Low-Energy ΞN and $\Lambda\Sigma$ Interactions*

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The low-energy ΞN and $\Lambda\Sigma$ interactions have been studied by the method of solving the coupled-channel Schrödinger equation. Using meson-exchange potentials to represent the outer regions and a phenomenological hard core to represent the inner regions of the interactions, the zero-energy scattering lengths and phase shifts as a function of energy have been computed for various choices of coupling parameters. No bound states in these systems are indicated for the most likely values of coupling parameters and core radii.

1. INTRODUCTION

ONE of the long-standing questions of the low-energy baryon-baryon interactions concerns whether or not any bound state of two baryons exists other than that of $T=0$ and $S=0$ (the deuteron). The fact that none have as yet been observed does not preclude the possibility of their existence, as their production cross sections would reasonably be expected to be quite small. Several authors[1–11] have discussed the possible dibaryon bound states and recently[12] an experimental search was made for one of $S=0$ (other than the deuteron), with negative results. Oakes[1] has argued for the possible existence of bound dibaryon states or resonances of strangeness $S=-2$, using considerations of unitary symmetry. As pointed out by Dalitz,[13] however, predictions of bound states or resonances based upon applying unitary symmetry directly to the scattering matrix elements neglect the important mass differences among the meson fields. These mass differences give rise to differences among the ranges of the forces for various baryon-baryon systems and thus to markedly different interaction strengths even if approximate equality of the Yukawa coupling strengths is assumed.

A particularly favorable two-baryon system to study for the existence of possible bound states or resonances is that of $S=-2$, $T=1$; here the ΞN threshold lies 35 MeV below the threshold for $\Lambda\Sigma$ production and 113 MeV below the $\Sigma\Sigma$ threshold. The relatively long range force due to one-pion exchange is a feature of all three systems, and would conceivably bring about formation of a loosely bound state analogous to the deuteron. The attractive force in the ΞN system will be enhanced by kaon coupling with the nearby closed $\Lambda\Sigma$ and $\Sigma\Sigma$ channels. For the $\Lambda\Sigma$ system, the closed $\Sigma\Sigma$ channel would be expected to have a similar effect.

It is the puropse of this calculation to examine the question of the possible existence of bound states or low-lying resonances in the $S=-2$, $T=1$ states of the ΞN and $\Lambda\Sigma$ systems, by determining the low-energy scattering behavior from the solution of the multichannel Schrödinger equation, using potentials derived from meson field theory. In the next section, the derivation of the potentials is discussed. In the final section, the results are summarized and discussed.

2. POTENTIALS

Starting with the nonrelativistic Hamiltonian density

$$\begin{aligned}H_{\mathrm{int}}=&(f_{NN}/\mu)(4\pi)^{1/2}(N^\dagger\sigma_i\boldsymbol{\tau}N)\cdot\nabla_i\boldsymbol{\pi}\\&+(f_{\Xi\Xi}/\mu)(4\pi)^{1/2}(\Xi^\dagger\sigma_i\boldsymbol{\tau}\Xi)\cdot\nabla_i\boldsymbol{\pi}\\&+(f_{\Lambda\Sigma}/\mu)(4\pi)^{1/2}(\Lambda^\dagger\sigma_i\boldsymbol{\Sigma}\cdot\nabla_i\boldsymbol{\pi}+\mathrm{H.c.})\\&+(f_{N\Lambda}/\mu)(4\pi)^{1/2}(N^\dagger\sigma_i\Lambda\nabla_iK+\mathrm{H.c.})\\&+(f_{\Xi\Lambda}/\mu)(4\pi)^{1/2}(\Xi^\dagger\sigma_i\Lambda\nabla_iK^G+\mathrm{H.c.})\\&+(f_{N\Sigma}/\mu)(4\pi)^{1/2}(N^\dagger\sigma_i\boldsymbol{\tau}\cdot\boldsymbol{\Sigma}\nabla_iK+\mathrm{H.c.})\\&+(f_{\Xi\Sigma}/\mu)(4\pi)^{1/2}(\Xi^\dagger\sigma_i\boldsymbol{\tau}\cdot\boldsymbol{\Sigma}\nabla_iK^G+\mathrm{H.c.})\\&+(f_{\Sigma\Sigma}/\mu)(4\pi)^{1/2}(\boldsymbol{\Sigma}^\dagger\sigma_i\chi\boldsymbol{\Sigma})\cdot\nabla_i\boldsymbol{\pi},\end{aligned}\tag{1}$$

the one-pion, one-kaon, and uncorrelated two-pion exchange contributions to the potentials were derived. The uncorrelated two-pion exchange was derived by the Brueckner-Watson procedure[14] as described by de Swart and Iddings.[15] The notation of Ref. 15 will be used here, except as otherwise noted. In addition to the one- and two-pion and the one-kaon exchange contributions which are considered here, effects due to the exchange of higher-mass mesons (e.g., the K^*) are also present, as well as higher-order exchanges of pions and kaons. In the approach adopted here we lump all of these effects into a phenomenological repulsive core, taken to be the same radius as that generally used in the analyses of the nucleon-nucleon problem (of the order 0.35 pion Compton wavelengths). If our results are not too critically

