Highly Polarized Neutrons from the s-Wave $T(d,n)^4He$ and $D(t,n)^4He$ Reactions*

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Formulas are presented in which the polarization of neutrons from the s-wave $T(d,n)^4He$ and $D(t,n)^4He$ reactions are expressed in terms of the projectile polarization and two real parameters which characterize the reaction. Neutron polarization limits are calculated from the presently available information about the range of possible values of these parameters. Several experiments are suggested by which these parameters can be determined. When this information is available, approximately 14-MeV neutrons with a large and accurately known polarization can be produced with the aid of a polarized ion source. The advantages, for this application, of a polarized ion source which can select a single nuclear substate are pointed out. The construction of a polarized neutron generator with useful intensities and large polarization appears to be feasible at the present time.

INTRODUCTION

It was first noted by Galonsky, Willard, and Welton[1] and by Goldfarb[2] that the s-wave $T(d,n)^4He$ reaction could be used as a tensor-polarization analyzer for low-energy deuterons, and that the neutrons which result from the reaction are in fact polarized.[3] These facts come about because a $J=\frac{3}{2}^+$ resonance occurs in the ^5He system only slightly above the $d+t$ mass; there results a total cross section for the $T(d,n)^4He$ reaction which reaches a maximum of about 5 b at a deuteron bombarding energy of only 107 keV. If it is assumed that the reaction proceeds wholly through the $J=\frac{3}{2}^+$ compound system, and that only incident s waves are involved, parameter-free expressions for the neutron angular distribution and polarization can be derived.[4] (These are, in fact, special cases of the formulas given below.)

Two recent developments have motivated the present discussion of the s-wave $T(d,n)^4He$ and $D(t,n)^4He$ reactions. First, it has been shown[5,6] that these reactions proceed partly through a $J=\frac{1}{2}^+$ compound system. Secondly, the rapid development of polarized-ion-source technology makes it appear possible that a practical source of highly polarized 14-MeV neutrons can be realized at the present time.

As will be shown in the following, 100% polarized neutrons may be obtained from a low-energy beam of deuterons 100% in the $m_I=1$ (or $m_I=-1$) state. The current developmental work at Los Alamos on a "metastable" type polarized negative ion source[7–10] is particularly interesting in this connection since the rf transition method which is planned for this source selects a single (arbitrarily chosen) deuteron spin state. The choice of $m_I=0$ deuterons results in unpolarized neutrons; this possibility should be a useful experimental feature. The beam from this source is expected to have an emittance and intensity competitive with conventional polarized positive ion sources.

A proposal by von Ehrenstein, Hess, and Clausnitzer,[11] in which deuterium atoms are passed through two inhomogeneous magnets and two rf transition regions, could also produce a pure $m_I=1$, 0, or -1 deuteron beam. The "flop in" type source of Hughes *et al.*[12] can also produce such a beam, but with low intensity.

CALCULATION

The cross-section and polarization expressions can be conveniently calculated from a general expression[13] in terms of the incoming and outgoing tensor moments;

$$T(q\bar{q})=\left(\frac{1}{2k'}\right)^2\frac{1}{2I+1}\left(\frac{2i+1}{2i'+1}\right)^{1/2}\sum(-1)^{I+I'-i-i'-s_2-s_2'}W(is_1is_2;Iq)W(i's_1'i's_2';I'q')$$
$$\times G_{\bar{q}}(J_1l_1s_1;Lq;J_2l_2s_2)G_{\bar{q}'}(J_1l_1's_1';Lq';J_2l_2's_2')\tfrac{1}{2}[R_1^*R_2+(-1)^{q+q'}R_2^*R_1]d_{\bar{q}'\bar{q}}{}^L(\theta)\exp(-i\bar{q}'\phi)\,T(q'\bar{q}'),\quad(1)$$

where the summation is over l_1', l_2', s_1', s_2', $J\pi_1$, $J\pi_2$, s_1, s_2, l_1, l_2, q', $\bar{q}'$, and L. This differs from the expression given in Ref. 13 only in that the normalization of the tensor moments differs by a factor $(2q+1)^{1/2}$. This change was made in order to conform with the now con-

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[1] A. Galonsky, H. B. Willard, and T. A. Welton, Phys. Rev. Letters 2, 349 (1959).

[2] L. J. B. Goldfarb, Nucl. Phys. 12, 657 (1959).

[3] As is well known, the low-energy $T(d,n)^4He$ and (mirror) $^3He(d,p)^4He$ reactions have similar properties. For definiteness, we will refer only to the $T(d,n)^4He$ reaction in this paper.

[4] An expression for the neutron angular distribution with a sign error on the term involving $T(21)$ has been used in several early experiments. See Ref. 6.

[5] L. Brown, H. A. Christ, and H. Rudin, Nucl. Phys. 79, 459 (1966).

[6] L. C. McIntyre and W. Haeberli, Nucl. Phys. A91, 369 (1967).

