Higher Symmetries for Vector-Meson–Baryon Couplings from Superconvergence Relations*†

GIOVANNI VENTURI‡
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois
(Received 15 June 1967)

Superconvergence relations for vector-meson–baryon scattering are saturated with the baryon octet (B) and decuplet (B^*). The sum rules are selected by the infinite-momentum limit and evaluated at zero momentum transfer. Using $SU(3)$ invariance at the vertices and retaining terms to all orders in μ^2/M^2 (μ is the vector-meson mass and M is the baryon mass), a nontrivial solution is obtained for all vector-meson–baryon couplings. For degenerate octet and decuplet masses, this solution is consistent with collinear $U(6)$ symmetry for the vertices and agrees with the corresponding result previously obtained by Oehme on neglecting corrections of order μ^2/M^2 at the **8–10** and **10-10** vertices. If the octet and decuplet mass splittings are retained in a subset of our relations, the deviations from collinear $U(6)$ invariance at the vertices are obtained. Consideration of the ρB scattering processes using only isospin invariance at the vertices and ρ dominance of the isovector form factors leads to broken collinear $U(6)$ relations between the magnetic moments of the baryons. The question of higher intermediate states in the saturation is discussed briefly.

I. INTRODUCTION

SUPERCONVERGENT sum rules for vector-meson–baryon scattering have been previously derived.[1] Complete saturation of these sum rules with an octet of spin-$\frac{1}{2}^+$ (B) and a deciment of spin-$\frac{3}{2}^+$ (B^*) particles gives a consistent set of relations for vector-meson–baryon couplings. Using $SU(3)$ invariance for the vertices as an input, it was shown that these relations have a unique solution corresponding to collinear $U(6)$ symmetry for the vertices. In obtaining this result, the following approximation was made: In the amplitudes involving **8–10** and **10–10** transitions, corrections of the order $O(\mu^2/M^2)$ (μ is the meson mass and M is the baryon mass) were neglected and an $M1$ transition used at the **8–10** vertex.

The purpose of this paper is to examine the results obtained from the superconvergence relations avoiding any approximation in the evaluation of the truncated sum rules. A larger number of equations is obtained, and, in general, the solutions for the extra terms in the general vertices are such as to make the relations, which were previously obtained to lowest order in μ^2/M^2, valid to all orders. Thus, for example, one obtains an $M1$ transition at the B^*BV vertex as a *solution* to our equations, as well as relations involving quadrupole vertices.

The sum rules are arrived at by using microscopic causality and unitarity. They are then selected with the help of the infinite-momentum limit and evaluated at $t=0$. Thus we will obtain solutions for our truncated sum rules which correspond to a collinear symmetry. The choice $t=0$ has been previously discussed in some detail.[2]

In Sec. II we examine the different equations obtained by truncating the sums over intermediate states with the octet and the deciment baryon states. The various relations are obtained both by considering the scattering of different particles and different helicity transitions for the initial and final baryon along the direction of infinite momentum.

In Sec. III we examine the relations between the diverse vector-meson–baron vertices which emerge from the sum rules obtained in Sec. II. We discuss the solutions and compare them with the collinear $U(6)$ predictions. Lastly, in Sec. IV, we consider the possible use of a subset of our sum rules for the purpose of relating symmetry breaking at the vertices to symmetry breaking in the masses. The particular case of ρ-meson scattering on the various members of the baryon octet is discussed briefly.

II. TRUNCATED SUM RULES

The relation between the matrix elements of an equal-time commutator of two local currents and the absorptive amplitude for a scattering process has been previously examined in some detail.[1,2] As a consequence of microscopic causality we have

$$\int d^4x\, e^{-iK\cdot x}\delta(x_0)\langle p_2|[V_\alpha{}^i(\tfrac{1}{2}x),V_\beta{}^j(-\tfrac{1}{2}x)]|p_1\rangle = \text{polynomial in } \mathbf{K}, \quad (2.1)$$

where $K=\frac{1}{2}(q_1+q_2)$, α and β are space-time indices, and i, j are $SU(3)$ or isospin indices. If we consider the infinite-momentum[3] limit of Eq. (2.1), that is, the limit

* Work supported in part by the U. S. Atomic Energy Commission.
† Part II of a thesis submitted to the Department of Physics, the University of Chicago, in partial fulfillment of the requirements for the Ph.D. degree.
‡ Present address: Physics Department, Rockefeller University, New York, New York.
[1] R. Oehme, Phys. Letters **21**, 567 (1966); **22**, 206 (1966); Phys. Rev. **154**, 1358 (1967).
[2] R. Oehme and G. Venturi, Phys. Rev. **159**, 1283 (1967); G. Venturi, *ibid.* **161**, 1438 (1967).
[3] S. Fubini and G. Furlan, Physics **1**, 229 (1965); R. F. Dashen and M. Gell-Mann, in *Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energy* (W. H. Freeman and Company, San Francisco, California, 1966); R. Oehme, Phys. Rev. **143**, 1138 (1966).

$p=\frac{1}{2}(p_1+p_2)\to\infty$, and project out the residues of the vector-meson poles at $q_i^2=-\mu_i^2$, we obtain a strong interaction sum rule

$$\lim_{q_i^2\to-\mu_i^2}(q_1^2+\mu_1^2)(q_2^2+\mu_2^2)\int_{-\infty}^{\infty}d\nu\, a_{ij}(\nu,q_1,q_2)$$

$$\equiv\int_{-\infty}^{\infty}d\nu\, a_{ij}(\nu,t)=0\,, \quad (2.2)$$

where $\nu=-2p\cdot K$, $t=-(q_1-q_2)^2$, and in our case $a_{ij}(\nu,t)$ is the absorptive part of an invariant coefficient in the spin-space decomposition of the amplitudes for $BV\to BV$, $BV\to B^*V$, and $B^*V\to B^*V$ scattering. The requirements for convergence of the sum rule Eq. (2.2) and the high-energy behavior of $a_{ij}(\nu,t)$ have been previously discussed.[2]

In general, one finds that our sum rules must be evaluated near $t=0$. Let us denote by ΔJ_P the helicity change for the initial and final baryon along the direction of infinite momentum. Then if we consider ΔJ_P odd transitions for the $VB^*\to VB$ reaction and use $SU(3)$ invariance at the vertices, we find $t=0$ as a requirement for consistency with our truncation in the intermediate-state sum.[2] Indeed, saturation for a finite interval in t requires an infinite set of particles with unlimited spin. We will return to this point in Sec. V.

Let us then consider our sum rule Eq. (2.2) for the various vector-meson–baryon scattering processes. For $t=0$ we may evaluate it by explicitly choosing $p_1=p_2=p$ and $q_1=q_2=K$ with $\mathbf{p}\cdot\mathbf{K}=0$ and take the limit $p\to\infty$.[3] We limit our intermediate-state sum to the baryon octet and decuplet and obtain different sum rules for the diverse values of ΔJ_P. Invariance under $SU(3)$ is assumed at the vertices. We define the following matrix elements:

$$\langle 8k(p_f)|V_\mu{}^j(0)|8i(p_i)\rangle$$
$$=i\bar{u}(p_f)\left\{\sqrt{3}\begin{pmatrix}8 & 8 & 8_f\\ i & j & k\end{pmatrix}[\gamma_\mu G_M{}^f(q^2)+iP_\mu F_2{}^f(q^2)]\right.$$
$$+(\sqrt{5}/3)\begin{pmatrix}8 & 8 & 8_d\\ i & j & k\end{pmatrix}$$
$$\left.\times[\gamma_\mu G_M{}^d(q^2)+iP_\mu F_2{}^d(q^2)]\right\}u(p_i)\,, \quad (2.3)$$