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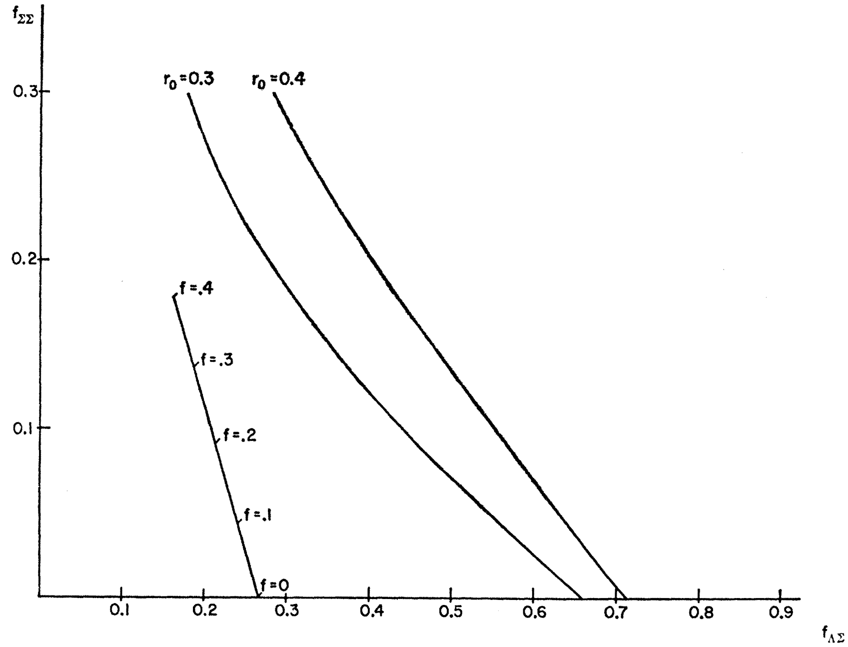

FIG. 1. The combinations of $f_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$ which give rise to bound $^1S_0$ $\Lambda\Sigma$ states. The straight line is the line of unitary symmetry, each point on the line corresponds to a certain value of f [the ratio $F/(F+D)$]. The region to the right of the curve for $r_0=0.3$ (0.4) is the bound-state region for a repulsive core radius $r_0=0.3$ (0.4). r_0 is measured in units of μ^{-1}.

dependent on the size of this core, we expect them to have some qualitative significance. This point is further discussed in Sec. 3.

The expressions for the $T=1$ potentials are as follows:

$$V(\Xi N,\Xi N)=f_{\Xi\Xi}f_{NN}V^{(2)}+(f_{\Xi\Xi}f_{NN})^2(5\,{}^{\mathrm{X}}V^{(4)}+{}^{\mathrm{II}}V^{(4)}),\quad (2)$$

$$V(\Lambda\Sigma,\Lambda\Sigma)=-P_xP_\sigma f_{\Lambda\Sigma}^2V_1^{(2)}+(f_{\Lambda\Sigma}f_{\Sigma\Sigma})^2[3({}^{\mathrm{II}}V^{(4)}+{}^{\mathrm{X}}V^{(4)})-2(-P_xP_\sigma)({}^{\mathrm{X}}V^{(4)}-{}^{\mathrm{II}}V^{(4)})],\quad (3)$$

$$V(\Sigma\Sigma,\Sigma\Sigma)=-f_{\Sigma\Sigma}^2V^{(2)}+2[f_{\Lambda\Sigma}^2f_{\Sigma\Sigma}^2({}^{\mathrm{X}}V^{(4)}+2\,{}^{\mathrm{II}}V^{(4)})]+f_{\Sigma\Sigma}^4\,{}^{\mathrm{II}}V^{(4)}-f_{\Lambda\Sigma}^4\,{}^{\mathrm{X}}V^{(4)},\quad (4)$$

