Electron Longitudinal Depolarization with Great Energy Loss

L. BRAICOVICH, B. DE MICHELIS, AND A. FASANA
Istituto di Fisica del Politecnico di Milano, Milano, Italy

and

Gruppo Nazionale di Struttura della Materia del Consiglio Nazionale delle Ricerche, Milano, Italy

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The average longitudinal depolarization of β particles of $^{90}Sr+^{90}Y$, which have lost a great fraction (up to 70%) of their energy in aluminum, has been measured in the energy ranges 0.35–1.1, 0.35–2.2, and 0.9–2.2 MeV. The results are discussed with reference to different theoretical pictures of the depolarization process; the results agree, within the errors, with the depolarizations calculated on the basis of the theory given by Braicovich.

I. INTRODUCTION

THE depolarizing effect of multiple scattering suffered by a longitudinally polarized beam of spin-$\frac{1}{2}$ particles has been studied when the effect of the energy loss of the particles is negligible. In this connection, several theoretical works[1–3] were carried out, and a satisfactory agreement between theory and experiment was found by Braicovich *et al.*,[4] in the case of electrons.

On the other hand depolarization with great energy loss has not yet been carefully investigated, although it is of considerable interest in the interpretation of some experiments, e.g., those in which the particles of the beam are slowed down before measuring polarization.[5] The main experiments performed in the field of depolarization with great energy loss are those carried out by Van Klinken *et al.*[6] with electrons and by Chinowsky *et al.*,[7] with positrons.

Although these experiments have given an initial phenomenological basis, they should not be considered a completely satisfactory test of the available theories of depolarization with great energy loss.

The first of these theories was developed by Bouchiat and Levy-Leblond,[8] who treated the problem in the small-angle approximation, and considered, in the calculation of the average polarization of the scattered beam, the depolarization due to the scatterings of the particles on the nuclei and to the purely Coulomb part of the electron-electron interaction; this assumption is equivalent to the substitution of $Z(Z+1)$ for Z^2 in the nuclear-scattering cross section. Bouchiat *et al.*'s calculation refers to geometrical conditions which allowed comparison with the depolarization observed by Chinowsky *et al.*, who found a rough agreement with the theory.

On the other hand there are many cases in which large angles must also be taken into account; as a consequence, the effect of the scatterings on the target electrons with large momentum transfer must be considered, as was pointed out by Iddings *et al.*[9] Obviously this effect cannot be treated with the substitution of $Z(Z+1)$ in place of Z^2 in the nuclear cross section.

Recently Braicovich[10] has suggested a theory of depolarization with great energy loss which is not affected by the preceding limitations; in this theory the average helicity of the beam emerging from the foil is calculated. The theory also considers, without limitation to small angles, the depolarizing effect of the collisions with the electrons and the lengthening of the path due to the lateral deflections suffered by the particles. Braicovich's theory is limited, in that it neglects the backscattering of the particles in the boundary conditions of the problem and assumes a one-to-one relation between path length and energy loss, i.e., it neglects straggling; it must be remembered that this last limitation also affects Bouchiat and Levy-Leblond's theory.

Braicovich's theory is valid both for electrons and positrons, and has been checked through an experiment carried out with electrons in order to avoid certain difficulties typical in measurements of positron polarization. This check may be considered sufficient as the difference in the sign of the charge is taken into account only through the values of the cross sections which are introduced in the general expressions given by Braicovich. The electrons were depolarized in an aluminum foil where a great fraction of their energy was lost. A low-Z material was chosen for the following reasons: the comparison between theory and experiment is more meaningful, as we shall point out further on, and the depolarization calculations are greatly simplified by the use of the first two Born approximations.

