Nuclear Structure of Na^{22}. III. Spins and Parities from the Linear Polarization of Gamma Rays*

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(Received 4 August 1967)

The linear polarizations of some γ rays resulting from the α bombardment of F^{19} have been measured. From these results, spins and parities of levels in Na^{22} have been assigned or verified as follows: 0.891 MeV ($J^{\pi}=4^+$), 1.528 MeV (5^+), 1.937 MeV (1^+), 2.211 MeV (1^-).

I. INTRODUCTION

TWO previous reports[1,2] have presented results of angular correlation, lifetime and level-energy measurements in Na^{22}. We describe in the present work measurements of the linear polarization of γ rays resulting from the de-excitation of levels in the nucleus Na^{22}. The results of the present work supplement previous investigations and have been incorporated in Fig. 1 of Ref. 2. Section II, below, discusses the theory of the detection of linearly polarized γ rays and the treatment of the experimental data. Sections III and IV describe experimental details and results.

II. THEORY: TREATMENT OF EXPERIMENTAL DATA

Fagg and Hanna,[3] in their comprehensive review of polarization measurements on nuclear γ rays, have given in a very convenient form all of the formulas needed to analyze linear polarization measurements of γ rays emitted from nuclear levels aligned in a nuclear reaction. The objects of this section are twofold: to discuss a method of treatment of the experimental data which has not been commonly used[4–8] and to elucidate those conditions under which the linear polarization is a maximum at angles to the axis of alignment other than 90°.

We will assume that the linear polarization of the γ ray emitted by an aligned nuclear state is measured by Compton scattering of the γ rays from a central scattering crystal into one or the other of two analyzer crystals, one of the analyzers being in the plane defined by the alignment axis and the scattering crystal (the reaction plane) while the other is perpendicular to this plane. Polarimeters of this type for the study of γ rays from nuclear reactions have been described by Suffert, Endt, and Hoogenboom,[5] Beck,[6] Willmes and Harris,[7] and Poletti.[9] Let the probability of emission of a γ ray from an aligned state be $W(\theta,\delta)$ where θ is the angle of emission of the γ ray with respect to the alignment axis (e.g., the beam axis if a cylindrically symmetrical detection geometry is used) and δ is the angle which the polarization vector $\mathbf{E}$ makes with the reaction plane. Then, following Frauenfelder and Steffen,[8] we define the *degree of polarization* at the angle θ by

$$P(\theta)=[W(\theta,0°)-W(\theta,90°)]/[W(\theta,0°)+W(\theta,90°)]. \quad (1)$$

If the γ ray is completely polarized in the reaction plane, then $P(\theta)=+1$ while if it is completely polarized perpendicularly to this plane $P(\theta)=-1$. For a detector which is not polarization sensitive,

$$W(\theta)=W(\theta,0°)+W(\theta,90°)=A_0\sum_k a_kP_k(\cos\theta),$$

where $a_0\equiv 1$, $P_k(\cos\theta)$ is the kth order Legendre polynomial and k takes the values 0, 2, 4$\cdots$. The differential cross section $\sigma_c(\varphi,\gamma)$ for Compton scattering of a polarized γ ray in which the polarization of the scattered photon is not detected is[8]

$$\sigma_c(\varphi,\gamma)=\tfrac{1}{2}r_0^2(k^2/k_0^2)(k_0/k+k/k_0-2\sin^2\varphi\cos^2\gamma), \quad (2)$$

where φ is the scattering angle, γ is the angle between the polarization vector $\mathbf{E}$ of the incident photon and the scattering plane, and $r_0=e^2/mc^2$, i.e., the classical electron radius. The initial energy of the γ ray is k_0 while k is the final energy of the scattered quantum,

$$k=k_0[1+(1-\cos\varphi)k_0/mc^2]^{-1}.$$

The asymmetry ratio R of a polarimeter (for a given scattering angle φ) is defined as the ratio of the cross sections for scattering into directions, respectively, parallel to and perpendicular to the polarization vector $\mathbf{E}$, as:

$$R=\bar{\sigma}_c(\phi,\gamma=0)/\bar{\sigma}_c(\phi,\gamma=90)\equiv\bar{\sigma}_c(0)/\bar{\sigma}_c(90). \quad (3)$$

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] E. K. Warburton, J. W. Olness, and A. R. Poletti, Phys. Rev. **160**, 938 (1967).

[2] A. R. Poletti, E. K. Warburton, J. W. Olness, and S. Hechtl, Phys. Rev. **162**, 1040 (1967).

[3] L. W. Fagg and S. S. Hanna, Rev. Mod. Phys. **31**, 711 (1959).

