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Two- and Three-Body Photodisintegration Cross Sections of ^{3}H and ^{3}He

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Electric-dipole photodisintegration cross sections of ^{3}H and ^{3}He into deuteron plus nucleon and into three nucleons are calculated using simple zero-range forms for the initial- and final-state wave functions. Final-state interactions between nucleon pairs in S states are taken into account. Shape fits to the experimental data are obtained, but the required normalization of the ground-state wave function is found to differ between two- and three-body breakup.

I. INTRODUCTION

THE photodisintegration reactions

$$^3\mathrm{H}+\gamma \rightarrow d+n\,, \tag{1a}$$

$$^3\mathrm{H}+\gamma \rightarrow p+n+n\,, \tag{1b}$$

$$^3\mathrm{He}+\gamma \rightarrow d+p\,, \tag{1a$'$}$$

$$^3\mathrm{He}+\gamma \rightarrow n+p+p\,, \tag{1b$'$}$$

in the low to medium energy range (0–40 MeV), occur predominantly via electric-dipole transition. The ground state of ^{3}H and of ^{3}He is thought[1] to be an almost pure $^2S_{1/2}$ state. An electric-dipole transition then gives a $^2P_{1/2,3/2}$ final state. The cross section is then given by

$$d\sigma=(2\pi)^2\left(\frac{e^2}{\hbar c}\right)|M_{fi}|^2E_\gamma\rho_f\,, \tag{2}$$

where $M_{fi}=(\Psi_f,(\boldsymbol{\varepsilon}\cdot\mathbf{r}_3)\Psi_i)$ is the transition amplitude with Ψ_i and Ψ_f as the initial and final nuclear wave functions. E_γ is the photon energy and $\boldsymbol{\varepsilon}$ its polarization vector. The coordinate of the odd particle (the proton in ^{3}H or the neutron in ^{3}He) is $\mathbf{r}_3=\frac{2}{3}\mathbf{r}'$ (see Fig. 1). ρ_f is the density of final states.

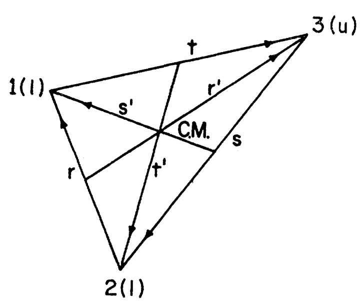

FIG. 1. Coordinates in the three-nucleon system. Vectors $\mathbf{r},\mathbf{s},\mathbf{t}$ are internucleon coordinates. Vectors $\mathbf{r}',\mathbf{s}',\mathbf{t}'$ locate a nucleon with respect to the center of mass of the remaining nucleon pair. Particles 1 and 2 are the like nucleons (protons in ^{3}He and neutrons in ^{3}H).

Since at present there is no general solution to the three-body Schrödinger equation for phenomenological two-body potentials (except in the special cases of Gaussian or separable potentials), most three-body ground-state wave functions are chosen from analytically convenient forms. A good test of these wave functions is provided by calculation of the cross sections of reactions (1), since the transition operator is known. As has been shown in Refs. 7, 8, and 9 for reactions (1a) and (1a$'$), there are several forms of the ground-state wave functions which yield fair agreement with the experimental photodisintegration cross sections. However, these same ground-state wave functions yield cross sections for reactions (1b) and (1b$'$) which are much larger than the experimental ones. The disagreement is even larger when the final-state interactions between the outgoing nucleons is included.

We seek, therefore, a three-nucleon ground-state wave function that predicts reasonable values for both two- and three-body breakup. In the case of the photodisintegration of the deuteron, a simple effective-range theory fits the experimental data up to 40 MeV fairly well.[2] This approximation works because the main contribution of the bound state to the electric-dipole matrix element comes from outside the range of nuclear forces where the asymptotic form of the wave function can be used. We attempt here a similar approach to the three-body photodisintegration.

In the following section we discuss an asymptotic form for the three-nucleon bound state. The final continuum states for two- and three-body breakup are shown in Sec. III. In Sec. IV we calculate the transition

[1] B. F. Gibson, Phys. Rev. 139, B1153 (1958).

[2] J. S. Levinger, *Nuclear Photodisintegration* (Oxford University Press, New York, 1960), p. 39.

amplitudes and cross sections. Comparison with the data and conclusions are in Sec. V. An Appendix gives useful relationships among the particle coordinates, momenta, and spin functions.

