Polarization of Protons from the D(d,p)T Reaction*

L. E. PORTER† AND W. HAEBERLI
University of Wisconsin, Madison, Wisconsin
(Received 27 July 1967)

The polarization of protons from the D(d,p)T reaction has been measured for eight deuteron energies between 2.1 and 14.1 MeV. For each deuteron energy, observations were made at nine laboratory angles between 10° and 70°. Scattering from helium was used as the polarization analyzer. For energies above 6 MeV, the polarization is positive at all angles with a maximum near 45°. The highest polarization was observed at 12.1 MeV where the peak value is about 0.5. The angular dependence of the polarization was fitted in terms of a sum over associated Legendre functions. To assure a smooth dependence of the coefficients on energy, they were represented by a power series (energy-dependent analysis). A comparison of the present results with measurements of the neutron polarization in the mirror reaction D(d,n)He^3 indicates that the proton polarization is larger by a factor of about 1.4.

1. INTRODUCTION

THE characteristics of the d-D reaction were studied very early because the reaction was widely used as a neutron source. The analysis of the reaction cross section by Konopinski and Teller[1] in 1948 indicated the presence of p waves with strong spin-orbit coupling between the deuterons. This led Wolfenstein to suggest[2] that the nucleons from the d-D reaction may be polarized. The idea was of considerable interest at the time since it offered the possibility of producing polarized neutrons with much higher intensity than could have been obtained by Schwinger's earlier proposal[3] of scattering an initially unpolarized beam of neutrons from helium. Indeed the earliest experiments which demonstrated the polarization of neutrons and protons from a nuclear reaction were done for the D(d,n)He^3 reaction[4] and for the D(d,p)T reaction.[5] In both cases the bombarding energy was below 1 MeV. The polarization effects were small and difficult to detect.

In recent years much additional work has been done on the polarization of neutrons from the D(d,n)He^3 reaction for deuteron energies between about 0.1 MeV and 20 MeV. The results of these measurements have been reviewed most recently by Barschall.[6] Because of charge symmetry of nuclear forces, one may assume that the polarization of the protons from the d-D reaction is very similar to that of the neutrons. However, no measurements have been reported of the proton polarization above 1.4-MeV deuteron energy. In the present experiment measurements were carried out between 2 and 14 MeV for laboratory reaction angles from 10° to 70°. Measurements below 1.4 MeV have been reported.[7–9] Generally only one proton emission angle was studied in those experiments.

The deuteron beam from a tandem electrostatic accelerator was used to bombard a deuterium gas target. The polarization of the protons was measured by scattering from helium because the analyzing power for p-α scattering is known rather accurately.

The angular dependences of the cross section $\sigma(\theta)$ and of the product $P(\theta)\sigma(\theta)$ were fitted with Legendre polynomials. The results of the calculations are discussed in Sec. 6 and compared to measurements of the neutron polarization from the d-D reaction in Sec. 7.

2. APPARATUS

The double scattering apparatus used has been described previously.[10,11] Basically it consists of a collimator for the deuteron beam, a deuterium gas target consisting of a very thin-wall (~3 μm) stainless steel cylinder which the beam traverses perpendicular to the axis, and a slit system to select reaction protons at one particular reaction angle. After scattering from a helium cell the reaction protons were detected with identical counter telescopes located at symmetric scattering angles. Each telescope contained a proportional counter and a CsI(Tl) scintillation counter. The scintillator pulses coincident with proportional counter pulses were recorded in a multichannel pulse-height analyzer. The usual laboratory angle of scattering from helium was 45°. For some measurements involving protons of very low energy the scattering angle was 115°. In this case a second proportional counter replaced the scintillation counter in order to reduce the sensitivity to γ rays.

