Symmetric Quark Model of Baryon Resonances*

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(Received 9 June 1967)

The symmetric quark model of baryon resonances is applied to the $(\mathbf{56},0^+)$, $(\mathbf{70},1^-)$, and $(\mathbf{20},1^+)$ supermultiplets. Using a systematic $SU(6)$ analysis, octet dominance, and dominance of two-body contributions to the mass operator, the Gürsey-Radicati mass formula for the $(\mathbf{56},0^+)$ is derived without use of perturbation theory. An equal-spacing relation is derived for the $SU(6)$-symmetric mass contribution for the $(\mathbf{56},0^+)$, $(\mathbf{70},1^-)$, and $(\mathbf{20},1^+)$ in ascending order, with the $(\mathbf{20},1^+)$ lying above 2 BeV. A detailed analysis of the $(\mathbf{70},1^-)$, which yields a quantitative fit for the spin-orbit-split negative-parity resonances, is made, with the result that the magnitude of the octet spin-orbit term is about six times greater than the singlet one. The masses and mixing amplitudes for all the resonances in the $(\mathbf{70},1^-)$ are calculated, and it is pointed out that because of the large mixings for $J^P=\frac{1}{2}^-$ and $\frac{3}{2}^-$, the Gell-Mann–Okubo mass formula cannot be expected to hold for these resonances, so that there is no sense in trying to group them into octets and decuplets.

1. INTRODUCTION

THE introduction of quarks[1] or other triplets as fundamental constituents of hadrons has given a new unifying point of view to particle physics. This notion of fundamental triplets achieved striking success when combined with the idea of approximate-spin and unitary-spin symmetry of the interactions relevant to the low-lying hadronic states.[2] This $SU(6)$ theory[3] has led to a number of striking results which can be obtained in an elementary way using the quark model[4]: classification of baryon and meson supermultiplets,[2] mass formulas for these multiplets,[2] magnetic moment ratios for baryons,[5] and scattering of hadrons at high energy.[6] The underlying idea is that the observed hadrons are bound states of the quarks or other triplets. Up to now, a naive bound-state model in which the triplets are assumed to move nonrelativistically has been surprisingly successful, even though at present the model has no fundamental justification. One of the surprising features which has appeared from the study of the baryons is that the $SU(6)$ wave function of the baryons is totally symmetric.[2] Since ordinarily the ground state of a composite system has all particles in the lowest s state, the symmetry under permutations of the $SU(6)$ wave function indicates an apparent symmetric statistics for the triplets in the ground state of the baryons. Evidence against an antisymmetric space wave function for the ground state of the baryons comes from the study of the saturation of triplets bound in hadronic states[7] and from the consideration of the electromagnetic form factors of the proton and the neutron[8]; further, the form factors provide specific evidence for a symmetric ground-state space wave function, since the proton form factors and the neutron magnetic form factor remain positive in the range of momentum transfers in which they have been measured[8–10] (up to 245 F^{-2}).[11]

Two proposals have been put forward to explain the apparent violation of Fermi statistics in the ground state of the baryons. One of these is that the quarks are parafermions of order three,[12] in which case three quarks can be in a symmetric state under permutations and the composite object which is a bound state of three such quarks is a Fermi particle. This theory seems to be equivalent to a theory of three indistinguishable Fermi triplets. (One could also have a theory with three distinguishable fractionally charged Fermi triplets.) For the paraquark theory the triplets must have the usual quantum numbers associated with the quarks, for ex-

* This work was reported in an invited talk at the meeting of the American Physical Society in Washington, D. C. in April, 1967.
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1 M. Gell-Mann, Phys. Letters **8**, 214 (1964); G. Zweig, CERN Reports 8182/TH.401 and 8419/TH.412, 1964, unpublished.

2 F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964); B. Sakita, Phys. Rev. **136**, B1756 (1964).

3 A. Pais [Rev. Mod. Phys. 38, 215 (1966)] and F. J. Dyson [*Symmetry Groups in Nuclear and Particle Physics* (W. A. Benjamin, Inc., New York, 1966)] contain reviews, reprints, and bibliographies.

4 R. H. Dalitz [in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, California, 1967), pp. 215–236] gave a survey of quark models in August, 1966.

5 M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964); B. Sakita, *ibid.* **13**, 643 (1964).

6 E. M. Levin and L. L. Frankfurt, JETP Pis'ma v Redaktsiyu **2**, 105 (1965) [English transl.: JETP Letters **2**, 65 (1965)]; H. J. Lipkin and F. Scheck, Phys. Rev. Letters **16**, 71 (1966).

7 O. W. Greenberg and D. Zwanziger, Phys. Rev. **150**, 1177 (1966).

8 A. N. Mitra and R. Majumdar, Phys. Rev. **150**, 1194 (1966).

9 R. E. Kreps and J. J. de Swart [University of Pittsburgh Report NYO-3829-4 (unpublished)] quote a theorem of L. K. Pandit and V. S. Mathur that the form factor must have at least one zero for any antisymmetric wave function. Kreps and de Swart argue that the zero in the form factor can be made to occur at arbitrarily large momentum transfer by proper choice of the antisymmetric wave function, and exhibit a 4-parameter antisymmetric wave function which leads to a form factor which fits the present data (see Ref. 10).

10 S. Ishida, K. Konno, and H. Shimodaira [Nuovo Cimento **46A**, 189 (1966)] found form factors which fit the present data using a one-parameter symmetric wave function.

11 W. Albrecht *et al.*, Phys. Rev. Letters **17**, 1192 (1966).

12 O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964). The supermultiplets $(2L+1)\,\dim SU(6)$ are labelled $[\dim SU(6), L^P]$ in the present article.

ample, fractional charge. The second proposal to explain the apparent violation of Fermi statistics is a three-triplet model[13] in which a new three-valued degree of freedom is introduced and there are nine fundamental particles. If one assumes that a new $SU(3)$ group called $SU(3)''$ acts on the new three-valued degree of freedom, then the lowest-lying baryons, those presently known, can be placed in the singlet of this $SU(3)''$ with an $SU(3)''$ wave function $\epsilon^{A''B''C''}$, A'', B'', $C''=1, 2, 3$. For this state each of the three $SU(3)''$ degrees of freedom occurs with equal probability and this degree of freedom is essentially averaged out. The quantum numbers of the triplets in these $SU(3)''$ singlet states are effectively replaced by their average values, which are those of the particles in the quark model. Thus this model has the attractive feature that the charges of the triplets are integral, but they have effectively fractional values in the observed baryons. [Similar remarks hold for the $SU(3)''$ singlet mesons.] The three-triplet model allows a construction of particles in the **10*** and other representations which cannot be obtained from the quark model using three-body states. Both the para-quark and three-triplet models introduce the number three explicitly as the maximum number of quarks[14] in a state which is symmetric under permutations. For most purposes in which the independent quarks do not appear singly these models are equivalent. We will refer to a model in which the three triplets in a baryon are symmetric under permutations as the *symmetric quark model.*