[4] M. Suffert *et al.* (Ref. 5) appear to have been the first authors to use this method in the analysis of results obtained from linear polarization measurements of γ rays from nuclear reactions. Recently F. Beck (Ref. 6) and H. Willmes and G. I. Harris (Ref. 7) have also used the same method of analysis. H. Frauenfelder and R. H. Steffen (Ref. 8) give the appropriate formulas for γ-γ angular correlations from a radioactive source.

[5] M. Suffert, P. M. Endt, and A. M. Hoogenboom, Physica **25**, 659 (1959).

[6] F. Beck, Ann. Phys. (Paris) **1**, 503 (1966).

[7] H. Willmes and G. I. Harris, Phys. Rev. **162**, 1027 (1967).

[8] H. Frauenfelder and R. H. Steffen, in *Alpha, Beta and Gamma-Ray Spectroscopy*, edited by Kai Siegbahn (North-Holland Publishing Company, Amsterdam, 1965), p. 1040 ff.

[9] A. R. Poletti, Phys. Rev. **153**, 1108 (1967).

Here $\bar{\sigma}(\gamma)$ denotes an average taken over the solid angle of the analyzer corresponding to mean angles φ and γ. In the case of the particular polarimeter which we have described, $\gamma+\delta=\frac{1}{2}\pi$ for the analyzer *perpendicular to the reaction plane* and for the analyzer *in the reaction plane*, $\gamma=\delta$. For a γ ray completely polarized in the plane ($\delta=0$), R is then the ratio of the (averaged) cross sections for scattering into the two analyzers.

Again following Frauenfelder and Steffen,[8] we define Q in terms of R:

$$Q=(R-1)/(R+1). \tag{4}$$

The quantity Q is actually one of the four Stokes parameters used to describe polarized photon beams. It is sufficient for our purposes, however, to look upon it as a calibration parameter of the polarimeter. From Eq. (2), for a point scatterer and point analyzers,

$$Q=(-\sin^2\varphi)/(k/k_0+k_0/k-\sin^2\varphi). \tag{5}$$

Q is always negative which reflects the fact that, from Eq. (2), the maximum cross section is for scattering perpendicular to the polarization vector of the incident photon. For a finite scatterer and finite analyzers Q, as implied by Eq. (3), will be less than the estimate given by Eq. (5). Its actual value for any given experimental conditions can be estimated by numerical integration,[10] Monte Carlo calculation,[11] or empirically.[9] For a particular γ ray whose linear polarization is being measured let us define, in agreement with Woods, Koički, and Koički,[11] the quantity

$$S(\theta)=(N_0-N_{90})/(N_0+N_{90}), \tag{6}$$

where N_0 and N_{90} are, respectively, the number of events recorded in the analyzer in the reaction plane and perpendicular to it. Further, since

$$N_0/N_{90}=[W(\theta,0)\bar{\sigma}(0)+W(\theta,90)\bar{\sigma}(90)]/\times[W(\theta,90)\bar{\sigma}(0)+W(\theta,0)\bar{\sigma}(90)],$$

$P(\theta)$, $S(\theta)$ and Q are related by

$$P(\theta)=S(\theta)/Q, \tag{7}$$

so that the degree of polarization is in all cases directly proportional to the difference of the counts recorded by the two analyzers. For a γ ray completely polarized in the reaction plane, $N_0/N_{90}=\bar{\sigma}_c(0)/\bar{\sigma}_c(90)=R$.

It is useful to give specific expressions for $P(\theta)$. These can easily be written down from the formulas given by Fagg and Hanna.[3] We give first the formulas appropriate for the detection of the linear polarization of a γ ray at any angle θ with respect to the alignment axis. For a mixed quadrupole/dipole transition

$$P(\theta)=\frac{\pm[\frac{1}{2}(a_2+b_2)P_2^{(2)}(\cos\theta)-\frac{1}{12}a_4P_4^{(2)}(\cos\theta)]}{1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)}, \tag{8a}$$

where

$$b_2=\frac{8a_2xF_2(12ba)}{3[F_2(11ba)-2xF_2(12ba)+x^2F_2(22ba)]}. \tag{8b}$$

The phase of the mixing ratio x in Eq. (8) and also in Eq. (9) below is as defined by Rose and Brink,[12] and for $E2/M1$ or $E3/M2$ mixtures [see Eq. (9)] is the same as that defined by Poletti and Warburton.[13] The radiation coefficient $F_k(LL'ba)$ is defined, for instance, in Ref. 13 and the plus sign in Eq. (8) is to be taken for $E2/M1$ mixtures, the minus sign for $M2/E1$ mixtures. The $P_k^{(2)}(\cos\theta)$ are associated Legendre polynomials defined, for instance, by Jahnke and Emde.[14] A tabulation of some of them is given by Fagg and Hanna.[3]

