Gravitational Superenergy as a Generator of Canonical Transformation

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The canonical transformation generated in linearized gravitation theory by the Robinson-Bel superenergy tensor is determined. The light this result sheds on the quantization program for the general theory of relativity is discussed.

I. INTRODUCTION

IN recent discussions[1] some doubt was cast on the possibility of obtaining a local quantization of general relativity. Briefly, the argument ran as follows. An examination of the procedure of field quantization for special relativistic field theories indicates that in the transition from the canonical group of the classical theory to the unitary group of the quantum theory it is essential that a particular subgroup of the canonical group, the subgroup corresponding to the space-time symmetries (i.e., the Poincaré group) be singled out for preferred treatment. The preferred treatment accorded the space-time group is that its structure be taken over intact into the quantum theory. The epistomological significance of this requirement and its evident relationship to the correspondence principle will be discussed elsewhere. For our purpose it is relevant to note that the space-time group has no preferred group-theoretic characterization within the context of the canonical group. It is the physics of the space-time continuum which singles it out. In the general theory of relativity the space-time group is the Einstein group (the group of four-dimensional curvilinear coordinate transformations). However, since this group effectively acts as a gauge group, the observables of the theory constrained to be gauge invariant, the Einstein group should not be preserved in the quantization of general relativity, but should in fact be eliminated. Having thereby eliminated the preferred space-time group, the possibility of a unique local quantization in conformity with the correspondence principle becomes doubtful. One is immediately led to consider nonlocal quantization schemes where a preferred space-time group is restored via boundary conditions on the physically permissible solutions (e.g., asymptotic flatness at infinity).

In this paper we wish to suggest a novel way of recovering much of the content of the space-time translation group without the difficulties attendant when merely resorting to gauge transformation. The essential observation is that, in linear field theories it is always possible to extend any first-order infinitesimal invariant transformation to higher differential orders such that the higher-order differentials are invariant transformations in their own right. In this fashion, starting from the space-time translations we can obtain proper canonical transformations. In Sec. II we shall illustrate our procedure for the simple case of the Klein-Gordon theory. The specific expressions obtained for the generators of canonical transformations are specializations of a general result found by Steudel[2,3] in a somewhat different connection. In Sec. III we shall determine the canonical transformation generated in linearized gravitation theory by the Robinson-Bel superenergy tensor. We shall find that the heuristic characterization of the superenergy as a second derivative of energy momentum can be made more specific by observing that it generates precisely the third derivative of the field variable. The implication for the full nonlinear gravitation theory of these results obtained in linear field theories will be discussed in Sec. IV.

II. GENERATION OF HIGHER-ORDER DIFFERENTIALS

For the free particle we can distinguish two different kinds of constants of the motion, both of which are necessary for the full determination of the classical trajectory. The first kind, which is typified by the momentum, but also includes the energy and angular momentum, is distinguished by the property that the time t does not enter explicitly into its definition. We shall call such constants of the motion "p type." The second kind of constants of the motion are the so-called "time-dependent constants," whose prototype is the initial position $x_0=x-vt$. Such constants of the motion we shall call "x type."

The extension of this dichotomy of constants of the motion to field theory is evident. The p-type constants are those whose density can be expressed solely in terms of the field variables, without the introduction of other geometric objects. For a homogeneous field, the energy-momentum is a typical example. The x-type constants of the motion in field theory have as their prototype the value of the field on some initial space-like hypersurface. For their definition they therefore require in an essential way an auxiliary geometric object, in this case in order to specify the initial space-

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[1] A. Komar, in Proceedings of the International Conference on Relativistic Theories of Gravitation, London, 1965, Vol. II (unpublished).

[2] H. Steudel, Nuovo Cimento **39**, 395 (1965).

[3] H. Steudel, Z. Naturforsch. **21a**, 1826 (1965).

like hypersurface. (It might be noted that in this case the angular momentum is now regarded as x type in view of the fact that the auxiliary object x^k is required for its definition.)