$$\langle 10k(p_f)|V_\mu{}^j(0)|10i(p_i)\rangle$$
$$=(\sqrt{6})\begin{pmatrix}10 & 8 & 10\\ i & j & k\end{pmatrix}i\bar{w}_\alpha(p_f)\{\gamma_\mu\delta_{\alpha\beta}H_M(q^2)$$
$$+\gamma_\mu(q_\alpha q_\beta/4m^{*2})\bar{H}_M(q^2)+iP_\mu[\delta_{\alpha\beta}H_3(q^2)$$
$$+(q_\alpha q_\beta/4m^{*2})\bar{H}_3(q^2)]\}w_\beta(p_i)\,, \quad (2.4)$$

where

$$F_2{}^i(q^2)=(1/2m)[G_M{}^i(q^2)-G_E{}^i(q^2)](1+q^2/4m^2)^{-1}\,,$$
$$H_3(q^2)=(1/2m^*)[H_M(q^2)-H_E(q^2)](1+q^2/4m^{*2})^{-1}\,, \quad (2.5)$$

and similarly for $\bar{H}_3(q^2)$. Also $P=p_i+p_f$, $q=p_i-p_f$ and in the above i, j, and k are $SU(3)$ indices and m^* and m are the mean masses of decuplet and octet, respectively. H_M, $\bar{H}_M$, etc., are, respectively, Sachs magnetic dipole and magnetic quadrupole form factors, etc.

We will also need the following general vertex:

$$\langle 10k(p_f)|V_\mu{}^j(0)|8i(p_i)\rangle$$
$$=\begin{pmatrix}8 & 8 & 10\\ i & j & k\end{pmatrix}\bar{w}_\alpha(p_f)[\delta_{\alpha\mu}D_1{}^V(q^2)+i\gamma_\mu q_\alpha(m+m^*)^{-1}$$
$$\times D_2{}^V(q^2)+q_\alpha P_\mu(m+m^*)^{-2}D_3{}^V(q^2)$$
$$+q_\alpha q_\mu(m+m^*)^{-2}D_4{}^V(q^2)]\gamma_5 u(p_i)\,. \quad (2.6)$$

For the above Eq. (2.6), conservation of the vector current implies

$$D_1{}^V(q^2)+D_2{}^V(q^2)+\frac{m^*-m}{m^*+m}D_3{}^V(q^2)$$
$$+\frac{q^2}{(m^*+m)^2}D_4{}^V(q^2)=0\,. \quad (2.7)$$

Moreover, the relation between the diverse form factors[4]

$$\left[1+\frac{q^2}{(m^*+m)^2}\right]^{-1}D_1{}^V(q^2)=-\frac{(m^*+m)}{2m^*}D_2{}^V(q^2)$$
$$=D_3{}^V(q^2)=-D_4{}^V(q^2)\,, \quad (2.8)$$

leads to the generalized $M1$ form for Eq. (2.6)

$$\langle 10k(p_f)|V_\alpha{}^j(0)|8i(p_i)\rangle=\begin{pmatrix}8 & 8 & 10\\ i & j & k\end{pmatrix}\frac{D_3{}^V(q^2)}{(m+m^*)^2}$$
$$\times\epsilon_{\alpha\beta\gamma\delta}P_\beta q_\gamma\bar{w}_\delta(p_f)u(p_i)\,. \quad (2.9)$$

In our calculations, however, we shall use the general form Eq. (2.6).

Since we project out the residues at the vector meson poles we need the following form-factor decompositions:

$$G_{E,M}{}^{f,d}(q^2)=g_{E,M}{}^{f,d}f_V\mu^2/(q^2+\mu^2)$$
$$+\varphi_{1,E,M}{}^{f,d}(q^2)\,, \quad (2.10)$$

$$D_i{}^V(q^2)=d_if_V\mu^2/(q^2+\mu^2)+\varphi_2{}^i(q^2)\,, \quad (2.11)$$

$$H_{E,M}(q^2)=h_{E,M}f_V\mu^2/(q^2+\mu^2)+\varphi_{3,E,M}(q^2)\,, \quad (2.12)$$

$$\bar{H}_{E,M}(q^2)=\bar{h}_{E,M}f_V\mu^2/(q^2+\mu^2)+\varphi_{4,E,M}(q^2)\,, \quad (2.13)$$

where in the above the $\varphi_i(q^2)$ are regular at $q^2=-\mu^2$. The g, d, and h are renormalized vector-meson–baryon coupling constants and f_V is a constant associated with the vector-meson pole. We note that in the homogeneous equations obtained from superconvergence relations the factors $\mu^2 f_V$ will cancel, and we are left with relations between the various coupling constants.

[4] R. Oehme, Phys. Letters 14, 518 (1965).

We first consider $VB\to VB$ scattering. For $t=0$ $(q_1=q_2=K)$, we have only $\Delta J_P=0$, and we obtain the following equations by employing the usual crossing-matrix technique: From **8** exchange in the t channel

$$\left(1-\frac{\mu^2}{4m^2}\right)^{-1}\left(g_E{}^fg_E{}^f-\frac{\mu^2}{4m^2}g_M{}^fg_M{}^f+\frac{5}{9}g_E{}^dg_E{}^d-\frac{5}{9}\frac{\mu^2}{4m^2}g_M{}^dg_M{}^d\right)=0. \quad (2.14)$$

Also from **8** exchange

$$\left(1-\frac{\mu^2}{4m^2}\right)^{-1}\left(2g_E{}^fg_E{}^d-\frac{\mu^2}{2m^2}g_M{}^fg_M{}^d\right)-\tfrac{1}{4}\Delta=0, \quad (2.15)$$

and from **10** exchange

$$\frac{4}{3}\left(1-\frac{\mu^2}{4m^2}\right)^{-1}\left(g_E{}^dg_E{}^d-\frac{\mu^2}{4m^2}g_M{}^dg_M{}^d\right)-\tfrac{1}{4}\Delta=0, \quad (2.16)$$

where

$$\Delta\equiv\frac{1}{3m^{*2}}[-\mu^2+(m^*-m)^2]d_1d_1+\frac{1}{(m^*+m)^2}d_2d_2\left[-\tfrac{4}{3}\mu^2+\frac{1}{3m^{*2}}(\mu^2+m^{*2}-m^2)^2\right]+\frac{1}{(m^*+m)^4}d_3d_3$$
$$\times\left[-\tfrac{4}{3}\mu^2+\frac{1}{3m^{*2}}(\mu^2+m^{*2}-m^2)^2\right][-\mu^2+(m^*-m)^2]+\frac{1}{m^*+m}d_1d_2\left[-\frac{2\mu^2m}{3m^{*2}}+\frac{2}{3m^{*2}}(m^{*2}-m^2)(m^*-m)\right]$$
$$+\frac{1}{(m^*+m)^2}d_1d_3\left[\frac{4m}{3m^*}-\frac{2m^2}{3m^{*2}}-\frac{2}{3}+\frac{2\mu^2}{3m^{*2}}\right](-\mu^2-m^{*2}+m^2)+\frac{1}{(m^*+m)^3}d_3d_2$$
$$\times\left[-\frac{8}{3}\mu^2(m^*-m)+\frac{2}{3m^{*2}}(m^*-m)(\mu^2+m^{*2}-m^2)^2\right]. \quad (2.17)$$

We must also consider our sum rule for the $VB^*\to VB$ reaction. We will have different sum rules for the diverse $\Delta J_P=h$. For h even (0 or 2) we have the following: From **8** exchange,

$$\tfrac{1}{3}\Gamma_d(h\text{ even})-\Phi(h\text{ even})=0. \quad (2.18)$$

From **10** exchange

$$-\tfrac{1}{3}\Gamma_d(h\text{ even})+\Gamma_f(h\text{ even})-\Phi(h\text{ even})=0. \quad (2.19)$$