[1] M. Mühlschlegel, and H. Koppe, Z. Physik **150**, 474 (1958).
[2] I. N. Toptygin, Zh. Experim. i Teor. Fiz. **36**, 488 (1959) [English transl.: Soviet Phys.—JETP **9**, 340 (1959)].
[3] G. Passatore, Nuovo Cimento **6**, 850 (1957); **18**, 532 (1960).
[4] L. Braicovich, B. De Michelis, and A. Fasana, Nucl. Phys. **63**, 548 (1965).
[5] L. Dick, L. Feuvrais, and M. Spighel, Phys. Letters **7**, 150 (1963).
[6] J. Van Klinken, K. Bulthuis, and R. J. Van Duinen, Nucl. Phys. **61**, 593 (1965).
[7] W. Chinowsky, D. Cutts, and R. Stiening, Nuovo Cimento **34**, 1431 (1964).
[8] C. Bouchiat, and J. M. Levy-Leblond, Nuovo Cimento **33**, 193 (1964).
[9] C. K. Iddings, G. L. Shaw, and Y. S. Tsai, Phys. Rev. **135**, B1388 (1964).
[10] L. Braicovich, Nuovo Cimento (to be published).

II. THE APPARATUS

A source of 100 mCi of $^{90}Sr+^{90}Y$ was employed; the active preparate was deposited on a backing of alumina and covered with a stainless-steel sheet 0.15-mm thick. The diameter of the active area was 12 mm.

The depolarizer foils were of aluminum 0.5, 1.0, and 1.5-mm thick. The foil was placed immediately in front of the source before the input collimator of the polarimeter, so that the incidence on the foil was diffuse; see Fig. 1(A). The theory allows the calculation of the average helicity of the whole emerging beam when the incidence on the absorber is normal, as in Fig. 1(B). Nevertheless, a comparison between theory and experiment is possible, as the successions of scatterings in the two cases are basically equivalent because the total deflection angles are the same. The effect of the difference in the weights pertaining to the total deflection angles in the two cases was negligible within the experimental accuracy.

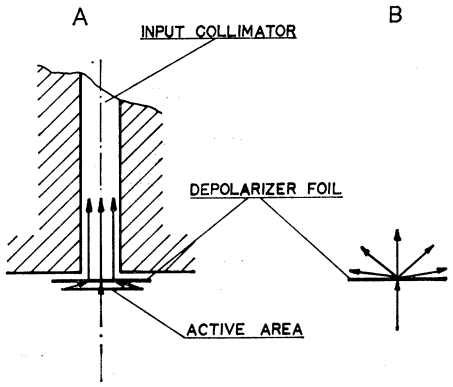

FIG. 1. (A) Geometrical arrangement of the source, the depolarizer foil and the input collimator of the polarimeter; (B) Geometrical conditions considered in the theory given in Ref. (10).

The polarimeter was based on the spin dependence of the Møller-scattering cross section and its features may be found in our preceding paper.[11]

The energy requirements on the two channels were adjusted to obtain the asymmetry measurements referring to electrons leaving the depolarizer foil, with energy in the three following ranges (i) 0.35–1.1 MeV, (ii) 0.35–2.2 MeV, and (iii) 0.9–2.2 MeV.

It must be remembered that the pair-collection efficiency $e(E)$, and the counting-rate asymmetry measured as a function of the energy of a beam with constant polarization (the so-called asymmetry efficiency), are different in the three preceding cases. This difference cannot be neglected in the discussion of the results of the present experiment. We have thus measured the pair-collection efficiency $e(E)$ of the instrument in the three cases using the method discussed in our previous paper[12]; the results are shown in Fig. 2 as a function of the energy of the electrons entering the polarimeter. In the cases mentioned, the asymmetry efficiencies η were calculated by a method similar to that of Geiger *et al.*[13]; the results are summarized in Fig. 3 as a function of the energy of the electrons. Besides the well-known energy and spin dependence of the Møller cross section, these asymmetry efficiencies also take into account the effect of multiple scattering in the analyzer foil.

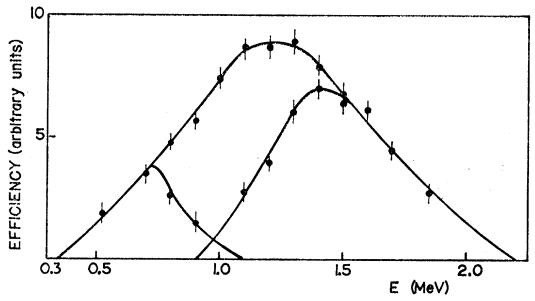

FIG. 2. Pair-collection efficiencies of the polarimeter as a function of the energy of the electrons entering the polarimeter. The efficiencies in the energy ranges 0.35–1.1, 0.35–2.2 and 0.9–2.2 MeV are shown.

