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Couplings of Baryon Resonances from Current Divergences*

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Using the partially conserved axial-vector current hypothesis and assuming the axial-vector current to remain an octet to a good approximation in broken SU_3, relations are obtained for the couplings of baryon resonances with the normal baryons and pseudoscalar mesons. Using a few observed decay widths as input, the strengths of the remaining couplings are estimated. The results are compared with the results of other authors using different approaches and with the sum rules following from a first-order symmetry breaking.

I. INTRODUCTION

IN a recent paper,[1,2] an estimate was made of ratios of pseudoscalar meson-baryon coupling constants in broken SU_3, assuming that the axial-vector currents retain their octet character to a good approximation in the presence of symmetry breaking,[3,4] and using the hypothesis of a partially conserved axial-vector current (PCAC). In this paper we start with these assumptions and obtain relations among the couplings of baryonic resonances to baryons and pseudoscalar mesons in broken SU_3. We consider in detail the $\frac{3}{2}^+$, $\frac{3}{2}^-$, and $\frac{5}{2}^+$ baryonic resonances.

Starting with the basic equations obtained by taking matrix elements of the PCAC relation, we use the observed widths of a few decay modes of the resonances to solve for the axial-vector renormalization constants entering the equations. These are then used for evaluating the remaining coupling constants of the resonances. The results for the coupling constants are compared, where possible, with the results obtained from other approaches.

In Sec. II we write the relations for the couplings of a $\frac{3}{2}^+$ decuplet baryon to a $\frac{1}{2}^+$ octet baryon and an octet pseudoscalar meson. By eliminating the axial-vector renormalization constants from the basic equations, sum rules are derived for the decuplet coupling constants in terms of the physical masses of the octet and decuplet baryons. Using the experimental values for the widths of three of the decay modes of the $\frac{3}{2}^+$ baryons, the renormalization constants for the couplings of the axial-vector current are deduced, and are used for evaluating the remaining coupling constants. The predicted coupling constants are compared with the results of other authors obtained by different methods.

In Secs. III and IV we consider the couplings of the $\frac{3}{2}^-$ and $\frac{5}{2}^+$ baryonic resonances (assuming each of these to form an octet) with the $\frac{1}{2}^+$ octet baryons and the octet pseudoscalar mesons. In Sec. V we summarize our conclusions.

A similar evaluation of the coupling strengths of meson resonances and the corresponding axial-vector renormalization constants will be reported elsewhere.

II. RELATIONS FOR DECUPLET COUPLINGS

We start by assuming the PCAC relation

$$\partial_\lambda \mathcal{A}_\alpha{}^\lambda(x) = C_\alpha \varphi_\alpha(x)\,, \tag{2.1}$$

where $\mathcal{A}_\alpha{}^\lambda(x)$ is an axial-vector current density, $\varphi_\alpha(x)$ is a pseudoscalar meson field operator, and α is an SU_3 octet index. In exact symmetry, C_α would be independent of α; in broken symmetry it will, in general, depend on α. We take the matrix element of each side of (2.1) between a $\frac{1}{2}^+$ octet baryon B and a $\frac{3}{2}^+$ decuplet baryon resonance B^*. Writing the matrix element of the axial-vector current $\mathcal{A}_\alpha{}^\lambda(0)$ as a sum of terms proportional to $g^{\lambda\rho}$, $\gamma^\lambda q^\rho$, $P^\lambda q^\rho$, and $q^\lambda q^\rho$, taken between a Rarita-Schwinger spinor $\bar{\psi}_\rho(p_f)$ and a Dirac spinor $u(p_i)$, we assume that the coefficient of each of these terms transforms like the (invariant) matrix element of an octet operator between an octet and a decuplet.

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[1] K. Raman, Phys. Rev. **149**, 1122 (1966); **152**, 1517(E) (1966).

[2] The method of obtaining relations for coupling constants from Goldberger-Treiman relations was first used by Riazuddin, Phys. Rev. **136**, B268 (1964) and by P. G. O. Freund and Y. Nambu, Phys. Rev. Letters **13**, 221 (1964). The latter authors also obtained values of the decuplet coupling constants. However, their detailed assumptions and results are different from ours.

