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Spectroscopic Factors for Pickup Reactions on Ca^{40}†

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The spectroscopic factors for the $d_{3/2}$, $s_{1/2}$, and $f_{7/2}$ pickup from Ca^{40} have been calculated in the shell model for two different forces, and the results were compared with distorted-wave Born-approximation (DWBA) analyses of recent experiments. The calculated spectroscopic factors for $f_{7/2}$ pickup were found to be considerably smaller than the DWBA results. Various reasons for this discrepancy are discussed.

1. INTRODUCTION

THE individual particle model[1] fails to predict precisely the properties of nuclei that are not near closed shells. This has been accepted for several years, and shell-model calculations away from closed shells have almost all been carried out using configuration mixing. Near closed shells, the individual particle model has been assumed to be accurate; that is, configuration mixing has not been considered to be very important.

For the ground state of Ca^{40} the individual particle model predicts that the $1s$, $1p$, and $1d$-$2s$ levels will be completely filled for both protons and neutrons, and that all the other levels will be empty. However, recent direct reaction measurements are in disagreement with this description. Pickup experiments[2–5] indicate that $1f_{7/2}$ particles are present in the ground state of Ca^{40}, and stripping experiments[3] indicate the presence of $d_{3/2}$ holes.

In this work, the spectroscopic factors for pickup are calculated by carrying out a configuration-mixing calculation for the Ca^{40} ground state and the relevant $A=39$ levels.

2. CALCULATIONS

The calculations are carried out by diagonalizing the nuclear Hamiltonian in a spherical harmonic-oscillator wave-function basis. The basis for the Ca^{40} ground state consists of the individual particle-model ground state and the states of the two-particle, two-hole configurations with the two particles in the $f_{7/2}$ shell and the holes in the $s_{1/2}$ and $d_{3/2}$ shells. The positive-parity $A=39$ configurations consist of the one-hole configuration and the configuration formed by adding two additional $d_{3/2}$ holes and two $f_{7/2}$ particles to the one-hole configuration. The negative-parity $A=39$ basis consists of configurations having two holes in the $s_{1/2}$ and $d_{3/2}$ shells and one particle in the $f_{7/2}$ shell.

These bases do not contain any components of the spurious states. The individual particle model ground configurations contain no spurious states because only one major L-S shell is filling at one time.[6] Higher excitations of $n\hbar\omega$ can be considered as generated by n repeated applications of particle-hole creation operators, which elevate the energy by $\hbar\omega$ with each application. A spurious state is generated by the center-of-mass operator $\mathbf{R}=(1/A)\sum_i \mathbf{r}_i$ operating on either a spurious or a nonspurious state.[6] States which are orthogonal to all the states generated by one or more applications of $\mathbf{R}$ contain no spurious components. In the case of levels near closed L-S shells, the lowest particle shell in the j-j shell model has the highest value of j. Since the operator $\mathbf{R}$ is a single-particle operator having angular momentum of only $1\hbar$, the single-particle matrix elements of $\mathbf{R}$ are nonzero only if $|\Delta j|\leq 1$. Thus, if all the particles in a configuration are in the first minor shell above the closure and none of the holes are in the lowest minor shell of the last closed L-S shell, then the configuration contains no spurious states.

Two forces are used in this calculation. One force is the central Gaussian force with the parameters of Gillet and Sanderson[7] ($W=0.175$, $M=0.575$, $H=0.100$, $B=0.250$, $V=-55$ MeV, and range 80% of the oscillator length parameter). The other force is the realistic Tabakin potential[8] with oscillator parameter $M\omega/2\hbar=3.5\ F^{-2}$. The single-particle energy splittings, shown in Table I, are averages of the separate neutron and

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proton values of Gillet and Sanderson.[7] While these forces and single particle energies are commonly used, they do not give a good fit to the negative parity spectra of Ca^{40} or $A=39$ in the conventional shell model.[9] Both forces are too weak to separate sufficiently states containing the same configurations, and a value of 7 MeV for the $d_{3/2}$-$f_{7/2}$ splitting brings all of the eigenvalues too high. The usual method of extracting single-particle energies, based on the assumption that states near closed shells are well represented by the extreme single-particle model, is inconsistent with recent calculations showing appreciable admixtures to the Ca^{40} and $A=39$ ground states.[10–12] To see what spectroscopic factors might be expected from a force and a single-particle energy which more nearly fit the spectrum, we have also done some calculations with a modified Tabakin force and with a value of 5 MeV for the $(d_{3/2})^{-1}(f_{7/2})$ single particle energy. The matrix elements of this augmented Tabakin force were uniformly increased by a factor of 1.356 to fit the separation between the lowest 3^- and 5^- states of Ca^{40}. While no fundamental significance is attached to this augmented Tabakin force, the results are similar, we believe, to what one would find with any reasonable force strong enough to fit the experimental energies.

