The fundamental question is how $T_0$ and $I$ determine the shape of the curve. In principle one has a method of determining independently $T_0$ and $I$ from ionization measurements; the position of the minimum and the depth of the trough will determine these two constants. It is difficult to estimate these values theoretically, and they are best determined experimentally. Even experimentally one is faced with a lack of una-

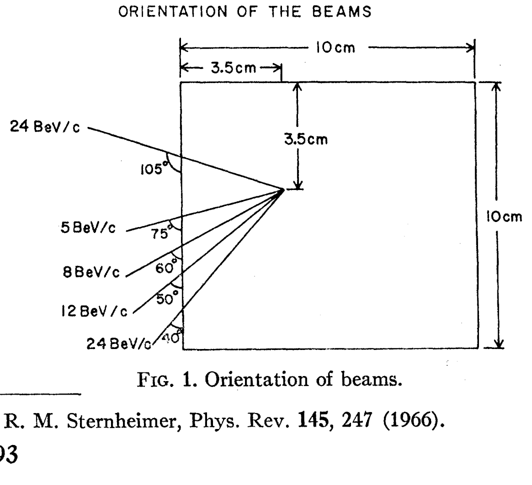

FIG. 1. Orientation of beams.

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[1] N. Bohr, Phil. Mag. **30**, 581 (1915).

[2] W. F. G. Swann, J. Franklin Inst. **226**, 598 (1938).

[3] E. Fermi, Phys. Rev. **56**, 1242 (1939); **57**, 485 (1940).

[4] G. C. Wick, Ricerca Sci. **11**, 274 (1940); **12**, 858 (1941); Nuovo Cimento **1**, 302 (1943).

[5] O. Halpern and H. Hall, Phys. Rev. **73**, 477 (1948).

[6] R. M. Sternheimer, Phys. Rev. **91**, 256 (1953).

[7] R. M. Sternheimer, Phys. Rev. **103**, 511 (1956); 88, 851 (1952).

[8] W. H. Barkas, *Nuclear Research Emulsions* (Academic Press Inc., New York, 1900), p. 383.

[9] R. M. Sternheimer, Phys. Rev. **145**, 247 (1966).

nimity regarding these constants. This may be due to the fact that these constants may have only a limited influence on the restricted energy loss and the role of one may be partially fulfilled by the other.

The earlier experimental difficulties in establishing the existence of a rise in grain density with increasing relativistic energies in a dense medium like a nuclear emulsion was due to a failure to distinguish between the restricted rate and the average rate of ionization loss. Pickup and Voyvodic[10] first reported a relativistic rise of ~10% and this was later supported by many authors.[10–17] Investigation of electron tracks in an emulsion established the existence of an ionization plateau at high energies. However these measurements were not extended down to the minimum of ionization and therefore could not yield the ratio $g_{\text{pl}}/g_{\text{min}}=$ (ionization at plateau)/(ionization at minimum). Shapiro[11] used cosmic rays and for $\gamma>100$ only electrons were used, while for the interval $10<\gamma<100$ both electrons and meson tracks were used. Michaelis and Violet[12] used machine-produced electrons at energies corresponding to the expected values at minimum and plateau, namely 2.93 and 293 MeV and found a ratio $g_{\text{pl}}/g_{\text{min}}$ of $1.08\pm0.01$. This value is probably underestimated because of the difficulties of excluding background grains when counting blobs on heavily scattered tracks. The level of development was also high, i.e., 26 blobs/(100 $\mu$) at plateau. The same was true in the work of Fleming and Lord[13] [27–35 blobs/(100 $\mu$)] in which they made comparisons between pions of various momenta and the electrons of average energy $34\pm10$ MeV secondary to $\mu$ decay. Alexander and Johnson[14] carried out an investigation using plates which had been exposed at the Bevatron and had been used for a careful investigation of the relative frequency of the decay of $K^+$ mesons. The secondary particles of the two-body decay modes $K_{\mu 2}$ and $K_{\pi 2}$ were compared with those of beam pions. The experimental data were made to fit the Bethe-Bloch formula[15,16] for "restricted ionization loss" for the mean ionization potential ($I$) value of $530\pm100$ eV and for the maximum energy transfer ($T_0$) value of $(2.9\pm0.5)\times10^4$ eV. Jongejans[17] used pions of energy $\sim36<\gamma<42$ and electron pairs with $65<\gamma<1100$. Stiller[18] used $\pi^-$ beams of nominal energy 450 MeV and also electron beams of energies 100, 210, 450, and 1000 MeV. He used an emulsion with a variety of mean grain diameters with the object of investigating the possible effect of emulsion grain diameter on the relativistic rise in ionization loss in the region from its minimum value to the Fermi plateau. His result indicates an increase of $g_{\text{pl}}/g_{\text{min}}$ with decreasing crystal size, which contradicted, in part, the earlier results of Stiller and Shapiro.[11] Buskirk[19] *et al.* used K-5 plates exposed to 16 GeV/$c$ $\pi^-$ and by using the technique of comparison with high-energy electrons from pairs caused by $\pi^0$ mesons produced in interactions by the primary particles, they got results which are in agreement with those of Jongejans[17] and with Johnston *et al.*,[14] who used G-5 emulsion and obtained the same $g_{\text{pl}}/g_{\text{min}}$ ratio of ~1.14. In general one can say that the ratio $g_{\text{pl}}/g_{\text{min}}$ varies between 1.06 and 1.14 while the plateau appears to be reached at $10<\gamma<200$.