$$V(\Xi N,\Lambda\Sigma)=-\sqrt{2}f_{\Xi\Lambda}f_{N\Sigma}V_{K1}^{(2)}-P_xP_\sigma\sqrt{2}f_{\Xi\Sigma}f_{N\Lambda}V_{K1}^{(2)},\quad (5)$$

$$V(\Xi N,\Sigma\Sigma)=2f_{\Xi\Sigma}f_{N\Sigma}V_{K2},\quad (6)$$

$$V(\Lambda\Sigma,\Sigma\Sigma)=\sqrt{2}[-f_{\Lambda\Sigma}f_{\Sigma\Sigma}V_2^{(2)}+(f_{\Lambda\Sigma}^3f_{\Sigma\Sigma}+f_{\Sigma\Sigma}^3f_{\Lambda\Sigma})\times({}^{\mathrm{II}}V^{(4)}-{}^{\mathrm{X}}V^{(4)})].\quad (7)$$

In these expressions,

$$V_i^{(2)}(r)=[V_\sigma^{(2)}(\mu_i'r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+V_T^{(2)}(\mu_i'r)S_{12}](\mu_i'/\mu)\mu_i',\quad (8)$$

where

$$\mu_1'=0.84\mu,\qquad \mu_2'=0.96\mu,\quad (9)$$

and

$$V_{Ki}(r)=[V_\sigma^{(2)}(\mu_{Ki}'r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+V_T(\mu_{Ki}'r)S_{12}]\times(\mu_{Ki}'/\mu)^2\mu_{Ki}',\quad (10)$$

where

$$\mu_{K1}'=0.89\mu_K,\qquad \mu_{K2}'=0.93\mu_K.\quad (11)$$

Equations (9) and (11) are derived according to a procedure previously discussed[15,16] in correcting for the mass differences between channels. P_x and P_σ are the space and spin exchange operators. The meaning of all of the other symbols appearing in the Lagrangian and the potentials has been given previously[16] and will not be repeated here.

3. RESULTS

The potentials given above, together with the phenomenological hard core, were inserted into the Schrödinger equation which was solved numerically to obtain the K matrix elements of interest. Whether or not bound states or resonances exist depends quite critically on the values one assumes for the various coupling constants. Only one, f_{NN}, is known with certainty; its value is 0.285. There is some evidence[17–19] that the $\Lambda\Sigma\Pi$ coupling is comparable in strength to the $NN\Pi$ coupling and that the $\Sigma\Sigma\Pi$ coupling is small, of order 0.1 or less. This would imply a small F/D ratio, [$f=F/(F+D)\leq 0.25$] if the relations of $SU(3)$ are assumed to exist between the pseudoscalar couplings of pseudoscalar mesons and baryons. Several authors[4,20–23]

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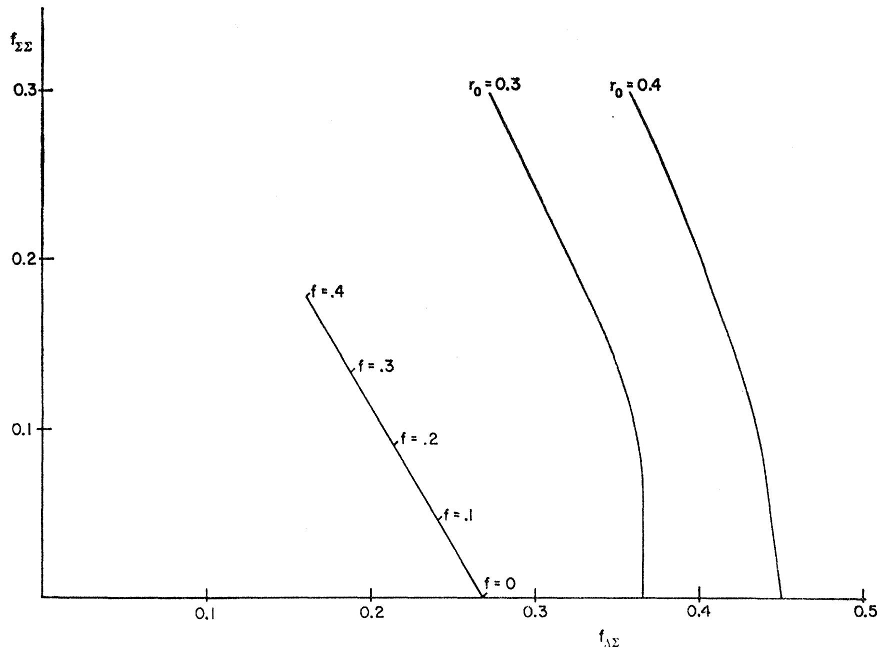