For a mixed octupole/quadrupole transition

$$P(\theta)=\frac{\pm[-\frac{1}{2}(a_2+c_2)P_2^{(2)}(\cos\theta)+\frac{1}{12}(a_4+c_4)P_4^{(2)}(\cos\theta)-(1/30)a_6P_6^{(2)}(\cos\theta)]}{1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)+a_6P_6(\cos\theta)}, \tag{9a}$$

where

$$c_2=\frac{a_2[xF_2(23ba)-(5/3)x^2F_2(33ba)]}{[F_2(22ba)-2xF_2(23ba)+x^2F_2(33ba)]}, \tag{9b}$$

and

$$c_4=\frac{3[4xF_4(23ba)+5x^2F_4(33ba)]}{5[F_4(22ba)-2xF_4(23ba)+x^2F_4(33ba)]}. \tag{9c}$$

In this case the plus sign is to be taken for $E3/M2$ mixtures, the minus sign for $M3/E2$ mixtures. Specializing Eqs. (8) and (9) to $\theta=90°$ and defining $P\equiv P(90°)$, we obtain for mixed quadrupole/dipole transitions

$$P=\pm\left[\frac{3(a_2+b_2)+(5/4)a_4}{2-a_2+\frac{3}{4}a_4}\right], \tag{10}$$

where b_2 is given in Eq. (8b). For pure dipole or quadrupole transitions, $b_2=0$. We emphasize that the plus sign in Eq. (10) applies for $M1$ or $E2$ radiation, the minus sign for $E1$ or $M2$ radiation.

For mixed octupole/quadrupole transitions

$$P=\pm\left[\frac{-3(a_2+c_2)-(5/4)(a_4+c_4)-\frac{7}{8}a_6}{2-a_2+\frac{3}{4}a_4-\frac{5}{8}a_6}\right], \tag{11}$$

where c_2 and c_4 are given in Eqs. (9b) and (9c). For pure quadrupole transitions c_2 and c_4 are zero while for pure octupole transitions $c_2=-5a_2/3$, $c_4=3a_4$. Note that in

[10] G. J. McCallum, Phys. Rev. **123**, 568 (1961).
[11] G. T. Wood, S. Koički, and A. Koički, Phys. Rev. **150**, 956 (1966).
[12] H. J. Rose and D. M. Brink, Rev. Mod. Phys. **39**, 306 (1967).
[13] A. R. Poletti and E. K. Warburton, Phys. Rev. **137**, B595 (1965).
[14] E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, Inc., New York, 1945).

Eq. (11) we take the plus sign for $M2$ or $E3$ radiation and the minus sign for $E2$ or $M3$ radiation.

Turning now to a closer consideration of Eq. (8), a numerical computer calculation easily gives the maximum value of $P(\theta)$ for particular values of a_2, a_4, and b_2. A greater insight can be obtained by noting that the denominator of Eq. (8) is merely the yield $W(\theta)$: this rarely varies with angle by more than a factor of 2, while the numerator generally varies more rapidly. Consider then just the numerator: if a_4 is zero or small compared to (a_2+b_2) then $P(\theta)$ will be a maximum at 90° since $P_2^{(2)}(\cos\theta)$ is maximum at this angle. If, however, (a_2+b_2) is small while a_4 is significant, then the important term is that in $P_4^{(2)}(\cos\theta)$ which has a maximum at approximately 41°. In this latter case the polarization effect would be a maximum for a detection angle of about 40°, the exact angle depending of course on the actual values of a_2, b_2, and a_4. An experiment involving such a situation will be discussed.

An interesting example involving Eq. (9) has occurred in the work of French and Newton.[15] In this case (the $E3$, $6.13 \rightarrow 0$ transition in O^{16}), the angular correlation was so strong that the variations of both the denominator and the numerator had to be considered. Although the polarization at 90° was a local maximum: $P(90) = -1$, the yield was only one-tenth of that at 62°, whereas $P(62°) = +1$. These workers therefore chose to work at 62°.

