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Linear Response of Nearly Free Electrons. II. Extreme Anomalous Skin Effect in Layer Lattices

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On the basis of the linear response function, calculated in paper I, the transverse conductivity $\sigma(\mathbf{q},\omega)$ for electrons in a layer lattice is derived and applied to evaluate the surface impedance $Z(0)$. If the Fermi surface lies completely inside the first Brillouin zone, we find only a very small correction to the free-electron result. If the Fermi surface intersects the boundary, we again obtain approximately the free-electron expression for Fourier coefficients V_1 of the lattice smaller than 0.1 eV, and a different function, with a slightly different power of the frequency and a sensitive dependence on V_1, if V_1 is larger than 0.1 eV. ω is assumed to be of the order of 10^{-4} eV.

I. INTRODUCTION

THE linear response function has been shown by many authors to be an important means to describe the properties of a many-particle system in a unified way.[1–3] In paper I of this series[4] we studied the linear response function for a system of electrons in a weak lattice potential. Starting from free electrons, we carried out a many-body perturbation treatment taking into account diagrams up to second order in the lattice potential. The resulting response function was discussed with respect to the dielectric constant, the pair-correlation function, and the stopping power.

In the present paper we apply the response function evaluated in I to the calculation of the transverse conductivity of electrons in a layer lattice neglecting electron-electron and electron-phonon interactions, i.e., assuming $\omega\tau\gg1$. A study of this type of lattice, aside from mathematical simplicity, seems to be worthwhile because for radiation incident perpendicularly on the surface the Fourier components of the external electromagnetic field have an influence on the electrons which is in a certain sense similar to that of the lattice. If the Fermi surface intersects the zone boundary and the Fourier coefficient of the lattice is not too weak (>0.1 eV for $\omega\approx10^{-4}$ eV), this lattice contribution can even be comparable in magnitude with the free-electron contribution. The surface impedance obtained from the lattice contribution alone is similar in structure to that calculated by Reuter and Sondheimer,[5] but has a slightly different frequency dependence and depends sensitively on the lattice potential.

Similar to layer lattices it is conceivable that also for other types of crystals a more detailed study of the higher-order lattice terms is required for a more complete understanding of the properties of the electrons. In this context, Harrison[6] points out that the discrepancies appearing in the construction of the Fermi surface of aluminum on the basis of experiments are expected to be mainly due to our present difficulties in understanding the anomalous skin effect.

II. ANOMALOUS SKIN EFFECT

In the theory of the anomalous skin effect, one is concerned with the transport properties of electrons near metal surfaces under the condition that the "free" path of an electron during one cycle of the external field or between any two scatterings with phonons or impurities is long compared to the skin depth. If we assume the metal surface to be in the x-y plane and an electromagnetic wave $(E,H_y,0)$ of frequency ω incident normally along the z axis, then this condition can be written as $m\omega/qk_F\ll1$, where q is the wave vector of the electromagnetic wave and k_F is the Fermi momen-

[1] A. J. Glick, in Proceedings of the International Spring School of Physics, Naples, 1960 (unpublished).
[2] A. J. Glick, Ann. Phys. (N. Y.) 17, 61 (1962).
[3] D. Pines, *Elementary Exhibitions in Solids* (W. A. Benjamin, Inc., New York, 1963).
[4] E. Gerlach and M. L. Glasser, Phys. Rev. 159, 511 (1967).
[5] G. E. H. Reuter and E. H. Sondheimer, Proc. Roy. Soc. (London) A195, 336 (1948).
[6] W. A. Harrison, *Pseudopotentials in the Theory of Metals* (W. A. Benjamin, Inc., New York, 1966).

tum of the electron. Now, taking q to be a typical wave vector of the decaying fields near the surface, we can put its reciprocal equal to the skin depth δ. Thus the condition reads $\delta \ll v_F/\omega$, in agreement with the above statement. A lower limit for the frequency is not required as we are going to consider scattering merely at a rigid lattice and thus have an infinitely large bulk conductivity and a vanishing normal skin depth. Furthermore we will use the condition $q \ll |k_F|$, which simply means that we do not consider field components which decay within a unit cell.