[3] This assumption is supported empirically by the SU_3 extension of the Adler-Weisberger relation. [See S. L. Adler, Phys. Rev. Letters **14**, 1051 (1965); W. I. Weisberger, *ibid.* **14**, 1047 (1965); L. K. Pandit and J. Schechter, Phys. Letters **19**, 56 (1965); A. Sato and S. Sasaki, Osaka University Report, 1965 (unpublished); C. A. Levinson and I. J. Muzinich, Phys. Rev. Letters **15**, 715 1965).] Theoretically, a motivation for it may be found by noting that in a broken $U(3)\times U(3)$ symmetry scheme one may derive a generalization of the Ademollo-Gatto theorem [M. Ademollo and R. Gatto, Phys. Rev. Letters **10**, 531 (1963); and C. Bouchiat and Ph. Meyer, Nuovo Cimento **34**, 1122 (1964)] which suggests that not only the vector current but also the axial-vector current remains a pure octet to the first order in the SU_3 symmetry breaking; see, e.g., J. Schechter and Y. Ueda, Phys. Rev. **144**, 1338 (1966); and G. S. Guralnik, V. S. Mathur, and L. K. Pandit, Phys. Letters **20**, 64 (1966). Possible deviations from the octet character of the axial-vector current and their consequences are under investigation.

[4] Recently, E. C. G. Sudarshan and N. Mukunda [Phys. Rev. **158**, 1424 (1967)] have formulated a stability principle which states that for many tensor operators, including an octet axial-vector current, the assumption that one component of the tensor operator is not renormalized to first order under a perturbation ensures that the complete tensor operator remains unchanged to first order. They further argue that if empirical evidence could be obtained for the nonrenormalization of a strangeness-conserving component of the axial-vector current to the first order in the SU_3 symmetry breaking, this would ensure the nonrenormalization of the whole axial-vector current to first order. This would support our basic assumption. We are obliged to these authors for a discussion of their work.

This gives the following[5,6]:

$$\langle B_f^*(p_f)|\,\mathcal{A}_\alpha^\lambda(0)\,|B_i(p_i)\rangle=[m_iM_f^*/E_iE_f]^{1/2}\mathcal{C}_{\alpha if}\bar{\psi}_\rho(p_f)\times(-i)\{G_A^*(q^2)g^{\lambda\rho}+F_A^*(q^2)\gamma^\lambda q^\rho+H_A^*(q^2)P^\lambda q^\rho+L_A^*(q^2)q^\lambda q^\rho\}u(p_i). \quad (2.2)$$

Here we have defined $P=(p_f+p_i)$ and $q=(p_f-p_i)$. $\mathcal{C}_{\alpha if}$ denotes an SU_3 Clebsch-Gordan coefficient.

For the matrix element of the meson source operator $j_\alpha(x)=(\Box^2+\mu_\alpha^2)\varphi_\alpha(x)$, we write the following:

$$\langle B_f^*(p_f)|\,j_\alpha(0)\,|B(p_i)\rangle=[m_iM_f^*/E_iE_f]^{1/2}f_{if\alpha}K_{if\alpha}^*(q^2)\bar{\psi}_\rho(p_f)q^\rho u(p_i). \quad (2.3)$$

From (2.1)–(2.3) we obtain, for $q^2\equiv(p_f-p_i)^2=0$, the relations[6]

$$\mathcal{C}_{\alpha if}[G_A^*(0)+(M_f^*-m_i)F_A^*(0)+(M_f^{*2}-m_i^2)H_A^*(0)]=d_\alpha f_{if\alpha}K_{if\alpha}^*(0). \quad (2.4)$$

Here, $K_{if\alpha}^*(q^2)$ is the form factor for the PBB^* vertex, normalized to $K_{if\alpha}^*(\mu_\alpha^2)=1$, and we have used the definition

$$d_\alpha=C_\alpha/\mu_\alpha^2. \quad (2.4a)$$

The relation (2.4) can give rise to a symmetry breaking in the PBB^* couplings $f_{if\alpha}$ through the dependence of the quantity

$$[G_A^*(0)+(M_f^*-m_i)F_A^*(0)+(M_f^{*2}-m_i^2)H_A^*(0)]/d_\alpha K_{if\alpha}^*(0) \quad (2.4b)$$

on the indices α, i, and f. If this quantity were independent of α, i, and f, then the PBB^* coupling constant would be the product of the Clebsch-Gordan coefficient $\mathcal{C}_{\alpha if}$ and a quantity independent of α, i, and f, so that the relative values of $f_{if\alpha}$ would be the same as with exact symmetry for the PBB^* coupling.