II. THE BOUND STATE

The assumption of a $^2S_{1/2}$ bound-state symmetric under interchange of any two particles fixes the form of the angular and spin functions.[3] The radial form is chosen by considering the action of the electric-dipole operator. The one-body $E1$ operator leaves one of the three nucleons in a p wave about the center of mass and the other two nucleons in a relative s wave. The operator and the radial p wave weight the large coordinate values of the bound wave function for this nucleon. If, say, particle 3 is far removed from the other two, an appropriate form of the bound state is $\varphi(r)e^{-\beta r'}/r'$. For simplicity we also take $\varphi(r)$ as a two-body zero-range wave function $e^{-\alpha r}/r$. Since we assume the wave function is symmetric in all three nucleons, it will have the same form in the three asymptotic regions characterized, respectively, by $r'\to\infty$, $s'\to\infty$, $t'\to\infty$. Specifically, we take

$$\Psi_i=\frac{N(4\alpha\beta)^{1/2}}{4\pi}\frac{e^{-\alpha r}}{r}\frac{e^{-\beta r'}}{r'}\chi_s(12,3) \tag{3}$$

for $r'\to\infty$, and the same function with the variables (s,s') or (t,t') for the regions $s'\to\infty$ and $t'\to\infty$, respectively. The parameters α and β are then subject to the condition

$$\alpha^2+\tfrac{3}{4}\beta^2=(m/\hbar^2)\epsilon, \tag{4}$$

where ϵ is the total binding energy of ^{3}H or ^{3}He, but the ratio α/β is considered a free parameter. N is introduced for normalization and to allow for an effective range correction. $\chi_s(12,3)$ is the total spin-$\frac{1}{2}$ eigenfunction antisymmetric in the pair 12. (See Appendix for definition and notation of spin states.)

III. FINAL STATES

In two-body breakup [reactions (1a) and (1a′)] the deuteron and nucleon are in a relative p wave. Since the doublet p-wave phase shift for nucleon-deuteron elastic scattering is small, we take the relative motion as an (antisymmetrized) plane wave of momentum $\hbar\mathbf{q}$.

$$\Psi_q=\frac{1}{\sqrt{2}}[\varphi(s)e^{i\mathbf{q}\cdot\mathbf{s}'}\chi_t(23,1)-\varphi(t)e^{i\mathbf{q}\cdot\mathbf{t}'}\chi_t(31,2)].$$

$\varphi(s)$ is the space part of the deuteron ground state and will be taken as

$$\varphi(s)=\left(\frac{2\gamma}{4\pi(1-\gamma r_0)}\right)^{1/2}\frac{e^{-\gamma s}}{s},$$

where $\gamma^2=m/\hbar^2$ (2.22 MeV) and r_0 is the triplet effective range. The doublet spin eigenstates χ have particles 2,3 and 3,1 coupled to a (symmetric) triplet, respectively.

In three-body breakup [reactions (1b) and (1b′)] there are two orthogonal final states $\Psi_{kp}^{(1)}$ and $\Psi_{kp}^{(2)}$ with $S=\frac{1}{2}$. These are eigenstates of the three-nucleon system belonging to the double continuum of energy eigenvalues describing the motion of three unbound nucleons characterized by momenta $\hbar\mathbf{k}$ and $\hbar\mathbf{p}$. When no interactions are present they become the plane-wave eigenstates $e^{i(\mathbf{k}\cdot\mathbf{r}+\mathbf{p}\cdot\mathbf{r}')}\chi_s(12,3)$ and $e^{i(\mathbf{k}\cdot\mathbf{r}+\mathbf{p}\cdot\mathbf{r}')}\chi_t(12,3)$. In the present case $\Psi_{kp}^{(1)}$ and $\Psi_{kp}^{(2)}$ must be taken with the incoming spherical wave asymptotic condition since they appear in the final state of a reaction.

In the transition amplitudes the electric-dipole operator selects the 1^- component of the space part of the final states. To include the s-wave final-state interactions among the nucleon pairs, we take the final states as

$$\Psi_{kp}^{(1)}=\chi_s(12,3)\left[e^{i\mathbf{k}_{12}\cdot\mathbf{r}}+f_s^{(-)}(k_{12})\frac{e^{-ik_{12}r}}{r}\right]e^{i\mathbf{p}_3\cdot\mathbf{r}'}+[-\tfrac{1}{2}\chi_s(23,1)f_s^{(-)}(k_{23})-\tfrac{1}{2}\sqrt{3}\chi_t(23,1)f_t^{(-)}(k_{23})]\frac{e^{-ik_{23}s}}{s}e^{i\mathbf{p}_1\cdot\mathbf{s}'}$$
$$+[-\tfrac{1}{2}\chi_s(31,2)f_s^{(-)}(k_{31})+\tfrac{1}{2}\sqrt{3}\chi_t(31,2)f_t^{(-)}(k_{31})]\frac{e^{-ik_{31}t}}{t}e^{i\mathbf{p}_2\cdot\mathbf{t}'}, \tag{5a}$$

