Mobility of Electrons in Compensated Semiconductors. I. Experiment*

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Ionized-impurity scattering mobilities were obtained from Hall-mobility measurements for a series of n-type germanium samples between 10 and 40°K. The specimens were prepared by transmutation doping, and contain a constant concentration of minority impurities ($N_{Ga}=2.9\times10^{15}$ cm^{-3}), while the compensation ratio $K=N_{minor}/N_{major}$ varies from 0.27 to 0.95. At the low temperatures considered, the number of electrons in the conduction band is very small, and the number of ionized impurities is essentially the same in all samples, $N_I\cong2N_{Ga}$. Under these conditions, the screening of scattering centers is due to ionized impurities rather than to electrons, and it is possible to study the dependence of this effect on the concentration of majority impurities.

I. INTRODUCTION

In the last twenty years a considerable number of papers[1–7] have been devoted to the study of ionized impurity scattering in semiconductors. These papers are mostly concerned with the understanding of the temperature and impurity concentration dependence of the electron mobility in germanium and silicon. Comparison is almost invariably made with the theoretical treatments of Conwell and Weisskopf,[8] Brooks and Herring,[9] and Dingle.[10] The agreement between experiment and theory is only fair in spite of the fact that some parameters (mostly the effective mass) are taken as adjustable. More sophisticated treatments[11] which take into account the anisotropy of the relaxation time do not seem to improve the agreement with the experimental observations.

The analysis of the experimental results, on the other hand, is always complicated by several factors: first, the difficulty of extracting the theoretically more tractable drift mobility from the usually measured Hall mobility; second, the uncertainty in subtracting the contributions of the other scattering processes which contribute to the relaxation rate; third, the difficulty in accurately determining the content of majority and minority impurities in the sample.

This research is an attempt to produce a systematic set of measurements which would minimize some of the above-mentioned difficulties and from which a more meaningful comparison with the theory could be made. As described in Sec. II, we have prepared a series of n-type Ge samples by "transmutation doping."[12] Such samples form a series in which the minority impurities have a constant concentration, and the majority impurities vary over a fairly wide range. This permits a systematic study of the dependence of the mobility on the majority impurity concentration. Moreover, the impurity content is in our case less uncertain than with the conventional doping procedures. We have also restricted our analysis to low temperatures (between 10 and 40°K) where ionized impurity scattering is the dominant scattering effect. We expect thus to minimize in this range of temperatures the uncertainties generally introduced in the evaluation and subtraction of the combined effects of phonon, neutral impurity, and electron-electron scattering. In this range of temperatures, too, the concentration of electron carriers as measured by the Hall coefficient should be more correct than at higher temperatures.[6] Also, the electrons are essentially frozen out of the conduction band and consequently the number of ionized impurities in all samples is essentially constant and equal to twice the number of minority impurities, i.e., $N_I\cong2N_{Ga}$.

Because of the absence of electrons, the most important screening effect is due to the distribution of ionized donors around the totally ionized acceptors. This screening effect increases markedly with the total number of available majority impurities; consequently we should expect better screening and higher mobilities with decreasing compensation ratios. Experimentally this effect is so strong that larger mobilities are obtained for those samples of our series containing a larger number of impurities than for others with lower dopings and compensations closer to one. The screening of ionized impurities more than compensates for the extra scattering due to the additional neutral impurities. This screening of scattering centers by scattering centers was predicted by Brooks and, to our knowledge, our measurements give the first clear and systematic evidence of such an effect. In Sec. II we discuss the sample preparation and the determination of the impurity content. In Sec. III we report the experimentally deter-

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[12] H. Fritzsche and M. Cuevas, Phys. Rev. **119**, 1238 (1960).

TABLE I. Impurity concentrations of the samples. $N_{Ga}(M)=2.90\times10^{15}$ cm^{-3}; $N_{As}(M)=9.32\times10^{14}$ cm^{-3}; $N_{Se}(M)=1.14\times10^{14}$ cm^{-3}. The ratio $\langle As/Ga\rangle$ and the compensation $K=0.40$ of the monitor sample are obtained from the yields of the radioactive decays of Ge^{70}, Ge^{74}, and Ge^{76} and the relative abundances as reported in Ref. 12.