* Work supported in part by the U. S. Atomic Energy Commission.
† Now at the University of Montana, Missoula, Montana.
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For reaction angles less than 45° the equipment had to be modified because otherwise the helium cell would have been exposed to charged particles originating from the beam entrance and exit spots of the target cell wall. At reaction angles of 10° and 20° the D(d,p)T cross section is so large that this foil background proved acceptable, and background measurements were made with the deuterium cell evacuated in order to determine the contribution of charged particles from the cell foil. For angles between 25° and 45°, however, the background was unacceptably high. A modified deuterium target cell was constructed of tantalum tubing. The geometry was such that charged particles from the beam-illuminated entrance foil could not reach the scattering cell for this range of reaction angles. The beam entered through a narrow window, traversed the target gas, and was stopped by the tantalum cell. The reaction protons left the cell through a narrow exit window. In addition to decreased background, this target cell provided improved angular resolution in the interval of rapidly changing polarization. Details of the experimental arrangement are given elsewhere.[12]

3. UNCERTAINTIES

The uncertainties in reaction and scattering angles are 0.2° for measurements at reaction angles between 25° and 40°, and are otherwise 0.1°. The larger value resulted from a larger uncertainty in positioning the modified target cell in the chamber.

The uncertainty in deuteron energy at the center of the reaction cell was less than 50 keV and arose from uncertainties in incident beam energy and in energy losses in the target gas and gas-retaining foil. The uncertainty in the average proton energy at the center of the scattering cell was less than 60 keV and stemmed from the same sources. Because no experimental data was available the energy loss in the gas-retaining foil[13] was measured in a separate experiment.

Four sources contributed to the overall uncertainty in proton polarization. The first and largest uncertainty was from counting statistics. The second uncertainty was that in analyzing power, which resulted from the uncertainties assigned to the p-α polarization measurements[10,11,14] and from the uncertainty in proton energy. The third source was the uncertainty in geometrical corrections which will be discussed in Sec. 4. The final source of uncertainty arose from possible errors in the background subtraction. A short discussion of background problems follows.

The primary method of measuring background was to place a tantalum strip between the deuterium and helium cells. The strip was sufficiently thick to stop all charged particles from the d-D reaction, so that the resulting background was attributed to neutrons and γ rays. For four-fifths of the measurements the number of background counts was between 3 and 20% of the total number of counts. Pulse-height spectra illustrative of the best and worst cases appear in Fig. 1. In the worst case the background was 50% of the total number of counts.

If the background subtraction is reliable, one would expect the pulse-height spectrum remaining after the subtraction to show a roughly symmetric peak dropping to zero on both sides of the peak. This was usually the case. However, occasionally the background did not completely subtract out on the left-hand side of the peak. When this was the case a corresponding uncertainty was assigned to the number of background counts in addition to the statistical uncertainty.

Target gas impurities were negligible sources of error. The purity of the helium used was 99.99%. The corresponding number for deuterium was 99.7% isotopic purity with less than five parts per million of impurities other than hydrogen. The gas-handling procedures could have permitted no more than 0.01% and 0.04% air impurity in the helium and deuterium, respectively.

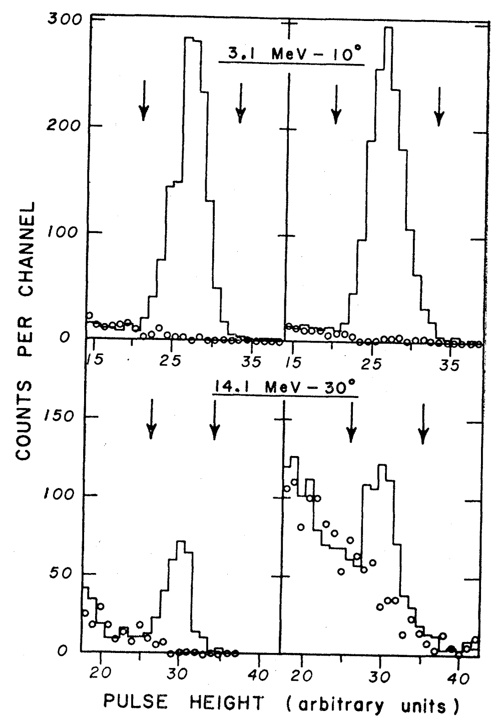

FIG. 1. Pulse-height spectra of D(d,p)T protons after scattering from a helium second target. The top spectra show the best, the bottom spectra the worst, background condition. The background (open circles) was generally measured by inserting a beam stop between first and second targets. Most data were similar to the top spectra. All counts in the pulse-height intervals marked by arrows were added.