Hence from Eqs. (2.18) and (2.19),

$$\Gamma_d(h\text{ even})=\tfrac{3}{2}\Gamma_f(h\text{ even}), \quad (2.20)$$

and

$$\Gamma_f(h\text{ even})=2\Phi(h\text{ even}). \quad (2.21)$$

For $\Delta J_P=0$, we have

$$\Gamma_i(h=0)=g_M{}^i\left[\frac{4}{m^*}(m-m^*)d_1+\frac{4}{m^*}(m-m^*)d_2-\frac{4\mu^2}{m^*(m+m^*)}d_2-\frac{4(m-m^*)^2}{m^*(m+m^*)}d_3-\frac{4\mu^2}{(m+m^*)^2}\left(2-\frac{m}{m^*}\right)d_3\right]$$
$$-\frac{1}{m}(g_M{}^i-g_E{}^i)\left(1-\frac{\mu^2}{4m^2}\right)^{-1}\left\{\frac{2}{m^*}d_1[-\mu^2-2m(m^*-m)]-\frac{2\mu^2}{m+m^*}d_2\left(1+\frac{2m}{m^*}\right)+\frac{4m}{m^*}(m-m^*)d_2+\frac{2}{(m^*+m)^2}d_3\right.$$
$$\left.\times\left[+\frac{2m^2\mu^2}{m^*}-\mu^2\left(3m+m^*+\frac{\mu^2}{m^*}\right)-(m^2-m^{*2})\left(-2m+\frac{2m^2}{m^*}-\frac{\mu^2}{m^*}\right)\right]\right\}, \quad (2.22)$$

where, of course, the index i refers to the symmetric (d) or antisymmetric (f) octet combinations. Also,

$$\Phi(h=0)=-\frac{\mu^2}{M^2}d_3\left[h_M\left(\tfrac{1}{3}+\frac{\mu^2}{3M^2}\right)+\frac{\mu^2}{6M^2}\bar{h}_M\left(1-\frac{\mu^2}{4M^2}\right)-\frac{\mu^2}{2M^2}(h_M-h_E)\left(1-\frac{\mu^2}{4M^2}\right)^{-1}-\frac{\mu^2}{4M^2}(\bar{h}_M-\bar{h}_E)\right]. \quad (2.23)$$

For $\Delta J_P=2$,

$$\Gamma_i(h=2)=\frac{2\mu^2}{m(m^*+m)}\left(1-\frac{\mu^2}{4m^2}\right)^{-1}d_2(g_M{}^i-g_E{}^i)-\frac{4\mu^2}{m^*+m}d_3\left[g_M{}^i-\frac{(m^*+m)}{2m}\left(1-\frac{\mu^2}{4m^2}\right)^{-1}(g_M{}^i-g_E{}^i)\right] \quad (2.24)$$

and

$$\Phi(h=2)=-\frac{\mu^2}{2M^2}d_3\left[\tfrac{2}{3}h_M+\frac{\mu^2}{3M^2}(h_M-h_E)\left(1-\frac{\mu^2}{4M^2}\right)^{-1}+(\mu^2/6M^2)(\bar{h}_M-\bar{h}_E)\right]. \quad (2.25)$$

Let us note that for Eqs. (2.23) and (2.25), we have used an $M1$ transition [Eq. (2.9)] at the B^*BV vertex, and we have taken octet and decuplet masses as degenerate with M the mean multiplet mass. The reason for this simplification will become apparent when, in the next section, we discuss the solutions to our equations.

One also has sum rules for ΔJ_P odd$(=1)$. In this case we have the following two equations:

$$\Gamma_d(h \text{ odd})=0 \tag{2.26}$$

and

$$\Gamma_f(h \text{ odd})=\Phi(h \text{ odd}). \tag{2.27}$$

For $\Delta J_P=1$ $(\frac{3}{2}\to\frac{1}{2})$ we have

$$\Gamma_i(h=\tfrac{3}{2}\to\tfrac{1}{2})=(m^*+m)^{-1}d_2[-4g_M{}^i+4(g_M{}^i-g_E{}^i)(1-\mu^2/4m^2)^{-1}] + \frac{1}{(m+m^*)^2}d_3\left\{(m-m^*)g_M{}^i+(g_M{}^i-g_E{}^i)\left(1-\frac{\mu^2}{4m^2}\right)^{-1}\left[-4(m-m^*)+\frac{2\mu^2}{m}\right]\right\} \tag{2.28}$$

and

$$\Phi(h=\tfrac{3}{2}\to\tfrac{1}{2})=\frac{1}{2M}d_3\left[-h_M\left(4-\frac{2\mu^2}{3M^2}\right)+\frac{\mu^2}{3M^2}\bar{h}_M\left(1-\frac{\mu^2}{4M^2}\right) +(4-5\mu^2/3M^2)(h_M-h_E)(1-\mu^2/4M^2)^{-1}-(\mu^2/3M^2)(\bar{h}_M-\bar{h}_E)\right]. \tag{2.29}$$

For $\Delta J_P=1$ $(\frac{1}{2}\to-\frac{1}{2})$,

$$\Gamma_i(h=\tfrac{1}{2}\to-\tfrac{1}{2})=d_1\left[-\frac{4}{m^*}g_M{}^i+\frac{2}{mm^*}(m^*+m)\left(1-\frac{\mu^2}{4m^2}\right)^{-1}(g_M{}^i-g_E{}^i)\right]-\frac{1}{m^*+m}d_2\left\{-4g_M{}^i+\frac{1}{m}(g_M{}^i-g_E{}^i)\left(1-\frac{\mu^2}{4m^2}\right)^{-1} \times\left[4m+\frac{2\mu^2}{m^*}-\frac{2}{m^*}(m^2-m^{*2})\right]\right\}+\frac{1}{(m^*+m)^2}d_3\left\{4g_M{}^i\left(\frac{m^2}{m^*}+m-2m^*-\frac{\mu^2}{m^*}\right)+\frac{1}{m}(g_M{}^i-g_E{}^i) \times\left(1-\frac{\mu^2}{4m^2}\right)^{-1}\left[-4m(m-m^*)+2\mu^2\left(2+\frac{m}{m^*}\right)-\frac{2}{m^*}(m^2-m^{*2})(m+m^*)\right]\right\}, \tag{2.30}$$

and

$$\Phi(h=\tfrac{1}{2}\to-\tfrac{1}{2})=\frac{1}{2M}d_3\left[h_M\left(-4-\frac{2\mu^2}{3M^2}\right)+\bar{h}_M\left(-\frac{\mu^2}{3M^2}+\frac{\mu^4}{12M^4}\right)+(h_M-h_E)\left(1-\frac{\mu^2}{4M^2}\right)^{-1} \times\left(4-\frac{\mu^2}{3M^2}+\frac{\mu^4}{3M^4}\right)+(\bar{h}_M-\bar{h}_E)\left(1-\frac{\mu^2}{4M^2}\right)^{-1}\left(\frac{\mu^2}{3M^2}+\frac{\mu^4}{12M^4}-\frac{\mu^6}{24M^6}\right)\right]. \tag{2.31}$$

Again, in Eqs. (2.29) and (2.31) we have used an $M1$ transition at the B^*BV vertex and taken the B and B^* masses as degenerate. The above are all the relations one can obtain from the $VB^*\to VB$ reaction sum rules by directly using $SU(3)$ invariance at the vertices and choosing $t=0$, as required by our truncation in the saturation.