	Procedure 1.			Procedure 2.				
Sample	N_{As} (orig.)	N_{As} (total)[a]	K[b]	$N_{As}+N_{Se}-N_{Ga}$	N_{As} (total)[c]	N_{Ga}	K[b]	$\bar{K}$
C1	2.15×10^{15}	3.08×10^{15}	0.942	0.208×10^{15}	3.00×10^{15}	2.90×10^{15}	0.966	0.954
C2	2.72×10^{15}	3.65×10^{15}	0.795	0.640×10^{15}	3.43×10^{15}	2.90×10^{15}	0.845	0.820
C3	3.02×10^{15}	3.95×10^{15}	0.734	1.04×10^{15}	3.83×10^{15}	2.90×10^{15}	0.757	0.745
C4	3.16×10^{15}	4.09×10^{15}	0.709	1.42×10^{15}	4.22×10^{15}	2.90×10^{15}	0.687	0.698
C5	5.53×10^{15}	6.46×10^{15}	0.449	2.89×10^{15}	5.68×10^{15}	2.90×10^{15}	0.510	0.480
C6	9.81×10^{15}	10.74×10^{15}	0.271	7.99×10^{15}	10.78×10^{15}	2.90×10^{55}	0.269	0.270

[a] $N_{As}(\text{total})=N_{As}(\text{orig.})+N_{Ga}(M)\langle As/Ga\rangle$. [b] $K=N_{Ga}(M)/N_{As}(\text{total})$. [c] $N_{As}(\text{total})=(N_{As}+N_{Se}-N_{Ga})-N_{Se}(M)+N_{Ga}(M)$.

mined values of the Hall coefficient and the mobility. Section IV is devoted to an analysis of the additional scattering contributions which, when subtracted, yield the contribution of the ionized impurities which we seek to study. A theoretical analysis of the results and a formulation of the scattering problem from a point of view different from previous authors is presented in the following paper.[13]

II. SAMPLE PREPARATION AND IMPURITY CONTENT

We have prepared a series of n-type Ge samples of varying compensation ratios but with a constant concentration of minority impurities by means of transmutation doping.[12] The starting material consisted of a series of germanium specimens containing between 2.15 and 9.81×10^{15} As impurities per cubic centimeter [N_{As} (orig.) of Table I]. After carefully determining the impurity content of these samples by measuring the Hall coefficients ($R=1/ne$) between 78 and 300°K, all samples were exposed to the same total flux of slow neutrons together with a piece of pure Ge, which served as a monitor. After about one year, which is equivalent to many halflives of the longest living radioactive isotope, (Ge^{71}), the samples were annealed at 400°C in order to remove radiation damage.

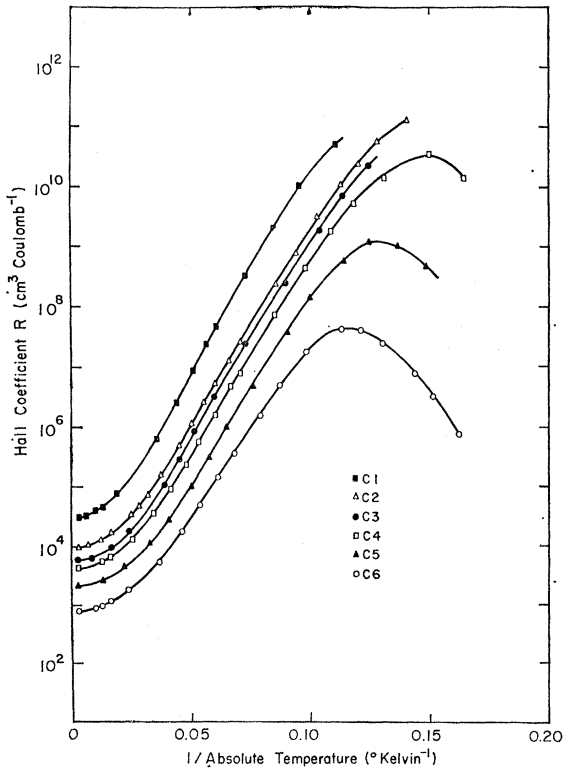

FIG. 1. Hall coefficient versus inverse temperature for six n-type germanium samples. The impurity concentrations are listed in Table I.