We abstracted this model from the properties of the ground state of the baryons. There are two possibilities to consider as candidates for the higher baryonic states: orbital excitation of quarks and $SU(6)$ excitation in which quark-antiquark pairs are added. It seems to be in keeping with the nonrelativistic picture used for the ground state to assume that quark-antiquark excitation is not important; in addition, it is attractive to use a model in which only three objects occur and to make use of the analogy of such a model to the systems studied in atomic and nuclear physics. In addition, with one exception, the Z_0^+ resonance which occurs in K^+N scattering,[15] there is no firm evidence for a resonance which does not fit into those $SU(3)$ multiplets which can be obtained from three quarks: namely, the singlet, octet, and decuplet. In this article we will study the orbital excitation model for the higher states.[12]

[13] Y. Nambu, in *Preludes in Theoretical Physics,* edited by A. de-Shalit, H. Feshbach, and L. Van Hove (North-Holland Publishing Company, Amsterdam, 1966), pp. 133–142; M. Y. Han and Y. Nambu, Phys. Rev. **139**, B1006 (1965); A. Tavkhelidze, in *Proceedings of the Seminar on High-Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965), pp. 763–779, and references cited therein.

[14] We will use the word "quark" for a particle which has an effectively fractional charge in the $SU(3)''$ singlet, as well as for a particle whose charge is really fractional.

[15] R. L. Cool *et al.*, Phys. Rev. Letters **17**, 102 (1966).

In order to restrict the numbeı of *a priori* possible mass formulas for the supermultiplets of hadronic states, we will assume that the terms in the mass formulas come from only two sources: one-body effects which can be associated with the masses of the quarks, and two-body effects which can be associated with two-body interactions among the quarks. (Our actual analysis makes use of a parametrization of the mass of the observed resonances in terms of one- and two-body operators and might not require so concrete an identification of the terms.) Within this framework we make the most general analysis which is compatible with approximate $SU(6)$ symmetry and octet dominance. In particular, our derivation of mass formulas does not make use of perturbation theory.

The baryon resonances are of special interest for two reasons. First, the interesting question of symmetric statistics arises only for the baryons; the mesons look the same in the symmetric-quark model as in the Fermi quark model. Secondly, the simplifying assumption that one- and two-body effects dominate holds no advantage for the mesons which are two-body systems.

We carry out our analysis using artificial single-particle space states for the quarks. This simplifying assumption makes it easier to handle the symmetrization of the states and probably does not lead to serious error provided that certain pitfalls, such as excitation of center-of-mass motion (the "spurious states" of nuclear physics) are avoided. We think that this simplifying assumption is appropriate for a phenomenological analysis such as we are making.

For the higher states, the $SU(6)$ approximate symmetry which has been so successful for the ground state should be augmented by an *L-S* coupling scheme,[12] which sometimes goes by the name $SU(6)\times O(3)$. We have adopted this classification of the higher states and have also used the *L-S* coupling point of view in the analysis of the mass operator. It is convenient to refer to the supermultiplets in this model by expressions (dim $SU(6)$, L^P). Thus the ground state is $(\mathbf{56},0^+)$ and the lowest negative-parity baryon resonances are $(\mathbf{70},1^-)$. We are mainly interested in these negative-parity resonances in this article; however, we will also consider the $(\mathbf{20},1^+)$ because the parameters necessary to derive a mass formula for this supermultiplet are already completely determined by the $(\mathbf{56},0^+)$ and the $(\mathbf{70},1^-)$.

To analyze the one- and two-body operators, we make systematic use of the $SU(6)$ approximate symmetry. From this point of view we can find the most general one- and two-body operators, and classify them according to their spin dependence and according to irreducible representations of $SU(6)$. Experience in analyzing the masses of baryons and mesons indicates that the lowest $SU(6)$ representations are most important,[16] and we will make use of this fact in choosing

[16] H. Harari and M. A. Rashid, Phys. Rev. **143**, 1354 (1966).

which operators to retain for our calculation.[17] Having found the relevant operators, we will normalize them in such a way that the parameters which are found from the experimental data can be compared. The values of the different parameters should satisfy certain physically reasonable conditions which can serve as constraints to be used in addition to the question of whether the data can be fitted.

We will classify the one- and two-body operators here, but defer to Secs. 2 and 3 a detailed derivation of the mass formulas. We label the operators $T_{\dim SU(6)}{}^{\dim SU(3)}$. Since a single particle is in a **6** of $SU(6)$, the most general operator acting on a single particle must be in

$$\mathbf{6}\times\mathbf{6}^*=\mathbf{1}+\mathbf{35}.$$

Only $S=0$ operators can act on a single particle and thus the only operators available are $T_1{}^1$ and $T_{35}{}^8$, which, of course, essentially come from the central mass and the mass splitting between the nonstrange and strange quarks. [We didn't need this argument to find such a simple result, but gave it to indicate how this $SU(6)$ analysis will work.] For the two-body operators the most general state is in

$$\mathbf{6}\times\mathbf{6}=\mathbf{21}_{\rm sym}+\mathbf{15}_{\rm anti}.$$

The most general operators[18] acting on a symmetric space state are in

$$\mathbf{21}\times\mathbf{21}^*=\mathbf{1}+\mathbf{35}+\mathbf{405}.$$

The $S=0$ operators here are $T_1{}^1$, $T_{35}{}^8$, $T_{405}{}^1$, and $T_{405}{}^8$. The first two operators cannot be distinguished from the corresponding one-body operators in a system with a fixed number of particles and thus do not contribute anything new. The four $S=0$ operators acting on spatially symmetric two-body states lead precisely to the Gürsey-Radicati mass formula for the $(\mathbf{56},0^+)$, as we will show in Sec. 2. The same parameters determined by the $(\mathbf{56},0^+)$ are used in the $(\mathbf{70},1^-)$. The parameters, when properly normalized, are reduced two-body matrix elements. For the $(\mathbf{70},1^-)$, we must also consider the $S=0$ operators acting on the antisymmetric space state; these occur in

$$\mathbf{15}\times\mathbf{15}^*=\mathbf{1}+\mathbf{35}+\mathbf{189}.$$

The $S=0$ ones are $T_1{}^1$, $T_{35}{}^8$, $T_{189}{}^1$, and $T_{189}{}^8$. The four new terms introduced here have parameters which must be found from the experimental data. The spin-orbit interactions can act only on the antisymmetric two-body space wave function, since the relative angular momentum is antisymmetric under the permutations of the two particles. Thus only the $S=1$ operators in $\mathbf{15}\times\mathbf{15}^*$ can contribute, namely $T_{35}{}^1$, $T_{35}{}^8$, and $T_{189}{}^8$. Tensor forces can only act on the symmetric space state and thus occur only in $T_{405}{}^1$ and $T_{405}{}^8$.