Let us now inquire into the possibility of p-type constants of the motion other than the energy-momentum. For simplicity we shall confine our discussion to the Klein-Gordon theory, the extension to homogeneous linear theories of higher spin being straightforward. The method we shall employ for the construction of constants of the motion and for the determination of the infinitesimal canonical transformations which they generate is that of the Lagrangian formalism.[4] The basic relation between the constant of the motion and the infinitesimal transformation which it generates is determined by the identity

$$C^\rho{}_{,\rho}+\lambda_a F_a\equiv 0\,, \tag{2.1}$$

where C^ρ is the conserved current density, a comma denotes differentiation, F_a are the Euler-Lagrange expressions whose vanishing constitute the field equations, and $\lambda_a=\delta\phi_a$ is the sought infinitesimal change in the field variable ϕ_a, generated by the constant of the motion

$$C=\int C^\rho dS_\rho\,. \tag{2.2}$$

We may summarize the content of Eq. (2.1) employing Poisson brackets

$$[\phi_a,C]=\lambda_a\,. \tag{2.3}$$

For the Klein-Gordon theory a complete set of constants of the motion may be obtained from a generating density of the form

$$C^\rho=\alpha\phi^{,\rho}-\alpha^{,\rho}\phi+D^{[\rho\mu]}{}_{,\mu}+f^\rho \tag{2.4}$$

where α is any solution of the Klein-Gordon equation, $D^{[\rho\mu]}$ is an arbitrary antisymmetric tensor, and f^ρ vanishes modulo the Euler-Lagrange equations. Although f^ρ does not contribute to the value of the constant its inclusion will enable us to satisfy the identity (2.1). ($D^{[\rho\mu]}$ generates nothing locally, but it becomes relevent when we consider asymptotic questions. We shall therefore neglect such terms in the remainder of this paper.) The set of constants obtained from the generating density of Eq. (2.4) is complete in the sense that if we select for α the singular solutions of the Klein-Gordon equation, $D(x-x')$, the constants of Eq. (2.2) are precisely the values of the field ϕ at each space-time point.

In order to obtain p-type constants of the motion we must select α such that it is a solution of the Klein-Gordon equation which involves no auxiliary geometric objects. If, in addition, we require that the generating density C^ρ be a local field, all that is available is the field ϕ itself and its derivatives. It is evident from Eq. (2.4) that for $\alpha=\phi$, C^ρ cannot generate anything. A somewhat more detailed investigation reveals that replacing α by any even-order derivative of ϕ does not yield a C^ρ which can satisfy Eq. (2.1). (The constants associated with the even-order derivatives of ϕ are x-type constants, closely associated with the multipole moments of the field Φ.) We must therefore confine our attention to the odd-order derivatives of ϕ. For example, if in Eq. (2.4) we take

$$\alpha=\tfrac{1}{2}\phi_{,\alpha} \tag{2.5}$$

and choose f^ρ judiciously, we may write the generating density thus:

$$C^\rho{}_\alpha=\tfrac{1}{2}(\phi_{,\alpha}\phi^{,\rho}-\phi_{,\alpha}{}^\rho\phi+\delta_\alpha{}^\rho\phi(\Box+m^2)\phi)\,, \tag{2.6}$$

whereupon

$$C^\rho{}_{\alpha,\rho}+\phi_{,\alpha}F\equiv 0\,, \tag{2.7}$$

where

$$F\equiv-(\Box+m^2)\phi=0 \tag{2.8}$$

is the Euler-Lagrange equation in our present case. Thus comparison with Eqs. (2.1) and (2.3) yields

$$\left[\phi,\int C^\rho{}_\alpha dS_\rho\right]=\phi_{,\alpha}\,. \tag{2.9}$$

We thereby recognize $C^\rho{}_\alpha$ as the usual energy-momentum tensor. (The angular momentum may be similarly obtained by selecting $\alpha=\frac{1}{2}(x^\alpha\phi^{,\beta}-x^\beta\phi^{,\alpha})$; however, it manifestly yields an x-type constant in view of the auxiliary object x^α employed in its definition.) There was nothing unique in our choice of Eq. (2.5). If, instead, we employ for α any odd-order derivative of ϕ, the computations go through identically as before. We obtain thereby new p-type constants of the motion closely akin to energy-momentum. Rather than generating the first derivative of the field ϕ, these new constants of the motion generate the higher odd-order derivatives. The interesting and important feature of these new constants is that although they are in a certain sense trivially related to the energy-momentum, they generate *proper* canonical transformations, that is, canonical transformations which do not lie in the space-time subgroup. The simplest example of such constants is obtained by considering the third derivative of ϕ:

$$\alpha=\tfrac{1}{2}\phi_{,\alpha\beta\gamma}\,. \tag{2.10}$$

Again choosing f^ρ adroitly, if we write for the generating density

$$\begin{aligned}C^\rho{}_{\alpha\beta\gamma}=\tfrac{1}{2}[&\phi_{,\alpha\beta\gamma}\phi^{,\rho}-\phi_{,\alpha\beta\gamma}{}^\rho\phi\\&-\tfrac{1}{3}\phi(\delta_\alpha{}^\rho F_{,\beta\gamma}+\delta_\beta{}^\rho F_{,\alpha\gamma}+\delta^\rho{}_\gamma F_{,\alpha\beta})\\&+\tfrac{1}{6}\delta_\alpha{}^\rho(\phi_{,\beta}F_{,\gamma}+\phi_{,\gamma}F_{,\beta})+\tfrac{1}{6}\delta_\beta{}^\rho(\phi_{,\gamma}F_{,\alpha}+\phi_{,\alpha}F_{,\gamma})\\&+\tfrac{1}{6}\delta_\gamma{}^\rho(\phi_{,\alpha}F_{,\beta}+\phi_{,\beta}F_{,\alpha})\\&-\tfrac{1}{3}F(\delta_\alpha{}^\rho\phi_{,\beta\gamma}+\delta^\rho{}_\beta\phi_{,\alpha\gamma}+\delta_\gamma{}^\rho\phi_{,\alpha\beta})]\,,\end{aligned} \tag{2.11}$$

[4] P. G. Bergmann and R. Schiller, Phys. Rev. 89, 4 (1953).

we find the identity

$$C^{\rho}{}_{\alpha\beta\gamma,\rho}+\phi_{,\alpha\beta\gamma}F\equiv 0. \tag{2.12}$$

As before, comparing Eq. (2.12) with Eqs. (2.1) and (2.3) we find

$$\left[\phi,\int C^{\rho}{}_{\alpha\beta\gamma}dS_{\rho}\right]=\phi_{,\alpha\beta\gamma}. \tag{2.13}$$

III. ROBINSON-BEL SUPERENERGY

The procedure for the construction of constants of the motion of the Klein-Gordon theory which we employed in the preceding section may easily be extended to linear field theories of higher spin.[5] It is evident that we again obtain generators of proper canonical transformations by considering higher derivatives of the field variables. However, rather than proceeding in the inductive fashion which we employed in the discussion of the Klein-Gordon theory, we shall be much more specific in this section. In the general theory of relativity a fourth-order completely symmetric tensor occurs whose covariant divergence vanishes modulo the Einstein field equations. This tensor, named the Robinson-Bel tensor, is formed from the Riemann tensor much as the Maxwell stress tensor is formed from the Maxwell field tensor.[6] Although it has two indices too many to understand it as an energy-momentum tensor, the Robinson-Bel tensor is clearly associated in some sense with the energy-momentum distribution, for it can effectively be obtained by averaging the Einstein energy-momentum pseudotensor over an infinitesimal region keeping the lowest-order terms which yield a covariant contribution.[7] The question naturally arises whether the Robinson-Bel tensor, dubbed the "superenergy," generates canonical transformations in its own right, and if so, whether they are related to the space-time translations generated by the energy-momentum.

In the preceding section we found a fourth-order tensor occurring in the Klein-Gordon theory which generates proper canonical transformations trivially related to the space-time translations. It immediately suggests that the Robinson-Bel tensor is the analog in gravitation theory of the tensor $C^{\rho}{}_{\alpha\beta\gamma}$ of Eq. (2.11), in the sense that both generate the third derivative of their corresponding field tensors. The remainder of this section shall be devoted to confirming this conjecture for the linearized, spin-two field theory which may be obtained from the Einstein theory by discarding all nonlinear terms in the field equations. In order to facilitate a comparison with the full nonlinear theory we shall modify our notation slightly. Heretofore, we used a comma to denote ordinary differentiation; we shall now employ the comma to denote covariant differentiation with respect to the flat Minkowski metric $\eta_{\mu\nu}$. (In the usual rectangular coordinate frame there will of course be no change.)