[7] G. G. Ohlsen and J. L. McKibben, Bull. Am. Phys. Soc. 12, 131 (1967); G. G. Ohlsen and J. L. McKibben, Los Alamos Scientific Laboratory Report No. LA-3725, 1967 (unpublished).

[8] G. P. Lawrence, J. L. McKibben, H. L. Daley, G. G. Ohlsen, and R. R. Stevens, Jr., Bull. Am. Phys. Soc. 12, 463 (1967).

[9] H. L. Daley, G. P. Lawrence, and J. L. McKibben, Bull. Am. Phys. Soc. 12, 463 (1967).

[10] J. L. McKibben, G. G. Ohlsen, and R. R. Stevens, Jr., Bull. Am. Phys. Soc. 12, 463 (1967).

[11] D. von Ehrenstein, D. C. Hess, and G. Clausnitzer, Phys. Letters 19, 114 (1965).

[12] V. W. Hughes, C. W. Drake, Jr., D. C. Bonar, J. S. Greenberg, and G. F. Pieper, Helv. Phys. Acta, Suppl. VI, 89 (1961).

[13] T. A. Welton, *Fast Neutron Physics, Part* II, edited by J. B. Marion and J. L. Fowler (Interscience Publishers, Inc., New York, 1963), p. 1317.

TABLE I. Spin-$\frac{1}{2}$ tensors.

	Spin operator representation	Matrix representation
$T(00)$	1	$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$
$T(11)$	$-\sqrt{2}(S_x+iS_y)$	$-\sqrt{2}\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$
$T(10)$	$2S_z$	$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

ventional spin-1 tensors first defined by Lakin[14] and further elucidated by Satchler.[15] The angle functions are those tabulated by Satchler.[15] The primed variables refer to the initial system and the unprimed variables refer to the final system; l is the orbital angular momentum, s is the channel spin, and $J\pi$ is the spin and parity of the intermediate system. R_1 and R_2 are the matrix elements associated with l_1', s_1', $J\pi_1$, l_1, s_1 and l_2', s_2', $J\pi_2$, l_2, s_2, respectively. The G functions (defined in Ref. 13) are finite sums involving Clebsch-Gordan and Fano X coefficients while the W functions are Racah coefficients. This formula assumes that

(a) The outgoing particle tensor moments are described in a right-handed coordinate system whose z axis coincides with the outgoing direction $\mathbf{k}/|\mathbf{k}|$ and whose y axis is in the direction $\mathbf{k}'\times\mathbf{k}/|\mathbf{k}'\times\mathbf{k}|$.

(b) The initial tensor moments are described in a right-handed coordinate system whose z' axis coincides with the projectile direction $\mathbf{k}'/|\mathbf{k}'|$ and whose y' axis is arbitrary.

(c) The angle θ and ϕ are the Euler angles α and β through which the initial coordinate system must be rotated in order to obtain the final coordinate system. Thus θ and ϕ are the usual polar and azimuthal scattering angles.

For reference, the spin-$\frac{1}{2}$ and spin-1 tensors as used here are given in Tables I and II. The general relation $T(q-\bar{q})=(-1)^{\bar{q}}T(q\bar{q})^{\dagger}$ can be used to find the tensors not explicitly given. (The dagger means complex conjugate for the spin-operator representation, and Hermitean conjugate for the matrix representation.)

For incident s waves, there are two matrix elements involved in the description of the reaction. We denote the matrix element for the channel spin-$\frac{3}{2}$ contribution by $R_{3/2}$ and for the channel spin-$\frac{1}{2}$ contribution by $R_{1/2}$.

TABLE II. Spin-1 tensors.

	Spin operator representation	Matrix representation
$T(00)$	1	$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$
$T(11)$	$-(\surd\frac{3}{2})(S_x+iS_y)$	$(\surd\frac{3}{2})\begin{pmatrix}0 & -1 & 0\\ 0 & 0 & -1\\ 0 & 0 & 0\end{pmatrix}$
$T(10)$	S_z	$(\surd\frac{3}{2})\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$
$T(22)$	$\frac{1}{2}\sqrt{3}(S_x+iS_y)^2$	$\sqrt{3}\begin{pmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$
$T(21)$	$-\frac{1}{2}\sqrt{3}(S_z[S_x+iS_y]+[S_x+iS_y]S_z)$	$(\surd\frac{3}{2})\begin{pmatrix}0 & -1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}$
$T(22)$	$(\surd\frac{1}{2})(3S_z^2-2)$	$(\surd\frac{1}{2})\begin{pmatrix}1 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & 1\end{pmatrix}$