III. EXPERIMENTAL RESULTS

The counting-rate asymmetry was measured in the different energy intervals, with and without the depolarizer foil inserted in the beam; the experimental procedure was the same as was used in our previous work.[4] The results of the measurements are collected in Table I; the measurements took about 1700 h to count the true coincidences and 3200 h to count the spurious coincidences.

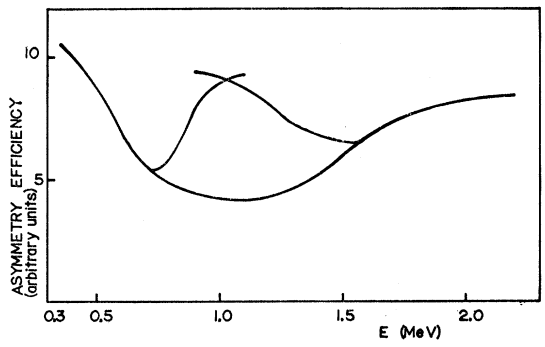

FIG. 3. Asymmetry efficiency of the polarimeter as a function of the energy of the electrons entering the polarimeter. The efficiencies in the energy ranges 0.35–1.1, 0.35–2.2, and 0.9–2.2 MeV are shown.

[11] L. Braicovich, B. De Michelis, and A. Fasana, Phys. Rev. 145, 952 (1966).

[12] L. Braicovich, B. De Michelis, and A. Fasana, Nucl. Phys. 82, 645 (1966).

[13] J. S. Geiger, G. T. Ewan, R. L. Graham, and D. R. Mackenzie, Phys. Rev. 112, 1684 (1958).

IV. ANALYSIS OF RESULTS

A. Method to Obtain Depolarization Values

Hereafter we will identify as "primary electrons" and "secondary electrons," respectively, the electrons before and after they have passed through the depolarizer foil. In the approximation in which straggling is neglected, there is a one-to-one correspondence between the energies of the primary and secondary electrons. As a result of the great energy loss, the asymmetries measured without the depolarizer foil do not belong to the spectrum of the primary electrons whose secondary electrons are detected when the foil is inserted. Obviously, it was not possible to arrange the energy requirements of the polarimeter so as to obtain, without the depolarizer foil, a direct measurement of the asymmetry belonging to the primary electrons; in any case the energy dependence of the instrumental efficiencies prevents such a precedure from being carried out. Consequently, an immediate comparison between the asymmetries measured with and without the depolarizer foil is meaningless and cannot give information on the depolarization suffered by the electrons. This depolarization is defined by the following expression:

$$d=1-(p_s/p_p), \tag{1}$$

where p_s and p_p are the longitudinal polarization of the secondary and of the primary electrons, respectively. The value of p_s is proportional to the measured asymmetry δ_s:

$$p_s=k\delta_s=k\int_{D_s}\eta(E)p_s(E)e(E)N_s(E)dE, \tag{2}$$

with normalization to 1 of the function $e(E)\ N_s(E)$; $p_s(E)$ and $N_s(E)$ are the helicity and the energy spectrum of the secondary electrons, respectively. The integration domain D_s is one of the three energy intervals accepted by the instrument. The value of p_p may be formally written analogously, with normalization to 1 of the function $N_p(E)$:

$$p_p=k\int_{D_p}\eta(E)p_0(E)N_p(E)dE, \tag{3}$$

where $p_0(E)$ is the helicity of the electrons emitted by the source, $N_p(E)$ is the primary electron spectrum corresponding to the secondary spectrum $e(E)N_s(E)$, and D_p is the integration domain over the primary energies corresponding to D_s.

TABLE I. Measured values of the counting-rate asymmetry with and without the depolarizer foil inserted in the beam.