The two-body dominance assumption leads to significant simplification. For example, from the standpoint of abstract $SU(6)$ the mass operator can occur in[16]

$$\mathbf{56}\times\mathbf{56}^*=\mathbf{1}+\mathbf{35}+\mathbf{405}+\mathbf{2695},$$

while from the two-body point of view the representation **2695** cannot occur. Similar simplifications occur for the states with $L>0$.

Section 2 gives a nonperturbative derivation of the Gürsey-Radicati mass for the $(\mathbf{56},0^+)$. Section 3 gives the analysis of the two-body operators for s- and p-wave particles; Sec. 4 gives the fit of the particles in the $(\mathbf{70},1^-)$, and Sec. 5 gives a summary and outlook for future work on this model.

2. DERIVATION OF THE GÜRSEY-RADICATI MASS FORMULA

We use a simple formalism with Bose operators to derive mass formulas in the symmetric-quark model. Let $a_\alpha{}^\dagger$ and a^α [$\alpha=Aa$, $A=1,2,3$ for $SU(3)$, $a=1,2$ for $SU(2)_S$,[19] where S stands for ordinary spin] be a set of Bose creation and annihilation operators for s-wave quarks.[20] In terms of these operators, the generators of $SU(6)$ are

$$I_\alpha{}^\beta=a_\alpha{}^\dagger a^\beta-\tfrac{1}{6}\delta_\alpha{}^\beta N,\quad N=a_\gamma{}^\dagger a^\gamma.$$

Relevant subgroups of $SU(6)$ are given in Table I, where $N=N_n+N_\lambda$ is the quark-number operator, $N_n=a_{\bar{A}c}{}^\dagger a^{\bar{A}c}$ and $N_\lambda=a_{3c}{}^\dagger a^{3c}$ are the nonstrange and strange quark-number operators, respectively, and S_n and S_λ refer to the corresponding quark spins. The reduction chains which occur are $SU(6)\to SU(3)\times SU(2)_S$, and $SU(6)\to SU(4)\times SU(2)_{S_\lambda}$ with $SU(4)\to SU(2)_I\times SU(2)_{S_n}$.[21]

TABLE I. Generators of relevant subgroups of $SU(6)$.

Subgroup	Generators
$SU(3)$	$I_A{}^B=I_{Ac}{}^{Bc}=a_{Ac}{}^\dagger a^{Bc}-\frac{1}{3}\delta_A{}^BN$
$SU(2)_S$	$S_a{}^b=I_{Ca}{}^{Cb}=a_{Ca}{}^\dagger a^{Cb}-\frac{1}{2}\delta_a{}^bN$
$SU(2)_I$	$\mathcal{I}_{\bar{A}}{}^{\bar{B}}=I_{\bar{A}}{}^{\bar{B}}-\frac{1}{2}\delta_{\bar{A}}{}^{\bar{B}}I_{\bar{E}}{}^{\bar{E}}=a_{\bar{A}c}{}^\dagger a^{\bar{B}c}-\frac{1}{2}\delta_{\bar{A}}{}^{\bar{B}}Y$
$U(1)_Y$	$Y=I_{3c}{}^{3c}=I_{\bar{E}c}{}^{\bar{E}c}=I_{\bar{E}}{}^{\bar{E}}=\frac{1}{3}(N_n-2N_\lambda)$
$SU(4)$	$I_{\bar{A}a}{}^{\bar{B}b}-\frac{1}{4}\delta_{\bar{A}}{}^{\bar{B}}\delta_a{}^bY$
$SU(2)_{S_n}$	$I_{\bar{A}a}{}^{\bar{B}b}-\frac{1}{2}\delta_a{}^bY$
$SU(2)_{S_\lambda}$	$I_{3a}{}^{3b}+\frac{1}{2}\delta_a{}^bY$

[17] We keep all operators for (spin) $S=0$, and those in the lowest $SU(6)$ irreducible (**35**) for $S=1$.

[18] Conservation of parity prohibits operators connecting symmetric- and antisymmetric-space states.

[19] We will always use small Greek letters for $SU(6)$, capital Latin letters for $SU(3)$, and small Latin letters for $SU(2)_S$. Later we will use capital barred Latin letters for $SU(2)_I$. Repeated indices are to be summed.

[20] S-wave quarks suffice for the Gürsey-Radicati mass formula. When we discuss the $(\mathbf{70},1^-)$ in Sec. 3, we will extend the formalism to include p-wave quarks. Particles with higher l and higher principal quantum number can be treated in a similar way.

[21] This formalism is that given by M. A. B. Bég and V. Singh, Phys. Rev. Letters **13**, 418 (1964), translated into second quantized form using Bose operators.

Some other relevant objects are the Casimir operators for various $SU(n)$ groups. For example,

$$C_2^{(6)}=I_\alpha{}^\beta I_\beta{}^\alpha,\quad C_2^{(3)}=I_A{}^B I_B{}^A,$$
$$C_2^{(2)}(S)=S_a{}^b S_b{}^a=2S(S+1),$$
$$C_2^{(2)}(I)=\mathcal{I}_{\bar{A}}{}^{\bar{B}}\mathcal{I}_{\bar{B}}{}^{\bar{A}}=I_{\bar{A}}{}^{\bar{B}}I_{\bar{B}}{}^{\bar{A}}-\tfrac{1}{2}Y^2=2I(I+1),$$

where we added a distinguishing letter in those cases where the same $SU(n)$ group enters more than once.

Now we derive the Gürsey-Radicati formula. We ignore the one-body operators since they do not give anything which will not occur in the two-body operators. The two-body operators all have the form $a_\alpha{}^\dagger a_\beta{}^\dagger M_{\gamma\delta}{}^{\alpha\beta}a^\gamma a^\delta$, where the coefficients M must be determined to make the operator lie in given $SU(6)$ and $SU(3)$ irreducibles, satisfy octet dominance,[22] and have $S=0$. The method of construction of these operators is elementary and so we will just list them in Table II, together with their form in terms of quantum numbers.[23] The numerical matrix $y_A{}^B$ is diagonal and has diagonal elements $\frac{1}{3}$, $\frac{1}{3}$, and $-\frac{2}{3}$. The Gürsey-Radicati formula for the $(\mathbf{56},0^+)$ results follows:

$$M=M_0+M_1Y+M_2C_2^{(3)}+M_3[I(I+1)-\tfrac{1}{4}Y^2]. \quad (1)$$

The Casimir operator $C_2^{(3)}$ can be eliminated in favor of $J(J+1)$ [$J=S$ in the $(\mathbf{56},0^+)$] to give a more familiar form of the Gürsey-Radicati mass formula.[24,25]

The Gürsey-Radicati mass formula predicts the following equalities:

$$N+\Xi=\tfrac{1}{2}(3\Lambda+\Sigma), \quad (2)$$

the Gell-Mann–Okubo formula for the octet;

$$\Omega-\Xi^*=\Xi^*-Y_1{}^*=Y_1{}^*-N^*=\Xi-\Sigma, \quad (3)$$

the equal spacing for the decuplet, and a relation between decuplet and octet. The average of the first three mass differences in Eq. (3) is 146 MeV and the last number is 124 MeV, so the best one can do in fitting the $(\mathbf{56},0^+)$ is to get within 11 MeV of all the masses.