It is quite possible that a certain portion of the measured ionization may consist of a contribution from $\delta$ rays superimposed on the primary ionization. The magnitude of this effect has been studied by Barkas and Patrick.[20] On the basis of a certain simple mechanism, they find for singly charged particles that the secondary grain density is given by an expression of the form $g_s=A/\beta^2$. Barkas found $A=3.9/100\ \mu$, a surprisingly high figure, one which would imply that ~25% of the observed grain density at minimum is of secondary origin.

Recently Sternheimer[9] has derived the relation for the ionization potential as $I/Z=(9.76\pm58.8)Z^{-1.19}$ eV; for AgBr, $I\sim434$ eV. To compare their experimental results with the theory, Shapiro and Jongejans used different values of the constants $I$ and $T_0$. Shapiro used the value $I=376$ for the ionization potential for two values of $T_0$: 2 and 5 keV. The calculated ratio $g_{\text{pl}}/g_{\text{min}}$ for $T_0=2$ keV is 1.152 and for $T_0=5$ keV is 1.137. They[11] found that the ionization loss ($g$) was not sensitive to the choice of $T_0$ between 2 and 5 keV. Jongejans's results were compared with the theory by using different values of $T_0$, i.e., 20, 100, and 500 keV, and for the ionization potential $I$ equal to 501, 377, and 574 eV. A least-squares fit showed that the theory for $I=501$ eV and $T_0=100$ keV gave the best result in fitting their data. This is in contradiction to the value of $T_0$ (~2 keV) which fits Shapiro's result. The apparent disagreement of the value of $T_0$ of Jongejans with that of Shapiro, along with the usage of different ionization potentials for the restricted energy loss theory, led us to attempt a definite measurement of $T_0$ and $I$ by removing the fundamental objections in previous experiments which are summarized as follows:

1. In a number of experiments which were performed previous to ours, the energy of the primary particles (cosmic-ray particles) was not known and it was determined by scattering measurements which would introduce an error ~15 to 20%. We must point out in the beginning that the over-all effect that we are

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[10] E. Pickup and L. Voyvodic, Phys. Rev. **80**, 89 (1950).

[11] B. Stiller and M. M. Shapiro, Phys. Rev. **87**, 682 (1952); **92**, 735 (1953).

[12] R. P. Michaelis and C. E. Violet, Phys. Rev. **92**, 511 (1953).

[13] J. R. Fleming and J. J. Lord, Phys. Rev. **92**, 511 (1953).

[14] G. Alexander and R. H. W. Johnston, Nuovo Cimento **5**, 363 (1957).

[15] H. A. Bethe, Z. Physik **76**, 293 (1932).

