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Deformed States in C^{12}†

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A calculation of energy levels in C^{12} using projected Nilsson wave functions was performed and the optimum deformation in each J state was found. The orthogonality between wave functions corresponding to different deformation was studied.

A CALCULATION of energy levels in C^{12} using projected Nilsson wave functions has been reported by Kurath and Picman[1]. In this work almost complete overlap was found between the wave functions solved exactly in the intermediate coupling shell model and the projected Nilsson wave functions with a deformation. This suggests that a projected Nilsson state with optimum deformation provides a good approximation for the wave functions and the expectation value of a Hamiltonian containing two-body interactions is a good approximation for the energy. This note reports results of a calculation using projected Nilsson wave functions in C^{12}.

The properties of states resulting from a single configuration were studied as a function of the deformation. The optimum value of the deformation was found and the orthogonality between wave functions corresponding to different deformation was studied.

In the calculation the configuration containing four holes in the No. 4 Nilsson orbit with $k=\frac{1}{2}$ is considered. All quantities are expressed as functions of the deformation strength D, which appears in the deformed potential defined as

$$H_D=-(\tfrac{4}{5}\pi)^{1/2}D\nu r^2Y_{20}(\omega)\,, \tag{1}$$

where ν is the oscillator frequency. The wave function $\Psi_N(D)$ corresponding to this configuration is projected into normalized eigenfunctions of J in the following way:

$$\Psi_N(D)=\sum_J N_J(D)\Psi_J(D)\,. \tag{2}$$

The projected and unprojected Nilsson energies are then calculated by the equations

$$E_J(D)=\langle\Psi_J(D),\mathcal{H}\Psi_J(D)\rangle \tag{3}$$

and

$$E_N(D)=\sum_J N_J{}^2(D)E_J(D)\,, \tag{4}$$

respectively, where $\mathcal{H}$ is a Hamiltonian containing two-body interactions.

In the numerical calculation all necessary quantities are taken from the work of Cohen and Kurath.[2] The energy difference between the $p_{3/2}$ hole and the $p_{1/2}$ hole states is given as 6.30 MeV. The two-body matrix elements denoted in Ref. 2 by (8–16) 2 BME are used. All energies are measured relative to the single $p_{1/2}$ hole level. As is seen in Table I and Table VI of Ref. 2, the term (8–16) 2 BME expresses the two-body matrix elements which are determined from the χ^2 fitting to energy levels and binding energies of the $1p$ shell nuclei between $A=8$ and 16. At the same time the values of the $1p_{3/2}$ and the $1p_{1/2}$ single-particle energies are determined. These values are consistent with the value, 6.30 MeV, for the energy difference between the $1p_{3/2}$ and the $1p_{1/2}$ hole levels.

The probabilities of three J components in the unprojected Nilsson wave function are given in Table I. At the spherical point only $J=0$ is possible while in a region of larger deformation the main part of the unprojected wave function consists of a $J=2$ component. This fact plays an important role in the results. The energies of the states Ψ_N and Ψ_J with $J=0$, 2 and 4 are shown in Fig. 1. Although negative deformations are somewhat more interesting, the figure also shows a region of positive deformations. Three interesting features appear in this figure. (1) Generally the opti-

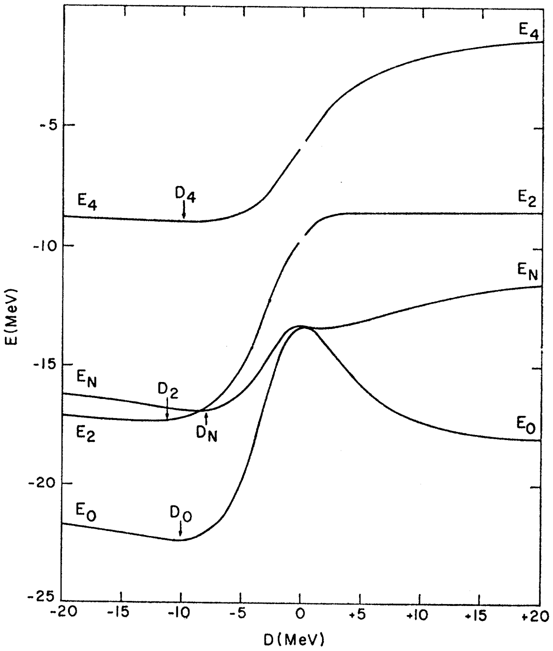

FIG. 1. The projected Nilsson energy E_J and the unprojected Nilsson energy E_N are plotted as functions of deformation. D_J and D_N are the optimum values of the deformation strength D.

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[2] S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965).