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4. GEOMETRICAL CORRECTIONS

In an experiment performed with point targets and point detectors and parallel reaction and scattering planes the proton polarization P is related to the measured left-right ratio r by

$$P=\frac{1}{P_s}\left(\frac{r-1}{r+1}\right), \tag{1}$$

where P_s is the p-α analyzing power for protons of energy E_s and laboratory scattering angle θ_s. To obtain acceptable counting rates in a double scattering experiment one has to permit relatively large spreads in energy and angle. The effect of finite aperture size on the measurements can of course be calculated if the angular and energy dependence of polarization and cross section for both targets is known. In the present case, the cross sections for both targets[15–18] and the polarization P_s for the second scattering[10,11,14] are known from previous experiments. For the polarization P from the first target, approximate values were obtained by neglecting finite geometry effects. Using these functions, the left-right ratio r of the double scattering experiment was calculated by numerical integration of the double-scattered intensity over slit dimensions, taking into account that the energy of the particles depends on the position of the target element in the target cell. The computed value of r differed from the measured one by a small amount. One then finds how large a change δP has to be made to the first target polarization to obtain agreement between calculated and measured asymmetries. The resulting δP ranged from -0.04 to $+0.033$. The calculation also took into account the angular spread caused by multiple scattering in the target foil windows. The spread in energy because of straggling in the foil was negligible compared to the spread from the gas target thickness. The over-all uncertainty in the correction is about ± 0.01. The largest contribution arises from the uncertainty of the cross section and polarization input data, a smaller amount from the approximation made in the treatment of multiple scattering. The corrections were calculated explicitly for two-thirds of the measurements. For the remainder the correction was obtained by interpolation. Additional details are given in Ref. 12.

5. EXPERIMENTAL RESULTS

The results of the measurements are listed in Table I and are displayed in Fig. 2 as a function of center-of-

TABLE I. Polarization of protons from the D(d,p)T reaction.