In (2.4b), $G_A^*(0)$, $F_A^*(0)$, and $H_A^*(0)$ are, by assumption, independent of α, i, and f. The dependence of (2.4b) on α, i, and f can arise from the dependence of m_i and M_f^* on i and f, the dependence of d_α on α, and the dependence of $K_{if\alpha}^*(0)$ on α, i, and f.

The variation of m_i and M_f^* with i and f is known from the physical masses. To obtain some information about the dependence of $d_\alpha K_{if\alpha}^*(0)$ on α, i, and f, we proceed as follows.

By taking matrix elements of (2.1) between spin-$\frac{1}{2}$ octet baryon states with masses m_1 and m_2 we obtain the relation

$$g_A^{(12\alpha)}(m_2+m_1)=d_\alpha G_{12\alpha}K_{12\alpha}(0). \quad (2.5)$$

We now assume that for given α, $K_{if\alpha}^*(0)$ and $K_{12\alpha}(0)$ are approximately equal and that they are largely independent of what the initial and final baryons are:

$$K_{if\alpha}^*(0)\approx K_{i'f'\alpha}^*(0)\approx K_{12\alpha}(0)\approx K_{1'2'\alpha}(0)\equiv K_\alpha(0), \quad (2.6)$$

that is, we assume that when the change in the form factors in going from $q^2=\mu_\alpha^2$ to $q^2=0$ is not too large, it is essentially determined, for a given α, by the value of μ_α^2 and does not depend sensitively on the masses and spins of the external baryons in the PBB and PBB^* vertices.

For given α, we can determine $d_\alpha K_{12\alpha}(0)$ if $g_A^{(12\alpha)}$ and $G_{12\alpha}$ in (2.5) are known. By using Cabibbo's theory of leptonic decays[7] (together with an analysis of the data on these decays), and assuming the values of the πNN and $\bar{K}N\Lambda$ coupling constants, one may obtain $d_\alpha K_{12\alpha}(0)$ for $\alpha=\pi$ and $\alpha=K$. Since there appear to be no reliable estimates available for a ηBB coupling constant, we cannot make a reliable estimate of $d_\eta K_{12\eta}(0)$ by this method.

For the πNN coupling constant, we assume $G_{\pi NN}^2/4\pi\approx 14.6$. For the $\bar{K}N\Lambda$ coupling constant, recent determinations by different workers do not agree; thus Lusignoli *et al.*[8] give the estimate $G_{\bar{K}N\Lambda}^2/4\pi=4.8\pm1.0$, while Rood[9] gives a larger value: $G_{\bar{K}N\Lambda}^2/4\pi=7.4\pm1.0$. (For comparison, we note that if one had assumed exact SU_3 symmetry for the PBB vertex and taken the d/f ratio for this vertex to be about 2, then using $G_{\pi NN}^2/4\pi\approx14.6$, one would obtain $G_{\bar{K}N\Lambda}^2/4\pi\approx13.5$.)

A considerable part of the difference between the estimates of Lusignoli *et al.* and of Rood seems to arise from the difference in their descriptions of low-energy $I=0$ KN scattering. The effective-range parametrization used by Rood for this amplitude appears to be better; however, he has neglected the K^--K^0 mass difference, the quantitative effect of which is not clear. There are several uncertainties in the input data used in the analyses, and at present it is not clear what is the best value for $G_{\bar{K}N\Lambda}^2/4\pi$.[10] We shall give the results following from the estimates of both Lusignoli *et al.* and Rood, stressing that the value of $d_K K_{12K}(0)$ will be uncertain to the extent that $G_{\bar{K}N\Lambda}$ is uncertain.

As shown in Ref. 1, if one assumes a value of about 2 for the d/f ratio of the matrix element of the axial-

[5] Here M^* and m denote the masses of a decuplet baryon B^* and an octet baryon B, respectively. P_α denotes a pseudoscalar meson with SU_3 octet index α.