[16] F. Floch, Z. Physik **81**, 363 (1933).

[17] B. Jongejans, Nuovo Cimento **16**, 625 (1959).

[18] B. Stiller, *Korpuscularphotographie* (Munchen, 1963), p. 542.

[19] F. R. Buskirk *et al.*, CERN 65-4 (unpublished).

[20] J. W. Patrick and W. H. Barkas, Nuovo Cimento **23**, 1 (1962).

looking for experimentally is an increase of about ~10 to 15% over the whole range. So our observational error has to be much less than this to make any meaningful contribution.

2. In some experiments[10–13,17–20] different particles were used with different energies rather than a single particle of different energies.

3. In almost all cases, different rather than the same pellicles were used for different particles with different energies. It is found that when different pellicles are processed, even together in the same solution under well-controlled conditions, a variation of a few percent in $g_{\min}$ among the several plates is observed.

4. Even when a single pellicle was used, different parts of the pellicle were used for the primary track. It is already known that the ionization changes not only from section to section of the pellicle but also from top to bottom of the same section of the pellicle.

Thus various experimental difficulties tend to obscure the rise in the ionization curve. Ideally, one would prefer to measure the rate of ionization loss over a wide range of velocities for a single type of particle accelerated to known energies. Also the tracks which are to be compared should be selected simultaneously in the same sample of emulsion and if possible at the same depth in the pellicle. Exactly this was achieved in our experiments.

## 2. EXPERIMENTAL TECHNIQUES

The nuclear emulsion used in this experiment is a part of a small stack of an Ilford G-5 emulsion with pellicle dimensions 10 cm×10 cm×600 $\mu$ exposed to four different primary beams of 5-, 8-, 12-, and 24-GeV/$c$ protons at the CERN proton synchrotron.[21] In the relativistic region in which one is interested here, a small difference in the ionization value is important, and thus one must take special precautions in the exposure. All the beams to be compared in this experiment were produced in the emulsion at the same temperature and within a time interval of a few hours.[21] This helped to keep the variation in sensitivity of the emulsion from top to bottom in the pellicle very low, and even this small variation was avoided[7] by picking up the tracks at the same depth in the emulsion, as explained later in the paper. The 5-GeV/$c$ beam has a mixture of protons and pions. The proton beam widths in the emulsion plane are approximately $1\frac{1}{2}$ cm, while for the 5-GeV/$c$ beam the width was much wider. The angles that the beams make in entering the emulsion with the edge of the plate are 75°, 60°, 50°, and 40° for the 5, 8, 12, and 24-GeV/$c$ beams, respectively. The 24-GeV/$c$ proton beam is inclined at two different angles, i.e., one at 40° and the other at 75° on the opposite side of the normal to the edge of the plate as shown in Fig. 1. The axes of the beams intersect at a common point 3.5 cm from the two adjacent sides of the plate; the point is also shown in Fig. 1. The 5-GeV/$c$ beam is much more intense than any other beam used in this experiment. All the beams are very flat with an average dip angle of approximately 0.3°.

TABLE I. Grain densities for all the beams with a maximum statistical error of ~0.4%.

| Beam momentum (GeV/$c$) | Grain density [grains/(100$\mu$)] | Grain density relative (to 5-GeV/$c$ proton) |
|---|---|---|
| 5 (proton) | 22.88 | 1.000 |
| 5 (pion) | 25.50 | 1.1145 |
| 8 (proton) | 23.32 | 1.0192 |
| 12 (proton) | 23.99 | 1.0485 |
| 24 (proton) | 24.97 | 1.0914 |

Blob counting was carried out using a Bausch and Lomb binocular microscope equipped with a special built-in scattering (rotating) stage. The plate was placed under the objective so that the intersection point of the beams was at the center of rotation of the state. The stage could be rotated until a particular beam was parallel to the $x$-axis movement of the stage. Since the beam was flat, a length of the track could be followed by observing $x$ movement alone. In order to pick up the other tracks, parallel to the particular beam, the plate could be moved in the $y$ direction up to about 7 or 8 mm on either side of the center point. The $x$ movement of the stage was limited to about 25 mm. For each of the four energy beams, a relatively small rectangular section of the plate approximately 20 mm by 15 mm was used. All the tracks used were picked up from almost the same depth in the emulsion so that the uncertainties in the variation in the emulsion itself are kept to a minimum.