Energy ranges (MeV)	$t_1=0.5$ mm	$t_2=1.0$ mm	$t_3=1.5$ mm	δ_0
0.35–1.1	3.75±0.34	3.99±0.43	3.96±0.55	3.55±0.31
0.35–2.2	3.67±0.28	3.33±0.35	3.54±0.65	3.51±0.26
0.9 –2.2			4.60±1.34	4.96±0.38

The value of the asymmetry δ_0 measured without the depolarizer foil is given by

$$\delta_0=\int_{D_s}\eta(E)p_0(E)e(E)N_0(E)dE, \tag{4}$$

with normalization to 1 of the function $e(E)N_0(E)$; N_0 is the energy spectrum of the electrons emitted by the source. By means of (1)–(4), the depolarization may be written in the form

$$d=1-C(\delta_s/\delta_0), \tag{5}$$

with

$$C=\int_{D_s}\eta(E)p_0(E)e(E)N_0(E)dE\Big/\int_{D_p}\eta(E)p_0(E)N_p(E)dE. \tag{6}$$

In the present work, we have used the method of obtaining the depolarization from the measured values of δ_s and δ_0 by means of formula (5) in which the value of C given by formula (6) is introduced. The values of the functions which appear in formula (6) and of D_p were obtained as follows.

The functions $e(E)$ and $\eta(E)$ were obtained as explained in Sec. II. The source spectrum $N_0(E)$ was measured with a magnetic spectrometer. The function $N_p(E)$ and the domain D_p may be obtained when the relation between the energies of secondary and primary electrons is established. With the aim of comparing the measured values of d with the theoretical ones obtained by neglecting the straggling, we have also neglected the straggling in the calculation of $N_p(E)$ and D_p. The relation between the energy of secondary and primary electrons was obtained from an auxiliary experiment in which a beam of monocromatic electrons was sent normally into an aluminum foil and the scattered electrons were detected by means of a scintillation spectrometer. As far as the knowledge of the function $p_0(E)$ is concerned, it must be noted that the helicity of the β radiation of the ^{90}Sr+^{90}Y was experimentally found[14] to be equal to v/c. The actual value of p_0 is lower than v/c owing to the depolarization in the source. Since the purpose of the experiment is to check a theory of depolarization, it seemed to us that the most natural way to obtain $p_0(E)$ was to write

$$p_0(E)=(v/c)(1-d_s) \tag{7}$$

and to calculate the depolarization in the source d_s by the theory which is to be checked.

[14] A. I. Alikhanov, G. P. Eliseiev, and V. A. Liubimov, Nucl. Phys. 7, 655 (1958).

On the basis of the preceding considerations, the values of G and consequently of d may be obtained. Within this method the measured depolarization in aluminum depends in principle on the theoretical description of the depolarization in the source; this fact must not be considered as a difficulty, since the depolarization in the source is small and only slightly affects the values of d in aluminum obtained in this way. The present method is further supported by the consistency between the measured values of δ_0 and those calculated with formula (4) by means of the calculated p_0 function.

From these considerations we adopted the above method rather than that of considering our experiment as one carried out with only one composite depolarizer foil consisting of the aluminum foil and of the cover of the source; it is clearly only a question of the choice of the more informative interpretation of the experiment since the two schemes are equivalent as far as checking the theory is concerned.

B. Primary Electrons Spectra

The relation between the energy of secondary and primary electrons was established by means of the separate experiment mentioned above. The measurements were taken as a function of the primary energy E, the total scattering angle θ, and the thickness t of the foil. From this we obtained an empirical plot of the functions $f(\theta,t,E)$ and $\Delta E=\Delta E(\theta,t,E)$, where f and ΔE represent the transmitted fraction and the average energy loss of the electrons. The following values of the independent variables were considered: $t=0.3$, 0.5, 0.8, 1.0, 1.3, and 1.5 mm; $\theta=0°$, 15°, 30°, 45°, 60°, and 75°; and $E_p=0.3$, 0.5, 0.7, 0.9, 1.2, 1.5, and 2.0 MeV.

The transmission of electrons was studied in such detail since the values of the functions f and ΔE are required in the development of the depolarization calculations as will become clear later. From the measured values of the functions f and ΔE, the average values of f and ΔE pertaining to the geometrical arrangement of the asymmetry measurements were obtained with suitable integration. On this basis the primary electron spectra of each asymmetry measurement were deduced in the approximation of neglecting the straggling. These spectra are shown in Fig. 4.