Briefly, the linearized gravitation theory may be summarized as follows: The basic field variable is a second-order symmetric tensor $h_{\mu\nu}$ which is subject to gauge transformations such that tensors which can be obtained from one another via the relation

$$h'_{\mu\nu}=h_{\mu\nu}-\gamma_{\mu,\nu}-\gamma_{\nu,\mu}, \tag{3.1}$$

where γ_{μ} is an arbitrary vector field, are understood to describe the same gravitational field. The curvature tensor, defined by

$$R_{\alpha\beta\gamma\delta}\equiv\tfrac{1}{2}(h_{\alpha\delta,\beta\gamma}+h_{\beta\gamma,\alpha\delta}-h_{\beta\delta,\alpha\gamma}-h_{\alpha\gamma,\beta\delta}), \tag{3.2}$$

is the local quantity of lowest differential order which is invariant under the gauge group, Eq. (3.1). If in addition we define the Christoffel symbols

$$\Gamma_{\beta\gamma}{}^{\alpha}\equiv\tfrac{1}{2}\eta^{\alpha\mu}(h_{\mu\beta,\gamma}+h_{\mu\gamma,\beta}-h_{\beta\gamma,\mu}), \tag{3.3}$$

the Ricci tensor

$$R_{\alpha\beta}\equiv\eta^{\mu\nu}R_{\alpha\mu\beta\nu}, \tag{3.4}$$

the Ricci scalar

$$R\equiv\eta^{\alpha\beta}R_{\alpha\beta}, \tag{3.5}$$

and the Einstein tensor

$$G_{\alpha\beta}\equiv R_{\alpha\beta}-\tfrac{1}{2}\eta_{\alpha\beta}R, \tag{3.6}$$

a Lagrangian for the linearized theory may be written

$$L=\eta^{\mu\nu}(\Gamma_{\mu\nu}{}^{\alpha}\Gamma_{\alpha\beta}{}^{\beta}-\Gamma_{\mu\alpha}{}^{\beta}\Gamma_{\nu\beta}{}^{\alpha}) \tag{3.7}$$

from which we can deduce the field equations

$$G_{\alpha\beta}=0. \tag{3.8}$$

As a consequence of the gauge group (3.1), one can confirm that the Einstein tensor satisfies the identity

$$\eta^{\mu\nu}G_{\alpha\mu,\nu}\equiv 0. \tag{3.9}$$

If we now define the Weyl tensor

$$\begin{aligned}C_{\alpha\beta\gamma\delta}\equiv R_{\alpha\beta\gamma\delta}&+\tfrac{1}{2}(\eta_{\alpha\delta}R_{\beta\gamma}+\eta_{\beta\gamma}R_{\alpha\delta}\\&-\eta_{\alpha\gamma}R_{\beta\delta}-\eta_{\beta\delta}R_{\alpha\gamma})+\tfrac{1}{6}(\eta_{\alpha\gamma}\eta_{\beta\delta}-\eta_{\alpha\delta}\eta_{\beta\gamma})R\\&\equiv R_{\alpha\beta\gamma\delta}+S_{\alpha\beta\gamma\delta},\end{aligned} \tag{3.10}$$

we may write the Robinson-Bel tensor thus:

$$T_{\alpha\beta\gamma\delta}=C_{\alpha\mu\gamma\nu}C_{\beta}{}^{\mu}{}_{\delta}{}^{\nu}+C_{\beta\mu\gamma\nu}C_{\alpha}{}^{\mu}{}_{\delta}{}^{\nu}-\tfrac{1}{8}\eta_{\alpha\beta}\eta_{\gamma\delta}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}. \tag{3.11}$$