Tracks were followed under a magnification of about 1500× from about 1 to 2 cm of length. None of our measurements made on a single track were less than 1 cm. In each beam more than 80 000 blobs were counted so that the statistical error in the blob count was ~0.4% for each case. A weighted average of the blob density per 100 $\mu$ was calculated for each beam. The 5-GeV/$c$ beam, which is a mixture of protons and pions, was separated, so as to give two distinct peaks corresponding to a peak blob density 17.8 and 19.4 for protons and pions, respectively. The blob density was then converted to grain density by the well-known method of O'Ceallaigh.[22] For the region of particular interest between the minimum and the plateau, the blob density is found to be proportional to the grain density so we used grain density in all our calculations. The values of mean grain density per 100 $\mu$ for all the beams are shown in Table I. The value of $\gamma$ (the total energy of the particle divided by its rest-mass energy) is a function of the particle velocity. For the 5-, 8-, 12-, and 24-GeV/$c$ protons and 5-GeV/$c$ pions the

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[21] We are very grateful to Professor W. M. Gibson of Bristol University, England for sending us the developed plates.

[22] C. O'Ceallaigh, CERN Report B. S. 11, 1954 (unpublished).

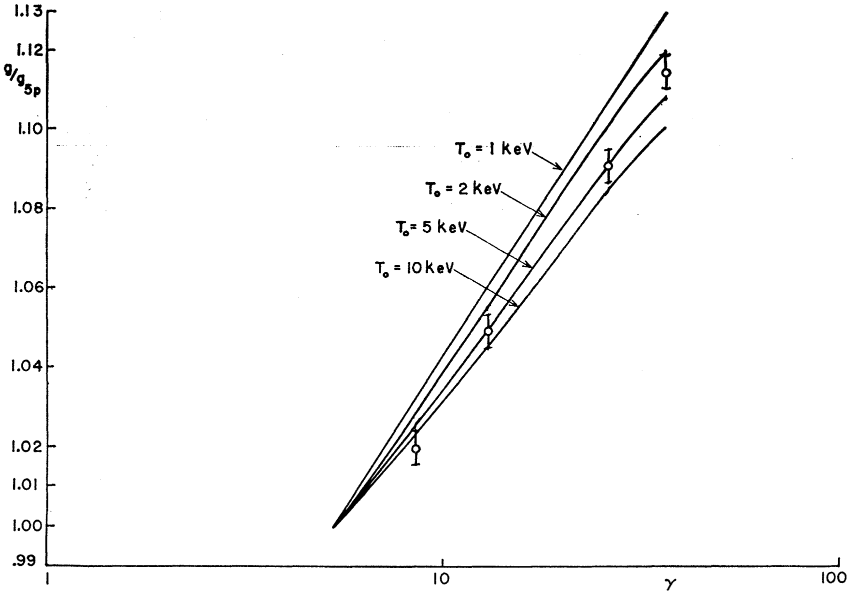

FIG. 2. Relative grain density as a function of $\gamma=(1-\beta^2)^{-1/2}$ for the mean ionization potential $I=434$ eV with $T_0=1$, 2, 5, and 10 keV. The errors shown for the experimental points are the statistical ones. All the experimental and the theoretical points are normalized to 5-GeV/$c$ protons.

values of $\gamma$ are 5.3, 8.6, 12.9, 25.7, and 35.8, respectively. Since we are discussing the ionization loss in terms of relative grain density $g^*$, the grain densities are normalized to the smallest $g$ value, i.e., 5-GeV/$c$ proton grain density. Relative grain densities are also shown in Table I.