E_d (MeV)	θ_{lab}[a]	10	20	25	30	35	40	50	60	70
2.08±0.03	θ	12.6	25.2	31.4	37.6	43.7	49.7	61.6		
	P	−0.011	−0.011	−0.043	−0.056	−0.010	−0.096	−0.106		
	ΔP	±0.020	±0.021	±0.025	±0.036	±0.035	±0.042	±0.053		
3.08±0.03	θ	13.0	26.0	32.4	38.8	45.1	51.3	63.5	75.3	
	P	+0.015	+0.014	−0.005	+0.076	+0.107	−0.069	−0.043	+0.050	
	ΔP	±0.022	±0.024	±0.027	±0.033	±0.057	±0.049	±0.046	±0.049	
4.09±0.02	θ	13.3	26.6	33.2	39.7	46.1	52.5	64.9	76.9	88.4
	P	+0.043	+0.074	+0.061	+0.068	+0.015	+0.046	−0.064	−0.006	+0.014
	ΔP	±0.022	±0.023	±0.028	±0.033	±0.035	±0.035	±0.038	±0.044	±0.041
6.09±0.02	θ	13.8	27.5	34.2	40.9	47.6	54.1	66.9	79.2	90.9
	P	+0.008	+0.040	+0.116	+0.186	+0.235	+0.191	+0.091	+0.041	−0.002
	ΔP	±0.029	±0.029	±0.024	±0.035	±0.036	±0.036	±0.031	±0.040	±0.037
8.09±0.02	θ	14.1	28.0	35.0	41.8	48.6	55.3	68.3	80.8	92.6
	P	+0.020	+0.128	+0.216	+0.365	+0.370	+0.239	+0.037	+0.027	+0.045
	ΔP	±0.018	±0.035	±0.035	±0.047	±0.035	±0.036	±0.031	±0.031	±0.035
10.09±0.02	θ	14.3	28.5	35.5	42.4	49.3	56.1	69.3	81.9	93.9
	P	−0.010	+0.117	+0.266	+0.455	+0.378	+0.302	+0.146	+0.121	−0.023
	ΔP	±0.024	±0.039	±0.037	±0.038	±0.040	±0.035	±0.042	±0.033	±0.043
12.09±0.02	θ	14.5	28.8	35.9	42.9	49.9	56.7	70.1	82.8	94.9
	P	+0.025	+0.150	+0.332	+0.571	+0.385	+0.249	+0.183	+0.031	−0.036
	ΔP	±0.030	±0.048	±0.043	±0.047	±0.042	±0.039	±0.041	±0.032	±0.032
14.10±0.02	θ	14.6	29.1	36.2	43.3	50.3	57.3	70.7	83.6	95.7
	P	+0.045	+0.251	+0.387	+0.441	+0.323	+0.205	+0.174	+0.077	−0.102
	ΔP	±0.036	±0.048	±0.050	±0.072	±0.072	±0.044	±0.043	±0.045	±0.049

[a] All angles are given in degrees. θ is the center-of-mass reaction angle.

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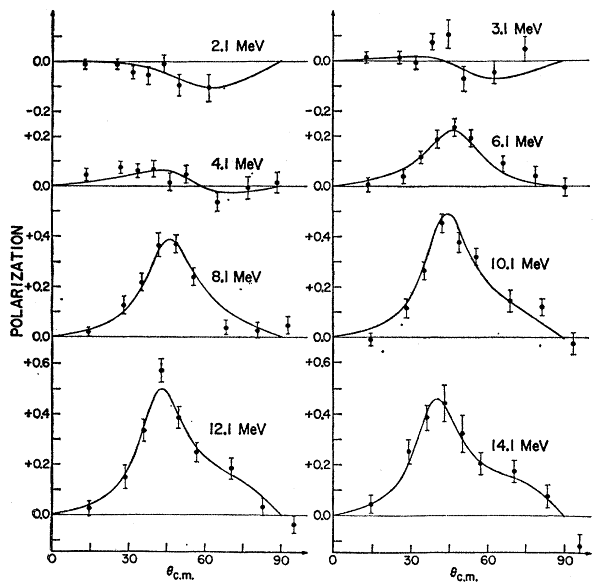

FIG. 2. Polarization of protons from the D(d,p)T reaction as a function of center-of-mass reaction angle. The curves are based on Eqs. (3) and (5) and the coefficients of Table II.

mass angle θ. The listed uncertainties ΔP contain all four of the contributions discussed in Sec. 3. The curves shown in Fig. 2 will be discussed in the following section.

6. ANALYSIS

The dependence of the center-of-mass differential cross section on center-of-mass reaction angle can in general be expressed by a sum of Legendre functions $P_k(\cos\theta)$:

$$\sigma(\theta)=\sum_{k=0}^{M} c_k P_k(\cos\theta). \qquad (2)$$

The largest value of k which has to be taken into consideration depends on the highest value of the angular momentum contributing to the reaction. According to the well known complexity rule,[19] M is no larger than either $2l_{\max}$, $2l_{\max}'$, or $2J_{\max}$, where $l_{\max}$ and $l_{\max}'$ are the largest values of incoming and outgoing orbital angular momentum present in the reaction, respectively, and $J_{\max}$ is the largest value of the total angular momentum of the compound nucleus. In the present reaction, because of the identity of target and incident particles, $c_k=0$ for odd k.