Exploiting the symmetries of the Riemann tensor, from Eq. (3.10) it is easy to obtain

$$C_{\alpha\beta[\gamma\delta,\epsilon]}=S_{\alpha\beta[\gamma\delta,\epsilon]} \tag{3.12}$$

(where the bracket denotes cycling the three subscripts), and

$$C_{\alpha\beta\gamma}{}^{\mu}{}_{,\mu}=S_{\alpha\beta\gamma}{}^{\mu}{}_{,\mu}+R_{\gamma\alpha,\beta}-R_{\gamma\beta,\alpha}. \tag{3.13}$$

Taking the divergence of Eq. (3.11), and employing

[5] A. Komar, Phys. Rev. **134**, B1430 (1964).
[6] M. Bel, *Théories Relativistes de la Gravitation* (Centre National de la Researche Scientifique, Paris, 1962), p. 119.
[7] F. A. E. Pirani, Phys. Rev. **105**, 1089 (1957).

Eqs. (3.12) and (3.13) we find

$$T_{\alpha\beta\gamma}{}^{\mu}{}_{,\mu}=S_{\alpha\sigma[\gamma\rho,\mu]}C_{\beta}{}^{\sigma\mu\rho}+S_{\beta\sigma[\gamma\rho,\mu]}C_{\alpha}{}^{\sigma\mu\rho} +C_{\alpha\sigma\gamma\rho}(S_{\beta}{}^{\sigma\mu\rho}{}_{,\mu}+R^{\sigma\rho}{}_{,\beta}-R^{\sigma}{}_{\beta}{}^{,\rho}) +C_{\beta\sigma\gamma\rho}(S_{\alpha}{}^{\sigma\mu\rho}{}_{,\mu}+R^{\sigma\rho}{}_{,\alpha}-R^{\sigma}{}_{\alpha}{}^{,\rho}). \quad (3.14)$$

Although the right-hand side of Eq. (3.14) evidently vanishes modulo the field equations (3.8), we cannot yet determine what the superenergy $T_{\alpha\beta\gamma}{}^{\mu}$ generates, for this equation is not of the form of Eq. (2.1). One discrepancy is that the comma on the left-hand side of the present equation denotes ordinary differentiation only in rectangular coordinates. If we wish to employ curvilinear coordinates we recall that the rectangular coordinates of Minkowski space are characterized up to a Lorentz transformation by an orthonormal quadruple of covariantly constant Killing vectors $\xi_a{}^{\mu}$. (Latin indices are used to denote the vectors of the quadruple.) We then project these basis vectors into the three indices of Eq. (3.14), thereby reducing covariant differentiation to ordinary differentiation. The second discrepancy with Eq. (2.1) is the usual situation whereby the right-hand side of Eq. (3.14) is not linear in the field equation (3.8). As in the Klein-Gordon example of the previous section we remedy this by an appropriate choice of f^{ρ}. Thus if we take

$$\xi_{abc}{}^{\alpha\beta\gamma}\equiv\xi_{(a}{}^{\alpha}\xi_{b}{}^{\beta}\xi_{c)}{}^{\gamma}, \quad (3.15)$$

where the parentheses denote complete symmetry, and define

$$T_{abc}{}^{\rho}\equiv\xi_{abc}{}^{\alpha\beta\gamma}(T_{\alpha\beta\gamma}{}^{\rho}-2C_{\alpha}{}^{\mu\rho}{}_{\beta}R_{\gamma\mu} +2C_{\alpha}{}^{\mu\nu}{}_{\beta}\delta_{\gamma}{}^{\rho}R_{\mu\nu}-2\eta_{\alpha\beta}C_{\gamma}{}^{\mu\nu\rho}R_{\mu\nu}), \quad (3.16)$$

we find, via Eq. (3.14), and the fact that the vectors $\xi_a{}^{\mu}$ are covariantly constant

$$T_{abc}{}^{\rho}{}_{,\rho}+2\xi_{abc}{}^{\alpha\beta\gamma}(C_{\alpha\mu\beta\nu,\gamma}+\eta_{\alpha\beta}C_{\gamma\mu\nu}{}^{\rho}{}_{,\rho} +\eta_{\alpha\mu}C_{\nu\beta\gamma}{}^{\rho}{}_{,\rho})G^{\mu\nu}\equiv 0. \quad (3.17)$$

