Measurements of the Polarization Parameters P and C_{nn} in $p\hat{p}$ Elastic Scattering between 500 and 1200 Mev*

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The polarization parameter P and the spin correlation parameter C_{nn} in proton-proton elastic scattering have been measured in an experiment with a polarized-proton target in the beam extracted from the synchrotron "Saturne." The angular distribution of P was measured at 7 energies between 0.5 and 1.2 GeV. The value of P shows a maximum of $+0.6$ at about 700 MeV for a scattering angle of 45° in the center-ofmass system. At 1.2 GeV the value of P is consistent with 0 for scattering angles $\gtrsim 70^{\circ}$ (momentum transfers $-t\gtrsim0.8$ (GeV/c)²]. The spin correlation coefficient C_{nn} was measured at three energies, 0.735, 0.978, and 1.19 GeV, by scattering a polarized-proton beam on the polarized target. The value of C_{nn} at 90° c.m. decreases from 0.7 to 0.4 as the energy increases from 0.735 to 1.19 GeV.

I. INTRODUCTION

'HE data on nucleon-nucleon scattering at intermediate energies have led to phase-shift analyses in the region of 700 MeV and at 970 MeV.¹ The solutions may be divided into two groups. One of them corresponds to high inelasticity at low angular momenta (central absorption); the other one corresponds to peripheral absorption.

Measurements of the polarization parameter P have been carried out, by various techniques, at several energies up to 735 MeV, at 970 MeU, and at higher energies.² The results have shown that the polarization reaches a maximum of $P \approx +0.6$ at about 700 MeV.

Measurements of the spin correlation coefficient C_{nn} measurements of the spin correlation coefficient C_{nn}
have shown³ that at 90[°] (c.m.) the value of $(1-C_{nn})_{90}$ ° is about 0.5 at 575 MeV and about 0.15 at 680 MeV. These results give an indication for the ratio of singletto-triplet scattering amplitudes at this angle.

⁸ H. E. Dost, University of California Radiation Laboratory
Report No. UCRL 11877, 1965 (unpublished); H. E. Dost et al.,
Phys. Rev. 153, 1394 (1967); B. M. Golovin, V. P. Dzhelepov,
R. Y.a. Zul'karneev, and Ts'ui Wa-Ch' (1963)].

We have measured the asymmetry parameter α (equal to the polarization parameter P) in protonproton scattering at seven energies between 500 and 1200 MeV with a polarized-proton target. At two of these energies (735 and 1194 MeV), we have extended the angular distribution to small angles by the technique of double scattering on hydrogen. At the energies 735, 970, and 1194 MeV, we have measured the angular distribution of the spin correlation coefficient C_{nn} with a polarized-proton beam incident on the polarized target.

II. WOLFENSTEIN PARAMETERS AND HELICITY AMPLITUDES

The transition matrix may be written in the following form, using the notation introduced by Wolfenstein':

$$
M(k, k') = BS + C(\sigma_1 + \sigma_2) \cdot N
$$

+
$$
\frac{1}{2}G[(\sigma_1 \cdot K)(\sigma_2 \cdot K) + (\sigma_1 \cdot P)(\sigma_2 \cdot P)]T
$$

+
$$
\frac{1}{2}H[(\sigma_1 \cdot K)(\sigma_2 \cdot K) - (\sigma_1 \cdot P)(\sigma_2 \cdot P)]T
$$

+
$$
N[(\sigma_1 \cdot N)(\sigma_2 \cdot N)]T.
$$

S and T are the operators projecting on the singlet state and the triplet state, respectively:

$$
S = \frac{1}{4} (1 - \sigma_1 \cdot \sigma_2),
$$

\n
$$
T = \frac{1}{4} (3 + \sigma_1 \cdot \sigma_2).
$$

 N , P , and K are unit vectors defined as follows:

$$
N = \frac{k_i \times k_j}{|k_i \times k_j|},
$$

\n
$$
P = \frac{k_i + k_j}{|k_i + k_j|},
$$

\n
$$
K = N \times P.
$$

⁴ L. Wolfenstein, Phys. Rev. 96, 1654 (1954).

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^{*}G. Bizard, J.F.Lemeilleur, and J.Yonnet from the University of Caen have participated in part of this experiment. '

Y. Hama and N. Hoshizaki, Progr. Theoret. Phys. (Kyoto) 31, 1162 (1964); Research Institute for Fundamental Physics (Kyoto), Report RIFP-47, 1965 (unpublished); Z. Janout, Yu. M. Kazarinov, F. Lehar, and A. F. Pisarev, Dubna Report No. E-2726, 1966 (unpublished).