C. Depolarization Values

The values of the depolarization d in aluminum obtained by the method described in Sec. IVA depend on the assumed theoretical description of the depolarization in the source; the depolarization due to the cover of the source was taken into account in all the approximations discussed later and marked a, b, c, d. The effect of the low-Z backing was neglected on the basis of the low backscattering coefficient and of the low depolarization due to backscattering on low-Z materials.[12] The

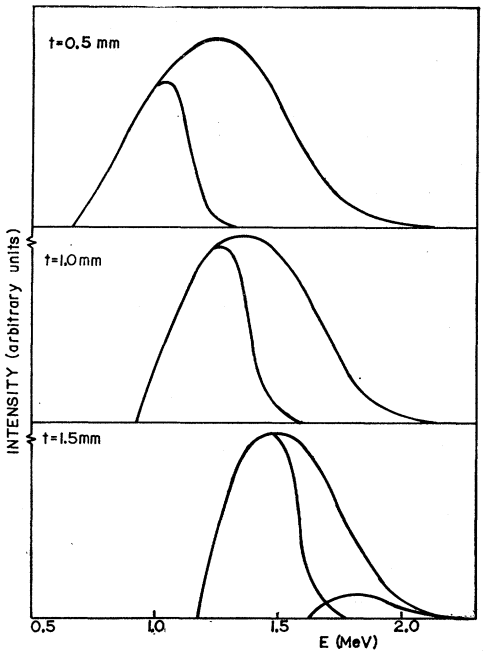

FIG. 4. The energy spectra of the primary electrons (see explanation in the text). At each thickness of the depolarizer foil the areas of the spectra are normalized to the coincidence counting rate of the polarimeter.

values of the coefficient C and of the depolarization d in aluminum foil are shown in Table II in all the approximations. As can be seen, the measured depolarizations are substantially independent on the assumption made in the calculation of the depolarization in the source, and the measured depolarization can undoubtedly be attributed to the effect of the aluminum foil.

In order to test the internal consistency of the experimental method, the depolarizations in aluminum were obtained with reference to different values of δ_0

TABLE II. Experimental values of the longitudinal depolarization in aluminum obtained by the method explained in Sec. IVA. The values of coefficients C given by formula (6) are given in parenthesis.

t (mm)	Energy ranges (MeV)	Approximation a	Approximation b	Approximation c	Approximation d
0.5	0.35–1.1	17.1±9.8 (1.1032)	15.6±10.0 (1.1167)	15.1±10.1 (1.1234)	14.4±10.2 (1.1319)
	0.35–2.2	9.7±10.0 (1.2156)	9.5±10.0 (1.2173)	9.3±10.0 (1.2234)	9.1±10.0 (1.2270)
1.0	0.35–1.1	13.9±11.4 (1.0705)	13.4±11.4 (1.0763)	13.4±11.4 (1.0765)	13.2±11.5 (1.0794)
	0.35–2.2	20.5±10.4 (1.1897)	19.8±10.4 (1.1949)	19.9±10.4 (1.1931)	19.8±10.4 (1.1942)
1.5	0.35–1.1	16.5±13.3 (1.0525)	15.6±13.4 (1.0569)	15.9±13.3 (1.0530)	15.9±13.3 (1.0531)
	0.35–2.2	17.8±16.4 (1.1592)	16.8±16.6 (1.1654)	17.1±16.5 (1.1613)	17.1±16.5 (1.1612)
	0.9 –2.2	11.1±26.9 (0.9640)	13.6±26.8 (0.9320)	14.1±26.5 (0.9263)	14.4±26.4 (0.9227)

and consequently of C in formulas (5) and (6). The values of d measured with the depolarizer foils 1.0 and 1.5-mm thick turned out to be independent within $\pm 2\%$ of the choice of δ_0, and this proves the usefulness of the method. In the case of the 0.5-mm foil, the choice of δ_0 affects the resulting d to a greater extent, owing to the particular shape of the primary spectra and of the $p_0(E)$ function. In this case, values of d which differ by 12% are obtained when different δ_0 are used. Nevertheless this uncertainty is lower than the statistical errors which are large owing to the extremely low coincidence counting rate (in some cases of the order of 20 cpm).