The last set of relations we obtain are the ones from $VB^*\to VB^*$ scattering. As usual, we shall limit our intermediate state sum to one-particle baryon octet and decuplet states and evaluate our sum rule at $t=0$. Here we shall directly use an $M1$ transition at the B^*BV vertex and take the B and B^* masses as degenerate. Again we will obtain different sum rules depending on the helicity transitions along the direction of infinite momentum for the B^* states. For $t=0$ we will only obtain equations for $\Delta J_P=h$ even and we have

$$(-\mu^2/18M^2)(1-\mu^2/4M^2)d_3d_3+\Theta(h \text{ even})=0. \tag{2.32}$$

For $\Delta J_P=0$ we have two cases, $\Delta J_P=0$ $(\frac{3}{2}\to\frac{3}{2})$,

$$\Theta(h=\tfrac{3}{2}\to\tfrac{3}{2})=\left(1-\frac{\mu^2}{4M^2}\right)^{-2}\left[h_M{}^2\left(-\frac{\mu^2}{9M^2}+\frac{\mu^4}{12M^4}-\frac{\mu^6}{36M^6}\right)+h_E{}^2\left(\frac{4}{3}-\frac{\mu^2}{M^2}+\frac{\mu^4}{9M^4}\right)-\frac{\mu^6}{144M^6}\bar{h}_M{}^2\left(1-\frac{\mu^2}{4M^2}\right)^2 +\bar{h}_E{}^2\frac{\mu^4}{36M^4}\left(1-\frac{\mu^2}{4M^2}\right)^2+h_Mh_E\frac{\mu^4}{9M^4}-\bar{h}_Eh_E\frac{\mu^2}{3M^2}\left(1-\frac{\mu^2}{3M^2}\right)\left(1-\frac{\mu^2}{4M^2}\right)+\bar{h}_Mh_M\frac{1}{36}\left(\frac{\mu^4}{M^4}-\frac{\mu^6}{M^6}\right) \times\left(1-\frac{\mu^2}{4M^2}\right)+\bar{h}_Eh_M\frac{\mu^4}{18M^4}\left(1-\frac{\mu^2}{4M^2}\right)\right], \tag{2.33}$$

and $\Delta J_P=0$ $(\frac{1}{2}\to\frac{1}{2})$,

$$\Theta(h=\tfrac{1}{2}\to\tfrac{1}{2})=\left(1-\frac{\mu^2}{4M^2}\right)^{-2}\Bigg[\left(1-\frac{\mu^2}{4M^2}\right)^2\left(4-\frac{4\mu^2}{9M^2}+\frac{4\mu^4}{9M^4}\right)h_M{}^2+\frac{\mu^4}{12M^4}\left(1-\frac{\mu^2}{4M^2}\right)^3\left(\frac{1}{3}-\frac{\mu^2}{3M^2}\right)\bar{h}_M{}^2+(h_M-h_E)^2$$
$$\times\left(4-\frac{5\mu^2}{3M^2}+\frac{7\mu^4}{9M^4}-\frac{\mu^6}{9M^6}\right)+\frac{\mu^4}{12M^4}\left(1-\frac{\mu^2}{4M^2}\right)^2\left(\frac{1}{3}-\frac{\mu^2}{3M^2}\right)(\bar{h}_M-\bar{h}_E)^2-\bar{h}_Mh_M\frac{\mu^2}{3M^2}\left(1-\frac{\mu^2}{4M^2}\right)\left(1-\frac{\mu^2}{M^2}+\frac{\mu^4}{3M^4}\right)$$
$$+h_M(h_M-h_E)\left(1-\frac{\mu^2}{4M^2}\right)\left(-8-\frac{8\mu^2}{9M^2}+\frac{4\mu^4}{3M^4}\right)+h_M(\bar{h}_M-\bar{h}_E)\left(1-\frac{\mu^2}{4M^2}\right)\left(\frac{\mu^2}{3M^2}-\frac{5\mu^4}{18M^4}+\frac{\mu^6}{9M^6}\right)-\frac{\mu^2}{M^2}\bar{h}_M(h_M-h_E)$$
$$\times\left(1-\frac{\mu^2}{4M^2}\right)\left(\frac{1}{3}+\frac{\mu^2}{3M^2}\right)\left(1-\frac{\mu^2}{3M^2}\right)-\frac{\mu^2}{M^2}(h_M-h_E)(\bar{h}_M-\bar{h}_E)\left(\frac{1}{3}-\frac{\mu^2}{3M^2}\right)\left(1-\frac{\mu^2}{4M^2}\right)\left(1-\frac{\mu^2}{3M^2}\right)$$
$$-\frac{\mu^4}{6M^4}\left(1-\frac{\mu^2}{4M^2}\right)^2\left(\frac{1}{3}-\frac{\mu^2}{3M^2}\right)\bar{h}_M(\bar{h}_M-\bar{h}_E)\Bigg]. \quad (2.34)$$

Lastly, for $\Delta J_P=2$ $(\frac{3}{2}\to-\frac{1}{2})$,

$$\Theta(h=2)=\left(1-\frac{\mu^2}{4M^2}\right)^{-2}\Bigg\{h_M{}^2\left(\frac{2\mu^2}{9M^2}-\frac{2\mu^4}{9M^4}-\frac{\mu^6}{18M^6}\right)+h_E{}^2\left(\frac{2\mu^2}{3M^2}-\frac{\mu^4}{9M^4}\right)+\bar{h}_M{}^2\left(1-\frac{\mu^2}{4M^2}\right)^2\frac{\mu^6}{144M^6}$$
$$-\bar{h}_E{}^2\frac{\mu^4}{36M^4}\left(1-\frac{\mu^2}{4M^2}\right)^2-\frac{\mu^4}{9M_4}h_Mh_E+\bar{h}_Eh_E\left(\frac{\mu^2}{3M^2}-\frac{7\mu^4}{36M^4}+\frac{\mu^6}{36M^6}\right)$$
$$+\bar{h}_Mh_M\left(1-\frac{\mu^2}{4M^2}\right)\left(-\frac{\mu^4}{12M^4}+\frac{\mu^6}{24M^6}\right)-\bar{h}_Eh_M\frac{\mu^4}{18M^4}\left(1-\frac{\mu^2}{4M^2}\right)\Bigg\}. \quad (2.35)$$

The above are all the relations one may obtain on considering the various vector-meson–baryon scattering processes and limiting the intermediate-state sum to the baryon octet and decuplet. Let us note that for finite small t our equations would have corrections of $O(t/M^2)$ or higher. Consistency to higher order in t may be obtained by including higher states in the saturation.

The equations obtained on using the $SU(3)$ density algebra,

$$[V_0{}^i(x),V_0{}^j(x')]_{x_0=x_0'}=if_{ijk}V_0{}^k(x)\delta^3(\mathbf{x}-\mathbf{x}'), \quad (2.36)$$

may be deduced from our previous equations by replacing $\mu^2\to-q^2$, $g_M\to G_M(q^2)$, $d_i\to D_i(q^2)$, and similarly for the remaining form factors. Moreover instead of zero for the right-hand side of Eqs. (2.14) and (2.15) we have $G_E{}^f(0)$ and $G_E{}^d(0)$, respectively. Also, Eq. (2.18) for $\Delta J_P=0$ would read

$$\tfrac{1}{3}\Gamma_d(q^2)-\Phi(q^2)=\frac{2(m^*-m)}{m^*}$$
$$\times\left[D_1{}^V(0)+D_2{}^V(0)+\frac{(m^*-m)}{m^*+m}D_3{}^V(0)\right], \quad (2.37)$$

with corresponding changes in Eqs. (2.20) and (2.21). Lastly, the right-hand side of Eq. (2.32) would be given by $2H_E(0)$ and $\frac{2}{3}H_E(0)$ for $\Delta J_p=0$ $(\frac{1}{2}\to\frac{1}{2})$ and $\Delta J_P=0$ $(\frac{3}{2}\to\frac{3}{2})$, respectively.