3. THE SPECTROSCOPIC FACTOR

For the reaction $X_{TJ}{}^{A}(a,b)W_{T'J'}{}^{A-1}$ the spectroscopic factor for picking up a nucleon from the j shell is defined by[13]

$$S(j)\equiv A\,|\langle(A,TJ)\,|\,(A-1,\,T'J')jTJ\rangle|^2,$$

where the portions of the wave functions in parentheses are antisymmetric, while the final particle on the right is not coupled antisymmetrically.

We have approximated the ground state of Ca^{40} by

$$\psi_{\mathrm{g.s.}}(\mathrm{Ca}^{40})=\alpha_0\psi_0 + \sum_{i\geq 1}\alpha_i\psi_{\mathrm{a.s.}}(j_{1i}{}^{-1}j_{2i}{}^{-1}(T_iJ_i)f_{7/2}{}^2(T_iJ_i)00),$$

the ground state $(3/2^+)$ or first excited state $(1/2^+)$ of $A=39$ by

$$\psi_{\mathrm{g.s.}}(A=39)=\beta_0(j)\psi_0(j^{-1})+\sum_{k\geq i}\beta_k(j) \times\psi_{\mathrm{a.s.}}(j_{1k}{}^{-1}j_{2k}{}^{-1}(T_kJ_k)f_{7/2}{}^2(T_k'J_k'')j^{-1}\tfrac{1}{2}j),$$

and an odd-parity $A=39$ state by

$$\psi^-(A=39)=\sum_{m=0}\gamma_m\psi_{\mathrm{a.s.}}(j_{1m}{}^{-1}j_{2m}{}^{-1}(T_mJ_m)f_{7/2}\tfrac{1}{2}j).$$

Table I. Single-particle energies of particle-hole configurations. These represent averages of the separate neutron and proton values used by Gillet and Sanderson (Ref. 7).

Configuration	Energy (MeV)
$(1d_{3/2})^{-1}(1f_{7/2})$	7.04
$(2s_{1/2})^{-1}(1f_{7/2})$	9.8
$(1d_{5/2})^{-1}(1f_{7/2})$	12.0

With wave functions of this form, the spectroscopic factors for the ground state pickup of a nucleon of specified charge are

$$S(d_{3/2})\cong 4\,|\alpha_0\beta_0(3/2)|^2,$$
$$S(s_{1/2})\cong 2\,|\alpha_0\beta_0(1/2)|^2,$$

and for the pickup to an odd-parity state

$$S(j)=1\,|\sum_{im}\Delta_{im}\alpha_i\gamma_m|^2,$$

where $\Delta_{im}=\delta_{j1ij1m}\delta_{j2ij2m}$. Terms of the type $\Delta_{ik}\alpha_i\beta_k$ have been neglected in the ground-state pickup as these terms are quite small. The results of these calculations are presented in Table II.

4. COMPARISON WITH DWBA RESULTS

The results of the calculations are shown in Table II with the results of distorted-wave Born-approximation (DWBA) analyses of experimental pickup measurements. The DWBA experimental spectroscopic factors are mostly considerably larger than the theoretical spectroscopic factors. The experimental values for $S(d_{3/2})$ for $\mathrm{Ca}^{40}(d,\mathrm{He}^3)\mathrm{K}^{39}$ and $\mathrm{Ca}^{40}(d,t)\mathrm{Ca}^{39}$ are obviously more than 20% too large. Assuming the large amount of mixing implied by the large $S(f_{7/2})$ to be correct, the discrepancy is even greater.

The theoretical and experimental spectroscopic factors $S(f_{7/2})$ differ by a factor of 3 to 10. There are several possible reasons for this disagreement. The

Table II. Spectroscopic factors $S(j)$ for Ca^{40} pickup to Ca^{39} and K^{39}. The experimental values are computed with the DWBA.

j		$d_{3/2}$	$s_{1/2}$	$f_{7/2}$
Energy level (MeV):	Ca^{39}	$\equiv 0.0$	2.42	2.74
	K^{39}	$\equiv 0.0$	2.53	2.81
Theoretical $S(j)$:				
Pure individual-particle model		4.0	2.0	0.0
Gaussian force		3.40	1.65	0.040
Tabakin force		3.40	1.62	0.049
Tabakin force, augmented[a]				0.094
Experimental	Ref.			
$\mathrm{Ca}^{40}(p,d)\mathrm{Ca}^{39}$	5	2.3		0.28
$\mathrm{Ca}^{40}(\mathrm{He}^3,\alpha)\mathrm{Ca}^{39}$	3	$\equiv 3.4$	1.25	0.53
$\mathrm{Ca}^{40}(d,\mathrm{He}^3)K^{39}$	4	4.98	1.93	0.50
$\mathrm{Ca}^{40}(d,t)\mathrm{Ca}^{39}$	4	4.84	1.83	0.43

[a] The calculation with the augmented Tabakin force employed a $(d_{3/2})^{-1}(f_{7/2})$ single-particle energy of 5 MeV, while the other calculations used the single-particle energies of Table I.