We can therefore conclude as in Eq. (2.3)

$$\left[h_{\mu\nu},\int T_{abc}{}^{\rho}dS_{\rho}\right]=2\xi_{abc}{}^{\alpha\beta\gamma}C_{\alpha\mu\beta\nu,\gamma}. \quad (3.18)$$

[Note that in obtaining Eq. (3.18) we discarded the last two terms in the parenthesis of Eq. (3.17) in view of the fact that it follows from Eq. (3.13) that these terms vanish modulo the field equations.]

Employing Eqs. (3.10), (3.8), and (3.2) we can rewrite Eq. (3.18)

$$\delta h_{\mu\nu}=\xi_{abc}{}^{\alpha\beta\gamma}(h_{\alpha\nu,\beta\mu\gamma}+h_{\beta\mu,\alpha\nu\gamma}-h_{\mu\nu,\alpha\beta\gamma}-h_{\alpha\beta,\mu\nu\gamma}). \quad (3.19)$$

Although Eq. (3.19) is not quite in the form which we conjectured, we still have at our disposal the gauge transformations of Eq. (3.1). Employing the identity equation (3.9), it is readily seen that the generator of the gauge transformation may be written

$$C^{\rho}=2\gamma_{\mu}G^{\mu\rho}. \quad (3.20)$$

We see that C^{ρ} vanishes modulo the field equations. Such a term may therefore be incorporated into the definition of $T_{abc}{}^{\rho}$ of Eq. (3.16) without altering its value. It is simply one more term in the expression for f^{ρ}. If we choose for γ_{μ}

$$\gamma_{abc\mu}=\xi_{abc}{}^{\alpha\beta\gamma}(h_{\mu\alpha,\beta\gamma}-\tfrac{1}{2}h_{\alpha\beta,\gamma\mu}), \quad (3.21)$$

it is easy to check that three of the terms on the right-hand side of Eq. (3.19) cancel and we are left with

$$\delta h_{\mu\nu}=-\xi_{abc}{}^{\alpha\beta\gamma}h_{\mu\nu,\alpha\beta\gamma}. \quad (3.22)$$

Had we worked in a rectangular coordinate system throughout, we could have dispensed with the projection tensor $\xi_{abc}{}^{\alpha\beta\gamma}$, and obtain at this point

$$\left[h_{\mu\nu},\int T_{\alpha\beta\gamma}{}^{\rho}dS_{\rho}\right]=-h_{\mu\nu,\alpha\beta\gamma}. \quad (3.23)$$

Apart from the minus sign, the analogy of Eq. (3.23) with Eq. (2.13) is complete. We have demonstrated that in linearized gravitation theory the Robinson-Bel superenergy generates the proper canonical transformation which is obtained by taking the second derivative of a translation. It is in this precise sense that we may regard the superenergy as the second derivative of the energy-momentum.

IV. CONCLUSION

Within the context of the linear theories discussed in the preceding sections the only motivation for singling out the generators of the third derivative of the field variables was that it provided the simplest example of p-type constants of the motion other than the energy momentum. There exist infinitely many such generating densities which we can obtain by considering arbitrarily high odd-order derivatives of the field variables.

Our principal interest concerns the quantization of general relativity, which is of course a nonlinear theory. It is by no means evident that the methods which we have developed for linear theories have any extension to nonlinear theories. In general, we would in fact expect the contrary. However, for the particular case of the superenergy we have cause for encouragement. For the recognition of the existence of the superenergy first occurred within general relativity.[6] Namely, there exists within the Einstein theory a tensor $T_{\alpha\beta\gamma\delta}$ [obtained by the obvious modification of the formulas (3.11) and (3.10) replacing $\eta_{\alpha\beta}$ by $g_{\alpha\beta}$] which has the following properties: It is completely symmetric on all indices, the trace vanishes on any pair of indices, $\xi^{\alpha}\xi^{\beta}\xi^{\gamma}\xi^{\delta}T_{\alpha\beta\gamma\delta}\geq 0$ for any timelike vector ξ^{α}, and, most significantly, when the Einstein field equations are satisfied, we have