As the electric field changes considerably over the skin depth, the changing influence of the field along the mean free path of the electron has explicitly to be taken into account. This can quite generally be expressed by the relation

$$j(\mathbf{r},t)=\int_{-\infty}^{\infty}\sigma(\mathbf{r}-\mathbf{r}',t)E(\mathbf{r}',t)d^3\mathbf{r}' \tag{1}$$

or

$$j(\mathbf{q},\omega)=\sigma(\mathbf{q},\omega)E(\mathbf{q},\omega), \tag{2}$$

where j is the current density and σ is the transverse conductivity. We symmetrize our problem in the usual way by introducing a current sheet at $z=0$. From Maxwell's equations we then obtain by a simple calculation[7]

$$-\mathbf{q}^2E(\mathbf{q})+(\omega/c)^2E(\mathbf{q})=-(4\pi i\omega/c^2)j(\mathbf{q}) +(2/\pi)^{1/2}(dE/dz)_{+0}, \tag{3}$$

which can be solved to give

$$E(z,t)=\frac{2}{\pi}\left(\frac{dE}{dz}\right)_{+0}\int_0^{\infty}\frac{e^{-iqz}dz}{-q^2+\omega^2/c^2+(4\pi i\omega/c^2)\sigma(\mathbf{q},\omega)}. \tag{4}$$

Thus for the surface impedance defined by

$$Z(0)=\frac{4\pi i\omega}{c^2}\frac{E(0)}{(\partial E/\partial z)_{+0}}, \tag{5}$$

we have

$$Z(0)=\frac{8\omega}{ic^2}\int_0^{\infty}\frac{dq}{q^2-\omega^2/c^2-(4\pi i\omega/c^2)\sigma(\mathbf{q},\omega)}. \tag{6}$$

The main task is now to calculate $\sigma(\mathbf{q},\omega)$, the transverse conductivity with respect to the incident light beam.

III. GENERAL REAL CONDUCTIVITY

Omitting local field effects,[8] the general conductivity tensor is given by[9]

$$\sigma_{\mu\nu}(\mathbf{q},\omega)=i\frac{e^2n}{m\omega}\delta_{\mu\nu}-\frac{1}{\Omega\omega} \times\int_0^{\infty}e^{-i\omega t}\langle\Psi_0|[j_\mu(\mathbf{q},t),j_\nu(-\mathbf{q},0)]|\Psi_0\rangle dt, \tag{7}$$

where n is the electron density, Ω the volume, $|\Psi_0\rangle$ the ground state, and j_μ the μ component of the current density operator. Equation (7) is equivalent to the zero-temperature Kubo formula for the electrical conductivity. For the Hamiltonian of the electrons we write

$$H=H_0+H_1, \tag{8}$$

where

$$H_0=\sum_{\mathbf{k}}\epsilon_{\mathbf{k}}a_{\mathbf{k}}^{+}a_{\mathbf{k}} \tag{9}$$

and

$$H_1=\sum_{\mathbf{k},\mathbf{K}}V_{\mathbf{K}}a^{+}_{\mathbf{k}+\mathbf{K}}a_{\mathbf{k}},(V_{\mathbf{K}}=V_{-\mathbf{K}}^{*},V_0=0). \tag{10}$$

$V_{\mathbf{K}}$ is the Fourier coefficient of the periodic lattice and $\epsilon_{\mathbf{k}}$ is the free-electron energy. Here we neglect the fact that the wave functions of the electrons vanish at the surface. This has been studied by Van Gelder,[10] who showed that the resulting error is only of the same order of magnitude as the effect of the surface irregularities.

We are especially interested in the real part of σ_{xx}, which we denote by σ:

$$\sigma=(1/\omega\Omega)\,\mathrm{Re}\left\{\int_0^{\infty}e^{-i\omega t}\langle\Psi_0|j_x(-\mathbf{q},0)j_x(\mathbf{q},t)|\Psi_0\rangle dt -\int_0^{\infty}e^{-i\omega t}\langle\Psi_0|j_x(\mathbf{q},t)j_x(-\mathbf{q},0)|\Psi_0\rangle dt\right\}. \tag{11}$$

Taking t in the first integral into the first operator we can write

$$\sigma=(1/\omega\Omega)\,\mathrm{Re}\{F(\mathbf{q},\omega)-F(-\mathbf{q},-\omega)\}, \tag{12}$$

where

$$F=\int_0^{\infty}e^{i\omega t}\langle\Psi_0|j_x(-\mathbf{q},t)j_x(\mathbf{q},0)|\Psi_0\rangle dt. \tag{13}$$

Equations (12) and (13) can be verified by using the fact that matrix elements with an even number of contractions—as those in which we are interested—contain the imaginary unit i and times only as products of the two. Furthermore we put $\mathrm{Re}F^{*}(\mathbf{q},\omega)=\mathrm{Re}F(\mathbf{q},\omega)$.