V. DEPOLARIZATION CALCULATIONS

In order to discuss the main features of the available theories on the depolarization, the calculations were made in the following four approximations which may be deduced from the work of Braicovich.[10]

(a) The energy loss is entirely neglected. The depolarization is calculated by means of a theory extremely similar in principle to that given by Passatore.[2] Only collisions on the target nuclei are considered and the length of the electron path is assumed equal to the thickness of the absorber. The equation of the average helicity p of the beam is

$$dp/dz = -\rho_n \epsilon_n p, \tag{8}$$

where

$$\epsilon_n = 2\int_{4\pi} \sigma_{hf} d\Omega, \tag{9}$$

ρ_n is the number of nuclei in the unit volume, and σ_{hf} is the cross section of the nuclear scattering in which the outgoing electron has helicity opposite to that of the incoming one. The coordinate z is taken normal to the foil.

(b) The energy loss is considered in the sense that each nuclear collision takes place at a lower energy than the preceding one. The path length is still equal to the thickness t of the foil. The equation is

$$dp/dE = -\rho_n \epsilon_n p (dE/dz)^{-1}. \tag{10}$$

This approximation is substantially equivalent to that in the theory of Bouchiat *et al.*,[8] in which the further approximation of considering $\sin\theta = \theta$ is made.

(c) The preceding approximation is improved by taking into account the depolarizing effect of the electron-electron collisions; the path length is still equal to t. The equation is

$$dp/dE = -(\rho_n \epsilon_n + Z\rho_n \epsilon_e) p (dE/dz)^{-1} \tag{11}$$

in accordance with Braicovich's paper.

The quantity ϵ_e refers to the electron-electron collisions and is analogous to ϵ_n.

(d) In addition to case (c), the lengthening of the trajectories of the particles is considered. The equation is

$$dp/dE = -(\rho_n \epsilon_n + Z\rho_n \epsilon_e) p (dE/ds)^{-1}, \tag{12}$$

where

$$ds = dz/\langle\cos\theta\rangle \tag{13}$$

is the differential of the "effective thickness" and

$$\langle\cos\theta\rangle = \int (\cos\theta) f d\Omega \Big/ \int f d\Omega, \tag{14}$$

where the integration is extended to the whole beam. The value of $\langle\cos\theta\rangle$ is a function of the energy and must be attributed to the average energy

$$\langle E\rangle = \int (E - \Delta E) f d\Omega \Big/ \int f d\Omega, \tag{15}$$

where f and ΔE are the functions defined in Sec. IV B, and E is the energy of the primary electrons. All these approximations are effected by the approximations of neglecting the effects of backscattering and of straggling on the depolarization.

The formula (13) is an approximate expression of $\langle\cos\theta\rangle$ in which the exact weighting function $fp(\theta)$, where p is the helicity at the angle θ, is replaced by f, as was suggested in the work of Braicovich.[10] The validity of this approximation has been confirmed by a separate experiment in which the average value of $p(\theta)$ in three angular regions was measured; it was thus ascertained that the value of $\langle\cos\theta\rangle$ is not affected by this approximation to an extent greater than 3%. This is satisfactory, since the depolarization increment due to the effective thickness never exceeded 20% with a resulting error lower than 0.6% in the final results.

The results of the depolarization calculations carried out in the four preceding cases are collected in Table III; it is clear that the different descriptions of the depolarization process give results which are extremely different one from another.

All these results are obtained by means of the following approximations which are not intrinsic to the depolarization theories.

TABLE III. Values of the longitudinal depolarization in aluminum, calculated on the basis of the four theoretical pictures explained in Sec. V.

t (mm)	Energy ranges (MeV)	Approximation a	Approximation b	Approximation c	Approximation d
0.5	0.35–1.1	4.459	7.158	10.242	11.938
	0.35–2.2	2.583	3.485	5.694	6.630
1.0	0.35–1.1	4.643	10.775	15.522	18.317
	0.35–2.2	3.390	6.936	10.493	12.377
1.5	0.35–1.1	4.265	13.264	19.189	22.592
	0.35–2.2	3.538	9.761	14.681	17.283
	0.9 –2.2	1.922	3.851	6.779	7.980

All nuclear scattering cross sections were calculated in the second Born approximation and the screening due to the atomic electrons was taken into account, following Nigam *et al.*,[15] by means of a simple exponential in which the screening parameter is given by $\mu\lambda_0$ with symbols of the Nigam paper. The choice of the value of μ is of little importance in our calculation and the value $\mu=1.8$ was chosen as suggested by Nigam.