and

$$\Psi_{kp}^{(2)}=\chi_t(12,3)e^{i(\mathbf{k}_{12}\cdot\mathbf{r}+\mathbf{p}_3\cdot\mathbf{r}')}+[\tfrac{1}{2}\sqrt{3}\chi_s(23,1)f_s^{(-)}(k_{23})-\tfrac{1}{2}\chi_t(23,1)f_t^{(-)}(k_{23})]\frac{e^{-ik_{23}s}}{s}e^{i\mathbf{p}_1\cdot\mathbf{s}'}$$
$$+[-\tfrac{1}{2}\sqrt{3}\chi_s(31,2)f_s^{(-)}(k_{31})-\tfrac{1}{2}\chi_t(31,2)f_t^{(-)}(k_{31})]\frac{e^{-ik_{31}t}}{t}e^{i\mathbf{p}_2\cdot\mathbf{t}'}. \tag{5b}$$

The s-wave part of the scattering amplitudes are

$$(f^{(-)}(k))^*=f(k)=\frac{e^{i\delta}\sin\delta}{k}=(k\cot\delta-ik)^{-1}, \tag{6}$$

with the phase shift given by the nucleon-nucleon s-wave[4] scattering length and effective range $k\cot\delta=a^{-1}+\frac{1}{2}r_0k^2$.

[3] G. Derrick and J. M. Blatt, Nucl. Phys. 8, 310 (1958). Isospin formalism will not be used. Antisymmetrization will be carried out only over the two like particles.

[4] In the calculation we have used

$a=-23.78$ F, $r_0=2.670$ F for n-p singlet, $a=$ 5.411 F, $r_0=1.749$ F for n-p triplet.

IV. CROSS SECTIONS

The transition amplitudes are calculated in the approximation where the ground-state wave function is replaced by the appropriate asymptotic forms discussed in Sec. II, i.e., in those terms where nucleon 3 is in the p wave relative to the center of mass of nucleons 1 and 2, the ground-state wave function is replaced by the form Eq. (3).[5]

Since in the two-body breakup there is no term in the continuum corresponding to r', we have only two terms:

$$M_q=\frac{N(\alpha\beta\gamma)^{1/2}}{4\pi^{3/2}(1-\gamma r_0)^{1/2}}\left[\int d^3sd^3s'\frac{e^{-\gamma s-i\mathbf{q}\cdot\mathbf{s}'}}{s}(-\boldsymbol{\varepsilon}\cdot\mathbf{s}'/3)\frac{e^{-\alpha s-\beta s'}}{ss'}\langle\chi_t(23,1)|\chi_s(12,3)\rangle\right.$$

$$\left.-\int d^3td^3t'\frac{e^{-\gamma t-i\mathbf{q}\cdot\mathbf{t}'}}{t}(-\boldsymbol{\varepsilon}\cdot\mathbf{t}'/3)\frac{e^{-\alpha t-\beta t'}}{ss'}\langle\chi_t(31,2)|\chi_s(12,3)\rangle\right]$$

$$=\frac{8\pi^{1/2}N}{\sqrt{3}(1-\gamma r_0)^{1/2}}\frac{(\alpha\beta\gamma)^{1/2}}{(\alpha+\gamma)}\frac{(-i\boldsymbol{\varepsilon}\cdot\mathbf{q})}{(q^2+\beta^2)^2}. \tag{7}$$

We have made use of the relations $\mathbf{r}_3=-\frac{1}{2}\mathbf{s}-\frac{1}{3}\mathbf{s}'=\frac{1}{2}\mathbf{t}-\frac{1}{3}\mathbf{t}'$ and noticed that contributions from $\boldsymbol{\varepsilon}\cdot\mathbf{s}$ and $\boldsymbol{\varepsilon}\cdot\mathbf{t}$ terms cancel. The spin matrix element is $\frac{1}{2}\sqrt{3}$.