TABLE II. Forms of the two-body operators.

Operator	Second quantized form	Form in terms of quantum numbers
$T_1{}^1$	$a_\alpha{}^\dagger a_\beta{}^\dagger a^\alpha a^\beta$	$N(N-1)$
$T_{35}{}^8$	$y_B{}^A a_{Ac}{}^\dagger a_\mu{}^\dagger a^{Bc}a^\mu$	$(N-1)Y$
$T_{405}{}^1$	$a_{Aa}{}^\dagger a_{Bb}{}^\dagger a^{Ba}a^{Ab}-(5/7)a_\mu{}^\dagger a_\nu{}^\dagger a^\mu a^\nu$	$C_2^{(3)}-(8/21)N^2-(16/7)N$
$T_{405}{}^8$	$y_B{}^A[a_{Aa}{}^\dagger a_{Cb}{}^\dagger a^{Ca}a^{Bb}-\frac{7}{8}a_{Ac}{}^\dagger a_\mu{}^\dagger a^{Bc}a^\mu]$	$I(I+1)-\frac{1}{4}Y^2-\frac{1}{6}C_2^{(3)}-(5/24)(N+3)Y$

3. ANALYSIS OF TWO-BODY FORCES FOR *s*- AND *p*-WAVE PARTICLES

With artificial single-particle space states, at first sight the following two-body parameters enter for *s*- and *p*-wave particles: *s*-*s*, $(s\text{-}p)_{\text{sym}}$, $(p\text{-}p)_{\text{sym},\,l=0}$, $(p\text{-}p)_{\text{sym},\,l=2}$, $(s\text{-}p)_{\text{anti}}$, and $(p\text{-}p)_{\text{anti}}$. Since with relative coordinates only two kinds of relative space states among *s*- and *p*-wave particles occur: $l=0$, which is symmetric in space, and $l=1$ which is antisymmetric in space, we must reduce the number of two-body parameters in some way. The simplest way to make this reduction is to identify all the $l=0$ cases and all the $l=1$ cases. The $l=2$ case does not contribute to the $(\mathbf{70},1^-)$ and $(\mathbf{20},1^+)$. This procedure has the virtues of being simple and of not introducing any more parameters than one would have in the more careful treatment with relative states. With this assumption the two-body parameters determined from the $(\mathbf{56},0^+)$ and the $(\mathbf{70},1^-)$ completely determine the $(\mathbf{20},1^+)$. The predictions about the $(\mathbf{20},1^+)$ which can then be made are a good check on this assumption; however, we expect the results for the $(\mathbf{70},1^-)$ to be more accurate than those for the $(\mathbf{20},1^+)$. Table III gives a summary of the parameters used and the resonances predicted.

To derive the mass formula for *s*- and *p*-wave particles, it is convenient to introduce Bose operators $c_{\alpha i}{}^\dagger$ and $c^{\alpha i}=(c_{\alpha i}{}^\dagger)^\dagger$, $i=0,1,2,3$, with

$$c_{\alpha i}{}^\dagger=a_\alpha{}^\dagger,\quad i=0$$
$$\quad\;\;=b_{\alpha i}{}^\dagger,\quad i=1,2,3,$$

where the $a^\dagger$'s create *s*-wave particles as before and the

TABLE III. Summary of the parameters used and the resonances predicted.

Term	Fixed by	No. of parameters	No. of resonances
Symmetric, $S'=0$	$(\mathbf{56},0^+)$	4	8 in **56**, $L=0^+$
Antisymmetric, $S=0$	$(\mathbf{70},1^-)$	4	{ 30 in **70**, $L=1^-$
Spin-orbit	$(\mathbf{70},1^-)$	2	{ 11 in **20**, $L=1^+$

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[22] Octet dominant operators either are a singlet under $SU(3)$ or lie in an octet and have $I=Y=0$.

[23] We give one sample of the manipulations leading from the second quantized form listed to the form in terms of the quantum numbers of the observed particles. Consider $T_{405}{}^8$. The irreducible **405** must be symmetric separately in upper and lower indices, and traceless:

$$(T_{405}{}^8)_{\alpha\beta}{}^{\gamma\delta}=a_\alpha{}^\dagger a_\beta{}^\dagger a^\gamma a^\delta-\tfrac{1}{8}(\delta_\alpha{}^\gamma\eta_\beta{}^\delta+\delta_\alpha{}^\delta\eta_\beta{}^\gamma+\delta_\beta{}^\delta\eta_\alpha{}^\gamma+\delta_\beta{}^\gamma\eta_\alpha{}^\delta)+(1/56)(\delta_\alpha{}^\gamma\delta_\beta{}^\delta+\delta_\alpha{}^\delta\delta_\beta{}^\gamma)\eta,$$

where $\eta=a_\mu{}^\dagger a_\nu{}^\dagger a^\mu a^\nu$ and $\eta_\alpha{}^\beta=a_\alpha{}^\dagger a_\mu{}^\dagger a^\beta a^\mu$. The $I=Y=0$ member of the $S=0$ octet is $Y_B{}^A(T_{405}{}^8)_{Aa,\,Cb}{}^{Ca,\,Bb}$, which simplifies to the form listed. The first term of the form listed is

$$y_B{}^A a_{Aa}{}^\dagger a_{Cb}{}^\dagger a^{Ca}a^{Bb}$$
$$=\tfrac{1}{2}y_B{}^A(a_{Aa}{}^\dagger a^{Ca}a_{Cb}{}^\dagger a^{Bb}-3\delta_b{}^a a_{Aa}{}^\dagger a^{Bb}+a_{Cb}{}^\dagger a^{Bb}a_{Aa}{}^\dagger a^{Bb})$$
$$=\tfrac{1}{2}y_B{}^A[I_A{}^C+\tfrac{1}{3}\delta_A{}^C N,\,I_C{}^B+\tfrac{1}{3}\delta_C{}^B N]_+-\tfrac{3}{2}Y.$$

The result listed is obtained by using the identity

$$I_A{}^C I_C{}^A=I_{\bar{A}}{}^{\bar{E}}I_{\bar{E}}{}^{\bar{A}}+[I_A{}^3,I_3{}^{\bar{A}}]_++(I_3{}^3)^2$$

and adding the second term.

[24] Using the Bose operators, it is straightforward to show that $C_2^{(3)}=C_2^{(2)}(S)+\frac{1}{6}N^2+N$ for symmetric representations of $SU(6)$. For $N=3$, this reduces to $C_2^{(3)}=2S(S+1)+\frac{9}{2}$, valid for the **56**. This last result was first given by Bég and Singh, Ref. 21.

[25] P. Federman, H. R. Rubenstein, and I. Talmi [Phys. Letters **22**, 208 (1966)] studied the **56** from the point of view of two-body dominance, but did not use the $SU(6)$ symmetry systematically.