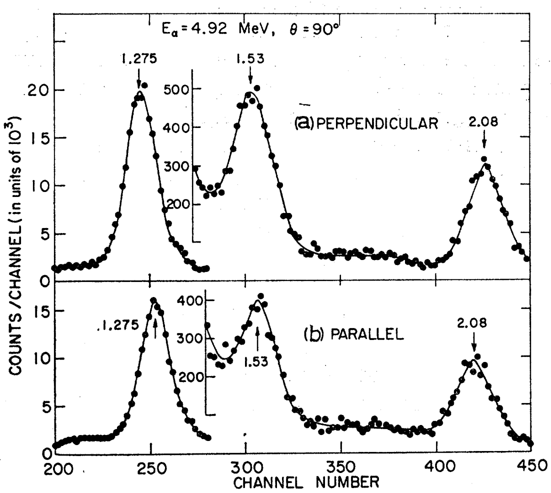

FIG. 1. Some results of an investigation of the linear polarization of γ rays from the $F^{19}(\alpha,n)Na^{22}$ and $F^{19}(\alpha,p)Ne^{22}$ reactions, resulting from α bombardment of a thick CaF_2 target. Gamma-ray full-energy peaks are labeled by their energies (in MeV). These spectra were measured with a Compton polarimeter placed at 90° with respect to the axis of alignment, using an α-bombarding energy of 4.92 MeV. This energy is below the threshold for production of the 2.21-MeV level in Na^{22} (1.55-MeV γ rays) and hence the peaks at 1.53 MeV are due entirely to the decay of the Na^{22} 1.53-MeV level. The other two peaks are due to the de-excitation of the first and second excited states of Ne^{22} at 1.275 and 3.357 MeV formed by the reaction $F^{19}(\alpha,p)Ne^{22}$. The perpendicular and parallel spectra were recorded simultaneously, thus a direct comparison of peak areas can be made in order to determine the degree of polarization. As expected for pure $E2$ transitions (whose angular distributions are given in Table II) both the 1.275- and 2.08-MeV γ rays are polarized mainly in the reaction plane so that there is more scattering into the crystal perpendicular to the reaction plane; cf. Fig. 1(a) and 1(b). This is also the case for the 1.53-MeV γ rays: It also is pure $E2$ in nature.

[15] A. P. French and J. O. Newton, Phys. Rev. 85, 1041 (1952).

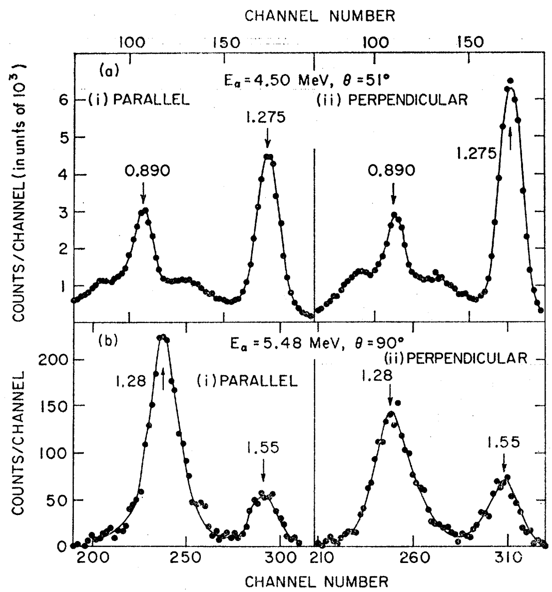

FIG. 2. (a) Ungated spectra recorded using the Compton polarimeter at an α-particle bombarding energy of 4.50 MeV with the polarimeter at an angle of 51° with respect to the axis of alignment. The 1.275-MeV γ ray from Ne^{22} is obviously strongly polarized, again in the reaction plane, while the 0.891-MeV γ ray arising from the ground-state decay of the Na^{22} 0.891-MeV level is polarized perpendicular to the reaction plane (the resolution of the parallel detection system is slightly worse than that of the perpendicular one, hence areas of peaks must be compared, not heights). (b) Spectra recorded as in Fig. 1, except that the α-particle bombarding energy is 5.48 MeV and the spectra are recorded in coincidence with 74-keV γ rays de-exciting the 657-keV level of Na^{22}. These 74-keV γ rays were detected in a 2-mm-thick ×5-cm-diam NaI(Tl) scintillator with its front face 2 cm from the target. The bombarding energy which was used was sufficient to excite the Na^{22} levels at 1.937 and 2.211 MeV, both of which decay to the 0.657-MeV level. The γ rays whose energies are, respectively, 1.28 and 1.55 MeV are pure dipole transitions: $J=1 \rightarrow J=0$. From Table II, it can be seen that they both have roughly the same anisotropy, hence the figure shows immediately that the two levels have opposite parity. Calibration of the polarimeter establishes the 1.937-MeV level to be of even parity in agreement with previous results; thus the 2.211-MeV level is of odd parity.