As we expect from the relation between the transverse and the longitudinal dielectric constants, our expression for σ in (11) is closely related to that derived for the linear response function

$$R(\mathbf{q},\omega)=-\frac{1}{\pi}\,\mathrm{Re}\int_0^{\infty}dt\,e^{i\omega t}\langle\Psi_0|\rho_{\mathbf{q}}^{+}(t)\rho_{\mathbf{q}}|\Psi_0\rangle \tag{14}$$

evaluated in I. We merely have to compare the current-density operator with the density-fluctuation operator:

$$j(\mathbf{q})=(e/m)\sum_{\mathbf{k}}k_xa_{\mathbf{k}}^{+}a_{\mathbf{k}-\mathbf{q}}, \tag{15}$$

$$\rho_{\mathbf{q}}^{+}=\sum_{\mathbf{k}}a_{\mathbf{k}}^{+}a_{\mathbf{k}+\mathbf{q}},$$

[7] C. Kittel, *Quantum Theory of Solids* (John Wiley & Sons, Inc., New York, 1963).
[8] S. L. Adler, Phys. Rev. **126**, 413 (1962).
[9] M. L. Glasser, Phys. Rev. **129**, 472 (1963).
[10] A. P. Van Gelder, Phys. Letters **21**, 18 (1966).

to obtain an explicit expression for the transverse conductivity $\sigma(\mathbf{q},\omega)$.

IV. SECOND-ORDER CALCULATION OF THE CONDUCTIVITY OF LAYER LATTICES

In the following we consider a layer lattice, the layers in the x-y plane, and electromagnetic radiation impinging perpendicularly on the layers, i.e., we assume

$$\mathbf{K}=(0,0,K) \quad \text{and} \quad \mathbf{q}=(0,0,q). \tag{16}$$

By this choice there are no contributions of the reciprocal lattice vectors to k_x in the current density (15), which arise from the contractions in (13).

Now from the zero order term R^0 calculated in I, we can immediately write down the free-electron contribution to $\sigma(q,\omega)$:

$$\sigma^0(q,\omega)=(\pi e^2/\omega\Omega m^2)\sum_{\mathbf{k}} k_x^2\delta(\omega-\epsilon_{\mathbf{k}+q}+\epsilon_{\mathbf{k}})n_{\mathbf{k}}^- n_{\mathbf{k}+q}^+, \tag{17}$$

where $n_{\mathbf{k}}^-=1-n_{\mathbf{k}}^+=\theta(k_F^2-k^2)$. Carrying out the sum over k we find the well-known result[9] for $\sigma^0(q,\omega)$:

$$\begin{aligned}
&0, \quad \text{when} \quad 2m\omega>q^2+2qk_F\\
&0, \quad \text{when} \quad q>2k_F \quad \text{and} \quad 2m\omega<q^2-2qk_F\\
&\frac{3\pi}{4}\frac{ne^2}{m}\frac{1}{qv_F}\left[1-\frac{q^2}{4k_F^2}-\frac{\omega^2}{q^2v_F^2}\right], \quad \text{when} \quad q<2k_F \quad \text{and} \quad 2m\omega<|q^2-2qk_F|\\
&\frac{3\pi}{16}\frac{ne^2}{m}\frac{k_F}{q\omega}\left[1-\left(\frac{\omega}{qv_F}-\frac{q}{2k_F}\right)^2\right]^2, \quad \text{when} \quad |q^2-2qk_F|<2m\omega<|q^2+2qk_F|.
\end{aligned} \tag{18}$$

These formulas were first obtained by Lindhard.[11] The first term in the third region is the Reuter-Sondheimer-Pippard expression for free electrons.

$\sigma^0(q,\omega)$ is a nondiverging real conductivity in the zero-frequency limit, though there is no energy-absorbing scattering mechanism. This is due to the "ineffectiveness," introduced by Pippard,[12] according to which the increase of a mean free path is compensated by a decreasing number of effective electrons.

The first-order contribution vanishes because of momentum conservation around the corresponding polarization loop.