## 3. RESULTS AND DISCUSSION

We are interested in the exact shape of the ionization curve as a function of velocity and this will depend upon the values of $T_0$ and $I$. In order to find the value of these two parameters, we used the experimental data for the ionization loss in terms of the relative grain density $g^*$, i.e., the grain density for different values of $\gamma$ normalized to the smallest $g$ value obtained from the primary beam which is here the 5-GeV/$c$ proton grain density. The theoretical values of $g^*$ for different $\gamma$'s were calculated from Eq. (1) using $I=434$ eV and the value of $C(\gamma)$ extrapolated from the table in Barkas's text.[8] The theoretical curves for $g^*$ versus $\gamma$ for different values of $T_0$ are plotted normalized to the 5-GeV/$c$ proton ($\gamma=5.33$) and are shown in Fig. 3 along with the experimental data. In Fig. 2 we notice that the majority of the observed points lie closest to the theoretical curve represented by $T_0\sim 5$ keV, although we cannot rule out any value from 2 to 10 keV. One gets similar results when the experimental points are normalized to 8-, 12-, and 24-GeV/$c$ protons. The fact that normalization to 5-GeV/$c$ pions and to 5-GeV/$c$ protons gives almost identical results near low $T_0$ values confirms the supposition that the curves are independent of the incident particle's identity. This is just what should be expected of the restricted ionization loss with which we are concerned here. In this region of $5<\gamma<36$, the shape of the curve is more sensitive to $T_0$ than the value of $I$ used. It should be noted that the values of $T_0\sim 2$ to 10 keV are in strong disagreement with Jongejans's previously quoted result of $T_0\sim 100$ keV and are in good agreement with the result of Shapiro, $T_0\sim 2$ to 5 keV. Also the relative ionization loss is somewhat sensitive to the choice of the $T_0$ value, especially at higher $\gamma$ values, which is in contradiction with the conclusion of Shapiro. A maladjustment of $T_0$, however, can be partially compensated by a large change in $I$.

TABLE II. Comparison of some values of $T_0$ as explained in the text.

| Author | $g_{\rm pl}/g_{\rm min}$ | $T_0$ (keV) |
|---|---|---|
| Alexander *et al.*[a] | 1.133±0.008 | 70 |
| Stiller *et al.*[b] | 1.14±0.03 | 2–5 |
| Fleming *et al.*[c] | 1.14±0.01 | 40 |
| Jongejans[d] | 1.129±0.010 | 100 |
| Michaelis *et al.*[e] | 1.097±0.010 | 500 |
| Stiller[f] | 1.07±1.13 | 1–5 |
| Buskirk *et al.*[g] | 1.14 | 100 |
| Barkas *et al.*[h] | 1.18 | 2 |
| Present work | 1.114±0.004<br>[$g_{5\text{-GeV}/c\text{ pion}}/g_{5\text{-GeV}/c\text{ proton}}$] | 2–5 |

[a] Reference 14.
[b] Reference 11.
[c] Reference 13.
[d] Reference 17.
[e] Reference 12.
[f] Reference 18.
[g] Reference 19.
[h] Reference 20.

Barkas and Patrick found a best fit to their data with Eq. (1) using $I=442$ eV and $T_0=2$ keV. Their value of $I$ (442 eV) does not significantly change the shape of the curve from that plotted in Fig. 2, $I=434$ eV and $T_0=2$ keV. However, in fitting their data to the theory they assumed that at minimum ionization 25% of the observed grain density was due to secondary ionization from $\delta$ rays between 2 and 22 keV. This secondary grain density is of the form $A/\beta^2$. But our results indicate 10% to be a more reasonable estimate of the secondary ionization.