The three-body breakup amplitudes (including the spin matrix elements) into the two symmetry states are given by

$$M_{kp}{}^{(1)}=\frac{N(4\alpha\beta)^{1/2}}{4\pi}\left\{\frac{2}{3}\int d^3rd^3r'\left[e^{-i\mathbf{k}_{12}\cdot\mathbf{r}}+f_s(k_{12})\frac{e^{ik_{12}r}}{r}\right]e^{-i\mathbf{p}_3\cdot\mathbf{r}'}(\boldsymbol{\varepsilon}\cdot\mathbf{r}')\frac{e^{-\alpha r-\beta r'}}{rr'}\right.$$

$$-\tfrac{1}{3}[\tfrac{1}{4}f_s(k_{23})+\tfrac{3}{4}f_t(k_{23})]\int d^3sd^3s'\frac{e^{+ik_{23}s}}{s}e^{-i\mathbf{p}_1\cdot\mathbf{s}'}(\boldsymbol{\varepsilon}\cdot\mathbf{s}')\frac{e^{-\alpha s-\beta s'}}{ss'}$$

$$\left.-\tfrac{1}{3}[\tfrac{1}{4}f_s(k_{31})+\tfrac{3}{4}f_t(k_{31})]\int d^3td^3t'\frac{e^{+ik_{31}t}}{t}e^{-i\mathbf{p}_2\cdot\mathbf{t}'}(\boldsymbol{\varepsilon}\cdot\mathbf{t}')\frac{e^{-\alpha t-\beta t'}}{tt'}\right\}$$

$$=\frac{N(4\alpha\beta)^{1/2}}{4\pi}\left\{\frac{2}{3}\left[\frac{4\pi}{\alpha^2+k_{12}^2}+f_s(k_{12})\frac{4\pi}{\alpha-ik_{12}}\right]\frac{-8\pi i}{(\beta^2+p_3^2)^2}(\boldsymbol{\varepsilon}\cdot\mathbf{p}_3)-\tfrac{1}{3}f_+(k_{23})\frac{4\pi}{\alpha-ik_{23}}\frac{-8\pi i}{(\beta^2+p_1^2)^2}(\boldsymbol{\varepsilon}\cdot\mathbf{p}_1)\right.$$

$$\left.-\tfrac{1}{3}f_+(k_{31})\frac{4\pi}{\alpha-ik_{31}}\frac{-8\pi i}{(\beta^2+p_2^2)^2}(\boldsymbol{\varepsilon}\cdot\mathbf{p}_2)\right\}, \tag{8a}$$

and

$$M_{kp}{}^{(2)}=\frac{N(4\alpha\beta)^{1/2}}{4\pi}\left\{\frac{1}{3}\frac{\sqrt{3}}{4}[f_s(k_{23})-f_t(k_{23})]\int d^3sd^3s'\frac{e^{-ik_{23}s}}{s}e^{-i\mathbf{p}_1\cdot\mathbf{s}'}(\boldsymbol{\varepsilon}\cdot\mathbf{s}')\frac{e^{-\alpha s-\beta s'}}{ss'}\right.$$

$$\left.-\frac{1}{3}\frac{\sqrt{3}}{4}[f_s(k_{31})-f_t(k_{31})]\int d^3td^3t'\frac{e^{-ik_{31}t}}{t}e^{-i\mathbf{p}_2\cdot\mathbf{t}'}(\boldsymbol{\varepsilon}\cdot\mathbf{t}')\frac{e^{-\alpha t-\beta t'}}{tt'}\right\}$$

$$=\frac{N(4\alpha\beta)^{1/2}}{4\pi}\left\{\frac{1}{4\sqrt{3}}f_-(k_{23})\frac{4\pi}{\alpha-ik_{23}}\frac{-8\pi i}{(\beta^2+p_1^2)^2}(\boldsymbol{\varepsilon}\cdot\mathbf{p}_1)-\frac{1}{4\sqrt{3}}f_-(k_{31})\frac{4\pi}{\alpha-ik_{31}}\frac{-8\pi i}{(\beta^2+p_2^2)^2}(\boldsymbol{\varepsilon}\cdot\mathbf{p}_2)\right\}, \tag{8b}$$

with the notation

$$f_+(k)=\tfrac{1}{4}f_s(k)+\tfrac{3}{4}f_t(k),$$
$$f_-(k)=f_s(k)-f_t(k).$$

The density of final states for two-body breakup is

$$\rho_2=\frac{d^3q}{dE(2\pi)^3}=\frac{2}{3}\frac{m}{\hbar^2}\frac{qd\Omega}{(2\pi)^3},$$

where q is related to the energy available E by $q^2=\frac{4}{3}(m/\hbar^2)E$. The density for three-body breakup is

$$\rho_3=\frac{d^3kd^3p}{dE(2\pi)^6}=\frac{1}{2(2\pi)^6}\left(\frac{m}{\hbar^2}\right)^3[W(2E-3W)]^{1/2}dWd\Omega_kd\Omega_p,$$

where W is the energy of the odd particle and E the total energy available for breakup.