TABLE I. The probabilities N_J^2 defined in Eq. (2) are listed as functions of deformation. D is the deformation strength in MeV units

N_J^2 \ D	−20	−16	−12	−8	−4	0	+4	+8	+12	+16	+20
0	0.21	0.21	0.22	0.26	0.46	1.00	0.71	0.53	0.45	0.41	0.39
2	0.57	0.58	0.58	0.58	0.48	0.00	0.28	0.43	0.49	0.52	0.53
4	0.22	0.21	0.20	0.16	0.06	0.00	0.01	0.04	0.06	0.07	0.08

mum values of the deformation strength associated with each J are different from each other and also different from the one given by the unprojected energy. (2) All energy curves are flat around their optimum positions indicating that a small perturbation can easily change the shape of the system. (3) The projected Nilsson energies with their optimum deformations agree with the exact values of Cohen and Kurath[2] while the ground state is about 5.4 MeV lower than the one obtained from the unprojected Nilsson energy with its optimum deformation. This clearly shows that in a Hartree-Fock calculation using Nilsson wave functions, projection into eigenstates of the total angular momentum[3] is necessary before the calculated binding energies can be compared with experiment.

To check the orthogonality of the wave function with respect to deformation, the following overlap integrals are calculated:

$$\langle\Psi_J(D),\Psi_J(D_J)\rangle\equiv 0_J(D)\,, \tag{5}$$

and

$$\begin{aligned}\langle\Psi_N(D),\Psi_N(D_N)\rangle&\equiv 0_N(D)\\&=\sum_J N_J(D)N_J(D_N)\langle\Psi_J(D),\Psi_J(D_N)\rangle\,,\end{aligned} \tag{6}$$

where D_J and D_N express the optimum values of the deformation strength given by the projected and unprojected energies, respectively.

The calculated overlap integrals are shown in Fig. 2. The figure shows that the unprojected Nilsson wave function is well localized in a region of negative deformation. The projected ones, however, are not, and especially the one for $J=0$ increases with increasing positive deformation. It is worthwhile to remember that, as is seen in Fig. 1, the projected Nilsson energy for $J=0$ decreases with increasing positive deformation. The lack of orthogonality shown in Fig. 2 could be explained by the fact that if the nuclear axis is fixed in a definite direction, which is the case of the unprojected Nilsson wave function, prolate and oblate shapes give entirely different density distributions and their overlap is very small. On the other hand, if the nuclear axis is rotated, which is the case of the projected wave functions, the two shapes will give similar density distributions and their overlap can be large. In some cases there can be two optimum positions in the unprojected energy of a given Nilsson configuration[4]; one in the positive and one in the negative deformation region. The unprojected wave functions could be almost orthogonal. This, however, does not mean that they correspond to two physically different states. The present result shows that if these two wave functions are projected into eigenstates of J, the projected wave functions with the same J could have a large overlap.

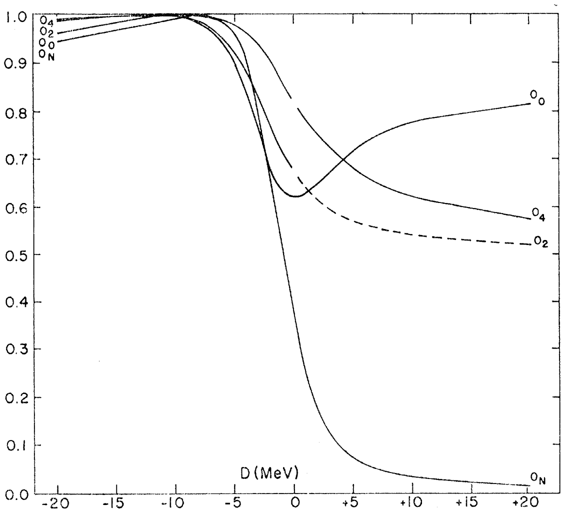

FIG. 2. The overlap integral O_J and O_N defined in Eq. (5) and (6) are plotted as functions of deformation. The solid line corresponds to a positive value while the dashed line corresponds to a negative value.

In this work the configuration was assumed to be constructed from two $1p_{3/2}$ and $1p_{1/2}$ single-particle states. If the configuration is allowed to include higher single-particle states such as $2s$, $1d$, $2p$ and $1f$, the results could be improved in the following two points. The energy curve could show a sharp minimum at the optimum deformation. The orthogonality between wave functions corresponding to different deformation could be improved. A confirmation of these improvements will be an interesting problem.

Discussions with Dr. S. Fallieros are highly appreciated.

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[4] W. H. Bassichis, A. K. Kerman, and J. P. Svenne (to be published).