III. EXPERIMENTAL

The experimental method, Compton polarimeter, and electronics which we have used have been described previously.[9] In essence, the polarimeter consisted of a 1.5-in. diam by 3-in. long NaI(Tl) crystal with its front face 10.3 cm from the target (Compton scatterer) and two 3×3-in. crystals which detected scattering parallel and perpendicular to the plane defined by the beam and the scattering crystal. Alpha particle beam energies of 4.92, 4.50, and 5.48 MeV together with a thick (1 mg/cm²) CaF_2 target were used to excite levels in the nuclei Na^{22} and Ne^{22} in the $F^{19}(\alpha,n)Na^{22}$ and $F^{19}(\alpha,p)Ne^{22}$ reactions, respectively.

TABLE I. Results of linear polarization measurements.

E_α (MeV)	Transition	$S(\theta)$[a]	$P(\theta)_{\mathrm{exp}}$[a,b]	$P(\theta)_{\mathrm{th}}$[a,c]	Comments
4.92	Ne^{22} 1.275 → 0	$-(0.127\pm0.005)$	$+(0.53\pm0.04)$	$+(0.54\pm0.03)$	$2^+\rightarrow0^+$
	Na^{22} 1.528 → 0	$-(0.155\pm0.020)$	$+(0.71\pm0.10)$	$+(0.65\pm0.09)$	$5^+\rightarrow3^+$ ($x=0.0$)
				$-(0.30\pm0.08)$	$3^+\rightarrow3^+$ ($x=-1.18\pm0.15$)
	Ne^{22} 3.357 → 1.275	$-(0.127\pm0.015)$	$+(0.73\pm0.13)$	$+(0.76\pm0.08)$	$4^+\rightarrow2^+$ ($x=0.0$)
4.50	Na^{22} 0.891 → 0	$+(0.004\pm0.012)$	$-(0.02\pm0.07)$	$-(0.02\pm0.07)$	$4^+\rightarrow3^+$ ($x=-2.6\pm0.5$)
		$+(0.124\pm0.010)$[d]	$-(0.45\pm0.06)$[d]	$-(0.47\pm0.03)$[d]	
	Ne^{22} 1.275 → 0	$-(0.131\pm0.008)$	$+(0.55\pm0.05)$	$+(0.51\pm0.03)$	$2^+\rightarrow0^+$
5.48	Na^{22} 1.937 → 0.657	$+(0.125\pm0.011)$	$-(0.52\pm0.05)$	$-(0.44\pm0.05)$	$1^+\rightarrow0^+$
	Na^{22} 2.211 → 0.657	$-(0.127\pm0.025)$	$+(0.59\pm0.10)$	$+(0.60\pm0.05)$	$1^-\rightarrow0^+$
				$-(0.60\pm0.05)$	$1^+\rightarrow0^+$

[a] Unless specified, $\theta=90°$.
[b] Calculated from $P(\theta)=S(\theta)/Q$ using the appropriate value for Q as described in the text.
[c] Calculated using the spins, parities, and mixing ratios listed in the last column, together with the angular distribution coefficients listed in Table II. The mixing ratios are from Ref. 1.
[d] $\theta=51°$.

We were mainly interested in the ground-state transitions from the Na^{22} 1.528-MeV level ($J^\pi=3^+$ or 5^+) and 0.891-MeV level ($J^\pi=4^+$) and in the transitions to the 0.657-MeV level ($J^\pi=0^+$) of Na^{22} from those at 2.211 and 1.937 MeV (both $J=1$). The experiments designed to investigate the linear polarization of the above four γ rays provided an internal calibration of the polarimeter. Representative spectra obtained using the polarimeter are shown in Figs. 1 and 2. It can be seen that all the γ rays are quite highly polarized, especially the 1.275-MeV γ ray from Ne^{22} [Fig. 2(a)].