III. HIGHER-SYMMETRY RESULTS FROM TRUNCATED SUM RULES

Let us begin by considering the equations obtained from $VB\to VB$ scattering and the $VB^*\to VB$ reaction. In particular, for the latter case we shall first consider only the equations which involve the B^*BV and BBV vertices. For this purpose, we define the following ratios:

$$\xi_E=g_E{}^d/g_E{}^f, \qquad \xi_M=g_M{}^d/g_M{}^f, \quad (3.1)$$

$$\alpha=g_M{}^d/(2m/\mu)g_E{}^f, \quad \delta_2=-d_2/d_3, \quad (3.2)$$

$$\delta^2=\frac{3}{16}\left(1-\frac{\mu^2}{4m^2}\right)\frac{\Delta}{(g_E{}^f)^2}. \quad (3.3)$$

Then from $VB\to VB$ scattering, Eqs. (2.14)–(2.16), we obtain

$$\xi_E-\alpha^2/\xi_M=\tfrac{2}{3}(\xi_E{}^2-\alpha^2), \quad (3.4)$$

$$\xi_E-\alpha^2/\xi_M=-(6/5)(1-\alpha^2/\xi_M{}^2), \quad (3.5)$$

$$\alpha^2-\xi_E{}^2=\delta^2. \quad (3.6)$$

From the $VB^*\to VB$ reaction, Eqs. (2.20) and (2.26) for $\Delta J_P=2$ and $\Delta J_P=1$ $(\frac{3}{2}\to\frac{1}{2})$, respectively, we obtain

$$\frac{2m}{\mu}\alpha\left[\delta_2+\frac{m}{m^*+m}\left(1-\frac{\mu^2}{2m^2}-\frac{m^*}{m}\right)\right]\left(1-\frac{3}{2\xi_M}\right)=(1-\delta_2)(\tfrac{3}{2}-\xi_E)\,, \tag{3.7}$$

$$\frac{\mu}{2m}\alpha(\delta_2-1)=\xi_E\left[\delta_2+\frac{m}{m^*+m}\left(1-\frac{m^*}{m}-\frac{\mu^2}{2m^2}\right)\right]. \tag{3.8}$$

We note that Eqs. (3.7) and (3.8) taken together lead to Eq. (3.4). From Eqs. (3.4) and (3.5) we have

$$\frac{\xi_E(1-\frac{2}{3}\xi_E)}{6/5+\xi_E}=\frac{\xi_M(1-\frac{2}{3}\xi_M)}{6/5+\xi_M}\,, \tag{3.9}$$

whence, for $\delta^2\neq 0$, we obtain

$$\xi_M=3(3-2\xi_E)/(6+5\xi_E)\,,$$
or
$$\xi_E=3(3-2\xi_M)/(6+5\xi_M)\,. \tag{3.10}$$

As for the choice $\delta^2=0$, we will later return to the possibility of alternative solutions to our equations. If we consider Eqs. (3.6) and (3.8) we obtain

$$\xi_E\left\{1-\tfrac{2}{3}\xi_E+\frac{8}{3}\frac{m^2}{\mu^2}\xi_E\left[\delta_2-\frac{m}{m^*+m}\left(1-\frac{m^*}{m}-\frac{\mu^2}{2m^2}\right)\right]^2\times\left(1-\frac{3}{2\xi_M}\right)\Big/(\delta_2-1)^2\right\}=0\,, \tag{3.11}$$

from which it follows that

$$\xi_E=0\,, \tag{3.12}$$

or

$$\delta_2=\left[1-\tfrac{2}{3}\xi_E\pm\frac{2m^2}{\mu(m^*+m)}\left(1-\frac{m^*}{m}-\frac{\mu^2}{2m^2}\right)\xi_E\right](1-\tfrac{2}{3}\xi_E\mp(2m/\mu)\xi_E)\,. \tag{3.13}$$

Lastly, on eliminating ξ_M from Eq. (3.4),

$$\alpha^2=(1-\tfrac{2}{3}\xi_E)^2. \tag{3.14}$$

Let us first examine the solution $\xi_E=0$; we shall later discuss the solution given by Eq. (3.13). From Eq. (3.9) for $\xi_E=0$ we obtain

$$\xi_M=\tfrac{3}{2}\,, \tag{3.15}$$

and from Eqs. (3.8) and (3.14),

$$\alpha^2=1\,, \tag{3.16}$$

$$\delta_2=1\,. \tag{3.17}$$

We may also examine the relations obtained from the $VB^*\to VB$ reaction for $\Delta J_P=0$ and $\Delta J_P=1$ $(\frac{1}{2}\to-\frac{1}{2})$. On substituting our solution given by Eqs. (3.12), (3.15)–(3.17), we obtain in both cases

$$[-\mu^2-2m(m^*-m)]d_1=\left[\frac{4m^2}{m+m^*}(m-m^*)-\frac{\mu^2(5m^2-m^{*2})}{(m^*+m)^2}+\frac{\mu^4}{(m+m^*)^2}\right]d_3\,. \tag{3.18}$$

Note that Eqs. (3.18) and (3.17), which yields $d_2=-d_3$, are not the results corresponding to a generalized $M1$ transition [Eq. (2.8)] which read

$$\left[1-\frac{\mu^2}{(m+m^*)^2}\right]^{-1}d_1=-\frac{(m+m^*)}{2m^*}d_2=d_3\,. \tag{3.19}$$

However, Eqs. (3.17) and (3.18) correspond to an $M1$ transition for $m=m^*$. We note that had we used the generalized $M1$ form Eq. (2.9), instead of the general vertex Eq. (2.6), in evaluating our sum rules, we would have obtained $m^*=m$ as a requirement for a nontrivial solution. The results given by Eqs. (3.12), (3.15), and (3.16) have been previously obtained by taking m^* and m masses degenerate, directly using an $M1$ transition at the B^*BV vertex, and retaining only terms to lowest order in μ^2/M^2.[1]

We now briefly comment on the possibility of alternative solutions to our equations obtained by choosing $\delta^2=0$ or Eq. (3.13). In the following for simplicity we shall take the B^* and B masses as degenerate with M the mean mass. Then on defining

$$\delta_1=(1-\mu^2/4M^2)^{-1}d_1/d_3 \tag{3.20}$$

from the $VB^*\to VB$ reaction, for both $\Delta J_P=0$ and $1(\frac{1}{2}\to-\frac{1}{2})$, we obtain

$$(1-\delta_2)\Big/\left(\delta_2-\frac{\mu^2}{4M^2}\right)=\left[3\delta_2-2\delta_1\left(1-\frac{\mu^2}{4M^2}\right)-1-\frac{\mu^2}{2M^2}\right]\Big/\left[-\delta_2\left(1+\frac{\mu^2}{2M^2}\right)+2\delta_1\left(1-\frac{\mu^2}{4M^2}\right)+\frac{3\mu^2}{4M^2}\right]. \tag{3.21}$$

The above relation is to be used in conjunction with our previous Eqs. (3.4)–(3.8). These equations do not have a unique solution. However, we must also consider the remaining equations, obtained from the $VB^*\to VB$ and $VB^*\to VB^*$ processes by employing the general form Eq. (2.6) for the VB^*B vertex. Then to lowest order in μ^2/M^2 the solution Eq. (3.12) is the only one consistent with a nontrivial solution for the B^*B^*V vertex. This may also be true to all orders in μ^2/M^2. Hence in the following we shall restrict ourselves to the solution Eq. (3.12), which is also consistent with pole dominance of the electromagnetic form factors.