² M. J. Longo, H. A. Neal, and D. E. Overseth, Bull. Am. Phys.
Soc. 10, 717 (1965); P. G. McManigal, R. D. Eandi, S. N. Kaplan, and B. J. Moyer, Phys. Rev. 137, B620 (1965); D. Cheng, University of California Radiation Laboratory Report No. UCRL 11926, 1965 (unpublished); D. Cheng *et al.*, Phys. Rev. (to be published); F. W. Betz, University of Ca

This expression for $M(k, k')$ is obtained assuming that the interaction is invariant under time reversal and under parity.

The above coefficients can be written as functions of the helicity amplitudes⁵:

$$
B = f_1 - f_2,
$$

\n
$$
C = -[i \sin\theta (f_1 + f_2 + f_3 + f_4) + i \cos\theta f_4],
$$

\n
$$
H = \frac{1}{2} (f_1 + f_2 - f_3 + f_4),
$$

\n
$$
N = \frac{1}{2} \cos\theta (f_1 + f_2 + f_3 - f_4) - \sin\theta f_5,
$$

\n
$$
G - N = f_3 + f_4.
$$

The amplitudes $f(\theta)$ are defined as follows:

$$
f_1(\theta) = \langle \theta, 0, ++ | M | 0, 0, ++ \rangle,
$$

\n
$$
f_2(\theta) = + + - - ,
$$

\n
$$
f_3(\theta) = + - + - ,
$$

\n
$$
f_4(\theta) = + - - + ,
$$

\n
$$
f_5(\theta) = - + + + + .
$$

 $|\theta, \varphi, \lambda_a, \lambda_b\rangle$ represents a state of two protons in their c.m. system, with the helicities λ_a and λ_b and with a momentum in the direction (θ, φ) .

The differential cross section for scattering of polarized protons on a polarized-proton target is

$$
d\sigma/d\Omega = (d\sigma/d\Omega)_0 \left[1 + P(\alpha + \beta) \cdot \mathbf{n} + C_{nn}(\alpha \cdot \mathbf{n}) (\beta \cdot \mathbf{n}) \right], (1)
$$

where α and β are the polarization vectors of the beam and of the target, respectively, and where n is the unit vector normal to the scattering plane. If both the incident protons and the target protons are polarized perpendicularly to the scattering plane, we have

> $\alpha = \alpha n$, $\alpha =$ polarization of the beam; $\beta = \beta n$, $\beta =$ polarization of the target.

The unpolarized cross section $(d\sigma/d\Omega)$ ₀, the polarization P, and the spin correlation coefficient C_{nn} are, in this case, expressed as functions of the Wolfenstein parameters or as functions of the helicity amplitudes by the following relations:

$$
\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{1}{2}\left[|f_1|^2 + |f_2|^2 + |f_3|^2 + |f_4|^2 + 4|f_5|^2\right],
$$

$$
\left(\frac{d\sigma}{d\Omega}\right)_0 P = 2 \text{ Re}C^*N = \text{Im}f_5(f_1 + f_2 + f_3 - f_4)^*,
$$

$$
\left(\frac{d\sigma}{d\Omega}\right)_0 (1 - C_{nn}) = \frac{1}{4}\left[G - N + B\right]^2 + \frac{1}{4}\left[G - N - B\right]^2
$$

$$
= \frac{1}{2}\left[\left|f_3 + f_4\right|^2 + \left|f_1 - f_2\right|^2\right].
$$

⁵ M. Jacob, thesis, University of Paris, 1961 (unpublished); M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. 120, 2250 (1960).

FIG. 1. The beam layout.

III. APPARATUS

This section describes the various parts of the apparatus used to measure α , P , and C_{nn} .

A. Beam Transport

The proton beam is extracted from the accelerator at the desired energy and brought to a first focus (Fig. 1). For the measurements of α the direct beam proceeds from there through a system with two-fold magnetic analysis, and is brought to a final (third) focus at the polarized target.

For the measurements of P and of C_{nn} , a first target (polarizer) is placed at the first focus and the same beam transport system is used to analyze the scattered protons and to produce a focus at the final target.