FIG. 2. The combinations of $f_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$ which give rise to bound ${}^3S_1$ $\Lambda\Sigma$ states. The straight line is the line of unitary symmetry; each point on the line corresponds to a certain value of f [the ratio $F/(F+D)$]. The region to the right of the curve for $r_0=0.3$ (0.4) is the bound-state region for a repulsive core radius $r_0=0.3$ (0.4). r_0 is measured in units of μ^{-1}.

have argued for a value $f=F/(F+D)$ of the order 0.3 or 0.4 (corresponding to $f_{\Lambda\Sigma}=0.187$ and 0.161, respectively).

First studied was the question of the existence of an isotriplet ΞN bound state. Table I gives the scattering lengths[24] at zero energy, calculated for a core radius $r_0=0.3\mu^{-1}$, and $f=0.0$, 0.3, 0.4. It is seen that for these couplings and core radius, not only is there no bound state, the interaction is repulsive. This conclusion was found to be not affected qualitatively by choosing different core radii (0.35, $0.4\mu^{-1}$). The core radii were taken to be the same in all channels. The effect of the long-range one-pion exchange contribution (OPEC) is clearly marked; it gives rise to a ***repulsive*** tail in the ${}^1S_0\,\Xi N$ potential and an *attractive* tail in the ${}^3S_1\,\Xi N$ potential. There is some uncertainty as to the size of the K couplings; there is some reason to believe that their values may be much smaller than those used here. The K couplings one obtains by applying unitary-symmetry relations to the ***pseudovector*** coupling constants are smaller than the ones used in the present paper by a factor of order μ/μ_K. Lusignoli *et al.*[25] and Zovko[26] using forward dispersion relations for the KN scattering, have argued for the smallness of the K couplings. A bound ΞN state would be even less likely if smaller K couplings were used, since the coupling to the closed channels produces an attractive effect.

A virtual bound state in the $\Lambda\Sigma$ system would be expected to give rise to a resonance in the ΞN scattering. This possibility was investigated by computing the ΞN scattering phase shifts as a function of energy. Table II gives the results, which do not indicate any such reso-

TABLE I. Singlet and triplet zero-energy scattering lengths for the ΞN S-wave scattering. r_0 is the radius of the repulsive core. a_0 and a_1 are the singlet and triplet scattering lengths, respectively. r_0, a_0, and a_1 are in units of μ^{-1}. $f=F/(F+D)$.

	$f=0.0$ $f_{\Xi\Xi}=-0.20$	$f=0.3$ $f_{\Xi\Xi}=-0.08$		$f=0.4$ $f_{\Xi}=-0.04$	
r_0	a_1	a_0	a_1	a_0	a_1
0.30	−0.77	0.35	0.22	0.34	0.27
0.35	−0.85	0.41	0.29	0.39	0.33
0.40	0.12	0.46	0.35	0.44	0.38

[24] The values of a_0 for $f=0.0$ are not shown in Table I because the solution breaks down for very strongly repulsive potentials [see Eq. (C3) of Ref. 28].

TABLE II. Singlet and triplet scattering phase shifts and coupling parameters for ΞN scattering, up to the $\Lambda\Sigma$ threshold. The ${}^1S_0$ scattering phase shift is denoted by δ_0; the triplet phase shifts for the mixture of S and D orbital angular-momentum states are denoted by δ_1 and δ_2. Reference 27 gives the definitions of δ_1, δ_2, and the coupling parameter ϵ. All the phase shifts and the coupling parameter are in degrees. $r_0=0.35\mu^{-1}$. $f=F/(F+D)$.

E (MeV)	δ_0, deg.	δ_1, deg.	ϵ, deg.	δ_2, deg.
		$f=0.0$		
15.0	−6.9	2.3	41.5	−2.2
30.0	−8.2	−5.1	−42.6	+2.7
45.0	−9.3	−7.6	−39.7	2.6
		$f=0.3$		
15.0	−12.9	−10.4	−4.5	−.03
30.0	−17.5	−15.1	−5.8	−.17
45.0	−21.0	−18.7	−6.2	−.36
		$f=0.4$		
15.0	−12.7	−11.4	−2.0	−.03
30.0	−17.6	−16.3	−2.6	−.12
54.0	−21.4	−20.1	−2.8	−.23