Let us note that the results which we have obtained so far, from the solution Eq. (3.12), have involved only the BBV and B^*BV vertices and do not require $m^*=m$. In examining the remaining equations, however, we shall for the sake of brevity choose $m^*=m$. Thus it may be that the remaining equations require $m^*=m$. Even if this is the case, as has been previously discussed,[2] it may still be meaningful to consider the results, for $m^*\neq m$, of a subset of the equations consistent with mass splittings.

Then, restricting ourselves to our physical solution Eq. (3.12) and taking B^* and B masses as degenerate, we have

$$\delta=\left(1-\frac{\mu^2}{4M^2}\right)\frac{\mu}{4M}\frac{d_3}{g_E{}^f}, \tag{3.22}$$

and from Eqs. (3.6) and (3.16),

$$\delta^2=\alpha^2=1. \tag{3.23}$$

We now examine the remaining equations obtained from the $B^*V\rightarrow VB$ reaction. We define

$$\eta_E=h_E/g_E{}^f,\quad \eta_M=h_M\Big/\left(\frac{2M}{\mu}g_E{}^f\right), \tag{3.24}$$

$$\bar{\eta}_E=\bar{h}_E/g_E{}^f,\quad \bar{\eta}_M=\bar{h}_M\Big/\left(\frac{2M}{\mu}g_E{}^f\right); \tag{3.25}$$

then, using our previous result for the B^*BV vertex ($M1$ transition for $m^*=m$), we have for $\Delta J_P=2$,

$$\frac{2M}{\mu}\frac{\alpha}{\xi_M}=\frac{2}{3}\left(1-\frac{\mu^2}{4M^2}\right)^{-1}\left\{\frac{2M}{\mu}\eta_M\left(1+\frac{\mu^2}{4M^2}\right)-\frac{\mu^2}{2M^2}\eta_E+\frac{\mu^2}{4M^2}\left(1-\frac{\mu^2}{4M^2}\right)\left(\frac{2M}{\mu}\bar{\eta}_M-\bar{\eta}_E\right)\right\}; \tag{3.26}$$

for $\Delta J_P=0$,

$$\frac{2M}{\mu}\frac{\alpha}{\xi_M}=\frac{2}{3}\left(1-\frac{\mu^2}{4M^2}\right)^{-1}\left[\frac{2M}{\mu}\eta_M\left(1-\frac{3\mu^2}{4M^2}-\frac{\mu^4}{4M^4}\right)+\eta_E\frac{3\mu^2}{2M^2}-\frac{2M}{\mu}\bar{\eta}_M\left(\frac{\mu^2}{4M^2}+\frac{\mu^4}{16M^4}-\frac{\mu^6}{32M^6}\right)+\bar{\eta}_E\frac{3\mu^2}{4M^2}\left(1-\frac{\mu^2}{4M^2}\right)\right]; \tag{3.27}$$

for $\Delta J_P=1$ ($\frac{3}{2}\rightarrow\frac{1}{2}$),

$$-4=\left(1-\frac{\mu^2}{4M^2}\right)^{-1}\left[-\frac{\mu^3}{3M^3}\eta_M-\eta_E\left(4-\frac{5\mu^2}{3M^2}\right)-\bar{\eta}_M\frac{\mu^3}{6M^3}\times\left(1-\frac{\mu^2}{4M^2}\right)+\bar{\eta}_E\frac{\mu^2}{3M^2}\left(1-\frac{\mu^2}{4M^2}\right)\right]; \tag{3.28}$$

and lastly for $\Delta J_P=1$ ($\frac{1}{2}\rightarrow-\frac{1}{2}$),

$$-4=\left(1-\frac{\mu^2}{4M^2}\right)^{-1}\left[\frac{\mu^3}{M^3}\eta_M-\eta_E\left(4-\frac{\mu^2}{3M^2}+\frac{\mu^4}{3M^4}\right)+\bar{\eta}_M\frac{\mu^3}{2M^3}\left(1-\frac{\mu^2}{4M^2}\right)-\bar{\eta}_E\left(\frac{\mu^2}{3M^2}+\frac{\mu^4}{12M^4}-\frac{\mu^6}{24M^6}\right)\right]. \tag{3.29}$$

Comparison of the above Eqs. (3.26)–(3.29) unambiguously leads to

$$\bar{\eta}_M=-2\eta_M/(1-\mu^2/4M^2), \tag{3.30}$$

and

$$\bar{\eta}_E=-2\eta_E/(1-\mu^2/4M^2). \tag{3.31}$$

On using the above relations in Eqs. (3.26) or (3.27), we obtain

$$\alpha/\xi_M=\tfrac{2}{3}\eta_M; \tag{3.32}$$

whence by our previous results

$$\eta_M=1, \tag{3.33}$$

and from Eqs. (3.28) or (3.29),

$$\eta_E=1. \tag{3.34}$$

Let us note that Eqs. (3.32) and (3.34) are essentially the equations previously obtained on restricting oneself to electric and magnetic vertices and retaining only terms to lowest order in μ^2/M^2.[1] It is quite remarkable that the contribution of the quadrupole terms is such as to make the previously obtained lowest-order equations valid to all orders. We have then found a solution for all vector-meson–baryon vertices occurring in the $VB^*\rightarrow VB$ reaction with our truncation.

The last set of equations we must consider are the ones obtained from $VB^*\rightarrow VB^*$ scattering. On using our relations for the quadrupole vertices Eqs. (3.30) and (3.31), we arrive at, for $\Delta J_P=0$ ($\frac{3}{2}\rightarrow\frac{3}{2}$),

$$-\tfrac{2}{3}\delta^2-\tfrac{1}{3}\eta_M{}^2+\eta_E{}^2=0; \tag{3.35}$$

for $\Delta J_P=0$ ($\frac{1}{2}\rightarrow\frac{1}{2}$),

$$-(2/9)\delta^2-(7/9)\eta_M{}^2+\eta_E{}^2=0; \tag{3.36}$$

and lastly for $\Delta J_P=2$ ($\frac{3}{2}\rightarrow-\frac{1}{2}$),

$$-\delta^2+\eta_M{}^2=0. \tag{3.37}$$

Again we find that the quadrupole contribution is such as to make the previously obtained lowest-order equations valid to all orders in μ^2/M^2. From the above, one immediately finds a solution

$$\delta=\eta_M=\eta_E, \tag{3.38}$$

in agreement with our previous results.

Let us then briefly summarize our results for the various vector-meson–baryon couplings for degenerate

B and B^* masses. We have

$$g_E{}^d=0, \quad g_M{}^d/g_M{}^f=\tfrac{3}{2}\,; \tag{3.39}$$

a relation between "electric" and "magnetic" couplings,

$$g_M{}^d=(2M/\mu)g_E{}^f, \tag{3.40}$$

also

$$g_E{}^f=(\mu/4M)(1-\mu^2/4M^2)d_3, \tag{3.41}$$

and an $M1$ transition at the B^*BV vertex

$$(1-\mu^2/4M^2)^{-1}d_1=-d_2=d_3. \tag{3.42}$$

For the B^*B^*V couplings, we have

$$h_M=g_M{}^d, \qquad h_E=g_E{}^f, \tag{3.43}$$

and

$$\bar{h}_M=-2h_M/(1-\mu^2/4M^2),$$

$$\bar{h}_E=-2h_E/(1-\mu^2/4M^2). \tag{3.44}$$

The above Eqs. (3.39)–(3.44) are the results obtained on the basis of collinear $U(6)$ symmetry for the vector-meson–baryon couplings.[4,5] The results Eqs. (3.39)–(3.41) and (3.43) were previously obtained by using an $M1$ transition at B^*VB vertices directly and retaining terms to lowest order in μ^2/M^2 in the evaluation of the absorptive parts. We have used general vertices, retained terms to all orders and have again obtained results consistent with collinear $U(6)$ symmetry.