In second order we again take over the expression for the response function calculated in I and insert there a factor k_x^2. We obtain

$$\begin{aligned}
\sigma^{(2)}(q,\omega)=(\pi e^2/\omega\Omega m^2)\sum_{\mathbf{k},\mathbf{K}}|V_K|^2k_x^2n_{\mathbf{k}}^-\{&[\delta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}+q+K}+\omega)n_{\mathbf{k}+q+K}^+-\delta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}+q}+\omega)n_{\mathbf{k}+q}^+]\\
&\times[(\epsilon_{\mathbf{k}+q+K}-\epsilon_{\mathbf{k}+q})^{-1}-(\epsilon_{\mathbf{k}+\mathbf{K}}-\epsilon_{\mathbf{k}})^{-1}]^2+\delta'(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}+q}+\omega)n_{\mathbf{k}+q}^+[(\epsilon_{\mathbf{k}+q+\mathbf{K}}-\epsilon_{\mathbf{k}+q})^{-1}-(\epsilon_{\mathbf{k}+\mathbf{K}}-\epsilon_{\mathbf{k}})^{-1}]\}.
\end{aligned} \tag{19}$$

This expression can be greatly simplified by noting that $\mathbf{q}$ and $\mathbf{K}$ are parallel. Using

$$(\epsilon_{\mathbf{k}+q+\mathbf{K}}-\epsilon_{\mathbf{k}+q})^{-1}-(\epsilon_{\mathbf{k}+\mathbf{K}}-\epsilon_{\mathbf{k}})^{-1}=-(qm/K)(q+k_z+K/2)^{-1}(k_z+K/2)^{-1} \tag{20}$$

and

$$\delta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}+q+\mathbf{K}}+\omega)=|m/(q+K)|\,\delta[(m\omega/(q+K))-(q+K)/2-k_z], \tag{21}$$

we can take the energy denominator out of the $\mathbf{k}$ sums and obtain

$$\begin{aligned}
\sigma^{(2)}(q,\omega)=(\pi e^2/\omega\Omega m^2)\sum_K|V_K|^2\Bigg\{&-\left(\frac{4qm}{K}\frac{1}{(K+2m\omega/q)^2-q^2}\right)^2\sum_{\mathbf{k}}k_x^2\delta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}+q}+\omega)n_{\mathbf{k}}^-n_{\mathbf{k}+q}^+\\
&+\left(\frac{4qm}{K}\frac{1}{[2m\omega/(q+K)]^2-q^2}\right)^2\sum_{\mathbf{k}}k_x^2\delta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}+q+\mathbf{K}}+\omega)n_{\mathbf{k}}^-n_{\mathbf{k}+q+\mathbf{K}}^-\frac{\partial}{\partial\omega}\frac{4qm}{K}\frac{1}{(K+2m\omega/q)^2-q^2}\\
&\times\sum_{\mathbf{k}}k_x^2\delta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}+q}+\omega)n_{\mathbf{k}}^-n_{\mathbf{k}+q}^+\Bigg\}.
\end{aligned} \tag{22}$$

[11] J. Linhard, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **28**, 1 (1954).
[12] A. B. Pippard, *The Dynamics of Electrons* (Gordon and Breach Science Publishers, Inc., New York, 1964).

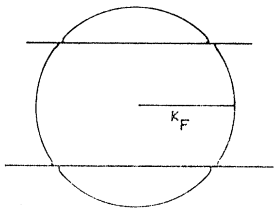

FIG. 1. Fermi surface for $2k_F>K$.

Expressing the $\mathbf{k}$ sums by σ^0, we can write

$$\sigma^{(2)}(q,\omega)=\sum_K |V_K|^2\left\{-\left(\frac{4qm}{K}\frac{1}{(K+2m\omega/q)^2-q^2}\right)^2\sigma^0(q,\omega)\right.$$
$$+\left(\frac{4qm}{K}\frac{1}{[2m\omega/(q+K)]^2-q^2}\right)^2\sigma^0(K+q,\omega)$$
$$\left.-\frac{\partial}{\partial\omega}\frac{4qm}{K}\frac{1}{(K+2m\omega/q)^2-q^2}\sigma^0(q,\omega)\right\}. \quad (23)$$

Now the following equations are deduced under conditions of the extreme anomalous skin effect, $m\omega/qk_F\ll 1$, $q\ll k_F$.