For two-body breakup, averaging over photon polarization gives $\langle|\boldsymbol{\varepsilon}\cdot\mathbf{q}|^2\rangle_{\text{pol}}=\frac{1}{2}q^2\sin^2\theta$, where θ is measured between the direction of photon propagation

[5] If the bound state were to be considered as the sum of three terms of the type shown in Eq. (3) then cross terms of the type $\psi_{kp}(r,r')(\boldsymbol{\varepsilon}\cdot\mathbf{r}_3)\psi_i(s,s')$ would have to be included in the matrix element. The spirit of this calculation is, however, to consider only the r',s',t' asymptotic directions.

and $\mathbf{q}$, and the differential cross section is

$$\frac{d\sigma_2}{d\Omega}=\frac{2^5N^2}{3^2}\left(\frac{e^2}{\hbar c}\right)\left(\frac{m}{\hbar^2}\right)\frac{\alpha\beta\gamma}{(1-\gamma r_0)(\alpha+\gamma)^2}\frac{E_\gamma q^3\sin^2\theta}{(q^2+\beta^2)^4}. \quad (9)$$

The differential cross section for three-body breakup is obtained from

$$|M_{kp}|^2=|M_{kp}^{(1)}|^2+|M_{kp}^{(2)}|^2$$

by expressing all momenta in terms of E, W, and ζ as described in the Appendix. The resulting expression after averaging over photon polarizations and integration over azimuthal angles is of the form

$$\frac{d^3\sigma_3}{dWd\zeta d(\cos\theta_p)}=\frac{(4\pi N)^2}{3}\left(\frac{e^2}{\hbar c}\right)\frac{m}{\hbar^2}\times[F(E,W,\zeta)\sin^2\theta_p+G(E,W,\zeta)]\times E_\gamma[W(2E-3W)]^{1/2}. \quad (10)$$

θ_p is the angle between the direction of the incident photon and the momentum $\hbar\mathbf{p}$ of the unlike nucleon. The functions F and G, associated, respectively, with the p wave and isotropic parts of the angular distribution of the unlike nucleon, are lengthy but easy-to-obtain combinations of the functions appearing in the expressions (8a) and (8b) for the transition amplitudes.

Integrations over ζ and W are done numerically to obtain the total cross section.

When only the final-state interaction between the like pair is considered, only the term containing $(\boldsymbol{\varepsilon}\cdot\mathbf{p}_3)$ appears in the amplitude $M_{kp}^{(1)}$ [Eq. (8a)] and $M_{kp}^{(2)}=0$.

Then

$$\frac{d^3\sigma_3}{dWd\Omega_pd\Omega_k}=\frac{8N^2}{9\pi^2}\left(\frac{\hbar^2}{m}\right)^2\left(\frac{e^2}{\hbar c}\right)\sin^2\theta\left(\cos\delta+\frac{\alpha}{k_{12}}\sin\delta\right)^2\times\frac{E_\gamma W[W(2E-3W)]^{1/2}}{[E-\frac{3}{2}W+(\hbar^2/m)\alpha^2]^2[2W+(\hbar^2/m)\beta^2]^4}.$$

This expression was previously derived in Ref. 6.

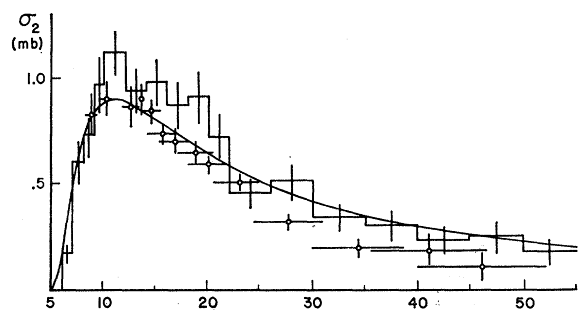

FIG. 2. The total cross section for $^3He(\gamma,d)p$. The histogram is from Ref. 8, the points from Ref. 7. The solid curve is the calculation with the parameters $\alpha=\gamma=0.232$ F^{-1}, $\beta=0.420$ F^{-1}, $N^2=1.82$.

[6] J. M. Knight, J. S. O'Connell, and F. Prats, Phys. Letters **22**, 322 (1966); **23**, 491 (1966).