Use of our results in conjunction with a vector-meson–pole model for electromagnetic form factor leads to the usual collinear $U(6)$ predictions for magnetic moments, etc.

IV. SUM RULES AND BROKEN SYMMETRIES

In Sec. III we have seen how the use of $SU(3)$ symmetry at vertices, with our truncation in the intermediate-state sum, has led to results consistent with the collinear $U(6)$ symmetry. We note, however, that Eq. (2.2) is valid for any individual scattering process. Thus, we can restrict ourselves to only isospin invariance at the vertices and employ the physical masses of the particles involved. Then on considering a particular scattering process, such as $\rho N\to\rho N$, and truncating our intermediate-state sum by limiting ourselves to nucleon and N^* intermediate states, we obtain relations between individual vertices involving the masses of ρ, N, and N^*. The relations satisfying the equations are chosen by requiring that they reduce to our previous results on going to the $SU(3)$ limit.

[5] P. G. O. Freund and R. Oehme, Phys. Rev. Letters **16**, 1085 (1965); K. J. Barnes, P. Carruthers, and F. Von Hippel, *ibid.* 14, 82, (1965); B. Sakita and K. C. Wali, Phys. Rev. **139**, B1355 (1965); A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **284A**, 146 (1965); R. Oehme, in *Preludes in Theoretical Physics* (North-Holland Publishing Company, Amsterdam, 1966).

The above procedure will lead to results corresponding to broken $U(6)$ symmetry. Higher intermediate states should also be included; however, one may hope that the inclusion of the lowest states, with physical masses and couplings, is sufficient as a first approximation. In practice one may expect that for complete saturation an infinite dimensional representation of some noncompact group is needed.

Let us examine the particular use of $\rho B\to\rho B$ scattering on using only isospin invariance. From each of the individual scattering processes which we may consider, we will be able to relate the "electric" and "magnetic" ρ vertices. Then on assuming ρ-meson pole dominance for the isovector form factor we will be able to relate the various magnetic moments. Other scattering processes such as $\rho B\to\omega B$, etc., may also be examined. However, for the case in which the masses of the initial and final vector mesons are different, our sum rules must be evaluated a suitable value of the momentum transfer, whereas in our case, as mentioned in Sec. II, it is sufficient to choose $t=0$.

Employing our usual procedure, evaluating our form factors at the ρ-meson poles, and limiting our intermediate state sum to baryon octet and decuplet states, we obtain for $\rho N\to\rho N$ scattering

$$\frac{25}{9}\frac{\mu_\rho{}^2}{2m_N}(g_{\rho NN}{}^M)^2-2(g_{\rho NN}{}^E)^2 +\frac{2}{3}\left(1-\frac{\mu_\rho{}^2}{4m_N{}^2}\right)\Delta_{N^*N\rho}=0, \tag{4.1}$$

where $\Delta_{N^*N\rho}$ is as defined by Eq. (2.14) with N^*, N, and ρ masses, and

$$\lim_{q^2\to-\mu_\rho{}^2}(q^2+\mu_\rho{}^2)\langle p(p_f)|V_\mu{}^\rho(0)|p(p_i)\rangle=\left(1-\frac{\mu_\rho{}^2}{4m_N{}^2}\right)^{-1} \times\bar{u}(p_f)\left(\frac{P_\mu g_{\rho NN}{}^E}{4m_N}-\frac{ir_\mu}{8m_N{}^2}\frac{5}{3}g_{\rho NN}{}^M\right)u(p_i), \tag{4.2}$$

where $r_\mu=\epsilon_{\mu\beta\gamma\delta}P_\beta q_\gamma\gamma_\delta\gamma_5$.

From $\rho\Sigma\to\rho\Sigma$ scattering we have

$$\frac{2}{9}\frac{\mu_\rho{}^2}{m_\Sigma{}^2}(g_{\rho\Sigma\Sigma}{}^M)^2+\frac{\mu_\rho{}^2}{6m_\Sigma{}^2}(g_{\rho\Sigma\Lambda}{}^M)^2-2(g_{\rho\Sigma\Sigma}{}^E)^2 -\frac{1}{12}\left(1-\frac{\mu_\rho{}^2}{4m_\Sigma{}^2}\right)\Delta_{\rho\Sigma\Sigma^*}=0, \tag{4.3}$$

where $\Delta_{\Sigma\rho\Sigma^*}$ is defined similarly to $\Delta_{N^*\rho N}$ and as a first approximation to $SU(3)$ symmetry breaking we have taken $m_\Sigma = m_\Lambda$. We also have

$$\lim_{q^2\to-\mu_\rho^2} (q^2+\mu_\rho^2)\langle\Sigma^+(p_f)\,|\,V_\mu{}^\rho(0)\,|\Sigma^+(p_i)\rangle$$

$$=\left(1-\frac{\mu_\rho{}^2}{4m_\Sigma{}^2}\right)^{-1}\bar{u}(p_f)$$

$$\times\left(\frac{P_\mu}{2m_\Sigma}g_{\rho\Sigma\Sigma}{}^E-\frac{ir_\mu}{4m_\Sigma{}^2}g_{\rho\Sigma\Sigma}{}^M\right)u(p_i)\,, \quad (4.4)$$

$$\lim_{q^2\to-\mu_\rho^2} (q^2+\mu_\rho^2)\langle\Sigma^+(p_f)\,|\,V_\mu{}^\rho(0)\,|\Lambda(p_i)\rangle$$

$$=-\frac{i}{\sqrt{3}}\left(1-\frac{\mu_\rho{}^2}{4m_\Sigma{}^2}\right)^{-1}\bar{u}(p_f)\frac{r_\mu}{4m_\Sigma{}^2}g_{\rho\Sigma\Lambda}{}^M u(p_i)\,. \quad (4.5)$$

Lastly from $\rho\Xi\to\rho\Xi$ scattering we obtain

$$\frac{\mu^2}{36m_\Xi{}^2}(g_{\rho\Xi\Xi}{}^M)^2-(g_{\rho\Xi\Xi}{}^E)^2-\frac{1}{6}\left(1-\frac{\mu_\rho{}^2}{4m_\Xi{}^2}\right)\Delta_{\Xi\Xi^*\rho}=0\,, \quad (4.6)$$

where $\Delta_{\Xi\Xi^*\rho}$ is defined similarly to the previous cases and

$$\lim_{q^2\to-\mu_\rho^2} (q^2+\mu_\rho^2)\langle\Xi^-(p_f)\,|\,V_\mu{}^\rho(0)\,|\Xi^-(p_i)\rangle$$

$$=-\bar{u}(p_f)\left(\frac{P_\mu}{4m_\Xi}g_{\rho\Xi\Xi}{}^E+\frac{ir_\mu}{24m_\Xi{}^2}g_{\rho\Xi\Xi}{}^M\right)$$

$$\times u(p_i)\left(1-\frac{\mu^2}{4m_\Xi{}^2}\right)^{-1}. \quad (4.7)$$

Note that in the above we have chosen the normalizations of the various coupling constants so that they are equal in the $SU(3)$ limit. On examining the above equations, we obtain the following broken collinear $U(6)$ symmetry relations between electric and magnetic couplings,

$$g_{\rho NN}{}^M=(2m_N/\mu_\rho)g_{\rho NN}{}^E\,, \quad (4.8)$$

$$g_{\rho\Sigma\Lambda}{}^M=g_{\rho\Sigma\Sigma}{}^M=(2m_\Sigma/\mu_\rho)g_{\rho\Sigma\Sigma}{}^E\,, \quad (4.9)$$

$$g_{\rho\Xi\Xi}{}^M=(2m_\Xi/\mu_\rho)g_{\rho\Xi\Xi}{}^E\,. \quad (4.10)$$

On assuming ρ-meson pole dominance for the isovector electric and magnetic form factors, which then implies $g_{\rho\Sigma\Sigma}{}^E=g_{\rho\Xi\Xi}{}^E=g_{\rho NN}{}^E$, we obtain the following relations between total magnetic moments:

$$3/5m_N(\mu_p-\mu_n)=(\sqrt{3}/m_\Sigma)\mu_{\rho\Sigma\Lambda}=3/5m_\Xi(\mu_{\Xi^0}-\mu_{\Xi^-})$$
$$=3/4m_\Sigma(\mu_{\Sigma^+}-\mu_{\Sigma^-})\,. \quad (4.11)$$

Of course, the above broken-symmetry relations are only a first approximation relating $U(6)$ symmetry breaking in the masses to symmetry breaking at the magnetic vertices. The effect of higher states in the intermediate state sum and deviations from pole dominance may be quite sizable. Let us, however, remember that Eq. (4.11) was obtained by using only isospin invariance and pole dominance.