$b^{\dagger}$'s create p-wave particles with $i=1, 2, 3$ corresponding to magnetic quantum number -1, 0, 1, respectively. The generators are similar to those given above, with the c's replacing the a's. For example, the generators of $SU(6)$ are

$$I_\alpha{}^\beta = c_{\alpha i}{}^\dagger c^{\beta i} - \tfrac{1}{6}\delta_\alpha{}^\beta N, \quad N = c_{\mu i}{}^\dagger c^{\mu i}.$$

The one-body operator is

$$M_n N_n + M_\lambda N_\lambda = (\tfrac{2}{3}M_n + \tfrac{1}{3}M_\lambda)N + (M_n - M_\lambda)Y,$$

where in a naive model M_n and M_λ would be the nonstrange and strange quark masses. The $S=0$ terms can again be manipulated into the form of quantum numbers; however now quantum numbers for both the $SU(3)$ and $SU(4)$ chains occur, and the observed resonances are, in general, not eigenstates in either chain. In Table IV we list the two-body operators in terms of quantum numbers. The second quantized forms from which they were derived can be written down easily in analogy with those given above for the (**56**,0^+). The new superscript on the left indicates the $SU(6)$ two-body state on which they act. The $S=0$ mass formula which results from the terms given in Table IV is

$$M = \sum M_i \mathfrak{N}_i T_i,$$

where the normalization factors

$$\mathfrak{N}_i = (T_{i,\max} - T_{i,\min} + 1)^{-1}$$

are chosen to make all the terms contribute in comparable ways in the two-body system; the M's are then reduced two-body matrix elements. Note that for $T=I_3$, the third component of the isospin, the factor becomes the standard $(2I+1)^{-1}$. These normalization factors are, in the order given in Table IV: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, 9/28, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{2}{5}$. Notice that for a symmetric representation the terms acting on the **21** reduce to those considered in Sec. 2, and the terms acting on the **15** vanish. For later numerical work it is convenient to write the $S=0$ mass formula in the form[26,27]

$$\begin{aligned} M = N_0 &+ N_1 Y + N_2[C_2^{(3)} + 2S(S+1)] \\ &+ N_3[I(I+1) - \tfrac{1}{4}Y^2 + S_n(S_n+1) - S_\lambda(S_\lambda+1)] \\ &+ N_4(C_2^{(6)} - 45/2) + N_5[Y - \tfrac{3}{4} + I(I+1) - \tfrac{1}{4}Y^2 \\ &- S_n(S_n+1) + S_\lambda(S_\lambda+1)] \\ &+ N_6[C_2^{(3)} - 2S(S+1) - \tfrac{9}{2}] \\ &+ N_7[C_2^{(4)} - 2S_\lambda(S_\lambda+1) - \tfrac{1}{4}Y^2 - 8Y - 15/2], \end{aligned} \quad (4)$$

where N_4 through N_7 inclusive vanish for the (**56**,0^+).

[26] Except for the terms with $C_2^{(6)}$, these terms occur in Bég and Singh, Ref. 21. Our two-body dominance assumption provides a rationale for the particular choice of $SU(6)$ irreducible operators made by Bég and Singh. We also relate different $(SU(6),L)$ supermultiplets. Bég and Singh [Phys. Rev. Letters **13**, 509 (1964)] discuss the **70** in abstract $SU(6)$ (without orbital excitation).

[27] A. W. Hendry [Nuovo Cimento 48, 780 (1967)] studied the **56**, **70**, and **20** in a way similar to Federman *et al.*, Ref. 25.

TABLE IV. The two-body operators in terms of quantum numbers.

Operator	Form in terms of quantum numbers
${}^{21}T_1{}^1$	$\frac{1}{2}[C_2^{(6)} + (7/6)N^2 - 7N]$
${}^{21}T_{35}{}^8$	$\frac{1}{4}[C_2^{(4)} - 2S_\lambda(S_\lambda+1) - \frac{1}{4}Y^2 - \frac{1}{3}C_2^{(6)}] + 2(\frac{1}{3}N - 1)Y$
${}^{21}T_{405}{}^1$	$\frac{1}{2}[C_2^{(3)} + 2S(S+1) - (5/7)C_2^{(6)}]$
${}^{21}T_{405}{}^8$	$\frac{1}{2}[I(I+1) - \frac{1}{4}Y^2 + S_n(S_n+1) - S_\lambda(S_\lambda+1)] - \frac{1}{12}[C_2^{(3)} + 2S(S+1)] - \frac{7}{32}[C_2^{(4)} - 2S_\lambda(S_\lambda+1) - \frac{1}{4}Y^2 - \frac{1}{3}C_2^{(6)}] + 21/4(N/3 - 1)Y$
${}^{15}T_1{}^1$	$\frac{1}{2}(-C_2^{(6)} + \frac{5}{6}N^2 + 5N)$
${}^{15}T_{35}{}^8$	$-\frac{1}{4}[C_2^{(4)} - 2S_\lambda(S_\lambda+1) - \frac{1}{4}Y^2 - \frac{1}{3}C_2^{(6)}] + (\frac{1}{3}N + 1)Y$
${}^{15}T_{189}{}^1$	$\frac{1}{2}[C_2^{(3)} - 2S(S+1)] - \frac{1}{10}C_2^{(6)}$
${}^{15}T_{189}{}^8$	$\frac{1}{2}[I(I+1) - \frac{1}{4}Y^2 - S_n(S_n+1) + S_\lambda(S_\lambda+1)] - \frac{1}{12}[C_2^{(3)} - 2S(S+1)] + \frac{1}{16}[C_2^{(4)} - 2S_\lambda(S_\lambda+1) - \frac{1}{4}Y^2 - \frac{1}{3}C_2^{(6)}]$

The N_i are related to the one- and two-body parameters introduced above by

$$\begin{aligned} \tfrac{2}{3}M_n + \tfrac{1}{3}M_\lambda - (21/8)({}^{21}M_1{}^1) + (45/8)({}^{15}M_1{}^1) &= N_0 - (45/2)N_4 - \tfrac{3}{4}N_5 - \tfrac{9}{2}N_6 - (15/2)N_7, \\ M_n - M_\lambda + \tfrac{2}{3}({}^{15}M_{35}{}^8) &= N_1 + N_5 - 8N_7, \\ ({}^{21}M_1{}^1) - ({}^{15}M_1{}^1) &= 4N_4 + (20/7)N_2 + (10/21)N_3 + (2/15)N_5 + \tfrac{4}{5}N_6 + \tfrac{4}{3}N_7, \\ ({}^{21}M_{35}{}^8) - ({}^{15}M_{35}{}^8) &= 12N_7 + (21/4)N_3 - \tfrac{3}{2}N_5, \\ ({}^{21}M_{405}{}^1) &= 10N_2 + (5/3)N_3, \\ ({}^{21}M_{405}{}^8) &= (56/9)N_3, \\ ({}^{15}M_{189}{}^1) &= 10N_6 + (5/3)N_5, \\ ({}^{15}M_{189}{}^8) &= 5N_5. \end{aligned}$$

Note that only the **405** and **189** two-body parameters can be separated individually from the one-body parameters.