Three parameters then enter the theory: $|R_{1/2}|$, $|R_{3/2}|$, and the difference in phase (denoted by δ) between $R_{1/2}$ and $R_{3/2}$. In order to write the expressions for cross section and polarizations more compactly, we define the quantities:

$$A=\frac{|R_{3/2}|^2}{12(k')^2},\quad a=\frac{|R_{1/2}|^2}{|R_{3/2}|^2},$$

$$b=\frac{R_{1/2}R_{3/2}^*+R_{1/2}^*R_{3/2}}{|R_{3/2}|^2}\equiv\frac{2\,\mathrm{Re}(R_{1/2}R_{3/2}^*)}{|R_{3/2}|^2}\equiv\frac{2|R_{1/2}|}{|R_{3/2}|}\cos\delta,\qquad(2)$$

$$ic=\frac{R_{1/2}R_{3/2}^*-R_{1/2}^*R_{3/2}}{|R_{3/2}|^2}\equiv\frac{2i\,\mathrm{Im}(R_{1/2}R_{3/2}^*)}{|R_{3/2}|^2}\equiv\frac{2i|R_{1/2}|}{|R_{3/2}|}\sin\delta.$$

Note that the ratios a, b, and c satisfy the relation $c=\pm(4a^2-b^2)^{1/2}$. The parameter A determines the total reaction cross section; we may choose $|R_{1/2}|/|R_{3/2}|$ and δ as the two real parameters which characterize the relative angular distribution and the polarization.

For a deuteron beam with the most general possible polarization, the resulting polarization and cross-section expressions are[16]

$$(1/A)(d\sigma/d\Omega)\langle\sigma_x\rangle=\sqrt{3}c[-\sin\theta\,\mathrm{Im}(e^{-2i\phi}T(22))+\cos\theta\,\mathrm{Im}(e^{-i\phi}T(21))]-(\surd\tfrac{2}{3})(a+\tfrac{1}{2}b-2)[\sqrt{2}\cos\theta\,\mathrm{Re}(e^{-i\phi}T(11))+\sin\theta T(10)],$$

$$(1/A)(d\sigma/d\Omega)\langle\sigma_y\rangle=\sqrt{3}c[\sin\theta\cos\theta\,\mathrm{Re}(e^{-2i\phi}T(22))-(2\cos^2\theta-1)\,\mathrm{Re}(e^{-i\phi}T(21))-(\surd\tfrac{3}{2})\sin\theta\cos\theta T(20)]+(\surd\tfrac{2}{3})(a+\tfrac{1}{2}b-2)[-\sqrt{2}\,\mathrm{Im}(e^{-i\phi}T(11))],\qquad(3)$$

$$(1/A)(d\sigma/d\Omega)\langle\sigma_z\rangle=(\surd\tfrac{2}{3})(a-b+1)[-\sqrt{2}\sin\theta\,\mathrm{Re}(e^{-i\phi}T(11))+\cos\theta T(10)],$$

$$(1/A)(d\sigma/d\Omega)=(a+2)+(b+1)[-\sqrt{3}\sin^2\theta\,\mathrm{Re}(e^{-2i\phi}T(22))+2\sqrt{3}\sin\theta\cos\theta\,\mathrm{Re}(e^{-i\phi}T(21))-\tfrac{1}{2}\sqrt{2}(3\cos^2\theta-1)T(20)].$$

[14] W. Lakin, Phys. Rev. **98**, 139 (1955).
[15] G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-2861, 1960 (unpublished).
[16] The coefficients in these expressions have been evaluated by computer techniques. G. G. Ohlsen and P. G. Young (to be published).

The spin-1 quantities $T(22)$, $T(21)$, etc., refer to expectation values of the relevant tensor; the spin-$\frac{1}{2}$ quantities have in all cases been expressed in terms of the expectation values of the Pauli matrices.

For a polarized deuteron beam produced in a nuclear scattering or reaction, $T(22)$, $T(21)$, $T(20)$, and $iT(11)$ are real if the y axis is chosen perpendicular to the reaction plane,[15] and the formulas accordingly simplify. However, for the present application, we need only consider the still simpler case of deuteron beams whose polarization can be described in terms of a single axis of symmetry. Since the $T(d,n)^4He$ reaction is assumed to involve incident s waves only, we may, without loss of generality, choose this quantization axis as our z axis. (Note that the quantization axis may or may not correspond to the direction of the deuteron velocity.) With this choice, only $T(10)$ and $T(20)$ are nonzero. The expressions given above then greatly simplify to

$$\begin{aligned}
(1/A)(d\sigma/d\Omega)\langle\sigma_x\rangle &= -(\sqrt{\tfrac{2}{3}})(a+\tfrac{1}{2}b-2)\sin\theta_s T(10)\,,\\
(1/A)(d\sigma/d\Omega)\langle\sigma_y\rangle &= -\tfrac{3}{2}\sqrt{2}c\sin\theta_s\cos\theta_s T(20)\,,\\
(1/A)(d\sigma/d\Omega)\langle\sigma_z\rangle &= (\sqrt{\tfrac{2}{3}})(a-b+1)\cos\theta_s T(10)\,,\\
(1/A)(d\sigma/d\Omega) &= (a+2)-(b+1)\\
&\quad\times[\tfrac{1}{2}\sqrt{2}(3\cos^2\theta_s-1)T(20)]\,,
\end{aligned}\tag{4}$$