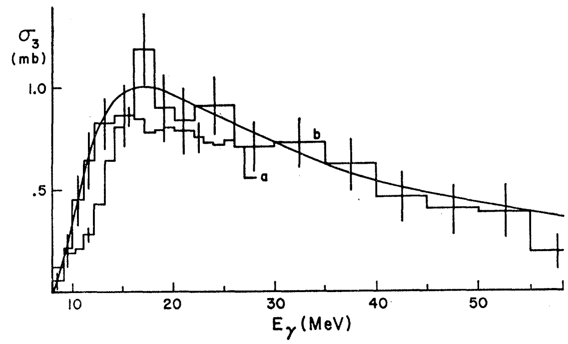

FIG. 3. The total cross section for $^3He(\gamma,n)2p$. Histogram (*a*) is the data of Ref. 9, histogram (*b*) of Ref. 8. The curve is the calculation with final state interaction between all three s-wave nucleon pairs with the parameters $\alpha=0.232$ F^{-1}, $\beta=0.420$ F^{-1}, $N^2=0.77$.

V. DISCUSSION

To evaluate σ_2 and σ_3 there are two parameters α/β and N^2 that must be fixed. The value of α/β is chosen by noticing that the peak of σ_2 occurs at $E=\frac{4}{3}(\hbar^2/m)\beta^2$. Using this to fix the values of $\beta=0.420$ F^{-1}, we find $\alpha=0.232$ F^{-1} by Eq. (4). These values are used for both σ_2 and σ_3. We find that we cannot fit σ_2 and σ_3 with a common value of N^2. The two-body breakup requires $N_2^2=1.82$ while for three-body breakup $N_3^2=0.77$. Using the more realistic Hulthén form $(e^{-\alpha r}-e^{-\alpha' r})/r$ in place of the zero-range form does not improve the N_2^2/N_3^2 ratio.

The total two-body breakup cross section is shown in Fig. 2 together with some of the experimental data[7,8] on ^3He. The total three-body breakup cross section is shown in Fig. 3 with the ^3He total-cross-section measurements of Refs. 8 and 9.

The energy distribution of the odd particle is shown in Fig. 4 at an excitation energy near the peak of the three-body cross section. This general shape agrees with the data of Ref. 8. The high-energy peaking is due to the attractive forces between the like nucleons in the final singlet s state. Figure 5 shows the contributions of the various final-state interaction terms to the total cross section. The singlet interaction between the like pairs is seen to give the largest addition to the plane-wave cross section. The triplet interaction between the unlike particles is small and subtracts from the plane-wave result. The angular distribution of the odd nucleon has a fairly constant ratio of isotropic term to $\sin^2\theta$ coefficient of about 7–8%. (The symmetric spin term contributes about 70% of the isotropic term.) The experiment ratio[8] is (3±3)%.

A justification can now be made of the approximation that the asymptotic forms of the bound and final-state p waves can be used in computing the $E1$ matrix

[7] J. R. Stewart, R. C. Morrison, and J. S. O'Connell, Phys. Rev. **138**, B372 (1965).

[8] V. N. Fetisov, A. N. Gorbunov, and A. T. Varfolomeev, Nucl. Phys. **71**, 305 (1965).

[9] H. M. Gerstenberg and J. S. O'Connell, Phys. Rev. **144**, 834 (1966).

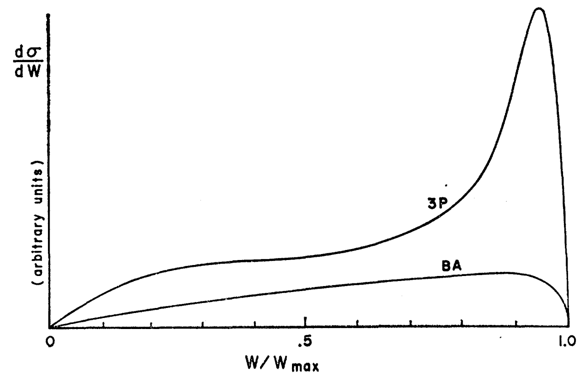

FIG. 4. The energy spectrum of the odd nucleon at the peak of the cross section. The curve labeled BA is the spectrum without final-state interactions (Born Approximation); the curve labeled 3P shows the effect of the three s-wave pairs.