For $2k_F<K$, the second term of (23) vanishes and thus we obtain only a small correction to the free-electron result, as one would expect.

$$\sigma_<^{(2)}(q,\omega)\approx 16m^2\sigma^0(q,0)\sum_K |V_K|^2/K^4. \quad (24)$$

For $2k_F>K$, where the Fermi surface intersects the zone boundaries, as shown in Fig. 1, we find that the main contribution to $\sigma^{(2)}(q,\omega)$ comes from the second term in (23):

$$\sigma_>^{(2)}(q,\omega)=\frac{16m^2}{q^2}\sum_K\frac{|V_K|^2}{K^2}\sigma^0(K,0). \quad (25)$$

This is different from Pippard's result, where $\sigma\sim 1/q$. In the following, we confine our attention mainly to this case.

The singularities in the second term of (23) and in (25) with respect to q arise from the fact that transitions to and from intersections of bands are only poorly described by the response function calculated in I (If they are not forbidden by conservation laws or the Pauli principle). These intersections are not resolved in our calculation to give a band gap, if they occur behind the expansion parameter V_K.

We remove this difficulty by replacing the free-electron energies for $q<q_0=mV_K/K$ by an energy expression which takes the band gap and the effective masses explicitly into account. For $k_z\approx -K/2$, we write $\epsilon_k'=\epsilon_{-K/2}\pm V_K\pm(\mathbf{K}/2+\mathbf{k})^2/2m^*$, etc. Carrying out this procedure for $q<q_0$ we obtain for the factor in the second term of (23)

$$\left(\frac{2m^*\omega}{V_K^2}\right)^2\left(1-\frac{q^2}{mV_K}\right),$$

where we ignored values of q which are smaller than ω (in atomic units). This factor no longer has a singularity and may be of the order 1 if $\omega\approx V_K^2$.

The imaginary part of the conductivity can be obtained by means of the Kronig-Kramers relations. It turns out that it is small compared to the real part,[13] once the singularity in the second term is removed as described above.

V. SURFACE IMPEDANCE

Now we insert $\sigma(q,0)$ into (6) to calculate the surface impedance of the layer lattice. To this end we have to estimate the relative magnitudes in the denominator of $Z(0)$.

$$q^2-\omega^2/c^2-4\pi i\omega\sigma(q,0)/c^2. \quad (26)$$

First we neglect the second term, since $v_F\ll c$. The remaining terms are displayed qualitatively in Fig. 2.

For $2k_F<K$, we expand the integrand with respect to $4\pi\omega|\sigma_<^{(2)}|$ and obtain after a short calculation

$$Z_<(0)=\frac{8}{3\sqrt{3}}\left(\frac{\pi k_F\omega^2}{3ne^2c^4(1+16m^2\sum_K|V_K|^2/K^4)}\right)^{1/3}$$
$$\times(1-i\sqrt{3}), \quad (27)$$

which is very close to the free-electron value.

For $2k_F>K$, we estimate the intersections q_1 and q_2 in Fig. 2 in atomic units:

$$(1)\quad q_1^2=\frac{4\pi\omega}{c^2}\frac{3\pi}{4}\frac{ne^2}{m}\frac{1}{q_1V_F},$$

which gives

$$q_1=\left(\frac{3\pi^2}{137^2}\frac{m\omega}{v_F}\right)^{1/3}\approx 10^{-3}$$
$$(m=1,\ v_F\approx 1,\ \omega\approx 3\times 10^{-6}); \quad (28)$$

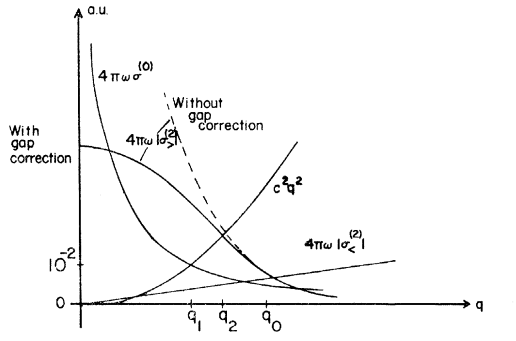

FIG. 2. Estimate of the order of magnitude of the various contributions to the integrand of $Z(0)$.