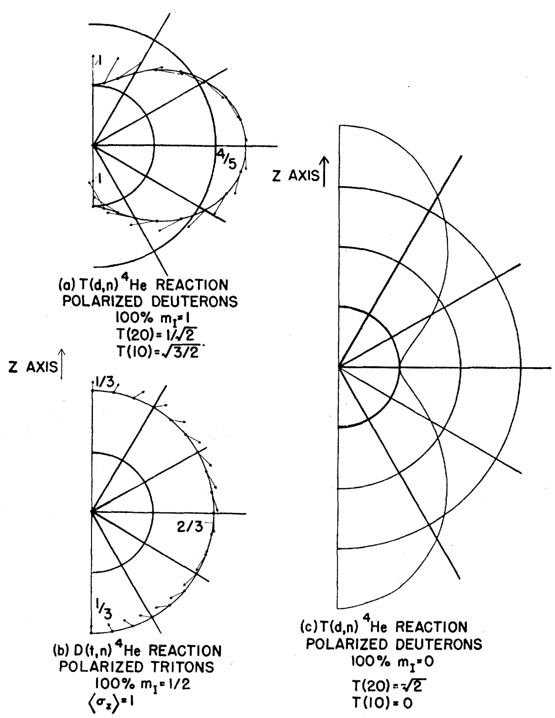

FIG. 1. Polar graphs of the yield and polarization of neutrons from the s-wave $T(d,n)^4He$ and $D(t,n)^4He$ reactions. The axis of quantization (z axis) is indicated. The assumed polarization of the incident beam is indicated for each graph. These curves correspond to a pure $J=\frac{3}{2}^+$ compound system; i.e., to $a=b=c=0$. The vectors have a length proportional to the neutron polarization; a few numerical values are indicated. The second semicircle corresponds to the unpolarized cross section.

where θ_s represents the angle between the outgoing neutron and the quantization axis.

In the case of a polarized triton beam, the general expressions may be written

$$\begin{aligned}
(1/A)(d\sigma/d\Omega)\langle\sigma_x\rangle &= \tfrac{1}{3}(a-2b+4)\\
&\quad\times[-\cos\theta(\langle\sigma_x\rangle\cos\phi+\langle\sigma_y\rangle\sin\phi)\\
&\qquad+\sin\theta\langle\sigma_z\rangle]\,,\\
(1/A)(d\sigma/d\Omega)\langle\sigma_y\rangle &= \tfrac{1}{3}(a-2b+4)\\
&\quad\times[\langle\sigma_x\rangle\sin\phi-\langle\sigma_y\rangle\cos\phi]\,,\\
(1/A)(d\sigma/d\Omega)\langle\sigma_z\rangle &= -\tfrac{1}{3}(a+4b-2)\\
&\quad\times[\sin\theta(\langle\sigma_x\rangle\cos\phi+\langle\sigma_y\rangle\sin\phi)\\
&\qquad+\cos\theta\langle\sigma_z\rangle]\,,\\
(1/A)(d\sigma/d\Omega) &= a+2\,.
\end{aligned}\tag{5}$$

If the z axis is again chosen to lie along the axis of quantization, we obtain

$$\begin{aligned}
(1/A)(d\sigma/d\Omega)\langle\sigma_x\rangle &= \tfrac{1}{3}(a-2b+4)\sin\theta_s\langle\sigma_z\rangle\,,\\
(1/A)(d\sigma/d\Omega)\langle\sigma_y\rangle &= 0\,,\\
(1/A)(d\sigma/d\Omega)\langle\sigma_z\rangle &= -\tfrac{1}{3}(a+4b-2)\cos\theta_s\langle\sigma_z\rangle\,,\\
(1/A)(d\sigma/d\Omega) &= a+2\,,
\end{aligned}\tag{6}$$

where θ_s is defined as before. In this case, the simpler formulas can describe the most general case, since spin-$\frac{1}{2}$ polarization can always be described in terms of a single axis. Note that no neutron-polarization component perpendicular to the "spin plane" is possible in the polarized-triton-beam case. (We shall use "spin plane" to mean that plane which contains the projectile quantization axis and the outgoing neutron direction.) For a polarized deuteron beam, the neutron-polarization component perpendicular to the spin plane is related to the components parallel to the spin plane through the equation $c=\pm(4a^2-b^2)^{1/2}$. Even in this case, however, the perpendicular component vanishes at $\theta_s=0°$, 90°, and 180°.

GENERAL REMARKS

In Fig. 1 the general behavior of the polarization and cross section is shown, in the form of a polar graph, for each of three cases:

(a) incident deuterons, all with $m_I=1$;
(b) incident tritons, all with $m_I=\frac{1}{2}$;
(c) incident deuterons, all with $m_I=0$.