The $SU(6)$-symmetric terms $\alpha + \beta C_2^{(6)}$, above, lead to equal spacing among the (**56**,0^+), the (**70**,1^-) and the (**20**,1^+) in increasing order.

The spin-orbit two-body operators cannot conveniently be put in the form of quantum numbers in the $SU(3)$ or $SU(4)$ chains because the spin-orbit operator breaks the $SU(6)$ L-S coupling model and has off-diagonal terms.[28] These terms can be evaluated either directly from their second quantized expressions, which we give below, or using standard $3j$ and $6j$ techniques. This is the only place in which the states in the (**70**,1^-) are needed; since these states are easily written down, we will not tabulate them here. We computed these independently by both methods as a check on the calculation. We kept the $S=1$ operators in the **35**, since this is the smallest irreducible of $SU(6)$ from which spin-orbit operators can be constructed. Recalling that spin-orbit operators act only on the **15**, and using the

[28] We keep the off-diagonal matrix elements of the spin-orbit operators in the (**70**,1^-); however, we drop the matrix elements of these operators to other supermultiplets (configuration mixing).

standard $l=1$ matrices,[29] we have[30]

$$T_{L\cdot S}{}^1=(b_{Aai}{}^\dagger a_\gamma{}^\dagger-b_{\gamma i}{}^\dagger a_{Aa}{}^\dagger)(b^{Abj}a^\gamma-b^{\gamma j}a^{Ab})\sigma_b{}^a\cdot l_j{}^i, \quad (5)$$

and

$$T_{L\cdot S}{}^8=y_B{}^A(b_{Aai}{}^\dagger a_\gamma{}^\dagger-b_{\gamma i}{}^\dagger a_{Aa}{}^\dagger)\times(b^{Bbj}a^\gamma-b^{\gamma j}a^{Bb})\sigma_b{}^a\cdot l_j{}^i. \quad (6)$$

We write the total spin-orbit operator

$$N_8T_{L\cdot S}{}^1+N_9T_{L\cdot S}{}^8, \quad (7)$$

and give these operators in the $SU(3)$ chain, in the Appendix.

The $S=2$ (tensor forces) act only on the symmetric space states and occur only in the **405** of $SU(6)$. We drop these terms here because there is not enough data at present to evaluate them. It is worth pointing out that only these terms resolve the degeneracy between the $J=\frac{1}{2}$ and $\frac{3}{2}$, Δ and Ω decuplet resonances; conversely, the splittings of those resonances will determine the octet-dominant tensor forces.

4. ANALYSIS OF DATA FOR RESONANCES IN THE (70,1⁻)

The first step in the analysis is to determine where negative-parity baryon resonances with given J, I, and Y quantum numbers can fit in the $(\mathbf{70},1^-)$. The possible placement of resonances is shown in Table V, where the expression under placement is $(\dim SU(3), \dim SU(2)_S)$. Since there are not yet enough uniquely placed resonances known experimentally, we used the following experimental indications to place some of the non-unique resonances: the quartet octet lies higher than the doublet octet, and the doublet octet lies higher than the singlet.[31] We used the table of Rosenfeld *et al.*[32] for the experimental data. Their table lists nine particles which they consider well determined, whose quantum numbers are such that they could fit in the $(\mathbf{70},1^-)$; however, two of these might be S-wave threshold effects. The remaining seven particles are listed in Table VI, assigned as described above. It should be emphasized that later

TABLE V. Possible placement of resonances in $(\mathbf{70},1^-)$.

Resonance	J^P	Placement	Unique?
N, Λ, Σ, Ξ	$\frac{5}{2}^-$	(8,4)	Yes
Δ, Ω	$\frac{1}{2}^-, \frac{3}{2}^-$	(10,2)	Yes
N	$\frac{1}{2}^-, \frac{3}{2}^-$	(8,2), (8,4)	No
Λ	$\frac{1}{2}^-, \frac{3}{2}^-$	(1,2), (8,2), (8,4)	No
Σ, Ξ	$\frac{1}{2}^-, \frac{3}{2}^-$	(8,2), (10,2), (8,4)	No

[29] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Publishing Company, Inc., New York, 1955), p. 146.

[30] These expressions are valid for the $(\mathbf{70},1^-)$ where the spin-orbit terms act on an $(s\text{-}p)_{\rm anti}$ space state.

[31] Strictly speaking, this holds only for the N and Λ for which the two octets mix (together with the singlet for the Λ) but the decuplet does not enter.

[32] A. H. Rosenfeld, *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

TABLE VI. Well-determined resonances in the $(\mathbf{70},1^-)$.

Resonance	J^P	Placement	Unique?
$\Lambda(1405)$	$\frac{1}{2}^-$	(1,2)	No
$\Lambda(1519)$	$\frac{3}{2}^-$	(1,2)	No
$N(1525)$	$\frac{3}{2}^-$	(8,2)	No
$N(1570)$	$\frac{1}{2}^-$	(8,2)	No
$N(1670)$	$\frac{5}{2}^-$	(8,4)	Yes
$\Delta(1670)$	$\frac{1}{2}^-$	(10,2)	Yes
$\Sigma(1768)$	$\frac{5}{2}^-$	(8,4)	Yes

on we will allow mixing among all the resonances that can mix.

For the symmetric space parameters we kept the parameters that are determined by the **56**. We determined the antisymmetric space parameters as follows. First we neglected spin-orbit forces and estimated the $S=0$ antisymmetric space parameters from the central masses of the L-S multiplets. We used $N(1525)$, $\Delta(1670)$, $\Sigma(1768)$, and estimated the fourth parameter by assuming $M_{189}{}^8/M_{189}{}^1\simeq M_{405}{}^8/M_{405}{}^1$. Since there are more resonances than parameters, we predicted the remaining particles from the parameters we had chosen, neglecting, of course, spin-orbit terms. These predictions, which took account of the $SU(3)$ mixing between the Λ's in (**1**,2) and (**8**,2), agreed rather well with the experimental data. We then estimated the two spin-orbit parameters from the large splitting between the singlet Λ's with spin $\frac{1}{2}$ and $\frac{3}{2}$, keeping the $SU(3)$ mixings which had been determined in the earlier step for these Λ's. We used these spin-orbit parameters to predict the spin-orbit splittings in the quartet octet; the results were encouraging but could be improved by adjusting the spin-orbit parameters, which we did. At this step of the calculation, we programmed all of the particles in the $(\mathbf{70},1^-)$ including diagonalizations of all the mixings [both $SU(3)$ and spin mixings]. We arranged the calculation so that inserting the six parameters to be varied gave the masses of all the isospin multiplets in the $(\mathbf{70},1^-)$. In order to find the best set of parameters, we introduced a figure of merit equal to the sum of the squares of the differences between the calculated and experimental masses for the seven well-established particles. We then searched, using the computer, for the values of those six parameters which minimized the figure of merit. During this search, we found no evidence for the existence of local minima; however, we did not attempt to exclude this possibility in a systematic way. The final figure of merit was 286, which means that in the least-squared sense the theoretical values are on the average 6.4 MeV from the experimental values. In fact, the worst fit particle, the $\Lambda(1520)$, was 11 MeV away from the theoretical value and contributed almost half of the total figure of merit. By comparison with the accuracy of the Gürsey-Radicati formula for the $(\mathbf{56},0^+)$ this fit is good. We want to emphasize again that *a priori* there is no

TABLE VII. Calculation versus experiment for **(70,1⁻)**. Asterisk stands for well-established input resonances; question mark stands for resonances less well established; superscript M stands for resonances mixed by more than 20%. The left columns are calculated masses. The right columns are experimental masses.