The concentrations of Ga acceptors and of As and Se donors are listed in Table I. The concentrations and compensation ratios K are obtained by two different procedures as listed in the Table. In the first procedure the As concentrations of the starting material N_{As}(orig.) are added to the relevant impurity concentrations of the monitor sample ($N_{Ga}(M)=2.9\times10^{15}$ cm^{-3}, $N_{As}(M)=9.32\times10^{14}$ cm^{-3}, $N_{Se}(M)=1.14\times10^{14}$ cm^{-3}). The impurity concentrations of the monitor sample were obtained from the compensation ratio $K=0.40$, which is determined by the relative abundances and capture cross sections of the various Ge isotopes, and from the Hall coefficient measurements between 77 and 300°K. The fact that each Se donor is doubly charged and hence compensates two acceptors has been taken into account. In the second procedure the Ga and Se concentrations of the monitor sample are used in conjunction with the Hall measurements between 78 and 300°K of the final samples, from which one obtains $N_{As}+N_{Se}-N_{Ga}$. These values are also listed in Table I. Both procedures yield the same K values to within a few percent. The average value $\bar{K}$ is used in all other calculations.

III. EXPERIMENTAL RESULTS

The Hall coefficient R, measured in a magnetic field of 7 kG, is plotted against the reciprocal temperature in Fig. 1. The onset of impurity conduction at low temperatures produces a decrease in the slopes of the Hall coefficient curves and gives rise to maxima when the contribution of the hopping process to the conductance is equal to the normal conductance of electrons in the

[13] L. M. Falicov and M. Cuevas, following paper, Phys. Rev. 164, 1025 (1967).

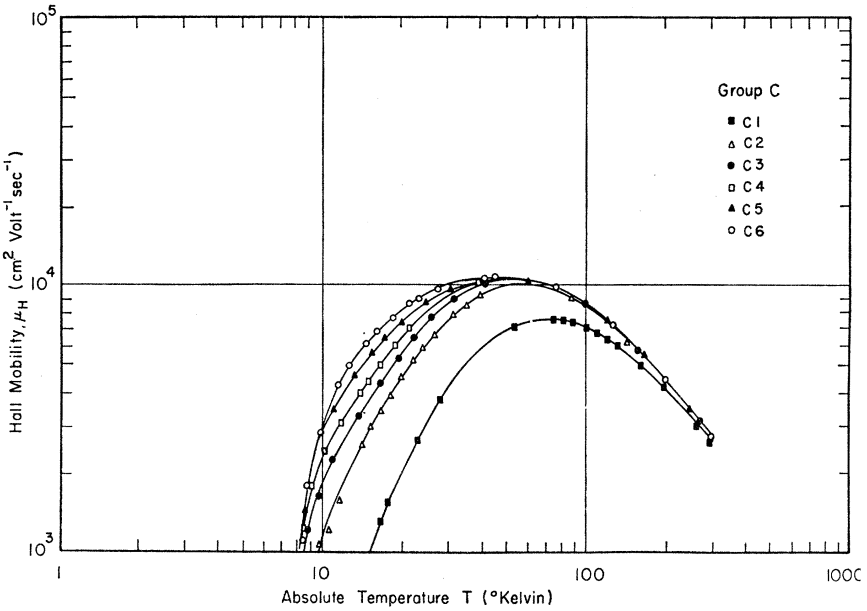

FIG. 2. Hall mobility versus absolute temperature curves for a series of n-type germanium samples with a constant minority impurity concentrations.