B. Polarized Target

The polarized-proton target consists of seven single crystals of $\text{La}_2\text{Mg}_3(\text{NO}_3)_{12}\cdot24\text{H}_2\text{O}$ (LMN) doped with 1% (solution) of Nd¹⁴². Its diameter and length are 25 and 33 mm, respectively. The polarization was on the average $|\beta|$ = 0.65. The polarization is measured by the enhancement of the nuclear magnetic resonance (NMR) signal of the proton. The method consists in measuring the variation of the ^Q value of a coil close to the crystal, as a function of the radio frequency. The signal at thermal equilibrium was measured with an error of $\pm 4\%$. The enhancement of the polarization is calculated from the enhanced NMR signal taking into account the absorption coefficient x'' and the dispersion coefficient x' of the crystal. For a Q meter tuned with a coaxial cable of length $\frac{1}{2}\lambda$, the approximate relation

$$
4\pi QX'' = [(V_0/V)^2 - (4\pi QX')^2]^{1/2} - 1
$$

is valid with an error of less than 1% . V/V_0 is the relative variation at the output voltage of the ^Q meter due to the increase of the polarization. The quantities $4\pi Qx''$ and $4\pi Qx'$ are computed by iteration from the above expression and from the relation between dispersion and absorption:

$$
\chi'(w) = \chi'(-\infty) + \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{\chi''(w')dw'}{(w'-w)},
$$

$$
[\chi''(-w') = -\chi''(w')].
$$

TABLE I. Experimental results for the asymmetry parameter a and the polarization parameter P .

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The relative error of the enhancement factor is of the order of $\pm 4\%$. The polarization of the target is therefore known with an error of about $\pm 6\%$.

Uniformity of the polarization throughout the crysta has been checked directly by aiming a collimated b beam, about 8 mm wide, on three different points of the target. The results of the three sets of measurements agree within statistics and show that the average 10% . The geometry of the radio-frequency coils and the polarization of the three zones differed by less than normal beam size tend to average out nonuniformity if it exists.

C. Detectors

For the three experiments, the arrangement of scintillation counters has been used to detect scattering on free protons in the polarized target (α and C_{nn}), or in a $CH₂$ target (P) . The target, the three counter arrays, and the analyzing magnet are shown in Fig. 2. The counters of the arrays P and Q provide constraints on the angular correlation of the two scattered protons for both polar and azimuthal angles. The magnet and the array R add a third constraint on the momentum of one of the two protons. The horizontal width of the individual counters was chosen such as to match the "irreducible width" of the hydrogen peaks which is mostly due to multiple scattering and differential energy loss in the target, and to finite target size. The coplanarity requirements were less stringent, in order to keep the number of counters and electronics circuits as small as possible. The background from scattering on bound protons was measured by runs with a dummy arget which contained no hydrogen.

D. Electronics

Three counters must be triggered in coincidence, one in each of the arrays P, Q, and R. If more than three counters are triggered, the event is supressed by an anticoincidence. Two of the four coplanarity have to be triggered in either one of the two expected configurations. If an event satisfies these criteria, the numbers of the corresponding counters are coded and stored in a memory. The system was able to register up to 800 events per burst, with a beam intensity of the order of 3×10^7 protons per burst of 0.4 sec.

E. Monitoring

The beam rate was too high for direct counting and too low for accurate monitoring by an ionization ch ber. A secondary monitor telescope counted the proton scattered by the target in the vertical plane which contains the polarization vector. Its counting rate is therefore independent of the polarization of the target and provides a relative measure of the number of protons interacting within the target. The correct normalization of successive runs was checked by means of those events which correspond to scattering on bound nucleons

FIG. 2. Experimental setup.

within the nuclei. Such events were recorded simulaneously with the hydrogen events. Their rate is also independent of the polarization of the target and proportional to the rate of incident protons.

IV. MEASUREMENTS OF α (ASYMMETRY WITH UNPOLARIZED BEAM AND POLARIZED TARGET)

The asymmetry parameter α is given by

$$
\alpha = \frac{(I_+)- (I_-)}{(I_+)+ (I_-)} \quad \text{or} \quad I_\pm = I_0 (1 \pm \alpha) \,,
$$

where $I_+(\theta)$ and $I_-(\theta)$ are counting rates proportional to the differential cross section for scatt unpolarized beam in the plane normal to the polarization of the target, in the case where the target polarization is ± 1 , and where one ignores the polarization of the scattered protons. On the other hand, the expression (1) shows that in this case $I_{\pm} = I_0(1\pm P)$ ($\alpha = 0, \beta = \pm 1$), thus $P = \alpha$. This equality results from having neglected, in expression (1) , those terms which are not invariant under time reversal.

One measurement consisted usually in six runs with alternate signs of the polarization β . A value for α can be deduced from any one of the three pairs by the relation

$$
a = \frac{I_+ - I_-}{|\beta_- I_+| + |\beta_+ I_-|}
$$

The angular distribution of α (or P) was measured with three overlapping positions of the counter arrays and the analyzing magnet. The results are shown in Fig. 3 and in Table I. The limit at small scattering angles corresponds to recoil protons of about 50 MeV in the laboratory.