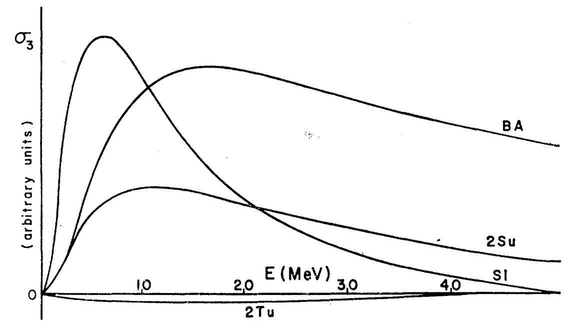

FIG. 5. Contribution of various terms to the total three-body cross section. Curve BA is for a plane-wave final state. Curve S1 is the contribution due to the singlet s-wave interaction between the like nucleon pair, 2 Su of the unlike singlet nucleon pairs, and 2 Tu of the unlike triplet nucleon pairs.

elements. Both the two- and three-body matrix elements have radial integrands in the primed coordinate of the form $r'^2 e^{-\beta r'} j_1(pr')$. This function (for p corresponding to the maximum values at the peaks of the two cross sections and $\beta = 0.420$ F^{-1}) has its maximum value at $r' = 5$ F, a value well outside the range of nuclear forces ($\lambda_\pi = 1.4$ F). Therefore, the electric-dipole matrix element will be insensitive at small distances to deviations of the true wave functions from the asymptotic forms.

The results of the approximation presented here parallel those obtained by Fetisov,[10] who has used for the ground state the wave function of the Pappademos type with the parameters fixed by Dalitz and Thacker[11] to fit the ^{3}H and ^{3}He rms radii and binding energies. Fetisov gets fits to σ_2 and σ_3 of about the quality of the ones obtained here, that is, σ_2 is $\sim 15\%$ too low and σ_3 is $\sim 30\%$ too high. However, this calculation included only the like nucleon pair interacting in the final state. Inclusion of all the interacting nucleon pairs would worsen the agreement.

The importance of the asymptotic region of the ground-state wave function in relation to the radii and binding energies has been emphasized by Dalitz and Thacker and recently its relevance for the photodisintegration problem has been studied by Fetisov.[12]

In relation to the three-body breakup we would like to point out that other workers[13] incorporate the final-state interactions by taking an incoherent sum of the like-nucleon-pair matrix element and the unlike nucleon pairs. This amounts in effect to assume that in the final state there are several independent three-particle channels which can be distinguished by the way a nucleon pair interacts. This does not seem to us to be the case, but rather that the final states of the three free particles are characterized solely by the momenta and spins of the outgoing particles.

The purpose of this paper has been to show that the electric dipole transition operator and the final-state wave functions, constructed in terms of interacting pairs, emphasize certain regions of the bound-state wave function. We have seen that the choice of simple asymptotic forms of the bound state in these regions lead to transition amplitudes that give reasonable estimates of both the two- and three-body photodisintegration cross sections. The true symmetric s-wave bound state will differ from our approximate form in regions of nucleon interaction not tested by the low-energy photodisintegration process and possibly by correlation effects among the three particles or other intrinsic three-body effects, but the forms of Eq. (3) seem to be valid in the region tested.

APPENDIX

We give here some useful relations between three-body coordinates, momenta, and spin functions.

Coordinates

In general, the positions of three particles are specified by nine coordinates. A useful decomposition is to use three of the coordinates to give the center of mass of the three particles, and three Euler angles to locate the plane passing through the three particles, leaving three internal variables. These last may be taken as either the interparticle coordinates r_{12}, r_{23}, r_{31}, or the particle coordinates as measured from the center of mass, r_1, r_2, r_3. Another useful set (because, unlike the first two sets, it does not have the implied delta function among the vectors) is one of the set $(r, r', \hat{r}\cdot\hat{r}')$, $(s, s', \hat{s}\cdot\hat{s}')$, $(t, t', \hat{t}\cdot\hat{t}')$. The following relationships hold among the coordinates:

$$\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j\,,$$

$$\mathbf{r} = \mathbf{r}_{12}\,,\quad \mathbf{s} = \mathbf{r}_{23}\,,\quad \mathbf{t} = \mathbf{r}_{31}\,,$$

$$\mathbf{r}' = \tfrac{3}{2}\mathbf{r}_3\,,\quad \mathbf{s}' = \tfrac{3}{2}\mathbf{r}_1\,,\quad \mathbf{t}' = \tfrac{3}{2}\mathbf{r}_2\,,$$

$$r_1^2 + r_2^2 + r_3^2 = \tfrac{1}{3}(r_{12}^2 + r_{23}^2 + r_{31}^2) = \tfrac{1}{2}r^2 + \tfrac{2}{3}r'^2.$$

[10] V. N. Fetisov, Phys. Letters **21**, 52 (1966).

[11] R. H. Dalitz and T. W. Thacker, Phys. Rev. Letters **15**, 204 (1965).

[12] V. N. Fetisov, Nucl. Phys. **A98**, 437 (1967).