An expression corresponding to Eq. (2) can be written[20] for the product of the polarization $P(\theta)$ of the outgoing protons or neutrons and the center-of-mass differential cross section $\sigma(\theta)$:

$$\pi(\theta)=P(\theta)\sigma(\theta)=\sum_{k=1}^{N} a_k P_k'(\cos\theta), \qquad (3)$$

where the P_k' are the associated Legendre functions:

$$P_k'(\cos\theta)=-\frac{d}{d\theta}P_k(\cos\theta)=+\sin\theta\frac{dP_k(\cos\theta)}{d(\cos\theta)}. \qquad (4)$$

Again for the present reaction $a_k=0$ for odd k, and the upper limit on N is the same as that on M.

The results of the present polarization measurements can be summarized conveniently by giving the values of the coefficients a_k for each bombarding energy. Initially the best-fitting coefficients, a_k and c_k, were determined at each energy separately. Satisfactory fits to the polarization data required $N=4$ up to 10 MeV and $N=6$ at 14.1 MeV. For reasons which are not understood a good fit at 3.1 MeV could not be obtained with $N=4$. In view of the behavior at the neighboring energies the use of additional terms in the expansion seems unjustified even through this obviously would improve the fit.

Since the D(d,p)T cross section and polarization show no rapid variation with energy (i.e., no resonances) one would expect the coefficients to vary smoothly with energy. In contrast to this expectation, our coefficients a_k fluctuated considerably. When a compromise smooth curve was drawn through the points and $\pi(\theta)$ was recalculated a poor fit to the data resulted. Clearly the problem arises from the fact that the experimental errors in the data cause large (but correlated) errors in the coefficients. The problem can be overcome by performing a so-called energy-dependent analysis in which the results at all energies are analyzed simultaneously by prescribing the energy dependence of the coefficients in analytic form.

In the present analysis the energy dependences of the coefficients a_k in Eq. (3) were expressed by power series in the laboratory deuteron energy E:

$$a_k(E)=\sum_{q=0}^{q_{\max}} a_{kq}E^q. \qquad (5)$$

The purpose of the analysis was to find the values of a_{kq} which best represent the data between 2.1 and 14.1 MeV. The experimental cross sections needed to obtain $\pi(\theta)$ in Eq. (3) were taken from Refs. 15–17. To obtain cross sections for the particular energies and angles for which the polarization measurements had been made the cross sections were interpolated with respect to angle by means of Eq. (2) and then with respect to energy by drawing free-hand curves through $\sigma(E)$ at a fixed angle. This can easily be done with sufficient accuracy because the relative errors in the cross section measurements are small compared to the errors in the polarization measurements.

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[20] A. Simon and T. Welton, Phys. Rev. **90**, 1036 (1953).

The determination of the coefficients a_{kq} was done by a computer program. A gradient search was used, i.e., in each step of the calculation all coefficients were adjusted simultaneously in the direction of most rapid decrease of the quantity

$$\chi^2=\sum_{i=1}^{Q}\left(\frac{\pi_{\text{calc}}-\pi_{\text{exp}}}{\Delta\pi}\right)_i^2, \qquad (6)$$

where $\pi_{\text{exp}}\pm\Delta\pi$ is the experimental value of $P(E,\theta)\times\sigma(E,\theta)$ and the sum extends over the measurements at Q different angles and energies. The resulting coefficients are listed in Table II. Terms up to $P_6'(\cos\theta)$ were used. The energy dependence of the coefficients is shown in Fig. 3. The solid lines in Fig. 2 show the quality of the fit to the data. The curves were calculated by dividing $\pi(\theta)$ obtained from Eqs. (3) and (5) by the interpolated experimental cross section $\sigma(\theta)$. In general the fit is very satisfactory except at 3.1 MeV. However, as mentioned already, difficulty at this energy was encountered even when the condition of energy continuity was not imposed.