conduction band. At higher temperatures the activation energies of the process are seen to be similar for all samples, the slightly smaller values shown by the less pure samples being due to the expected decrease of donor binding energies. In Fig. 2 the Hall mobility $\mu_H = R/\rho$ (where ρ is the resistivity) is plotted as a function of the temperature. Our analysis is restricted to the low temperature range ($\sim$10 to 40°K) where ionized impurity scattering is predominant. Since the magnetic field used in these measurements was 7 kG and at the temperatures of interest the scattering times were long enough, the high field limit in the measurements of the Hall coefficient has been attained, and consequently the difference between the Hall mobility and the drift mobility can be neglected.

The steep descent of the curves at the low temperature end is again due to the onset of impurity conduction.

At the highest temperatures ($T > 80$°K), the Hall mobility curves of Fig. 2 show the behavior which is characteristic of predominantly acoustical and optical phonon scattering. In the region of interest(10–40°K), both effects (phonon and hopping impurity conduction) are small.

IV. EVALUATION OF μ_I

The extraction of μ_I, the mobility due to ionized impurity scattering, from the total mobility μ in the temperature region of interest was carried out in two successive steps. Firstly, the inverse mobility was expressed in the following form,[1]

$$\frac{1}{\mu} = \frac{1}{\mu_N} + \frac{1}{F}\left(\frac{1}{\mu_L} + \frac{1}{\mu_{I0}}\right), \tag{4.1}$$

where μ_N is the contribution due to neutral impurity scattering, μ_L the contribution due to lattice scattering, and μ_{I0} the contribution due to ionized impurity scattering. The factor F in (4.1) takes into account the simultaneous presence of two scattering processes (phonon scattering and ionized impurity scattering) with different energy dependence. F was calculated by Johnson and Lark-Horovitz.[14] The correction is not necessary for neutral impurity scattering because this process is energy-independent.[15]

In the second step the contribution to the mobility due to ionized impurity scattering μ_I is written in the form

$$\mu_{I0} = \mu_I G, \tag{4.2}$$

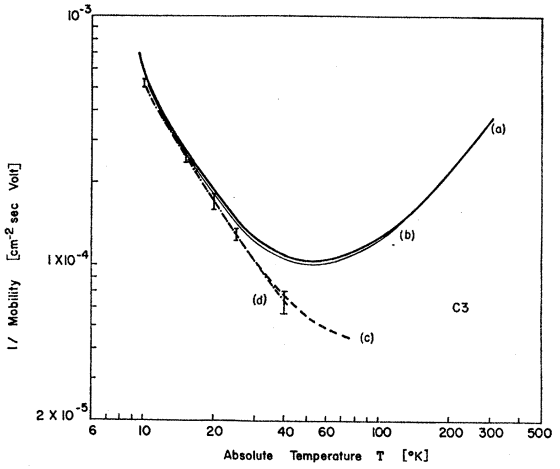

FIG. 3. Inverse mobility versus absolute temperature for sample C3: (a) inverse of measured Hall mobility, (b) inverse mobility after subtracting the neutral impurity scattering contribution, (c) experimentally determined ionized impurity scattering which includes electron-electron collisions, (d) experimentally determined values of the inverse mobility due only to ionized impurity scattering.

[14] V. A. Johnson and K. Lark-Horovitz, Phys. Rev. 82, 977 (1951).

[15] C. Erginsoy, Phys. Rev. 79, 1013 (1950).

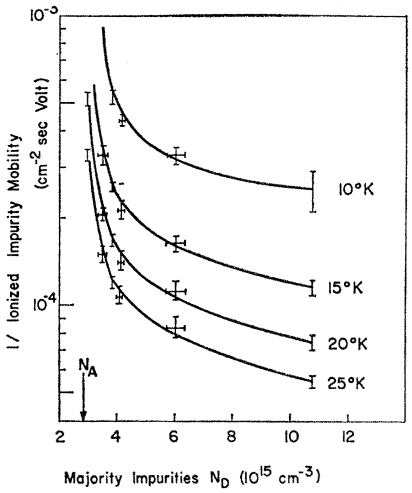

FIG. 4. Inverse of ionized impurity mobilities as obtained from experiment after applying necessary corrections versus majority impurity concentration for 10, 15, 20, and 25°K. The minority impurity concentration N_A, which is the same for all samples, is indicated by the arrow on the horizontal scale.

where the factor G is the correction due to electron-electron scattering as calculated by Appel,[16] and μ_{I0} (the "experimental" value) is obtained from (4.1).

