project and gave advice and encouragement throughout its duration. Thanks are also due to the members of Professor Knight's research group, particularly Richard Noer and William Hines. Dr. Gareth Thomas allowed me to use his electron microscope and Larry Ernst instructed me in its use. Dr. James McAlear made available the ultramicrotome with which samples were sectioned, and my success with this instrument was due to the patience and guidance of Paula Finch and Ann Boardman. I was assisted in a small-angle x-ray scattering experiment through the availability of equipment and instruction by Professor Robert Wilde and Professor Robert Hewitt of the University of California at Riverside. George Gordon performed the large angle x-ray diffraction experiments for me. Dr. Robert Lindquist and Dr. Robert Lewis of the California Research Corporation gave a great deal of time and effort to produce Sample "L". Finally, it is a pleasure to acknowledge helpful discussions with Dr. James Garland, Dr. Joachim Appel, and Dr. Robert Hammond.

PHYSICAL REVIEW VOLUME 163, NUMBER 2 10 NOVEMBER 1967

Motion of Magnetic Flux through Superconducting Strips*

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(Received 26 June 1967)

A torsion pendulum was used to measure mechanically the energy dissipation occurring in strips and vacuum-deposited films of indium as a function of the velocity with which they moved through a magnetic field normal to their surfaces. No dissipation was observed either above or below T_c when the strips moved through a uniform field. Below T_c, energy dissipation occurred in both the strips and the films when they moved through localized fields, and it was an order of magnitude greater than in the normal state. The energy loss below T_c contains a part proportional to the velocity and a velocity-independent part. An analysis of the velocity-dependent part of the losses indicates good agreement with the theoretical considerations of Bardeen and Stephen.

INTRODUCTION

In recent years, numerous experiments[1–4] on the electrical resistance of superconducting strips in a magnetic field have been interpreted in terms of a motion of the magnetic field lines in a direction perpendicular to their length and somewhat perpendicular to the current. This motion is understood to be opposed by a constant force associated with the pinning of magnetic flux to pinning centers of an undetermined nature, plus a viscous resistance proportional to the velocity. Since all of these reported experimental results have been electrical measurements, and the motion of the flux lines has been only postulated to explain them, it seemed worthwhile to undertake a series of experiments in which the flux motion is mechanically induced. This paper is a report of one such set of measurements which shows both types of force postulated and shows a viscous force close to that calculated by Bardeen and Stephen.[5]

The experiment consisted in mounting two strips of indium radially on a phenolic disc $2\frac{7}{8}$ in. in diam. The disc was mounted as a torsion pendulum in a helium Dewar. Two pendulums were used. The periods of 23.0 and 19.6 sec were similar but the moments of inertia and the torsion constants were quite different. Two magnets, wound on nonmagnetic (glass) cores with superconducting wire, were then arranged as shown in Fig. 1 to provide a concentrated magnetic field through which the superconducting strips moved as the pendulum swung. The endpoints of each swing were observed so that the loss of energy could be determined from the difference in the squares of the endpoints and the value of the torsion constant.

It would be desirable, or course, to have a sharply discontinuous field, but in the absence of that possibility the field intensity $H(r)$ in the gap was measured as a function of the distance r from the center and could be well represented by $H(r) = H_0 \exp(-\alpha r^2)$, with $\alpha = 3.81$ cm^{-2}. This approximate form was then used in in the analysis because of its mathematical convenience.

Four different pairs of strips were used. Two were evaporated films $\frac{7}{8}$ in. long by $\frac{1}{8}$ in. wide. They were estimated to be respectively 10^3 and 8.5×10^3 Å thick, but the estimates may be in error by as much as a factor of 2. Two thicker strips were cut from sheet and were 0.0127 and 0.0508 cm thick, respectively.

* Supported in part by a grant from the National Aeronautics and Space Administration.

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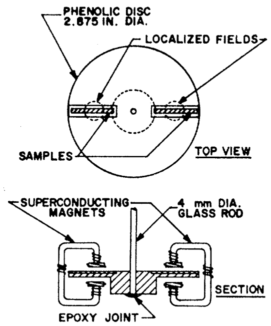

FIG. 1. Sketch of the arrangement of the torsion pendulum and the magnets in the equilibrium position.

RESULTS AND DISCUSSION

A number of preliminary observations[6] reported earlier served to identify the effects to be observed and to indicate how the apparatus could be improved to give more quantitative results. The improvements were principally in the design of the magnets to get a more concentrated field in the gap, and in the use of two different pendulum systems to provide a wider range of forces in moving the strips through the magnetic fields. The nature of the results was the same as in the earlier experiments, but the quantitative results reported here are all from the later series of measurements.