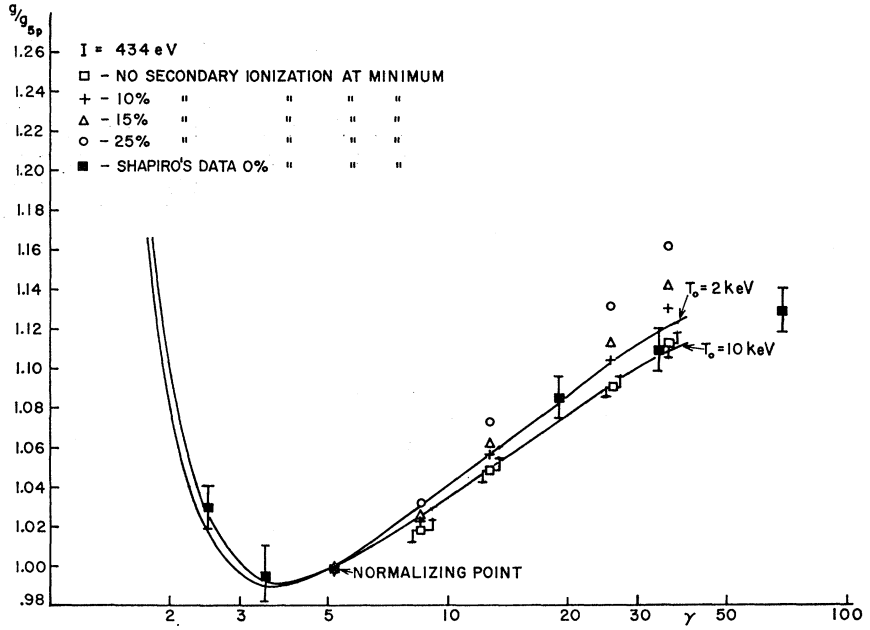

FIG. 3. Relative grain density as a function of $\gamma=(1-\beta^2)^{-1/2}$ for the mean ionization potential $I=434$ eV, with $T_0=2$ and 5 keV. Data are shown for no secondary ionization at minimum and for 10%, 15%, and 25% secondary ionization at minimum. Shapiro's data with 0% secondary ionization at minimum are also shown.

The result of our work described above is best summarized in Fig. 3 where the values of $g^*$ are plotted as a function of $\gamma$ for tracks gathered from a single pellicle. The theoretical curve with $I=434$eV and $T_0=2$ keV is essentially the best fit to the data of Barkas and Patrick with the 25% contribution (of the form $A/\beta^2$) due to secondary $\delta$ rays subtracted out. For comparison purposes we have also plotted the experimental points from Shapiro's work, normalized to 1 at $\gamma=5.33$. The agreement of our data with those of Shapiro and with theory ($T_0\sim 2$–10 keV) puts us in strong disagreement with Jongejans and Barkas (before correction). To illustrate our disagreement with the latter we have also plotted our data (with no error loss) after subtracting out various percentage contributions due to secondary ionization of the form $A/\beta^2$. The best fit to the theory $I=434$ eV, $T_0=2$ keV (corresponding to energy lost in a single AgBr crystal) is given by our data with about a 0–10% correction due to secondary ionization. The difference in $I$ between our value (434 eV) and that used by Jongejans (500 eV) only partially compensates for the difference in value of $T_0$ from 2 to 100 and we are in disagreement with Jongejans's previously quoted result of $T_0\sim 100$ keV.

Differential experimental results on $g_{\mathrm{pl}}/g_{\mathrm{min}}$ are given in Table II. Our results for the $T_0$ value 2–5 keV agree with Shapiro's if one accepts the proposal of Messel and Ritson[23] that in the energy-loss calculation the maximum transferable energy $T_0$ should be replaced by the energy of a $\delta$ ray with a range about equal to the grain size; for the emulsion this is about 2–5 keV. Of course a value of $T_0$ in excess of $\sim$30 keV would lead to the production of a $\delta$ ray of length 7 $\mu$.

We point out that in the assignment of errors, we used the statistical error as $100/\sqrt{N}$ %, where $N$ is the total number of blobs of the tracks determining the point. The variation in relative grain density $g^*$ from the mean value of the relative grain density $\bar{g}^*$ for each energy beam is expressed in terms of its variance as

$$\sigma_{g^*}{}^2=\sum_{i=1}^{n}(g_i^*-\bar{g}^*)^2/\sqrt{n},$$

where $n$ is the total number of tracks for each energy beam. The observational variance due to imperfect counting procedure on the same track (counted twice) is denoted by $\sigma_{\mathrm{obs}}{}^2$ and is related to the variance in statistical error as well as to the variance in relative grain density $\sigma_{g^*}$ as

$$\sigma_s{}^2=\sigma_{g^*}{}^2-\sigma_{\mathrm{obs}}{}^2.$$

For the error resulting in $\bar{g}^*$, we have to divide $\sigma_s$ by the square root of the number of tracks used in the determination of $\bar{g}^*$, i.e.,

$$\sigma_s'=\sigma_s/n.$$

Taking the same number of tracks in each of the four

TABLE III. Determination of errors (grains/100$\mu$).

