

we obtain

$$U = \frac{h^2}{8ma^2} \sum_{s,t,u} \sum_{j=1}^{\infty} z^j (s^2 + t^2 + u^2) e^{-\mu j(s^2 + t^2 + u^2)}$$

$$\equiv \frac{3h^2}{8ma^2} \sum_{j=1}^{\infty} z^j \left[\sum_s s^2 e^{-\mu j s^2} \right] \left[\sum_t e^{-\mu j t^2} \right]^2. \quad (\text{A4})$$

The sums involved here can be evaluated by using Poisson's formula¹⁴

$$\sum_{s=-\infty}^{\infty} e^{-\mu j s^2} = \left(\frac{\pi}{\mu j} \right)^{1/2} \sum_{s=-\infty}^{\infty} e^{-\pi^2 s^2 / \mu j}, \quad (\text{A5})$$

and the one resulting from (A5) by a differentiation with respect to (μj) , namely,

$$\sum_{s=-\infty}^{\infty} s^2 e^{-\mu j s^2} = \left(\frac{\pi}{\mu j} \right)^{1/2} \sum_{s=-\infty}^{\infty} \left[\frac{1}{2} - \frac{\pi^2 s^2}{\mu j} \right] e^{-\pi^2 s^2 / \mu j}. \quad (\text{A6})$$

From (A5) and (A6) it follows that, in the limit $\mu \rightarrow 0$,

$$\sum_{s=1}^{\infty} e^{-\mu j s^2} \equiv \frac{1}{2} \left[\left(\frac{\pi}{\mu j} \right)^{1/2} \sum_{s=-\infty}^{\infty} e^{-\pi^2 s^2 / \mu j} \mp 1 \right]$$

$$\simeq \frac{1}{2} [(\pi/\mu j)^{1/2} \mp 1], \quad (\text{A5}')$$

¹⁴ See D. Menzel, *Fundamental Formulas of Physics*, (Dover Publications, Inc., New York, 1960), p. 77. This is also referred to as the θ -function transformation: see R. H. Fowler and H. Jones, Proc. Camb. Phil. Soc. 34, 573 (1938).

and

$$\sum_{s=1}^{\infty} s^2 e^{-\mu j s^2} \equiv \frac{1}{2} \sum_{s=-\infty}^{\infty} s^2 e^{-\mu j s^2} \simeq \frac{1}{4} [\pi/(\mu j)^3]^{1/2}. \quad (\text{A6}')$$

Accordingly, (A4) becomes

$$U \simeq \frac{3}{2} (V/\lambda^3) kT g_{5/2}(z) \mp \frac{1}{4} (A/\lambda^2) kT g_2(z), \quad (\text{A7})$$

where $\lambda = h/(2\pi m kT)^{1/2}$ and the function

$$g_n(z) = \sum_{j=1}^{\infty} j^{-n} z^j.$$

The free energy of the system can be calculated by means of the formula

$$F = -T \int_0^T \frac{U}{T^2} dT, \quad (\text{A8})$$

whence we finally get

$$F = (-V/\lambda^3) kT g_{5/2}(z) \pm \frac{1}{4} (A/\lambda^2) kT g_2(z). \quad (\text{A9})$$

For $T < T_\lambda$, $z \simeq 1$; then, for $n > 1$, $g_n(z) \simeq \zeta(n)$. The second term in (A9) thereby gives precisely the expression (2.11) for F_s/A .

Errata

Effect of Molecular Redistribution on the Non-linear Refractive Index of Liquids. R. W. HELLWARTH [Phys. Rev. 152, 156 (1966)]. We have discovered an important error in the sign of a term calculated in the Appendix. As a result, the question raised in this paper as to the accuracy of the Kirkwood superposition approximation for calculating certain configurational averages for liquids can now be answered: The approximation gives the wrong sign at liquid densities for the nonlinear free-energy term and also for other quantities whose signs are known on general grounds. Specifically, in Eq. (A17), for I_3 the factor $(l^2 + 2\pi^2/3)$ should be $(l^2 - 2\pi^2/3)$. Also, in (A21) the factor 0.5066 should be replaced by 0.922 and in (A22) the factor -0.117 should be replaced by -0.467 . We have re-examined the small correction K_4 and ascertained that its sign is negative, so that it contributes, though very little, to the worsening of the failure of the superposition approximation. With these correc-

tions, the superposition approximation gives, instead of the existing Eq. (38),

$$U_{22}/(NE^4 \rho^2 \alpha^4) = (l^2 + 2\pi^2/3)\Delta - 2\pi^2/3$$

$$+ \pi^2(32 + 136\Delta + 167\Delta^2)/(45\rho\tau)$$

$$+ 1.13\rho\tau(1 + K_4/4). \quad (\text{38})$$

From the definition (28b), U_{22} must be positive. For symmetric molecules ($\Delta = 0$), the right-hand side is seen to fail to stay positive as the density ρ increases beyond $1.6/\tau$, where $\tau/8$ is the molecular volume. Normal liquid densities run between $2/\tau$ and $3/\tau$. The above change in U_{22} requires that the expression (41) for n_2 be altered to

$$n_2 = \frac{\pi\rho\alpha^2\beta}{n(1 - 4\pi\rho\alpha/3)^4} \left[\Delta + \frac{4\pi^2\rho\alpha^2}{45\tau} (32 + 136\Delta + 167\Delta^2) \right.$$

$$\left. + \frac{8\pi^2\rho^2\alpha^2(\Delta - 1)}{3} + 4.51\alpha^2\rho^3\tau \right], \quad (\text{41})$$

which should also always be positive. The first two columns of Table I, which were calculated with the wrong Eq. (41), have no significance, except to indicate roughly the relative magnitude of the factor $\rho\alpha^3\beta n^{-1}(1-4\pi\rho\alpha/3)^{-4}$, which will exist even when a more accurate method than the superposition approximation is found.