[13] S. Nakajima, Progr. Theoret. Phys. (Kyoto) 23, 694 (1960).

$$(2)\quad q_2^2=\frac{4\pi\omega}{c^2}\frac{16m^2}{q_2^2}\frac{2|V_K|^2}{K^2}\sigma(K,0),$$

which gives

$$q_2=\left(\frac{96\pi^2}{137^2}\omega|V_K|^2\right)^{1/4}=2\times10^{-2}|V_K|^{1/2}$$

$$[K=1,\ \sigma(K,0)=3\pi/4]; \quad (29)$$

$$(3)\quad q_0=|V_K|. \quad (30)$$

From Fig. 2 and the estimates of q_1, q_2, and q_0, we find the following results: For $q_2<q_1$, that is, when the energy gap $\Delta=2|V_K|$ is smaller than 0.1 eV and $\omega\approx10^{-4}$ eV, we have the well-known free-electron surface impedance. Then, for Δ about 0.1 eV and somewhat larger we expect the pure lattice contribution $\sigma_<^{(2)}$ to be dominating without much influence of the gap. For still larger values of Δ, the surface impedance finally will be mainly determined by the gap correction and the free-electron term. We are essentially interested in the intermediate range, i.e., in the influence of the pure lattice part. Inserting now (25) into (6), we obtain

$$Z_>(0)=\frac{8\omega}{ic^2}\times\int_0^\infty\frac{dq}{q^2-64\pi im^2\omega c^{-2}q^{-2}\sum_K|V_K|^2\sigma(K,0)/K^2}. \quad (31)$$

Small and large values of q apparently do not contribute to the integral. Evaluation of the integral gives

$$Z_>(0)=\left(\frac{\pi^3\omega^3}{c^6m\sum_K|V_K|^2\sigma(K,0)/K^2}\right)^{1/4}e^{-3\pi i/8}. \quad (32)$$

Our formula for the surface impedance of layer lattices for $2k_F>K$ and $\Delta\gtrsim0.1$ eV is similar in structure to that obtained by Reuter and Sondheimer. The order of magnitude of the impedance is expected to be the same as for free electrons. The frequency dependence of (32) is slightly different, which could be a good criterion in an experimental investigation. If the power of ω turns out experimentally for a layer lattice to be $\frac{3}{4}$, then (32) can be used as a means for determining $|V_K|$. This result could be of interest for a number of layer lattices, as, for instance, AuSn, which was studied by several authors.[14–16]

We expect that our results (27) and (32) still hold if we include electron-electron interactions, since it has been shown[9,17] at least for free-electron gas that these interactions do not contribute to the transverse conductivity in the zero-frequency limit and thus to the surface impedance.

ACKNOWLEDGMENTS

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APPENDIX

In a private communication, Glasser pointed out that it is possible to obtain $\sigma(\mathbf{q},\omega)$ for a layer lattice exactly without expanding it with respect to the lattice potential. One starts from the following expression for the conductivity[9]:

$$\sigma(\mathbf{q},\omega)=(2\pi e^2/m_x^2\omega\Omega)\sum_{\mathbf{k}}k_x^2\delta(\epsilon_{\mathbf{k}+\mathbf{q}}-\epsilon_{\mathbf{k}}-\omega)n_{\mathbf{k}}^-n_{\mathbf{k}+\mathbf{q}}^+, \quad (33)$$

and inserts $\epsilon_{\mathbf{k}}=k_x^2/2m_x+k_y^2/2m_y+E(k_z)$. In the zero-frequency limit, expansion of (33) with respect to small $\mathbf{q}$ in general yields a $1/q$ dependence of σ, which is in agreement with Pippard's expression. But in the case of nearly free electrons ($2k_F>K$) and for certain types of nonmonotonic shapes of the band structure (regions filled with electrons at the center of the zone and, separated from that, at the boundaries) a $1/q^2$ dependence is expected, which gives rise to the peculiar behavior of the surface impedance as discussed in Sec. V.

[14] J. P. Jan and W. B. Pearson, Phil. Mag. 8, 911 (1963).
[15] J. P. Jan, W. B. Pearson, A. Kjekshus, and S. B. Woods, Can. J. Phys. 41, 2522 (1963).
[16] D. J. Sellmyer and P. A. Schroder, Phys. Letters 16, 100 (1965).
[17] E. A. Stern, University of Maryland Physics Department, Technical Report No. 258, 1962 (unpublished).