The error includes the statistical error and a $\pm 6\%$ variable error in the measurement of the polarization other). of the target (variable from one set of points to the

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FIG. 4. Coefficients of Legendre polynomials for $P/\sin\theta_{\rm c.m.}$. The full circles correspond to this experiment. Curves are a manual fit of the results.

At each energy we have fitted functions of the form

$$
\frac{P}{\sin\theta_{\text{e.m.}}} = \sum_{n} C_n P_n(\cos\theta_{\text{e.m.}}).
$$

The coefficients C_1 , C_3 , and C_5 as functions of energy are shown in Fig. 4. The figure also represents fits to data obtained by other groups. The lowest-order term is predominant, with a maximum at 700 MeV. The coefficient C_3 shows no strong energy dependence; C_5 changes sign at about 1000 MeV.

V. MEASUREMENT OF P AT SMALL ANGLES (DOUBLE SCATTERING ON HYDROGEN)

For the measurements of α with a polarized target of LMN, discrimination against background from bound protons was possible only by making use of the angular correlation of the scattered protons. This method fails at small angles where the recoil proton has an energy of less than about 50 MeV in the laboratory.

At two of the seven energies, we have extended the angular distribution to smaller angles, by measuring the polarization P of protons scattered from liquid hydrogen. The polarization is measured by the asymmetry of a second scattering on hydrogen, at a larger angle where the analyzing power, i.e., the value of α , is known from the measurements with the polarized target.

A target of liquid hydrogen is placed at the first focus of the extracted beam. Two deflecting magnets up-

TABLE II. Experimental results for polarization parameter P' as obtained from double scattering on hydrogen.

T_p (MeV)	$\theta_{\rm n.m.}$	
735	(6.7 ± 2) ° $(18.3+2)$ °	$0.118 + 0.16$ $0.367 + 0.032$
1194	(13.8 ± 2) ° $(21.7+2)$ °	0.124 ± 0.033 $0.224 + 0.03$

stream of this point are adjusted to make the beam strike this target at a small angle with respect to the axis of the subsequent system. This system accepts protons scattered within an interval of $\Delta\theta_{lab}=\pm1^{\circ}$, including the width and the divergence of the incident beam. The momentum of the scattered protons is defined by a slit at the intermediate image with a resolution of $\Delta p / p = \pm 0.01$. A final focus is produced at the second target.

The second target is a cylinder of $CH₂$, which replaces the polarized target. Scattering on free protons is detected by the same arrangement of counter arrays. The angle of the second scattering was chosen to be in the region of $\theta_{\text{c.m.}} \simeq 45^{\circ}$ where the analyzing power is largest.

The number of protons incident on the first target was monitored by a secondary emission chamber. The effective direction of the incident protons was measured and checked by film exposures showing the "shadow" of two markers placed at either side of this target. Background of inelastic hydrogen scatters (pion production) was eliminated by the magnetic analysis (first target) and by the coincidence requirements (second target). Background from the Mylar walls of the first target and from the edges of the first collimator was measured by runs with no hydrogen. Background from bound protons within the carbon nuclei of the second target was from 5 to 10% only, and was determined by a smooth extrapolation from the off-angle coincidences.

The value of P is given by

$$
P(\theta_1) = \frac{1}{\alpha(\theta_2)} \frac{(I_+ - I_-)}{(I_+ + I_-)},
$$

where $I_{\pm}=I_0(1\pm P\alpha)$ are the counting rates proportional to the differential cross sections at the second scattering angle θ_2 , for the scattering angles $+\theta_1$ and $-\theta_1$ at the first target (reversal of the polarization of the scattered beam).

The results are given in Table II. They are represented on Fig. 3, together with the measurements of α , and have been included in the fits of Legendre polynomials.

VI. MEASUREMENT OF C_{nn} (POLARIZED BEAM ON POLARIZED TARGET)

The spin correlation coefficient C_{nn} has been measured by scattering a polarized beam on a polarized target, both polarizations being normal to the scattering plane.

Errors are mainly statistical.

A beryllium target was placed at the first focus of the extracted beam. The primary beam strikes this target at an angle of $\pm 9^{\circ}$ with respect to the axis of the subsequent beam transport system which analyses the scattered protons and produces a final focus at the polarized target (see Sec. V). The polarization of the beam (a) is deduced from a measurement with $\beta = 0$ (unpolarized target) if P is known from previous experiments. At 735 MeV, the scattered beam had a polarization of $\alpha = 0.392 \pm 0.014$, in agreement with existing data on p -Be scattering. At 978 MeV and at 1190 MeV the polarization was, respectively, 0.28 ± 0.028 and 0.31 ± 0.07 . The intensity was about 10⁶ protons per burst. The reversal of the beam polarization can be assumed since special care was taken to ensure that the beam was produced by a first scattering at exactly the same angle to the left (for one sign), or to the right (for the other sign).