[13] V. N. Fetisov, J. Nucl. Phys. (U.S.S.R.) **4**, 720 (1966) [English transl.: Soviet J. Nucl. Phys. **4**, 513 (1967)].

The coordinates have the following properties under interchange of the like particles 1 and 2:

$$P_{12}\mathbf{r}=-\mathbf{r}, \quad P_{12}\mathbf{r}'=\mathbf{r}';$$
$$P_{12}\mathbf{s}=-\mathbf{t}, \quad P_{12}\mathbf{s}'=\mathbf{t}';$$
$$P_{12}\mathbf{t}=-\mathbf{s}, \quad P_{12}\mathbf{t}'=\mathbf{s}'.$$

Momenta

Let $\hbar\mathbf{k}_1, \hbar\mathbf{k}_2, \hbar\mathbf{k}_3$ be the momenta of the three particles relative to the center-of-mass system with the condition $\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3=0$. We then define

$$\mathbf{k}_{ij}=\tfrac{1}{2}(\mathbf{k}_i-\mathbf{k}_j),$$
$$\mathbf{p}_i=\mathbf{k}_i.$$

These momenta can be expressed in terms of the energy of the odd particle $W=\hbar^2p_3^2/2m$, the total energy available for breakup E, and ξ the cosine of the angle between $\mathbf{p}_3$ and $\mathbf{k}_{12}$:

$$k_{12}^2=\frac{m}{2\hbar^2}(2E-3W),$$

$$p_1^2=\frac{m}{\hbar^2}\{E-W-\xi[W(2E-3W)]^{1/2}\},$$

$$k_{23}^2=\frac{m}{\hbar^2}\{\tfrac{1}{4}E+\tfrac{3}{4}W+\tfrac{3}{4}\xi[W(2E-3W)]^{1/2}\},$$

$$p_2^2=\frac{m}{\hbar^2}\{E-W+\xi[W(2E-3W)]^{1/2}\},$$

$$k_{31}^2=\frac{m}{\hbar^2}\{\tfrac{1}{4}E+\tfrac{3}{4}W-\tfrac{3}{4}\xi[W(2E-3W)]^{1/2}\}.$$

The three-particle phase-space density can be expressed in terms of E and W by means of the Jacobian

$$dkdp=\tfrac{1}{2}m\frac{dWdE}{[W(2E-3W)]^{1/2}}.$$

Plane waves in all three particles can be expressed in one of the following equivalent forms:

$$e^{i(\mathbf{k}_1\cdot\mathbf{r}_1+\mathbf{k}_2\cdot\mathbf{r}_2+\mathbf{k}_3\cdot\mathbf{r}_3)}=e^{i(\mathbf{k}_{12}\cdot\mathbf{r}+\mathbf{p}_3\cdot\mathbf{r}')}$$
$$=e^{i(\mathbf{k}_{23}\cdot\mathbf{s}+\mathbf{p}_1\cdot\mathbf{s}')}=e^{i(\mathbf{k}_{31}\cdot\mathbf{t}+\mathbf{p}_2\cdot\mathbf{t}')}.$$

Spins

$\chi_s(12,3)$, $\chi_t(12,3)$ are total spin-$\tfrac{1}{2}$ eigenfunctions with the spins of the nucleon pair 12 coupled, respectively, to a singlet (antisymmetry in the pair 12), to a triplet (symmetry in the pair 12). Specifically, for $s_z=+\tfrac{1}{2}$,

$$\chi_s(12,3)=\frac{1}{\sqrt{2}}[\alpha(1)\beta(2)-\beta(1)\alpha(2)]\alpha(3),$$

$$\chi_t(12,3)=\frac{1}{\sqrt{6}}[\alpha(1)\beta(2)+\beta(1)\alpha(2)]\alpha(3) - (\sqrt{\tfrac{2}{3}})\alpha(1)\alpha(2)\beta(3),$$

where α means spin up, β spin down.

The following spin matrix elements are used in the photodisintegration calculation:

$$\langle\chi_s(12,3)|\chi_s(12,3)\rangle=1,$$
$$\langle\chi_s(23,1)|\chi_s(12,3)\rangle=-\tfrac{1}{2},$$
$$\langle\chi_t(23,1)|\chi_s(12,3)\rangle=-\tfrac{1}{2}\sqrt{3},$$
$$\langle\chi_s(31,2)|\chi_s(12,3)\rangle=-\tfrac{1}{2},$$
$$\langle\chi_t(31,2)|\chi_s(12,3)\rangle=\tfrac{1}{2}\sqrt{3},$$

where the subscript s means the two nucleons are in the (antisymmetric) singlet state and the subscript t means they are in the (symmetric) triplet.