One difficulty encountered in the present energy dependent analysis was that convergence to a good solution was very slow. A similar energy dependent analysis of the D(d,p)T cross sections and of D(d,n)He^3 polarization and cross-section results was attempted but again convergence was very slow. We suspect that the problem in part is that the power series of Eq. (5) was a poor choice. Since the actual polarization and cross section change fairly rapidly at low energies but very slowly at the higher energies the expansion should be in terms of functions $f(E)$ which have that property.

The coefficients c_{kq} giving the energy and angular dependence of the D(d,p)T cross section are given in Table III. The c_{kq} are related to the c_k as the a_{kq} are related to the a_k in Eq. (5). These coefficients give an acceptable representation of the cross section for deuteron energies between 3 and 14 MeV. Even Legendre functions through P_{10} were used. The power series which was used to represent the energy dependence of the coefficients included terms up to E^3.

7. DISCUSSION

The Legendre expansion of the cross section [Eq. (2)] requires[17] $M=6$ below 6 MeV and up to $M=10$ below 14 MeV. In contrast, the polarization can be represented reasonably well with $N=4$ below 10 MeV and $N=6$ below 14 MeV. In general (i.e., if spin-orbit forces act for all angular momenta involved) one would expect $M=N$. The present data suggest, therefore, that as one increases the bombarding energy and a new partial wave enters the reaction, the splitting takes place a few MeV above the point where the new partial wave becomes evident in the cross section. A similar behavior has been noted for the D(d,n)He^3 reaction,[21] but in this case fewer terms may possibly be needed in the polarization expansion because the accuracy of the polarization data is much less than that of the cross section data.

For bombarding energies of several MeV the Coulomb effects in the exit channel are small, so that one would expect the same magnitude of polarization for protons from the D(d,p)T reaction as for neutrons from the D(d,n)He^3 reaction. In order to compare the two sets of data, the D(d,n)He^3 cross sections[15,17,22] and polarizations[21,23] were represented by a least-squares fit of Eqs. (2) and (3) to the data. Points taken from the resulting smooth curves through the data every 5° were plotted as a function of deuteron energy. The

TABLE II. Coefficients a_{kq} which best represent the measured D(d,p)T proton polarization between 2 and 14 MeV. The units of a_{kq} are mb/(MeV)^q.

k	a_{k0}	$10a_{k1}$	10^2a_{k2}	10^3a_{k3}
2	−0.782	2.690	−1.148	0
4	−0.034	0.794	−1.065	0.416
6	0	−0.017	0.061	0

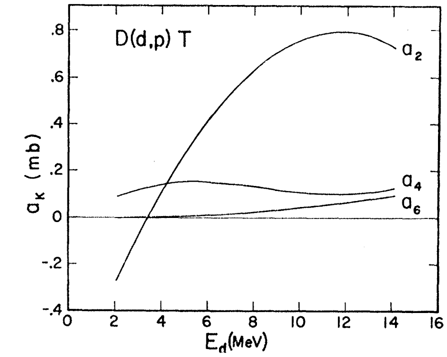

FIG. 3. Energy dependence of the coefficients a_k in Eq. (3).

TABLE III. Coefficients c_{kq} which represent the measured D(d,p)T cross section between 3 and 14 MeV. The units of c_{kq} are mb/(MeV)^q.

k	c_{k0}	c_{k1}	c_{k2}	c_{k3}
0	5.30	9.90	−14.62	5.55
2	7.91	6.29	−6.78	1.80
4	−2.54	54.12	−53.77	16.26
6	−0.52	3.84	12.04	−6.77
8	0.0	−0.92	5.80	−2.47
10	0.0	0.23	−0.57	0.65

[21] F. O. Purser, Jr., J. R. Sawers, Jr., and R. L. Walter, Phys. Rev. **140**, B870 (1965).