The magnitude and direction of the neutron polarization is indicated by the vectors. In these graphs the (relatively small) effects of the $J=\frac{1}{2}^+$ channel are neglected; i.e., it is assumed that $a=b=c=0$. Note that the values of the incident-beam polarization tensors are quoted on the graph and that the 0°, 90°, and 180° neutron polarization values are given. Substitution of $m_I=-1$ for $m_I=1$ merely reverses the direction of all neutron polarization vectors. It is again emphasized that all angles are measured with respect to the quantization axis. Except for kinematic effects, to be discussed later,

TABLE III. 0° and 90° polarization and intensity for various deuteron spin populations [for $a=b=c=0$].

Method of production	Population $m_I=1$	$m_I=0$	$m_I=-1$	$\sqrt{2}T(20)$	$(\sqrt{\frac{2}{3}})T(10)$	Neutron polarization P_n 0°	90°	Relative yield per incident deuteron 0°	90°
$2\rightarrow 6$ rf transition	$\frac{2}{3}$	0	$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{1}{3}$	4/15	1	$\frac{5}{2}$
$3\rightarrow 5$ rf transition	$\frac{1}{3}$	$\frac{2}{3}$	0	-1	$\frac{1}{3}$	$\frac{1}{9}$	4/9	3	$\frac{3}{2}$
$3\rightarrow 6$ rf transition	$\frac{2}{3}$	$\frac{1}{3}$	0	0	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	2	2
$\pm m_F\rightarrow \pm m_F$ weak-field low-frequency transition	0	$\frac{1}{3}$	$\frac{2}{3}$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	2	2
Adiabatic reduction of magnetic field to zero	4/9	4/9	$\frac{1}{9}$	$-\frac{1}{3}$	$\frac{1}{3}$	1/7	12/33	7/3	11/6
Select $m_I=1$	1	0	0	1	1	1	$\frac{4}{5}$	1	$\frac{5}{2}$
Select $m_I=0$	0	1	0	1	0	0	0	4	1
Select $m_I=-1$	0	0	1	1	-1	-1	$-\frac{4}{5}$	1	$\frac{5}{2}$

the projectile direction has no significance in an s-wave reaction.

For a polarized ion source which can produce only mixtures of the cases illustrated, the neutron polarization and yield can be obtained by a suitable average of the quantities shown on the graphs. (The neutron polarization direction is unaffected by the magnitude of the projectile polarization; also, the same directions are involved for polarized deuteron or triton beams.) In Table III the ideal neutron polarization and yield expected for various types of deuteron polarization are given. In this table the reaction is again assumed to proceed wholly through a $J=\frac{3}{2}^+$ intermediate system. The notation describing the method of production is that of, for example, Ref. 11.

Ideally, one would like to produce a beam of ~14 MeV neutrons with a precisely known polarization. To do this, in general, one needs to know the polarization of the deuteron or triton beam, and the values of the parameters a and b.

At $\theta_s=0°$ or 180° and for a beam of deuterons 100% in the $m_I=1$ state, the neutron polarization (P_n) is 100% regardless of the values of a and b. However, as may be seen from Fig. 2, a slight dependence enters if the deuteron polarization falls below this ideal value. In Fig. 2 it is assumed that the beam polarization is such that the $m_I=-1$ and $m_I=0$ population will be equal; i.e., $T(20)=\sqrt{3}T(10)$. Such a deuteron beam may be thought of as a combination of a pure $m_I=1$ component (fraction f) and an unpolarized component (fraction $1-f$). Because of the relatively large cross section for the $m_I=0$ contribution at $\theta_s=0°$ or 180°, the neutron polarization decreases rapidly as the deuteron polarization decreases; thus, in this case, the deuteron polarization would need to be accurately known. Finally, the neutron polarization is longitudinal and would need to be rotated with a transverse magnetic field spin-precession device.

At $\theta_s=90°$ and for a beam of deuterons 100% in the $m_I=1$ state, the neutron polarization (which is now transverse to the neutron direction and in the reaction plane) depends strongly on the parameters a and b (see Fig. 2). The dependence on the beam polarization is much less rapid than is the case at 0° or 180°. At the zeros of $P_2(\cos\theta_s)$ (54.7° and 125.3°) the neutron polarization depends linearly on the deuteron polarization.

If polarized tritons are used (see Fig. 3), the neutron polarization depends linearly on the beam polarization at all angles; further, there is sensitivity to the parameters a and b at all angles.