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nance for a wide range of f values. The $\Lambda\Sigma\pi$ coupling is the most important coupling in determining the strength of the $\Lambda\Sigma$ interaction. In an effort to see what values of $f_{\Lambda\Sigma}$ would give rise to such a bound $\Lambda\Sigma$ state, $f_{\Lambda\Sigma}$ was increased to very large (unreasonably large) values. With the procedure used[27] in the numerical solution of the Schrödinger equation, the very strong potentials thus generated caused problems in the numerical solution of a type which has been described elsewhere.[28] However, reasoning from the weakness and short range of the K couplings (as evidenced by the negligible effect of the closed $\Lambda\Sigma$ and $\Sigma\Sigma$ channels on the ΞN scattering[29]), it was decided to neglect the ΞN channel entirely to study the $\Lambda\Sigma$ interaction. This was done, and the results are shown in Tables III and IV and in Figs. 1 and 2. In Figs. 1 and 2 the calculated results are presented in the form of two curves corresponding to $r_0=0.3\mu^{-1}$ and $r_0=0.4\mu^{-1}$, respectively. Points in the $f_{\Lambda\Sigma}$, $f_{\Sigma\Sigma}$ plane to the left of a curve give rise to an unbound $\Lambda\Sigma$ state; points to the right of the curve give rise to bound $\Lambda\Sigma$ states. The third curve represents the line of unitary symmetry; points on that curve correspond to various values of $f=F/(F+D)$ as indicated. Figures 1 and 2 show that neither triplet nor singlet bound S states are possible without increasing $f_{\Lambda\Sigma}$ to unrealistically large values, values not possible for any combination of F and D couplings. This is true for both core radii chosen.

TABLE III. Singlet and triplet zero-energy scattering lengths for the S-wave $\Lambda\Sigma$ scattering. $f=F/(F+D)$ ratio. Core radius $r_0=0.3\mu^{-1}$. a_0 and a_1 are the $^1S_0$ and $^3S_1$ scattering lengths, respectively, in units of μ^{-1}.

	$f=0.0$ $f_{\Lambda\Sigma}=0.268$ $f_{\Sigma\Sigma}=0.0$	$f=0.3$ $f_{\Lambda\Sigma}=0.187$ $f_{\Sigma\Sigma}=0.134$	$f=0.4$ $f_{\Lambda\Sigma}=0.161$ $f_{\Sigma\Sigma}=0.79$
a_0	$-.11$	$+.018$	$+.033$
a_1	$-.13$	$+.13$	$+.15$

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TABLE IV. Singlet and triplet scattering phase shifts and coupling parameters for $\Lambda\Sigma$ scattering up to the $\Sigma\Sigma$ threshold. $r_0=0.3\mu^{-1}$. $F/(F+D)$ ratio $f=0$ ($f_{\Lambda\Sigma}=0.268$, $f_{\Sigma\Sigma}=0.0$). E_{lab} in MeV. Phase shifts and coupling parameters in degrees.

E_{lab}	δ_0	δ_1	ϵ	δ_2
5.0	-0.2	2.5	29.1	-0.9
10.0	-2.5	3.4	37.7	-2.5
15.0	-4.6	3.7	40.9	-4.1
25.0	-8.4	3.6	44.0	-6.8
35.0	-11.5	3.2	-44.0	-9.1
45.0	-14.2	2.6	-42.4	-11.2
55.0	-16.6	2.0	-41.0	-13.1
65.0	-18.8	1.3	-40.0	-14.8
75.0	-20.8	0.6	-39.0	-16.5

Table III gives the values of the scattering lengths for various choices of $f=F/(F+D)$, for a hard-core radius of $0.3\mu^{-1}$. The behavior of these scattering lengths shows that in both $^3S_1$ and $^1S_0$ states, there is an attractive tail outside the repulsive core, which, in conjunction with the effect of the closed $\Sigma\Sigma$ channel, leads to a net weakly repulsive force for all but the very lowest points on the line of unitary symmetry.

Table IV gives the $^1S_0$ and $^3S_1$ $\Lambda\Sigma$ phase shifts as a function of energy for $f=0.0$, 0.3, and 0.4. As is seen, no resonance is found for any of the couplings. Again these results are qualitatively the same for small ($\pm0.05\mu^{-1}$) variations in r_0.

4. SUMMARY

On the basis of the above results, it seems quite unlikely that there should exist a bound ΞN state or a virtually bound $\Lambda\Sigma$ state, for reasonable values of an inner repulsive core radius, and coupling constants given by $SU(3)$. In fact, the low-energy ΞN interaction is indicated as repulsive [except for pure D-type $SU(3)$ couplings, where it is weakly attractive].

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