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theoretical calculations employing the Gaussian and Tabakin forces suffer, we believe, from the use of a force which is too weak and from an incorrect choice for the $d_{3/2}$-$f_{7/2}$ single-particle energy splitting. To test the effect of a stronger force, we adjusted the Tabakin force to give the correct energy difference between the 3^- and 5^- states of Ca^{40}. This amounted to a 36% increase in the size of the Tabakin matrix elements and resulted in a spectroscopic factor of 0.064. When the $d_{3/2}$-$f_{7/2}$ splitting was reduced from 7 MeV to 5 MeV, as some evidence indicates it should be[10,14] the spectroscopic factor was increased to 0.074. When both these adjustments were made, the spectroscopic factor rose to 0.094, a total increase of 90%.

No reasonable choice of parameters for this calculation would give a spectroscopic factor as high as the experimental value, but the "experimental" results from the DWBA analysis of pickup experiments are not free from theoretical uncertainties. The application of direct reaction theory to reactions where the incident nucleus is not a single nucleon may not be well justified.

To extract the spectroscopic factor in the DWBA, the particle that is picked up is considered to be in a single particle bound eigenstate of a potential well. Direct reactions are extremely sensitive to the long range part of the wave function of this particle. This long range part of the wave function is strongly influenced by the shape of the potential well at the nuclear surface and by the binding energy. The binding energy used in the DWBA analysis is the "separation" energy of the picked up particle. It is proper to use the separation energy as the binding energy in a pure individual particle model description, but the very presence of a nonzero $S(f_{7/2})$ is not consistent with a pure individual particle model picture. The separation energy actually binds the $f_{7/2}$ nucleons below the $d_{3/2}$ nucleons. If the standard DWBA analysis is modified to use an "effective" binding energy which places the $f_{7/2}$ particles at about the correct energy above the $d_{3/2}$ levels, the experimental spectroscopic factor is reduced considerably.[4,5] For example, in the case of $Ca^{40}(p,d)Ca^{39}$ the $f_{7/2}$ spectroscopic factor is reduced to 0.14 for an effective binding of about one half the separation energy.[5] Some other sources of uncertainty in DWBA analyses are discussed in Ref. 4. When these ambiguities in the DWBA analysis are taken into account, the discrepancy between the two spectroscopic factors is seen to be less serious.

Two calculations of the ground state of Ca^{40} have been reported recently. In both cases some effect of four-particle, four-hole states was included in an approximate basis. No spectroscopic factors were found since only $A=40$ states were considered. In the first of these calculations Gerace and Green[11] mixed deformed two-particle, two-hole and four-particle, four-hole configurations with the usual spherical ground state in an attempt to find a better explanation of the even-parity states of Ca^{40}. While their calculation is obviously very different from ours, it is interesting that their value of 80% for the purity of the ground state agrees with the result obtained from the augmented Tabakin force. In the second calculation, Gillet and Sanderson[12] have improved and enlarged their calculation of Ca^{40} to include the ground state. They find an occupation number for the $f_{7/2}$ shell which is much larger than indicated by this work or by the calculation of Gerace and Green. This large mixing is a result of the use of the random-phase approximation which always results in greater ground-state mixing than the ordinary shell model.

5. CONCLUSION

The calculated spectroscopic factors for pickup of an $f_{7/2}$ nucleon from the ground state of Ca^{40} appear to be too small. This results from using forces which are too weak and a $d_{3/2}$-$f_{7/2}$ splitting which is too large.

The experimental spectroscopic factors resulting from the DWBA analysis, on the other hand, are clearly too large. The direct reaction theory needs to be modified to take account of the mixed nature of actual nuclear wave functions.

With a force and single-particle energies chosen to fit actual spectra of the $A=40$ and $A=39$ nuclei, the theoretical values of $S(f_{7/2})$ are in the vicinity of 0.1. While experiments interpreted by conventional DWBA analysis give 0.3–0.5, it is likely that a more satisfactory direct-reaction theory will result in a better agreement.

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