MEASUREMENTS

To make the best use of the $T(d,n)^4He$ reaction as a polarized neutron source, one needs to know the projectile polarization and the parameters a and b. Many independent measurements which determine some combination of the quantities of interest are possible. The following is a description of some of the measurements that could be made. Although extreme angles such as 0° and 90° are usually used in the discussion it is clear that similar expressions can be written for arbitrary

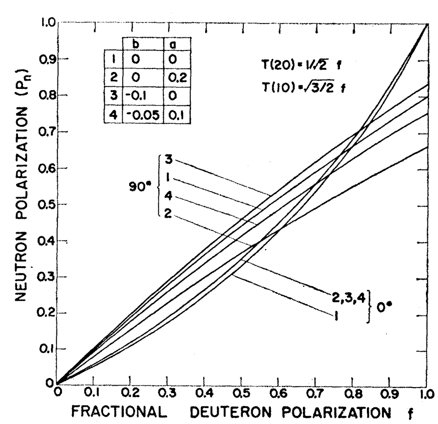

FIG. 2. Graph of the s-wave $T(d,n)^4He$ reaction neutron polarization versus deuteron polarization at 0° and 90°. The values of the parameters are chosen so that $g=(1+b)/(1+\frac{1}{2}a)\approx 0.9$. The parameter f is the fraction of the incident deuterons which is 100% polarized in the $m_I=1$ or -1 state; the fraction $1-f$ is unpolarized background.

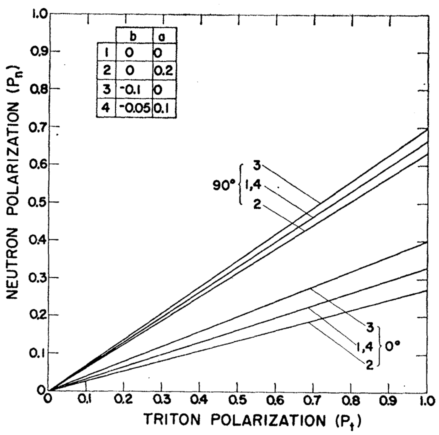

FIG. 3. Graph of T$(d,n)^4$He reaction neutron polarization versus triton polarizations at 0° and 90°. The values of the parameters are as in Fig. 3.

angles. It is also clear that $\theta_s=180°$ can be substituted for $\theta_s=0°$ in the discussion.

(a) Measurement of the $\theta_s=0°$ to $\theta_s=90°$ yield rates. We have, for polarized deuterons:

$$\frac{\sigma(0°)}{\sigma(90°)}=\frac{1-gT(20)/\sqrt{2}}{1+gT(20)/2\sqrt{2}}, \quad g=\frac{1+b}{1+\frac{1}{2}a}. \qquad (7)$$

It is known[5] that g has the approximate value 0.9. The values of a and b chosen for illustration in Figs. 2 and 3 are consistent with this known fact; thus, the difference between the extreme curves may be taken as an indication of the present neutron polarization uncertainty. Note that there is no *a priori* reason why g must be less than unity. This is somewhat surprising since one might naively suppose that the effect of a $J=\frac{1}{2}^+$ contribution would be to make the over-all angular distribution more isotropic.

(b) Measurement of the angle between the outgoing neutron polarization and its velocity. This quantity is independent of the projectile polarization. For polarized deuterons, we obtain

$$\tan\theta_{\text{spin}}=\frac{\langle\sigma_x\rangle}{\langle\sigma_y\rangle}=-\left(\frac{a+\frac{1}{2}b-2}{a-b+1}\right)\tan\theta_s. \qquad (8)$$

For polarized tritons, we obtain

$$\tan\theta_{\text{spin}}=\frac{\langle\sigma_x\rangle}{\langle\sigma_y\rangle}=-\left(\frac{a-2b+4}{a+4b-2}\right)\tan\theta_s. \qquad (9)$$

For the expected range of values for the parameters, the spin directions will be altered by as much as 5°–10° with respect to the directions shown in Fig. 1. Such an experiment would make use of an accurately calibrated neutron spin-precession device and a neutron polarimeter. Only relative (transverse) polarization would need to be determined by the neutron polarimeter.

(c) Measurement of the out-of-plane neutron polarization. For a polarized deuteron beam, the out-of-plane neutron polarization is given by

$$\langle\sigma_y\rangle=\frac{-\frac{3}{2}\sqrt{2}c\sin\theta_s\cos\theta_s T(20)}{(a+2)[1-\frac{1}{4}g\sqrt{2}(3\cos^2\theta_s-1)T(20)]}. \qquad (10)$$

The parameter c could possibly be best determined by measuring a ratio of $\langle\sigma_y\rangle$ to $\langle\sigma_x\rangle$ or $\langle\sigma_z\rangle$, using a neutron spin-precession device where necessary.

(d) Measurement of the in-plane neutron polarization. The polarization of the $\theta_s=0°$ neutrons could be measured absolutely, with, for example, n-^{4}He scattering. For an $m_I=\pm1$ deuteron beam, $P_n(0°)$ depends only slightly on the parameters a and b; thus, measurement of this quantity would essentially constitute a measurement of the deuteron polarization. The polarization of the $\theta_s=90°$ neutrons for a deuteron beam (or the neutrons at any angle for a triton beam) depends strongly on a and b. [A measurement of $P_n(90°)$ for a deuteron beam with a relatively low vector polarization has been reported.[17] The accuracy of this measurement is not sufficiently high to distinguish between the values of the parameters a and b which are consistent with the approximately known value of g.]