To be sure of the nature of the observed energy loss, observations were made with the 0.0127-cm and the 0.0508-cm thick strips in a uniform magnetic field parallel to the axis of the pendulum. This was done at both 4.2 and 3.0°K. No energy loss associated with the magnetic field was observed in either case. The only damping observed was that due to internal friction in the torsion fiber and the viscosity of the helium gas in which the system was suspended.

The result in the normal state is easily understood. The eddy currents flow from one end of the strip to the other, but are so small as to produce an unobservable damping.

The result in the superconducting state would be expected from the symmetry of the situation. And yet on the picture of flux lines passing through the strip it is less obvious. One must then conclude that if such "lines" or "bundles" of lines pass through the strips in certain spots, these bundles are not at all fastened to anything on the solenoid which produces the field.

An interesting observation in this connection was made when a uniform field was produced by a solenoid surrounding the whole apparatus and with the small magnets still in place. Even though no current was sent through these magnets, the superconductivity of the windings distorted the field enough so that an energy loss was observed. It was necessary to remove them completely before a true observation in a uniform field could be made. A relatively small departure from uniformity leads to an energy loss.

[6] W. V. Houston and D. R. Smith, Phys. Rev. Letters 16, 516 (1966).

Measurements made at 4.2°K on the indium films and the thicker strips with the localized fields showed no damping on the films but a damping of the thicker strips clearly attributable to eddy currents. No attempt was made to calculate the eddy current damping to be expected since the shape of the field and the shape of the strip make it complicated. The observed loss was roughly proportional to the square of the field, as is to be expected. The absence of observed eddy current damping in the thin films is due to the high resistance of such films.

At 3.0°K, well below the critical temperature for indium, the behavior is strikingly different from that in the normal state. If the pendulum has enough energy as the strip approaches the magnetic field, the strip is driven through the field and comes out on the other side with a reduced energy. As the amplitude of the pendulum falls off, there comes a point where the film or thicker strip cannot be driven through the field and it bounces back. This behavior can be understood in terms of a simplified model for the potential energy of the system.

The energy in the torsion fiber is $\frac{1}{2}Kx^2$. The energy associated with the magnetic field in the superconducting material is given by $-\mathbf{M}\cdot\mathbf{H}$ integrated over the volume of the strip. The magnetization may be idealized and approximated by the expression

$$-4\pi M = H_1(H_c - H)/(H_c - H_1), \tag{1}$$

where H_1 is that field below which the Meissner effect is perfect. For a sphere $H_1 = \frac{2}{3}H_c$, and for an oblate spheroid it would be smaller. For thin strips, such as those used in this work, it would be quite small. The expression is, of course, valid only for $H_1 < H < H_c$.

Assuming the above form for M, and the field in the magnet gap to be given by $H = H_0 \exp(-\alpha r^2)$, where r is the distance from the axis, the potential energy of the system is

$$V = (\tfrac{1}{2}Kx^2) + (1/2\pi\alpha)^{1/2}(\tfrac{1}{2}td)[H_1H_0^2/(H_c - H_1)] \times [2^{1/2}(H_c/H_0)\exp(-\alpha x^2) - \exp(-2\alpha x^2)], \tag{2}$$

when t is the thickness and d is the width of the superconducting strips; x is the distance from the axis of the field to the center line of the strip.

When K is large enough, the first term dominates and $x = 0$ is a minimum. If, however,

$$K < (\alpha/128\pi)^{1/2}[H_1H_c^2/(H_c - H_1)]td, \tag{3}$$

the center point, $x = 0$, is a maximum, and for a range of values of H_0 there are minima on either side of it. Thus one condition for a position of equilibrium away from the center of the magnetic field is that both α and H_1 be large enough. No independent estimate of H_1 has been made for the strips used. H_1 should be expected to be quite small and to increase with increasing thickness of the strips. The qualitative behavior of the system was quite in accord with this analysis of it.

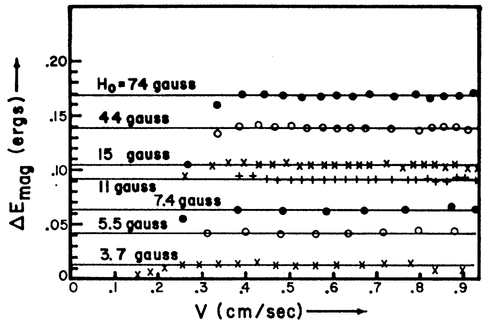

FIG. 2. Energy dissipation per swing as a function of the speed with which the 8500 Å films at 3°K passed through the field. The energy dissipation is independent of the speed but increases with the field intensity.