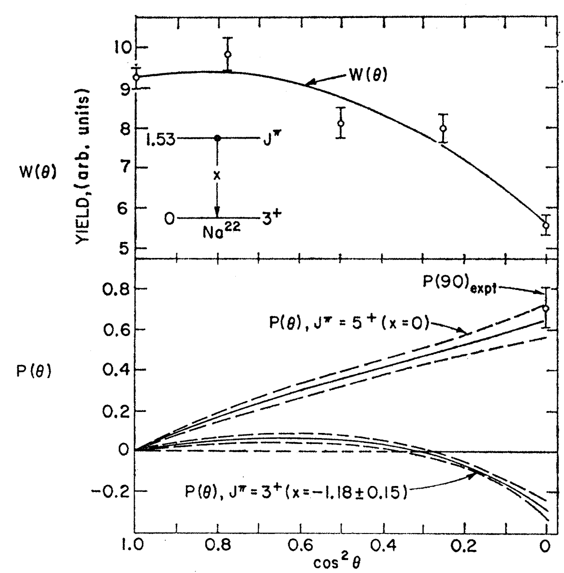

FIG. 3. Results of angular distribution and linear polarization measurements on the decay of the 1.53-MeV level in Na^{22}. The angular distribution can be equally well-fitted for spin assignments of $J=5$ or 3 to the 1.53-MeV level. The single linear polarization measurement at $\theta=90°$ is consistent only with the $J^\pi=5^+$ assignment, $J^\pi=3^\pm$ and 5^- all being rejected. The two bands drawn in the lower half of the figure are the polarizations calculated (using the values of the Legendre coefficients a_2 and a_4 derived from the angular distribution at the top of the figure) for $J^\pi=5^+$ with a pure $E2$ transition to the 3^+ ground state and $J^\pi=3^+$ with a mixed $E2/M1$ transition ($x=-1.18\pm0.15$) to the ground state. For odd-parity assignments to the 1.53-MeV level the sign of $P(\theta)$ is reversed [see Eq. (8)].

At a bombarding energy of 4.92 MeV, the levels at 1.275 and 3.357 MeV (2.082-MeV γ ray) in Ne^{22} and 1.528 MeV in Na^{22} gave full-energy peaks which were clearly resolved in a NaI(Tl) detector. The measurement of the linear polarization of the three γ rays from these transitions involved only a measurement of the anisotropy of the Compton scattering of these γ rays at $\theta=90°$ and a measurement of their angular distribution using an 8-cc Ge(Li) detector. The two Ne^{22} lines, since they are known[16] to arise from pure $E2$ transitions, were used to calibrate the polarimeter. The levels at 2.211 and 1.937 MeV in Na^{22}, which de-excite by 1.554- and 1.280-MeV γ rays, were populated at a beam energy of 5.48 MeV. In this case, however, the de-exciting γ rays could only be resolved in a NaI(Tl) detector if they were recorded in coincidence with the 74-keV γ ray de-exciting the 0.657-MeV level of Na^{22}. Since the 0.657-MeV level has $J=0$, it is still sufficient to measure the angular distribution of each γ ray using the Ge(Li) detector; furthermore $J=0$ for the 0.657-MeV level implies that both the 1.280- and 1.554-MeV γ rays are pure dipole transitions, and hence they can both be used to assist in the calibration of the polarimeter.

At 4.50-MeV bombarding energy, the Na^{22} 0.891-MeV level was strongly excited. The linear polarization of the 891-keV γ ray from this level was therefore measured at this energy and provides a good example of the use of Eq. (8) above. At $\theta=90°$, the polarization predicted for a mixing ratio of $x\simeq-2.6$ (as required by experiment[1]) is small and very sensitive to the actual value of x, both in magnitude and sign. Hence, no significant information on the parity of the level could be obtained. However, at angles between $\theta=30°$ and $\theta=60°$ the polarization is large and the distinction between even and odd parity for the 0.891-MeV level is very clear. This is evident from the results shown in Fig. 2 for $\theta=51°$.

[16] C. Broude and M. A. Eswaran, Can. J. Phys. **42**, 1300 (1964); **42**, 1311 (1964).

TABLE II. Angular distribution coefficients for γ rays resulting from α bombardment of F^{19}.

E_α (MeV)	Transition	a_2	a_4	χ^2
4.92	Ne^{22} 1.275 → 0	$+(0.341\pm0.015)$	$-(0.142\pm0.018)$	1.32
	Na^{22} 1.528 → 0	$+(0.389\pm0.050)$	$-(0.162\pm0.056)$	1.55
	Ne^{22} 3.357 → 1.275	$+(0.427\pm0.040)$	$-(0.130\pm0.046)$	1.47
4.50	Na^{22} 0.891 → 0	$+(0.541\pm0.020)$	$+(0.270\pm0.020)$	0.08
	Ne^{22} 1.275 → 0	$+(0.370\pm0.010)$	$-(0.319\pm0.010)$	0.43
5.48[a]	Na^{22} 1.937 → 0.657	$-(0.34\ \pm0.05)$	...	...
	Na^{22} 2.211 → 0.657	$-(0.50\ \pm0.05)$	...	...

[a] The values of a_2 quoted for this bombarding energy have been obtained by averaging the measurements of the present work with those quoted in Ref. 1 for the same bombarding energy.