A measurement of the 0°–90° neutron-polarization ratio could be made without an absolute polarization measurement. For a polarized deuteron beam, this would determine

$$\frac{\langle\sigma_z(0°)\rangle}{\langle\sigma_x(90°)\rangle}=\left(\frac{a+\frac{1}{2}b-2}{a-b+1}\right)\left(\frac{1+gT(20)/2\sqrt{2}}{1-gT(20)/\sqrt{2}}\right), \qquad (11)$$

while for the polarized triton beam it would determine

$$\frac{\langle\sigma_z(0°)\rangle}{\langle\sigma_x(90°)\rangle}=\frac{a-2b+4}{a+4b-2}. \qquad (12)$$

In addition to the measurements proposed above, experiments unrelated to the reactions under discussion could be used to determine the projectile polarization. [As noted above, for deuteron projectiles, $P_n(0°)$ depends only slightly on the reaction parameters. Thus, if the deuteron polarization could be sufficiently well determined, it would not be necessary to determine a and b in order to obtain neutrons of well-known polarization.]

For the metastable source, it would appear likely that the beam polarization can be determined by atomic-physics techniques.[7] For any type of polarized ion source, the beam polarization can be determined by acceleration and scattering from a suitable analyzer. For deuterons, ^{4}He might be a useful analyzer, although

[17] F. Seiler, E. Baumgartner, W. Haerberli, P. Huber, and H. R. Striebel, Helv. Phys. Acta 35, 385 (1962).

some uncertainties remain.[18] For tritons, no calibrated analyzing reaction is presently available.

DISCUSSION

It should be remembered that the parameters a, b, and c are functions of energy. Since measurements as described above would almost certainly involve the use of a thick target, an energy average of the parameters would be measured unless "differential" bombarding energy techniques were used or unless neutron energies were accurately measured. However, the same averages enter into all the expressions, for a given projectile energy. For a target sufficiently thick to stop the projectile beam, these averages are of the form

$$u_{\mathrm{av}}=\int_{E_0}^{0}\frac{\sigma_0(E)u(E)dE}{\epsilon(E)}\bigg/\int_{E_0}^{0}\frac{\sigma_0(E)dE}{\epsilon(E)}, \qquad (13)$$

where $u=a$, b, or c, and where $\sigma_0(E)$ is the total cross section for the reaction, $\epsilon(E)$ is the relevant differential energy loss function, and E_0 is the center-of-mass (c.m.) bombarding energy. It is interesting to note that, since $\epsilon(E)$ is a function of projectile velocity, these expressions are independent of which particle is used as projectile. This is also true of the total yield function if the conversion from the c.m. to the laboratory system is neglected.

If the projectile energy is raised beyond 100 or 200 keV, the laboratory angle (θ_{lab}) of the neutrons begins to differ significantly from the center of mass angle (θ). (Recall that the axis defined by $\theta=0$ corresponds to the projectile direction and that the axis defined by $\theta_s=0$ corresponds to the quantization direction. Both of these systems are "c.m." coordinate systems.) If $\langle\sigma_x\rangle$, $\langle\sigma_y\rangle$, and $\langle\sigma_z\rangle$ describe the neutron polarization in the (θ) c.m. coordinate system, the outgoing polarization in the laboratory system is given by

$$\begin{aligned}\langle\sigma_x\rangle_{\mathrm{lab}}&=\cos(\theta-\theta_{\mathrm{lab}})\langle\sigma_x\rangle+\sin(\theta-\theta_{\mathrm{lab}})\langle\sigma_z\rangle,\\ \langle\sigma_y\rangle_{\mathrm{lab}}&=\langle\sigma_y\rangle,\\ \langle\sigma_z\rangle_{\mathrm{lab}}&=-\sin(\theta-\theta_{\mathrm{lab}})\langle\sigma_x\rangle+\cos(\theta-\theta_{\mathrm{lab}})\langle\sigma_z\rangle.\end{aligned} \qquad (14)$$

For example, in the case of 600-keV, $m_I=+1$ deuterons with the quantization axis parallel to the direction of motion, the neutron polarization becomes transverse at about 83.4° (lab) instead of at 90° (lab). Thus, a spread in bombarding energies, such as one obtains with a thick target, will result in a spread of spin directions at a given laboratory angle. This effect vanishes at $\theta=\theta_{\mathrm{lab}}=0°$ (or 180°); this is a possible significant argument in favor of using 0° (or 180°) neutrons rather than 90° neutrons. On the other hand, the energy spread of the neutrons emitted near 90° is small compared to the spread at small or large angles. It is clear that these effects would have to be accounted for in the extraction of a, b, and c from experiments of the type described above.