TABLE III. Experimental results for the spin correlation parameter C_{nn} .

$T_{p, {\rm lab}}$ (MeV)		735			978			1190	
	$\theta_{\rm c.m.}$	C_{nn}	δ_{Cnn}	$\theta_{\rm c.m.}$	C_{nn}	δ_{Cnn}	$\theta_{c.m.}$	C_{nn}	$\pmb{\delta}_{Cnn}$
	92.1	0.88	0.21	77.4	0.79	0.17	92.2	0.44	0.32
	88.8	0.55	0.34	73.4	0.69	0.16	88.6	0.43	0.39
	85.5	0.88	0.29	70.0	0.39	0.27	85.0	0.50	0.27
	82.0	0.65	0.25	68.6	0.71	0.47	81.1	0.46	0.21
	78.5	0.83	0.23	64.3	0.44	0.27	77.1	0.37	0.22
	74.8	0.47	0.16	60.0	0.46	0.26	73.1	0.59	0.22
	71.0	0.66	0.19	55.6	0.62	0.20	69.0	0.38	0.15
	66.7	0.51	0.10	51.2	0.61	0.16	65.0	0.31	0.21
	62.3	0.45	0.09	46.7	0.72	0.15	60.2	0.47	0.14
	58.4	0.50	0.09	42.0	0.70	0.19	55.6	0.57	0.12
	53.9	0.43	0.09				50.8	0.43	0.11
	49.6	0.38	0.06				46.0	0.36	0.11
	45.3	0.53	0.06				41.2	0.48	0.10
	40.9	0.35	0.09				36.2	0.35	0.10
	35.5	0.46	0.11						

According to Eq. (1) we can determine C_{nn} if we measure the counting rate for at least two sets of values for α and β . The values of P, α , and β are known (P from the previous experiment). In fact, we used four values of counting rates corresponding to different values of the polarization of the beam and of the target.

We have measured the angular distribution of C_{nn} at three energies. The results are given in Fig. 5 and in Table III. The value of C_{nn} at 90° in the c.m. system decreases from about $+0.7$ at 735 MeV to about $+0.4$ at 1194 MeV.

VII. CONCLUSION

The existence of a maximum of the polarization parameter P (P \approx 0.6) at an energy of about 700 MeV is confirmed by our experiment. Our results at 609 MeV are consistent with those of Coignet et al. at 595 MeV, and slightly lower than those of Cheng et al.; at 500 MeV they are also lower than those of Cheng et al.²

The spin correlation coefficient C_{nn} at 90° c.m. also seems to reach a maximum $(C_{nn}\approx 0.7)$ at about 700 MeV.

Note that this energy is close to the threshold for production of the isobar $N^*_{3/2,3/2}(1236)$. The total inelastic cross section increases from $\sigma_{\text{tot inel}} = 3$ mb at 500 MeV to $\sigma_{\text{tot inel}} = 23$ mb at 800 MeV. At these energies, all aspects of the inelastic reaction are well explained by the production of the 3,3 isobar.⁶ A phase-shift analysis of the elastic scattering data should vield inelasticity parameters consistent with those deduced from the isobar model.7

The quantity $P(\partial \sigma/\partial \Omega)$ has been expanded in associated Legendre polynomials of the first order, P_n ¹ (Fig. 6). The presence of a P_4 ¹ term shows a strong

⁶ S. Mandelstam, Proc. Roy. Soc. (London) A244, 491 (1958);

E. Ferrari and F. Selleri, Nuovo Cimento 27, 1450 (1963).

7 U. Amaldi, Jr., R. Biancastelli, and S. Francaviglia, Nuovo Cimento 47A, 85 (1967).

FIG. 6. Coefficients obtained by Legendre-polynomial fit:
 $I_0P = \sum C_n P_n!$. Curves are a manual fit to the results.

interference between the P wave and the F wave above 500 MeV. A P_8^1 term denotes the presence of H wave above 900 MeV.

At 1200 MeV the values of P between 75° and 90° are close to zero. The results at higher energies⁸ also show very low polarization at high momentum transfer.

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M. J. Longo, H. A. Neal, and O. E. Overseth, Phys. Rev. Letters I6, 536 (1966).