IV. RESULTS

From a preliminary analysis of the counting rates recorded in the two 3×3-in. analyzer crystals, an approximate evaluation of Q (see Sec. III) was obtained using the observed polarizations for the Ne^{22} 1.275- and 2.082-MeV γ rays. This evaluation was nevertheless sufficiently accurate to verify the even-parity assignment to the Na^{22} 1.937-MeV level, determine that the parity of the Na^{22} 2.211-MeV level is odd, and further, that the spin of the 1.528-MeV level, which was previously known to be 3^+ or 5^+, is $J=5$. For a $J=5$ assignment to the 1.528-MeV level the known[1] lifetime determines that the mixing ratio for the ground-state transition is $|x|<0.003$ i.e., the transition is essentially quadrupole. These three transitions in Na^{22}, as well as the two γ rays from Ne^{22}, could thus all be used to help calibrate the polarimeter. Additional calibration data were obtained from the N^{14} 1.64-MeV and F^{18} 2.10-MeV γ rays studied in a previous experiment.[9] The seven calibration points were fitted with a linear relationship given by $-Q=a_0+a_1E_\gamma$. If E_γ is in MeV then $a_0=0.340\pm0.053$ and $a_1=-(0.079\pm0.035)$. This fit is a purely phenomenological one; however, it quite well represents the variation of Q with energy. The value of χ^2 obtained in the least-squares fit was 0.42. The values of the calibration parameter Q obtained by this "bootstrap" method have been incorporated in the results which we give in Table I. With the help of this more precise calibration, we can most firmly reject a 3^- assignment to the 1.528-MeV level as well as the possibilities of 3^+ and 5^-: thus $J^\pi=5^+$ for this level (see Fig. 3).

As mentioned previously, a measurement of the linear polarization of the 891-keV γ ray at $\theta=90°$ did not differentiate between even and odd parity assignments to the 891-keV level. It was therefore necessary to measure the polarization at an angle other than 90°. Because of this somewhat unusual situation and as a check on the isotropy of response of the polarimeter, the linear polarization of the γ ray was measured at a number of angles which included $\theta=0°$. The linear polarization of the 1.275-MeV γ ray from the first excited state of Ne^{22} was measured simultaneously, and since the $E2$ character of this γ ray is well established,[16] the measurement provided a useful check on the correct

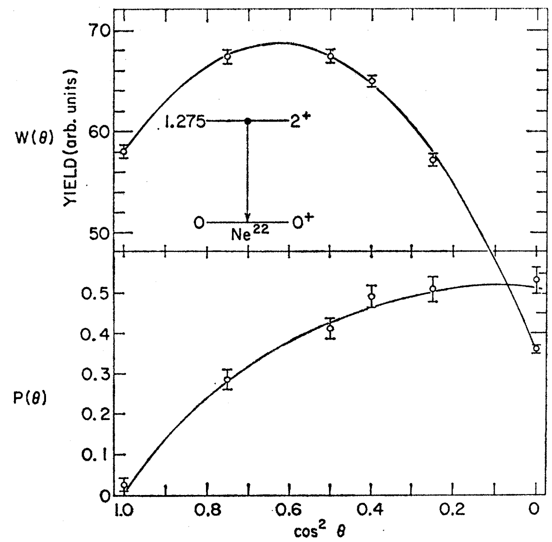

FIG. 4. Results of angular distribution and linear polarization measurements on the decay of the 1.275-MeV level of Ne^{22} (see caption to Fig. 3 for details). The angular distribution $W(\theta)$ was obtained at the same time as the polarization, $P(\theta)$, by adding the counts recorded in the two analyzer crystals: $W(\theta)=N_0+N_{90}$.

functioning of the experimental apparatus. It also provides an instructive example of the usefulness of Eq. (8). Figure 4 shows the angular distribution of the 1.275-MeV γ ray at $E_\alpha=4.50$ MeV as well as the linear polarization $P(\theta)$. The solid curve through the angular distribution points shows a least-squares fit with a function of the form $A_0[1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)]$ while the curve in the lower part of the figure is obtained from Eq. (8), where we set $b_2=0$ and take the plus sign in front of the numerator since the transition is pure $E2$. The very good agreement between the experimentally measured polarization $P(\theta)$ and that calculated from the coefficients a_2 and a_4 (see Table II) gives us confidence that the polarimeter was working correctly.