	^4P				^2P	
	$J=\frac{5}{2}$	$\frac{3}{2}$	$\frac{1}{2}$		$\frac{3}{2}$	$\frac{1}{2}$
				Ω	2031	2031
				Ξ	1851^M, 1815?	1850^M
Ξ	1853	1705^M	1701^M	Σ	1759^M	1692^M
Z	1763, 1768*	1572^M	1612^M	Δ	1676	1676, 1670*
Λ	1763	1808	1815			
N	1672, 1670*	1744	1784	Ξ	1954^M, 1933?	1799^M
				Σ	1798^M	1827^M
				Λ	1635, 1682?	1705^M, 1670?
				N	1520, 1525*	1563, 1570*
				Λ	1531, 1519*	1402^M, 1405*

guarantee that one could fit the intricate spin-orbit splitting of the particles with only two spin-orbit parameters. Table VII lists the calculated masses along with the well-established experimental particles and some other particles that are not as well-established.[33]

TABLE VIII. Mixing amplitudes for resonances in the **(70,1⁻)**. The N's are essentially unmixed.

Particle (mass) (J)	Amplitudes: (S,SU(3))		
	($\frac{1}{2}$,1)	($\frac{1}{2}$,8)	($\frac{3}{2}$,8)
Λ(1402)($\frac{1}{2}$)	0.76	−0.65	0.02
Λ(1705)($\frac{1}{2}$)	−0.62	−0.71	0.35
Λ(1815)($\frac{1}{2}$)	0.21	0.27	0.94
Λ(1531)($\frac{3}{2}$)	−0.99	0.14	0.08
Λ(1635)($\frac{3}{2}$)	−0.10	−0.94	0.33
Λ(1808)($\frac{3}{2}$)	0.12	0.32	0.94
	($\frac{1}{2}$,10)	($\frac{1}{2}$,8)	($\frac{3}{2}$,8)
Σ(1612)($\frac{1}{2}$)	−0.63	0.03	0.78
Σ(1692)($\frac{1}{2}$)	0.70	0.47	0.54
Σ(1827)($\frac{1}{2}$)	0.35	−0.88	0.32
Σ(1572)($\frac{3}{2}$)	−0.67	0.37	0.64
Σ(1759)($\frac{3}{2}$)	0.72	0.54	0.44
Σ(1798)($\frac{3}{2}$)	0.19	−0.75	0.63
Ξ(1701)($\frac{1}{2}$)	−0.57	−0.31	0.76
Ξ(1799)($\frac{1}{2}$)	0.35	0.74	0.57
Ξ(1850)($\frac{1}{2}$)	0.74	−0.60	0.31
Ξ(1705)($\frac{3}{2}$)	−0.70	0.06	0.71
Ξ(1851)($\frac{3}{2}$)	0.60	−0.50	0.63
Ξ(1954)($\frac{3}{2}$)	0.39	0.87	0.31

[33] V. S. Bhasin, D. L. Katyal, and A. N. Mitra [Phys. Rev. **161**, 1546 (1967)] also studied the **(70,1⁻)** in the symmetric quark model. Although they used the ideas of two-body and octet dominance, they did not try to find the most general mass operator with these properties, and did not make an $SU(6)$ analysis of their operators. They did not find a quantitative fit to the well-established resonances. They assigned N(1570) to **(8,4)** citing Dalitz's analysis, Ref. 4, and the decay analysis of Mitra and Ross, Ref. 34, while we assign it to **(8,2)** on the basis of its mass. The **(8,4)** assignment for N(1570) depresses the predicted masses of the other resonances.

Note that none of the undiscovered resonances have mass less than 1550 MeV. We want to call particular attention to the following predictions: a $\Sigma(\frac{3}{2}^-)$ at 1572 MeV, an accidentally degenerate $\Lambda(\frac{5}{2}^-)$ at the same mass as the known $\Sigma(\frac{5}{2}^-)$ at 1768 MeV, and $\Xi(\frac{1}{2}^-)$ and $\Xi(\frac{3}{2}^-)$ at 1701 and 1705 MeV, respectively. This calculation at the same time produced the mixing amplitudes which are tabulated in the $SU(3)$ basis[34,35] in Table VIII. We remind the reader that the mixing probabilities are the squares of these numbers. The mixing amplitudes can be compared with experiment using branching ratios for various decays.

Our analysis resulted in values of the mass parameters which we introduced. Those for the antisymmetric space terms are generally comparable in magnitude but smaller than the similar parameters for the symmetric space terms. The results are[36]

$$({}^{21}M_1{}^1)-({}^{15}M_1{}^1)=-291.5\text{ MeV},$$

$$({}^{21}M_{35}{}^8)-({}^{15}M_{35}{}^8)=-22.9\text{ MeV},$$

$$M_{405}{}^1=199\text{ MeV},\quad M_{405}{}^8=121\text{ MeV},$$

$$M_{189}{}^1=36.8\text{ MeV},\quad M_{189}{}^8=-103\text{ MeV}.$$

The spin-orbit parameters, which for convenience are normalized using the diagonal terms of their contributions to the observed resonance states in the $SU(3)$ basis, are

$$M_{L\cdot S}{}^1=35.4\text{ MeV},\quad M_{L\cdot S}{}^8=-189\text{ MeV}.$$

It is interesting to note that the octet term has opposite sign and is about six times larger in magnitude than the singlet term.

We emphasize that the strong mixture among $J^P=\frac{1}{2}^-$ and $\frac{3}{2}^-$ resonances in the **(**$SU(3)$, $SU(2)_S$**)** basis implies that the Gell-Mann–Okubo mass formula will not be valid, and that it makes no sense to try to group these particles into octets or decuplets.