The following formulae and theories were employed in these calculations: The neutral impurity scattering contribution was calculated making use of Erginsoy's equation[15]

$$\mu_N=\frac{e^3 m_D^*}{20\hbar^3\kappa}\frac{1}{N_N}, \tag{4.3}$$

where m_D^* is the density of states effective mass, N_N is the density of neutral impurities, and κ is the dielectric constant of germanium. It was not thought necessary in our case to use Sclar's[17] more sophisticated expression for neutral impurity scattering which includes a weak energy dependence, because for all samples and at all temperatures considered, $\mu_N \gg \mu$. As a representative example of the size of this correction we have plotted in Fig. 3 the inverse Hall mobility as obtained from the experiment (curve *a*) together with the inverse of the mobility after subtracting the neutral impurity scattering contribution (curve *b*). The largest corrections for neutral impurity scattering occur for sample C6 and amount at most to 30% of the measured Hall mobility.

At relatively high temperatures ($T>77$°K) lattice scattering is due both to acoustical and optical phonons and gives rise to a $T^{-1.75}$ dependence.[18] Below 77°K though, acoustical phonons dominate the scattering giving rise to a weaker dependence, $T^{-1.5}$. Since we are concerned here with temperatures well below 77°K, we compute the phonon scattering by[19]

$$\mu_{PA}(T)=2.8\times10^7 T^{-1.5}\ \text{cm}^2V^{-1}\,\text{sec}^{-1}. \tag{4.4}$$

The use of the function F of Eq. (4.1) to subtract the phonon scattering contribution should give quite accurate results in our case since, as pointed out by Debye and Conwell,[1] the error incurred is negligible when either $\mu_L \gg \mu_I$ or $\mu_L \ll \mu_I$. Curve *c* of Fig. 3 represents the inverse of the mobility thus corrected for neutral impurity and phonon scattering.

We consider now the effects of electron-electron collisions. These encounters affect the mobility of the carriers because they tend to distribute the momentum acquired by the electrons in the electric field randomly among the different velocity groups. When the scattering mechanism is such as to lead to a nonuniform distribution, electron-electron collisions give rise to a net transfer of momentum from electrons which dissipate momentum less efficiently to those which dissipate it more efficiently. The size of the effect depends on the relative frequency of electron-electron and electron-impurity encounters and on the velocity dependence of the relaxation time.

We estimate the reduction in mobility produced by electron-electron scattering making use of Appel's[16] theory, developed for one single band and where the pair interaction between conduction electrons is described by a shielded Coulomb potential. To first order then the factor G takes the form

$$G=\frac{1}{3.25}\left[1+\frac{\frac{5}{2}-M_1/M_0}{M_2/M_0-(M_1/M_0)^2+2^{-3/2}nL_1/N_I M_0}\right] \tag{4.5}$$

with the definition of the symbols M_0, M_1, M_2, and L_1 as given in Ref. 16. As seen from Fig. 3, curve *c*, the corrections are indeed very small at low temperatures since $n \ll N_I$.

Figure 4 represents the final value for ionized impurity scattering as a function of the majority impurity concentration N_D for 10, 15, 20, and 25°K. This figure shows a decrease in ionized impurity scattering as majority impurities are added. This decrease results from the greater effectiveness of mutual screening of the constant concentration of positive and negative ionized impurities caused by the larger number of available majority impurity sites. An interpretation of these results is made in the following paper.[13]

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[19] Reference 7, p. 110.