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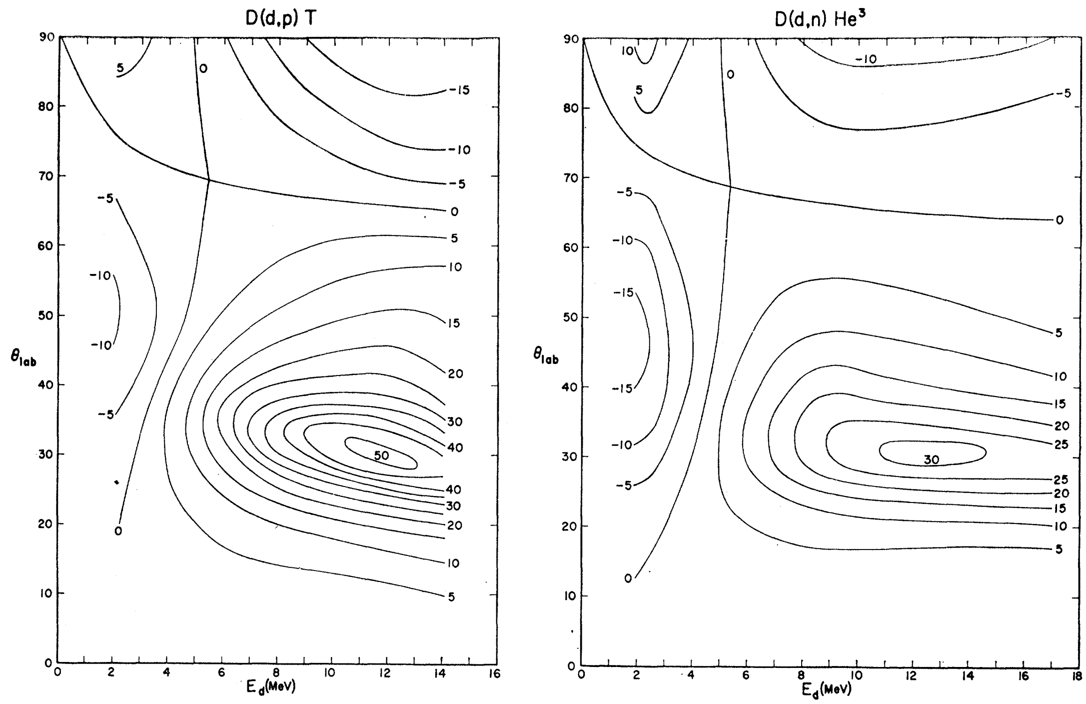

FIG. 4. Curves of constant polarization (in %) of nucleons from the D(d,p)T and the D(d,n)He^3 reactions as a function of laboratory deuteron energy and laboratory reaction angle (in degrees).

data were then smoothed with respect to energy by drawing free-hand curves, taking into account that different points had different weight depending on how close in angle an actual measurement had been made. In Fig. 4 the resulting polarization contour lines for the (d,n) reaction are compared with the corresponding values for the (d,p) reaction. The latter were calculated in the same way as the solid lines of Fig. 2. The angles and energies in Fig. 4 are given in the laboratory system. The comparison clearly shows the great similarity between the two reactions that one expects from charge symmetry of nuclear forces. However, there are clear indications that the proton polarization is systematically larger than the neutron polarization. Above 6 MeV the proton polarization is on the average about 1.4 times as large as the neutron polarization. As was pointed out by Barschall,[6] there are several other cases where one might expect the same results for neutron and proton polarizations on the basis of charge independence, but where the polarization is higher for protons than for neutrons.

ACKNOWLEDGMENTS

We wish to thank Dr. L. W. Morrow, Dr. L. C. McIntyre, and Dr. W. G. Weitkamp for assistance in performing the experiment, and Paul Kristof for extensive help with the calculations. We also wish to thank Dr. R. I. Brown for several helpful discussions.