It is interesting to examine the predictions of Eq. (4.11). Using the proton and neutron magnetic moments as input, we obtain $\mu_{\rho\Sigma\Lambda}\approx 2.0$ nuclear magnetrons. Also, on using the experimental value[6] $\mu_{\Sigma^+}=4.3\pm1.5$ we obtain $\mu_{\Sigma^-}=-0.3\pm1.5$ nuclear magnetons.

V. CONCLUSIONS

We have examined a set of collinear superconvergence relations for the absorptive parts of the amplitudes for the processes $VB\to VB$, $VB^*\to VB$, and $VB^*\to VB^*$. The sum rules are selected by the use of the infinite-momentum limit. We limit our intermediate-state sum to single-particle states of the baryon octet and decuplet.

The resulting equations lead to a nontrivial solution relating all vector-meson–baryon couplings. For degenerate B and B^* masses, the solution is consistent with the collinear $U(6)$ symmetry for the vertices. Thus, for example, one obtains an $M1$ transition at the B^*BV vertex on taking the B and B^* masses as degenerate. If one retains the mass splitting between octet and decuplet, collinear $U(6)$ symmetry breaking at the vertices and in the masses are related. Relations between the various VBB and VB^*B vertices have been obtained while retaining the octet-decuplet mass splitting. However, the relations involving the remaining vertices, for the sake of brevity, have been obtained keeping the B^* and B masses degenerate.

Assuming vector-meson pole dominance for the form factors and using our results for the vector-meson–baryon vertices, one can obtain relations between form factors consistent with the collinear $U(6)$ predictions. Thus, one may obtain relations between electric and magnetic form factors of octet and decuplet, form factors for vector-meson B^* production, and electric and magnetic quadrupole form factors of the decuplet.

Symmetry breaking at the vertices and in the masses may be related by considering the diverse $\rho B\to\rho B$ scattering processes and using only isospin invariance at the vertices. On assuming ρ-pole dominance for the isovector form factors, relations between the various isovector magnetic moments of the baryons in broken $SU(3)$ symmetry are obtained. These results are only a first approximation, because deviations from pole dominance and contributions from higher states may be quite sizable.

Let us note a few points about the inclusion of higher intermediate states. It has been previously pointed out[2] for the case of mesons that states should not be introduced singly but in *sets* corresponding, for example, to higher representations of a rest symmetry like

[6] A. McInturff and C. Roos, Phys. Rev. Letters 13, 246 (1964).

$U(6)\otimes U(6)\otimes O(3)$ with collinear $U(6)\otimes O(2)$ invariance for the vertices. These sets then *separately* saturate the sum rule for $t=0$ and the results for the lowest multiplet remain unchanged.[7] Similarly for the baryons it may be possible to introduce higher states in sets corresponding to higher representations of some suitable group containing $U(6)\otimes U(6)$ so that each set separately satisfies the superconvergence relation for $t=0$.

In general, the saturation of our sum rules with particles of definite mass and spin corresponds to a power-series expansion of the integrals over absorptive parts around $t=0$.[8] The exact saturation for a finite interval in t requires, of course, an infinite set of particles with unlimited spin. In order to saturate our sum rules for small finite values of t, we may try to use the sequence of states which saturate the forward superconvergence relations. The nonforward superconvergence relation will certainly imply stringent additional restrictions on the mass spectrum and on the vertices.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor R. Oehme for suggesting the problem, continued guidance, and stimulating discussions.

[7] Recent calculations by Oehme, as well as by Freund and Rotelli, for the case of mesons, indicate that this is indeed what happens if higher representations of $U(6)\otimes U(6)\otimes O(3)$ are included (private communication). See also P. G. O. Freund, R. Oehme, and P. Rotelli, Phys. Rev. (to be published).

[8] For a detailed discussion of these points, see Ref. 2.

PHYSICAL REVIEW VOLUME 163, NUMBER 5 25 NOVEMBER 1967

Infinite Multiplets and Crossing Symmetry.* I. Three-Point Amplitudes

C. Fronsdal and R. White
Department of Physics, University of California, Los Angeles, California
(Received 25 May 1967)

This paper investigates what remains of crossing symmetry in theories that are conventional local field theories in all but one respect: that infinite irreducible representations of the homogeneous Lorentz group are used. Only vertex functions are studied here; results for scattering amplitudes will be reported in a sequel. It is found that: (i) Form factors for scattering ($t<0$) and form factors for annihilation ($t>4m^2$) are strongly related to each other by the requirement that the interaction Lagrangian density be local, but they are *not* connected by analytic continuation. (ii) In the case of half-integral-spin fields, the empirical fact that the parities of particles and antiparticles are opposite makes it necessary to use a pair of conjugate irreducible representations, rather than a single unitary irreducible representation. An analog of the Dirac equation allows one to avoid parity doubling and to ensure a proper physical interpretation, provided that quantization is carried out with anticommutators.

I. INTRODUCTION

LOCAL field theory possesses a number of "good" properties of a general sort, such as microcausality and crossing symmetry; and some "bad" specific properties, for example, the fact that the first Born approximation is an extremely poor representation of experimental form factors. Infinite-component field theories were first introduced because of the ease with which they can accommodate internal symmetries of the type of $SU(6)$, but even apart from $SU(6)$ they turned out to have considerable intrinsic interest. In these theories the first Born approximation to the form factors is remarkably similar to the best parametric fits to experimental data.[1] On the other hand, it is not clear that their general properties are satisfactory.[2] In a previous paper[3] it has been shown that locality, in the dual sense of a local Lagrangian density and local commutation relations, can be satisfied, and that the conventional relation between spin and statistics is at least favored. The purpose of the present paper is to show precisely what are the crossing properties of a sample infinite-component "local" field theory.

The conclusions that have been reached here, with regard to vertex functions, are as follows. The requirement that the Born approximation be given by a local interaction Lagrangian density implies that scattering and annihilation form factors are strongly related to each other. However, the form factors for the two channels are *not* related to each other by analytic continuation in the invariant momentumt ransfer. This does not mean that analyticity is lacking, but only that the analytic continuation of a vertex function from negative to positive values of the invariant momentum transfer has no direct physical significance. In the case of half-integral-spin theories, it is found (as first pointed out to

* Supported in part by the National Science Foundation.
[1] See, e.g., G. Cocho, C. Fronsdal, H. Ar-Rashid, and R. White, Phys. Rev. Letters **17**, 275 (1966).
[2] See, e.g., E. Abers, I. T. Grodsky, and R. E. Norton, Phys. Rev. **159**, 1222 (1967).
[3] C. Fronsdal, Phys. Rev. **156**, 1653 (1967).