The electron-electron scattering cross sections were calculated with reference to the Feynmann diagrams of the lowest order.

The knowledge of the so-called incoherent scattering function, which takes into account the binding of the atomic electrons, is necessary in the calculations of ϵ_e, as has been pointed out by Braicovich; the values of this function calculated for the Thomas-Fermi model of the atom were used.[16]

VI. DISCUSSION

The measured depolarizations are compared with the calculated ones by plotting the results of the measurements against those of the calculations; we have used this representation since in the present experiment the various measurements were carried out with reference to different values of several parameters, such as energy, thickness and energy loss in the depolarizer foil, and to different shapes of the primary spectra. Consequently, the measurements cannot be expressed as a function of a parameter and the values of this function cannot be compared with the calculated ones. In the chosen representation, the best fit line was calculated and a straight line was chosen, owing to the presence of large statistical errors which prevent a more refined fitting of the points. This straight line must pass through the origin with no uncertainty, since the theory gives no depolarization at zero thickness and the statistical error of the asymmetry measurement without the depolarizer foil is taken into account in the error of the depolarizations obtained by means of the method given in Sec. IV A.

The comparison between theory and experiment is made with reference to the four approximations specified in the preceding paragraph; this comparison is given in Figs. 5–8. Although the experimental errors are large, the difference in the usefulness of the various approximations is clear.

The first approximation is completely unsatisfactory, as was to be expected, since the energy loss of the electrons is entirely neglected. The second approximation explains about 60% of the observed depolarization, but a satisfactory result can be obtained only if the depolarizing effect of the electron-electron collisions is considered. It is relevant that this contribution is larger than the fraction $1/Z$ of the nuclear contribution

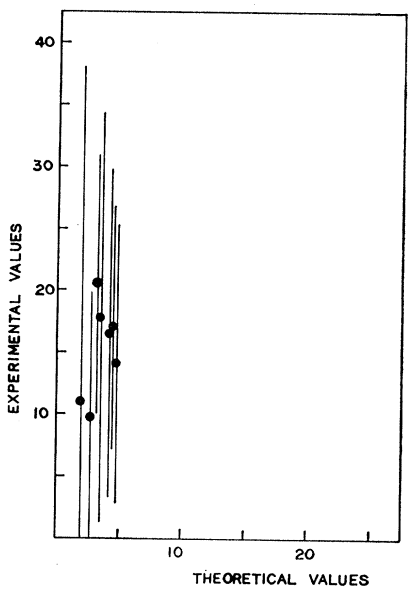

FIG. 5. Measured depolarization in aluminum against calculated ones. The approximation a is used in the depolarization calculations. The slope of the best-fit line is 4.06±0.40.

owing to the effect of the collisions with large momentum transfer. The electronic contribution is particularly evident, since the depolarizing material has a low atomic number and the second approximation may be better at higher Z, at least from this point of view. The difference between approximations c and d (the effect of lateral deflections), is relatively small and a thorough discussion of this last approximation is difficult, owing

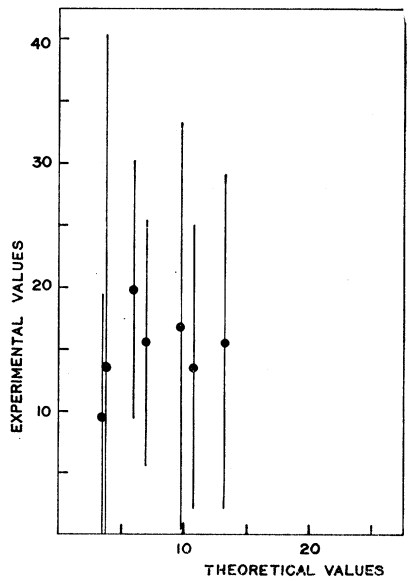

FIG. 6. Measured depolarization in aluminum against calculated ones. The approximation b is used in the depolarization calculations. The slope of the best line is 1.70±0.26.