When the energy of the pendulum was sufficient to drive the strips through the magnetic field, a loss of energy was observed quite different from that observed at 4.2°K. It can be described as due to a constant resisting force plus a viscous force proportional to the velocity. Both forces act over a fixed distance, the width of the strip.

For the thin films the constant force was dominant. Figure 2 shows the energy loss per swing as a function of the average speed in passing through the field for the 8500 Å film. Figure 3 shows the loss as a function of magnetic field for the two films. It is clear that the force is not an eddy current damping, proportional to the square of the field. No such damping was observed at 4.2°K. Furthermore, the thinner and less perfect film showed the greater energy loss. This can be understood in terms of "pinning centers" from which the flux lines must be pulled away. There are more such centers in the less perfect film but in each case there is an approach to saturation as the centers are more nearly exhausted.

For the thicker strips, a velocity proportional resistance as well as a constant force was observed. Figure 4 shows the energy loss as a function of speed for a number of fields and Fig. 5 shows the coefficient of proportionality to the speed as a function of the field.

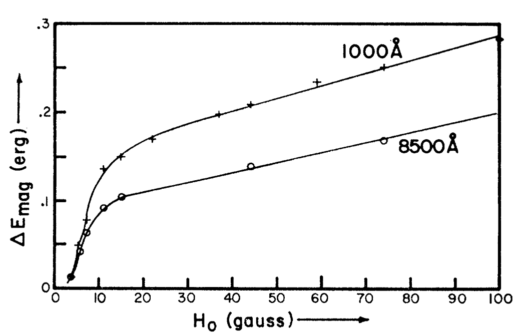

FIG. 3. Energy dissipation per swing as a function of the localized field strength for the 1000 and the 8000 Å films. The difference between the curves is attributed to the relative qualities of the films.

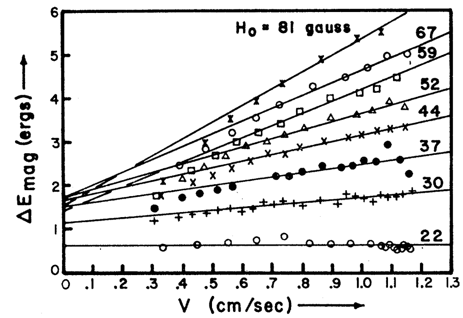

FIG. 4. Energy dissipation per swing as a function of speed for the 0.0508-cm strips at 3°K for various field strengths.

The velocity-dependent part of the energy loss, ΔE_1, may be described in terms of a viscosity coefficient η. If the flux is pictured as confined to bundles of circular cross section, the energy loss due to one bundle passing through the indium strip is given by

$$\Delta E_1 = \eta v_L t d, \tag{4}$$

when v_L is the speed at which the strip passes through the field, t is the thickness of the strip, and d is the width, the distance over which the force acts.

Since indium is a superconductor of the first kind, it is to be expected that the flux will pass through the strip in "spots" of radius much larger than the penetration depth λ. In these spots the field will be the critical field H_c. The total flux is then

$$\Phi = \pi H_c \sum_i a_i^2 = \pi H_c A^2, \tag{5}$$

where the a_i are the radii of different flux bundles. These are probably of the same order of magnitude, but are certainly not all the same. The sum is over the "spots" which pass over the strip as it moves through the field.

The total flux through which the strips must pass is given by

$$\Phi = H_0 \int_0^{r_1} \exp(-\alpha r^2) r dr = (\pi/\alpha)[H_0 - H_1], \tag{6}$$

where $H_1 = H_0 \exp(-\alpha r_1^2)$. H_1 is the field below which

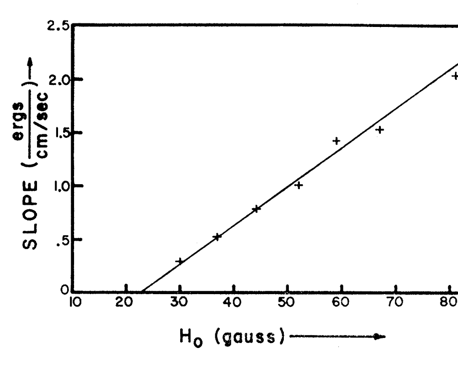

FIG. 5. Slopes of the energy dissipation-speed curves as a function of the field strength for the 0.0508-cm strips at 3°K.