| Beam momentum (GeV/$c$) | $\sigma_g{}^*$ | $\sigma_s$ | $\sigma_s'$ | $\sigma_{\mathrm{obs}}$ |
|---|---|---|---|---|
| 5 $p$ | 0.027 | 0.018 | 0.004 | 0.020 |
| 5 $\pi$ | 0.035 | 0.030 | 0.006 | 0.018 |
| 8 $p$ | 0.051 | 0.044 | 0.007 | 0.026 |
| 12 $p$ | 0.043 | 0.038 | 0.006 | 0.020 |
| 24 $p$ | 0.035 | 0.032 | 0.006 | 0.015 |

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[23] H. Messel and D. Ritson, Phil. Mag. 41, 1129 (1950).

energy beams we summarize the results for $\sigma_s$ in Table III. Thus we see that the observational error is as large as the statistical error in grain-counting a single track 1 cm long.

In the above discussion it has been assumed that the ionization of singly charged particles depends only on $E/mc^2$, and not on their nature or the sign of their charge. This assumption is necessary in the absence of measurements covering the whole energy range with a single type of particle. There are practical difficulties in the way of achieving such a complete coverage, which come nearest to being solved in the case of electrons; they are easily identified and are available with a wide range of energies from zero to hundreds of MeV. Unfortunately, near the region of minimum ionization, multiple scattering becomes very large. Consequently it becomes very difficult to measure the track length in the emulsion, to avoid accidental background grains lying close to the track, and to avoid jumping from one trace to another at points where tracks cross; finally, multiple scattering introduces a high probability that the particle will leave the emulsion after only a comparatively short distance. For these reasons, measurements on electrons below and in the vicinity of the minimum ionization ($2<\gamma<6$) require very great care, and it has been found exceedingly difficult to use electrons in this region. On the other hand, electrons are very useful for higher-energy values where we cannot as yet have heavy but singly charged machine-made particles.

We must remember that the mechanism of production of developed centers in silver halide crystals is exceedingly complex and very imperfectly understood. So far in our discussion, we have not taken into consideration the different kinds of emulsions and the different development techniques among the different experiments. It is just possible that the shape of the experimental curve and the value of the observed $g_{\mathrm{pl}}/g_{\mathrm{min}}$ ratio may depend upon the degree of development and also on the type of emulsion used since the size of the underdeveloped crystals varies from emulsion to emulsion. It is also possible that the temperature of various emulsions used in the previous experiments might not have been sufficiently well under control with the result that the sensitivity of the emulsion used might be different at the times of injection of various particles and might vary from region to region in the same emulsion stack. The question of the possible effects of differential fading may also give rise to some serious problems.

It seems clear that there is a need for further careful work in the low- as well as in the high-energy range with different kinds of emulsion under various development conditions. We should not forget that in these experiments the precision required is very high, as we are dealing here with very small variations. The results of such experiments will throw some light on the problem of the magnitude and the velocity dependence of the secondary ionization.

We also point out that from the recent theoretical work of Tsytovich[24] it appears that by considering the second-order approximation of perturbation theory he was led to the conclusion that radiative corrections may become important at values of $\gamma>10^2$. Hence, according to this author, the first-order approximations which take account of the effects of polarization by previous workers may fail in the extreme relativistic region. For the purpose of comparison of ionization of electron energy in the range of $100<\gamma<10^5$ we shall try to make exposures at the Standford linear accelerator of high-energy electrons with different kinds of emulsions under various conditions of their development.

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[24] V. N. Tsytovich, Dokl. Acad. Nauk SSSR **144**, 310 (1962) (English transl.: Soviet Phys.—Doklady) **7**, 411 (1962)].