A further point which Fig. 4 clearly illustrates is that if statistical accuracy is likely to be a limitation and the polarization at one point, *only*, is to be measured, it would be much better in this particular case to detect the linear polarization at an angle of approximately 50° rather than the usual 90°. The degree of polarization at this angle is only 10% less than at 90° while the counting rate as shown by the angular distribution has increased by almost a factor of 2. For the 0.891-MeV γ ray significant polarizations were observed for angles from 30° to 60° with a maximum at about 51° in agreement with an even-parity assignment to the level. This is illustrated in Fig. 5. For an odd-parity assignment, the measured polarization would have had the opposite sign but the same magnitude [see Eq. (8)]. The reason the polarization is not a maximum at $\theta=90°$ can be seen from Eq. (8). For the transition $J=4$ to $J=3$ with a mixing ratio $x=-2.6$ and the angular distribution shown in Fig. 5, (a_2+b_2) is very small so

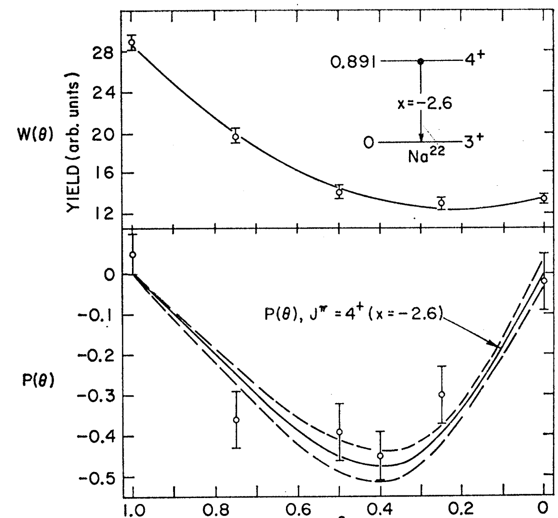

FIG. 5. Results of angular distribution and linear polarization measurements on the decay of the 0.891-MeV level of Na^{22} (see caption to Fig. 3 for details). The measurement of $P(\theta)$ at $\theta=90°$ does not distinguish between odd and even parity because the polarization is almost zero at this angle. At angles between 30° and 60°, however, the γ rays are highly polarized and a possible odd-parity assignment is completely rejected.

that the polarization is determined mainly by the term in $P_4^{(2)}(\cos\theta)$. The measured angular distributions whose coefficients were used to calculate the quantities $P(\theta)_{\text{exp}}$ in Table I are summarized in Table II.

V. DISCUSSION

From the point of view of the nuclear shell model, the nucleus Na^{22} with six particles in the $(2s,1d)$ shell is an extremely complex one. As a consequence, applications of the shell model to Na^{22} have been restricted thus far to attempts to predict the energy spectrum of the states arising from the $(2s,1d)^6$ configuration.

The usual shell model calculations which consider the residual interactions between particles which are bound in some common potential well (as opposed to effective interaction least-squares fits) have so far not met with great success. For instance, in the calculations reported by Arima,[17] the ground state is predicted to be $J^\pi=1^+$ and not 3^+ as it should be. However, preliminary results of the Oak Ridge group[18] look rather more promising and ultimately should yield predictions for γ-transition strengths as well.

On the other hand, apart from identifying the possible bands[19] as $K^\pi=0^+$, $T=0$; $K^\pi=0^+$, $T=1$; and $K^\pi=3^+$, $T=0$, no definitive attempt has been made, using the collective model, to explain the energy spectrum of Na^{22}. In spite of this, however, the collective model at present gives the best qualitative understanding of the energy spectrum and decay modes. On this model, the states at 0.00, 0.89, 1.53, and perhaps the one at 3.71 MeV can be identified as the 3^+, 4^+, 5^+, and (6^+) members of a rotationlike band based on the $K^\pi=3^+$, $T=0$ ground state. The states at 0.66, 1.95, and 4.08 MeV can be identified as the 0^+, 2^+, and 4^+ members of a band based on the $K^\pi=0^+$, $T=1$ state at 0.66 MeV and the states at 0.58 and 1.98 MeV are perhaps the 1^+ and (3^+) members of the $K^\pi=0^+$, $T=0$ band. The observed decay properties of the 3.53-MeV level are not inconsistent with the hypothesis that it is the 5^+ member of this same band. There are still a number of low-lying states not explained by this scheme, for instance the 1^- 2.21-MeV level. This state could be the first member of an odd-parity band arising from a single-particle excitation of the O^{16} core. Other possible members of this band could be the 2.57- and 3.06-MeV levels; but this is, of course, highly speculative.

The few pieces of evidence which we now have on the structure of Na^{22} reveal an interesting complexity. Much further experimental and theoretical work remains to be done.

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