TABLE I. Observed values of η/a^2.

T (°K)	t (cm)	H_c (G)	(η/a^2) (dyn sec cm^{-4})
3.0	0.0508	67.4	1.1×10^3
2.5	0.0508	141.5	7.7×10^3
3.0	0.0127	67.4	1.0×10^3

the Meissner effect is perfect. The value of A^2 is then

$$A^2=\sum_i a_i^2=(H_0-H_1)/\alpha H_c. \qquad (7)$$

Let η_i be the viscous drag per unit length of the flux bundle i and assume that $\eta_i/a_i^2=(\eta/a^2)$ is independent of a. Then the total energy loss when the strips pass through the magnetic field is

$$\Delta E_i=v_L 2td(\eta/a^2)\sum a_i^2$$
$$=v_L 2td(\eta/a^2)(H_0-H_1)/\alpha H_c. \qquad (8)$$

The slope of the curve in Fig. 5 gives $(2td/\alpha H_c)(\eta/a^2)$. The observed values of η/a^2 are given in Table I.

An understanding of the viscosity η can be obtained by following the procedure used by Bardeen and Stephen for the case of a superconductor of the second kind. As applied to a superconductor of the first kind, such as indium, a bundle of flux can be considered as passing through a cylindrical hole of radius a of normal material. Using the ideal case of a discontinuous transition between the normal and the superconducting material, the electric field, outside of the normal material and necessary to stop the supercurrents ahead of the moving cylinder and to start them behind it, is given by

$$\mathbf{E}=(m/e)\mathbf{v}_L\times\text{curl }\mathbf{v}_s-\text{grad}[(m/e)\mathbf{v}_L\cdot\mathbf{v}_s], \qquad (9)$$

where $\mathbf{v}_L$ is the velocity of the flux bundle relative to the material and $\mathbf{v}_s$ is the velocity of the electrons carrying the supercurrent around the hole. The latter can be evaluated in terms of H_c, the critical field assumed in the normal material. At the normal superconducting surface practically all this field is tangent to the surface. The field normal to the surface makes only a small correction to the field induced inside the normal region by the relative motion of the flux and the material. This field is then $-\mathbf{v}_L\times H_c/c$, and the power dissipation in the normal material is

$$P_1=\pi a^2 t\sigma E^2=\sigma\pi a^2 v_L^2 tH_c^2/c^2, \qquad (10)$$

where σ is the conductivity. The work done in moving the normal spot across the width of the strip is then

$$W=\sigma\pi a^2 tdv_L H_c^2/c^2=\eta v_L td \qquad (11)$$

or

$$(\eta/a^2)=\sigma\pi H_c^2/c^2=3.50\times10^{-21}\sigma H_c^2. \qquad (12)$$

It has not been possible to compare this result accurately with the observations because σ is not known for these particular strips. At temperatures near 3°K the resistance is largely residual resistance and is strongly dependent on the crystal imperfections as well as chemical impurities. However, an order-of-magnitude comparison has been made by computing the conductivities necessary to give the results of Table I. Averaging the first and last values together gives σ (3.0°K) = 4.45×10^{19} sec^{-1}. The other value gives σ (2.5°K) = 11.0×10^{19} sec^{-1}. If then the resistance is described as $\rho_0+\rho_1 T^5$, the ρ_0 and ρ_1 can be determined and the resistivity at 3.4°K determined to be 2.35×10^{-20} sec. The resistivity of indium is given by the tables in the Handbook of Chemistry and Physics (Chemical Rubber Publishing Company) to be 8.37×10^{-18} sec. The resistance ratio $\rho(3.4)/\rho(273)=0.0028$. An observation by Meissner and Voigt[7] in 1930 gave 0.0032. This agreement is enough to suggest the correctness of the analysis of Bardeen and Stephen. Although the thermodynamic critical field for In at 3.0°K is about 60 G, the damping measurements extended to values of H_0 well above 60 G. It could be that all the flux for $H>H_c$ passes through the strip in one large normal spot, and the remaining flux passes through in smaller spots with $H=H_c$. However, the measurements leave the radii of the normal spots undetermined.

The picture of lines of magnetic flux usually attributed to Faraday, and made much more significant by the discovery of flux quantization, is a useful model in many situations. A little consideration, however, and the almost completely to be expected results of the experiments in a uniform field, suggest that it needs to be used with discretion. As has been pointed out by others,[8] it is the rate of change of the field at a point in the material that is significant.

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[8] M. J. Stephen and H. Suhl, Phys. Letters **13**, 797 (1964).