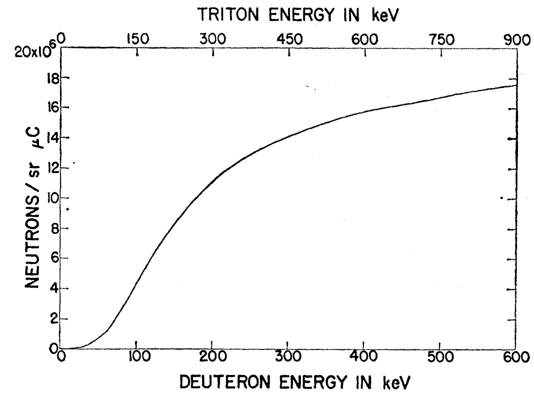

FIG. 4. Neutron yield per microcoulomb for unpolarized projectiles. The lower energy scale is for deuteron projectiles and the upper scale is for triton projectiles. This figure can be used in conjunction with Table III and Fig. 1 to estimate the yield from polarized projectiles. Conversion from the center-of-mass to the laboratory system is neglected.

It should be noted that the effect of a p-wave contribution will become significant for sufficiently high bombarding energies. However, for laboratory deuteron bombarding energies (E_d) less than ~1 MeV, the unpolarized c.m. cross-section ratio[19] $\sigma(180°)/\sigma(0°)$ varies approximately as $1-0.26E_d^2$. Thus, at 100 keV the maximum deviation from isotropy is ~0.25%. Since, as pointed out in Ref. 6, the same squared matrix elements and cross terms are involved in both the description of the polarization effects and the unpolarized cross section, effects of the same *order of magnitude* are expected in each case. However, it is clear that for the most accurate work, or for extension to energies higher than ~200 keV, this question would need to be seriously investigated. Experimental techniques similar to those suggested above in connection with the determination of a, b, and c could probably be utilized.

Finally, Fig. 4 shows a curve of neutron yield versus projectile energy for an unpolarized beam current of 1 μA and for a thick tritium-zirconium or deuterium-zirconium target.[19] This curve is approximate and does not take into account the small difference in yield at different angles with respect to the projectile direction. (This dependence arises solely from the c.m. to laboratory conversion and is, for example, less than ±6% for a deuteron energy of 600 keV.) For a beam of $m_I=\pm1$ deuterons with energy 150 keV and intensity 1 μA, a neutron flux of about 4.2×10^6/sr sec is obtained in the direction parallel to the deuteron quantization axis, and a flux of about 10.5×10^6/sr sec is obtained in the

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[19] J. D. Seagrave, Los Alamos Scientific Laboratory Report No. LAMS-2162, 1958 (unpublished).

perpendicular direction. For a polarized triton beam, a neutron flux of about 8.4×10^6/sec is obtained at any angle. Such fluxes compare very favorably with those available from the commonly used polarized neutron-source reactions. In addition, of course, polarization values attainable by the present technique are much higher. Yields could be greatly enhanced by the use of more sophisticated targets. For example, liquid-helium-cooled differentially pumped targets, D_2O ice, or T_2O ice targets might be considered. In addition, it is quite possible that polarized ion currents substantially higher than 1 μA will be realized in the next few years.

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Study of the (d,p) Reaction in the $1p$ Shell*

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Angular distributions for the (d,p) reaction leading to bound states in the $1p$ shell have been obtained at $E_d=12$ MeV for all stable targets. Spectroscopic factors obtained in distorted-wave Born-approximation (DWBA) analyses with average parameters are in surprisingly good agreement with those obtained in the shell-model calculations of Cohen and Kurath. The oscillating structure of the angular distributions at backward angles tends to be qualitatively reproduced by the DWBA calculations, although the amplitudes of the oscillations and the magnitudes of the backward cross sections are very sensitive to details of the calculations. J-dependent effects, similar to those found in heavier nuclei, but with some complications, are also found here.

I. INTRODUCTION

THE study of (d,p) reactions on light nuclei dates back to the early days of plane-wave stripping theory.[1] The (d,p) reactions on $1p$-shell nuclei were then studied[2] at several energies in the cyclotron energy range $\gtrsim 8$ MeV. The main purpose of these investigations was to assign the orbital angular momenta l of the transferred neutrons. Consequently, most of these early investigations were restricted to forward angles, since backward angles were not expected to contain any useful information. Similarly, many of the experiments reported only relative-cross sections, or rather poorly determined absolute ones, since the plane-wave Born-approximation (PWBA) stripping theories predict absolute yields which are too large by one to two orders of magnitude. The early work was summarized in the review article by Macfarlane and French.[3]

Since the introduction of the DWBA stripping theory, the interest has tended to shift to heavier nuclei, although a larger number of (d,p) and (d,n) experiments on light nuclei at $E_d \lesssim 6$ MeV have also been interpreted by the DWBA with varying degrees of success. There have been relatively few studies of (d,p) reactions on $1p$-shell nuclei at energies above 6 MeV,[4–11] and most of these were on Be^9, B^{10}, and C^{12} targets and led to ground states. As a consequence, the data were rather incomplete at these energies at which compound-nucleus effects might become relatively unimportant. This longtime neglect of the direct reactions on light nuclei probably reflects general misgivings regarding the applicability of optical-model potentials and the DWBA in light nuclei. It therefore seems somewhat

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