We have looked for other obvious failures of the Kirkwood approximation and have found several. Kirkwood [J. Chem. Phys. 4, 592 (1963)] calculated corrections to the Clausius-Mossotti formula which, in the limit of small polarizability α , would be exact for all densities if one used exact correlation functions instead of the superposition approximation. These corrections change from positive to negative as the density increases beyond around $2/\tau$. This contradicts the demonstration in the present paper that the Clausius-Mossotti formula is a lower bound for the dielectric constant. Another typical failure of the superposition approximation occurs in the calculation of the positive quantity $\langle \sum_{\alpha\beta\gamma} D_{xx}^{\alpha\beta} D_{xx}^{\beta\gamma} \rangle / N - \langle \sum_{\alpha\beta} D_{xx}^{\alpha\beta} \rangle^2 / N^2$ for which it gives [with Eq. (37)] $64\pi^2\rho^2/(45\rho\tau) - 2\pi^2\rho^2/3$, a negative result when $\rho > 32/15\tau$. The failure cannot be traced to the fact that with Eq. (A1), $\int p_4 d\mathbf{r}_4 / V$ deviates slightly from p_3 , etc.

The following typographical errors should be noted. A minus sign should be inserted before one side of (3). In the exponent of the right-hand side of Eq. (24), " $\mu_2 - \mu_1$ " should read " $u_2 - u_1$ ". In the second line of the first paragraph, second column, of p. 161, " $\frac{1}{4}\tau$ " should read " $\frac{1}{8}\tau$ ". The term $4\langle u_2 \rangle^2$ in Eq. (A19) and the left-hand side of Eq. (A23) should be divided by $\alpha^4 E^4$.

The conclusion is reached that there is no consistent usable approximation scheme for computing the configuration averages required to estimate the contribution of molecular redistribution to the nonlinear index n_2 of liquids. However, Eq. (41), which comes from the Kirkwood superposition approximation, should be accurate for molecular densities below those at which the inconsistencies discussed above appear.

Collisional Depolarization of the Rb 5p and Cs 6p Doublets. ALAN GALLAGHER [Phys. Rev. 157, 68 (1967)]. Burkhard Brehm has pointed out a very direct explanation of the lack of $J = \frac{1}{2}$ depolarization in the adiabatic approximation: In the adiabatic approximation one can evaluate the interaction at each fixed atomic position, then integrate the "stationary interaction" matrix elements throughout the collision. But if one considers the interatomic axis as a "stationary" axis of quantization, then

the $C_{\infty v}$ symmetry of the system requires that the alkali m_J are eigenvalues of the total Hamiltonian, and $\langle m_J | H | m_{J'} \rangle = \delta_{m_J m_{J'}} f(|m_J|)$. Consequently, the matrix of the interaction between only $m_J = \pm \frac{1}{2}$ states will be a multiple of the unit matrix, and it remains a multiple of the unit matrix under any rotation. Thus the interaction is diagonal between the laboratory frame m_J state when $J = \frac{1}{2}$, but not for $J > \frac{1}{2}$ because $f(|m_J|)$ is not independent of $|m_J|$.

Density, Coefficient of Thermal Expansion, and Entropy of Compression of Liquid He⁴ under Pressure below 1.4°K, C. BOGHOSIAN AND H. MEYER [Phys. Rev. 152, 200 (1966)]. Table I. The density $\rho(O, P)$ extrapolated to 0°K should read

$P = 10.9$	15.2	20.0	24.5	atm
$\rho = 0.1603$	0.1648	0.1691	0.1728	g/cm ³

Table V. The compressibility $k_T(T=0)$ expressed in 10^{-2} atm⁻¹ should be

$P = 15$	20	24	atm
$k_T(O) = 0.59$	0.52	0.46	

The other values are unaffected.

The temperature dependence of the compressibility [$k_T(T, P) - k_T(O, P)$] is expressed in 10^{-6} atm.

Acoustic Wave Mode in a Weakly Ionized Gas, UNO INGARD AND MICHAEL SCHULZ [Phys. Rev. 158, 106 (1967)]. In Eq. (13) $\Omega_n = (\omega_{ni} + \omega_{ne})/\omega$ should read $\Omega_n = \omega_{ni} + \omega_{ne}$. In Eq. (20) a factor v_n is missing on the right-hand side. In Eq. (22) a factor v_n is missing and a factor ω^2 should be removed on the right-hand side. Four lines below Eq. (22) $\omega_{in} \simeq c_n \sigma_{in} N_n$ should read $\omega_{in} \simeq c_n \sigma_{in} N_n$. In the expression for R in the Appendix, x should be replaced by δ .

Theory of Transport Processes in Nearly Ferromagnetic Fermi Liquids, MICHAEL J. RICE [Phys. Rev. 159, 153 (1967)]. The following algebraic error has been detected: the symbol \bar{I} should be absent from the numerator of Eq. (3.3). Consequently \bar{I}^3 appearing in Eqs. (3.13) and (3.18) should be replaced by \bar{I}^2 , and \bar{I}^2 in Eqs. (4.8) and (A.4) by \bar{I} . Since \bar{I} is always close to unity these replacements cause insignificant change in the corresponding numerical calculations.

Because of a numerical error made in evaluating Eq. (2.22) the expression (3.18) for α_D should contain an extra factor of $\frac{1}{4}$. Consequently the calculated value of α_D should be $\alpha_D = 2.075 \times 10^5$ cgs units.