$$T_{\alpha\beta\gamma}{}^{\mu}{}_{;\mu}=0\,, \tag{4.1}$$

where the semicolon denotes covariant differentiation. When the Riemannian manifold permits Killing vector fields ξ^{μ}, we return to a situation strikingly similar to that of the linearized theory. For if we define

$$T^{\rho}=(\sqrt{g})\xi^{\alpha}\xi^{\beta}\xi^{\gamma}T_{\alpha\beta\gamma}{}^{\rho}\,, \tag{4.2}$$

then if follows from Eq. (4.1) that

$$T^{\rho}{}_{,\rho}=0\,, \tag{4.3}$$

where a comma again denotes ordinary differentiation. That is, we have again obtained a true constant of the motion which generates a proper canonical mapping closely related to that of the linearized theory.

In general there do not exist Killing fields in the solutions of the Einstein field equations. However, the fact that Eq. (4.1) remains valid gives rise to the expectation that the superenergy generates proper canonical transformation in the full nonlinear theory, closely related to the third derivative of the metric (as computed in some preferred coordinate system). Investigation of this conjecture is currently being pursued. The significance of an affirmative conclusion to this investigation for the quantization program has been indicated in the introduction to this paper. For the relationship between the space-time translations and the proper canonical transformations generated by the superenergy is conspicuous.

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Search for an Electron-Proton Charge Inequality by Charge Measurements on an Isolated Macroscopic Body*†

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A new method is reported for testing the electrical neutrality of matter containing an equal number of protons and electrons. A small iron spheroid was magnetically suspended in a uniform, horizontal electric field in such a manner that it was possible to measure electric deflecting forces small enough to detect 0.03 proton charge on the spheroid. An upper limit to the charge difference between the proton and electron, defined by $f=1+$(electron charge)/(proton charge), was found to be $|f|\leq 0.8\times10^{-19}$. It was necessary to assume: (neutron charge) = (electron charge) + (proton charge). Values of f in the range $0.8\times10^{-19}<|f|<2.8\times10^{-19}$ were excluded, and the probability that $|f|>0.8\times10^{-19}$ is not greater than 0.2. A by-product of the measurements was the finding that the iron spheroids contained less than 1 quark in 2.5×10^{18} nucleons. The measurements also permitted an estimate that the absolute electric charge on 2-eV photons is less than 10^{-16} proton charge.

I. INTRODUCTION

THE equality of the magnitudes of the electric charges of the proton and electron is an empirical discovery which remains as one of the fundamental mysteries of atomic physics. The very great experimental precision of this equality rests on measurements made during the last forty years,[1–4] although most of these measurements have been made in the last decade, the best of these being those by Hillas and Cranshaw (1959),[2] whose limiting accuracy sets an upper bound on the equality of 2 parts in 10^{21}.

Stimuli, other than curiosity, to experiments to look for a charge inequality between the proton and the electron have come, at various times, from suggestions that, if present, it might explain: (1) the magnetic field of the earth,[1] (2) the expansion of the universe,[5] (3) baryon conservation.[6] Items (1) and (2) are precluded by several experiments.[1–4] Any charge inequality, however small, would be sufficient to account for baryon conservation, if charge conservation is assumed.

In order to avoid deceptions arising from the systematic errors in a particular experimental method it is important to have several different experimental

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1 A. Piccard and E. Kessler, Arch. Sci. Phys. et Nat. **7**, 340 (1925).

2 A. M. Hillas and T. E. Cranshaw, Nature **184**, 892 (1959); *ibid.*, **186**, 459 (1960).

3 J. G. King, Phys. Rev. Letters **5**, 562 (1960).

4 J. C. Zorn, G. E. Chamberlain, and V. W. Hughes, Phys. Rev. **129**, 2566 (1963).

5 R. A. Lyttleton and H. Bondi, Proc. Roy. Soc. (London), **A252**, 313 (1959).

6 G. Feinberg and M. Goldhaber, Proc. Natl. Acad. Sci. U. S. **45**, 1301 (1959).