[34] A. N. Mitra and M. Ross [Phys. Rev. **158**, 1630 (1967)] studied decays of the **(70,1⁻)** in the symmetric quark model. We will make a few comments about such decays, even though we have not made a systematic analysis of them. Mitra and Ross find a kinematic effect enhancing S-wave decay into high-mass mesons (κ and η over π) which provides a mechanism to break $SU(3)$ in decays; however, they do not introduce an explicit octet-dominant $SU(3)$-violating term in the decay interaction. We suggest that such $SU(3)$-violating terms be allowed, and that their magnitude, relative to the $SU(3)$-conserving ones be determined when the experimental data is sufficient. The spin-orbit terms which we found give one example where the (octet-dominant) $SU(3)$-violating terms are larger than the $SU(3)$-conserving ones. (See footnote 35.)

[35] G. B. Yodh [Phys. Rev. Letters **18**, 810 (1967)] analyzed the decays of Λ(1519) assuming strict $SU(3)$ symmetry and found a conflict between the singlet assignment and the experimental branching ratios. Our analysis suggests that an $SU(3)$-violating term in the decay interaction is necessary to resolve this conflict [Yodh's interpretation (b)].

[36] The N_i introduced in Eqs. (4) and (7) are, in order (and in MeV): 997.5, −168.1, 16.69, 19.45, −79.95, −20.51, 7.1, −12.98, 2.08, and −26.99.

5. SUMMARY AND OUTLOOK

We studied the symmetric quark model of baryons (in which the baryons are composites of three quarks with wave functions symmetric in the visible quantum numbers) with orbital excitation of quarks using approximate phenomenological $SU(6)$ symmetry with spin-orbit coupling to resolve the degeneracy among resonances with the same L and S but different J. To reduce the number of possible mass formulas, we assumed dominance of two-body contributions to the mass operator in addition to the octet-dominance which is usually a part of an approximately $SU(3)$-symmetric theory. We showed that octet- and two-body dominance yield the Gürsey-Radicati mass formula for the $(\mathbf{56},0^+)$, thereby giving a derivation of this formula free of perturbation theory. The same principles, together with dominance of spin-orbit operators lying in the lowest possible $SU(6)$ irreducible, lead to a ten-parameter unified mass formula for the 49 isospin multiplets lying in the $(\mathbf{56},0^+)$, $(\mathbf{70},1^-)$, and $(\mathbf{20},1^+)$. The large $SU(6)$ symmetric terms in this formula obey an equal spacing rule which places the $(\mathbf{20},1^+)$ above 2 BeV, so that the $N(1400)$ resonance cannot be placed in the $(\mathbf{20},1^+)$. We found a quantitative fit for the seven well-established negative-parity baryon resonances which can be placed in the $(\mathbf{70},1^-)$, and predicted the remaining resonances in this supermultiplet. For this fit the octet spin-orbit term was (in magnitude) about six times as large as the singlet one (and opposite in sign).

A table of higher supermultiplets in the symmetric quark model with orbital excitation was given earlier.[12] Here we emphasize that the $(\mathbf{56},1^-)$ is spurious and does not occur in the quark model, so that the $(\mathbf{56},2^+)$ is the next **56** above the $(\mathbf{56},0^+)$, just as in the Regge theory. Several authors have pointed out that the $\Sigma(2035)$ and $\Delta(1920)$ fit in the ${}^4D_{7/2}$, and $\Sigma(1910)$, $\Lambda(1820)$ and $N(1688)$ fit in the ${}^2D_{5/2}$ of this supermultiplet. Only tensor forces split the degenerate J multiplets here. It seems premature to make a detailed study of the mass formula for this case.

We point out an interesting regularity among nucleon resonances which deserves further study. It is well-known that there are two series of resonances of opposite parity: $\Delta((\frac{3}{2}+2n)^+)$, $n=0, 1, \cdots 4$, and $N((\frac{3}{2}+2n)^-)$, $n=0, 1, \cdots 3$, where the number in parentheses is J. In the symmetric quark-quark model these opposite parity series can be united into one family. Place the Δ's in the series $(\dim SU(6), L^P) = (\mathbf{56}, (2n)^+)$, $n=0, 1, \cdots 4$, and the N's in the series $(\mathbf{70}, (2n+1)^-)$.[37] Then a plot of M^2 versus L shows that both series lie on a single straight line with equation

$$M^2 = 1.46 + 1.09L \text{ in } (\text{BeV})^2.$$

This regularity suggests that something like the exchange degeneracy which was found for mesons[38] may also occur for baryons.

When more supermultiplets have been analyzed, a systematic analysis of the reduced two-body parameters classified by $SU(6)$ may be useful in revealing regularities in the interactions.

APPENDIX: SPIN-ORBIT OPERATORS

We list in Tables IX and X the (real symmetric) matrices of the spin-orbit operators $T_{L\cdot S^1}$ and $T_{L\cdot S^8}$ in the $SU(3)$ chain. The properly normalized spin-orbit operator is

$$(1/17)M_{L\cdot S^1}T_{L\cdot S^1} + (1/7)M_{L\cdot S^8}T_{L\cdot S^8},$$

with

$$M_{L\cdot S^1} = 17N_8, \quad M_{L\cdot S^8} = 7N_9.$$

TABLE IX. Matrix elements of $T_{L\cdot S^1}$.

	$J=\frac{5}{2}$	$\frac{3}{2}$	$\frac{1}{2}$		$J=\frac{3}{2}$	$\frac{1}{2}$
(8,4)	6	-4	-10			
(10,2)		0	0	$\langle 8,4\vert T_{L\cdot S^1}\vert 8,2\rangle$	$-\sqrt{10}$	-2
(8,2)		2	-4			
(1,2)		4	-8			

TABLE X. Matrix elements of $T_{L\cdot S^8}$. $T_{L\cdot S^8}=0$ on the Δ and Ω states. $T_{L\cdot S^8}=\frac{1}{3}T_{L\cdot S^1}$ on the nucleon states.

Basis states		$J=\frac{3}{2}$			$J=\frac{1}{2}$	
$\Lambda(8,4)$	$-\frac{2}{3}$	$-\frac{2}{3}\sqrt{10}$	$-\frac{1}{3}\sqrt{10}$	$-5/3$	$-\frac{4}{3}$	$-\frac{2}{3}$
$\Lambda(8,2)$		--	$5/3$		$\frac{4}{3}$	$-10/3$
$\Lambda(1,2)$			0			0
$\Sigma(8,4)$	$\frac{2}{3}$	$\frac{2}{3}\sqrt{10}$	$-\sqrt{10}$	$5/3$	$\frac{4}{3}$	-2
$\Sigma(8,2)$		$\frac{2}{3}$	-1		$-\frac{4}{3}$	2
$\Sigma(10,2)$			0			0
$\Xi(8,4)$	$\frac{3}{2}$	$-\frac{1}{3}\sqrt{10}$	$-\sqrt{10}$	$5/3$	$-\frac{2}{3}$	-2
$\Xi(8,2)$		$-\frac{4}{3}$	-1		$8/3$	2
$\Xi(10,2)$			0			0

[37] We ignore complications due to admixture of spurious states here.

[38] R. C. Arnold, Phys. Rev. Letters 14, 657 (1965).