[15] B. P. Nigam, M. K. Sundaresan, and Ta-You Wu, Phys. Rev. 115, 491 (1959).

[16] G. W. Grodstein, Natl. Bur. Std. (U. S.) Circ. No. 583 (1957).

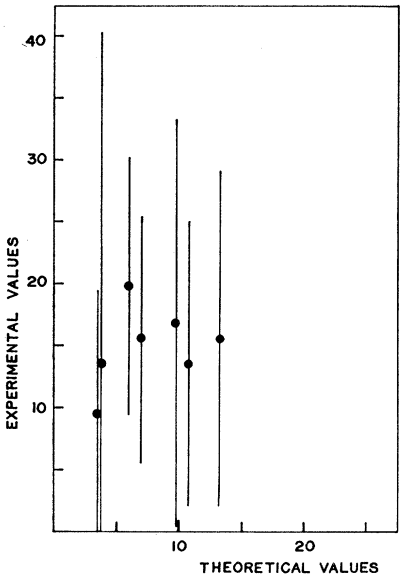

FIG. 7. Measured depolarization in aluminum against calculated ones. The approximation c is used in the depolarization calculations. The slope of the best-fit line is 1.17±0.17.

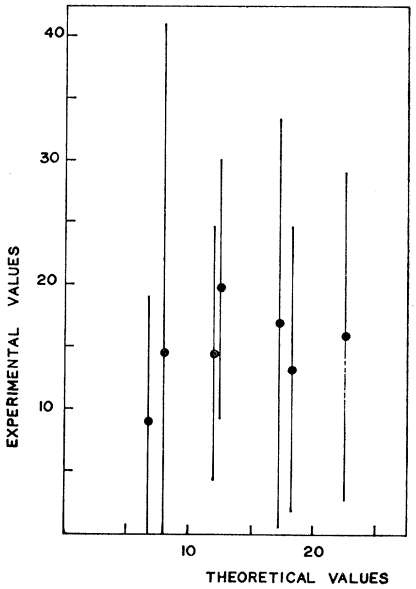

FIG. 8. Measured depolarization in aluminum against calculated ones. The approximation d is used in the depolarization calculations. The slope of the best-fit line is 0.98±0.14.

to the large errors, although it may be seen that a better agreement is obtained in the last approximation.

Special discussion is required by the fact that all these depolarization calculations are affected by the limitation of neglecting backscattering in the boundary conditions. Approximations *c* and *d* are satisfactory within the experimental errors and this is due to the fact that the backscattering coefficient of the aluminum is small. Moreover, this fact has an important consequence. In effect, if the backscattering is neglected, the boundary condition at the surface of the foil where the electrons are incident is:

$$I(\theta)p(\theta)=I_0p_0\delta(\theta) \qquad (16)$$

with

$$0<\theta<\pi.$$

On the basis of our previous work[12] and of the available backscattering coefficients, we may say that, in the case of a thick aluminum target, the average value of $(Ip)/(I_0p_0)$ in the region $\frac{1}{2}\pi<\theta<\pi$ is of the order of 0.15 instead of 0, as required by condition (16). Furthermore, in the case of lead, the value of $(Ip)/(I_0p_0)$ is of the same order since the depolarization is high, although the value of I is large owing to the large backscattering coefficient. Consequently, we may argue that the approximate boundary condition (16), which is good in the case of low Z materials, may be also useful in the case of high-Z materials.

We may thus conclude that the effect of the energy loss on depolarization cannot be neglected when the thickness of the absorber is so high that a considerable fraction of the energy of the electrons is lost (more than 25%). The effect of energy loss may be taken into account by means of Braicovich's theory in which both the nuclear and electronic contributions to depolarization are considered. These effects are considerably more important than the effect of the lateral deflections of the particles. Thus, in many cases, the depolarization calculations may be greatly simplified by avoiding the calculation of the effective thickness which is laborious and often requires the knowledge of the results of separate *ad hoc* experiments.

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