[6] The independent couplings in (2.2) can be chosen in different ways. However, the general explicit dependence on the masses m and M of the matrix element of $\partial_\mu\mathcal{A}^\mu(x)$ between a spin-$\frac{1}{2}^+$ baryon B described by a Dirac spinor and a spin-$\frac{3}{2}^+$ baryon B^* described by a Rarita-Schwinger spinor is of the form shown on the left-hand side of Eq. (2.4). Our assumption is that when the matrix element of $\partial_\mu\mathcal{A}^\mu(x)$ is written in this form, then the couplings so defined (corresponding to the G_A^*, F_A^*, and H_A^* terms) are the ones that may be assumed to behave, to a good approximation, as invariant matrix elements of an octet operator.

[7] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1960).

[8] M. Lusignoli, M. Restignoli, G. A. Snow, and G. Violini, Phys. Letters **21**, 229 (1966).

[9] H. Rood, CERN Report, 1966 (unpublished).

[10] We are grateful to Professor G. A. Snow for a discussion of the questions arising in the determination of $G_{\bar{K}N\Lambda}^2$. We are informed by him that further work on this is being done by Lusignoli *et al.* Recently, a determination of $G_{\bar{K}N\Lambda}^2$ has been made by J. K. Kim, who obtains the considerably larger value $(1/4\pi)G_{\bar{K}N\Lambda}^2\simeq16.0$. See the note added in proof at the end of the paper for a discussion of this.

vector current between baryons, then with $G_{\bar{K}N\Lambda}^2/4\pi \approx 4.8$ one obtains a value of about 1.78 for the ratio $d_K K_K(0)/d_\pi K_\pi(0)$; with $G_{\bar{K}N\Lambda}^2/4\pi \approx 7.4$, one obtains a value of about 1.43. In exact SU_3, with degenerate meson masses, this ratio would be unity. Thus it is seen that a considerably large symmetry breaking can arise from the factor $d_\alpha K_\alpha(0)$ in (2.4b), if the input value of $G_{\bar{K}N\Lambda}^2/4\pi$, used in determining $d_K K_K(0)$ through (2.5), deviates considerably from its value in exact SU_3 symmetry.

One may further examine how much of the symmetry breaking would arise from the deviation of d_K/d_π from unity and how much from the ratio $K_K(0)/K_\pi(0)$. In Ref. 1 it has been shown[11] how, using Cabibbo's theory of leptonic decays and a recent analysis of the data on leptonic decays, one may obtain the estimate

$$R \equiv d_K/d_\pi \approx 1.27. \tag{2.7}$$

Then, using the estimates of $d_K K_K(0)/d_\pi K_\pi(0)$ discussed earlier, we obtain

$$K_{\bar{K}N\Lambda}(0)/K_{\pi NN}(0) \approx 1.4 \tag{2.8a}$$

when one assumes $G_{\bar{K}N\Lambda}^2/4\pi \approx 4.8$, and

$$K_{\bar{K}N\Lambda}(0)/K_{\pi NN}(0) \approx 1.15 \tag{2.9a}$$

when one assumes $G_{\bar{K}N\Lambda}^2/4\pi \approx 7.4$. If we neglect the deviation of $K_{\pi NN}(0)$ from unity, we would obtain

$$K_{\bar{K}N\Lambda}(0) \approx 1.4 \tag{2.8b}$$

and

$$K_{\bar{K}N\Lambda}(0) \approx 1.15 \tag{2.9b}$$

for $G_{\bar{K}N\Lambda}^2/4\pi \approx 4.8$ and 7.4, respectively.

The estimates given above show that assuming that the axial-vector current is an octet to a good approximation, one may obtain for the PBB^* coupling constants a fairly large deviation from the values in exact SU_3 symmetry. With our assumptions, the magnitude of this deviation will be determined by the mass splitting in the baryon decuplet (and octet) and by the extent to which $d_\alpha K_\alpha(0)$ depends on α.

To determine $G_A^*(0)$, $F_A^*(0)$, and $H_A^*(0)$, we write Eq. (2.4) for three different choices of B_f^* and B_i:

$$\begin{pmatrix} 1 & \delta_1 & 2\delta_1\bar{m}_1 \\ 1 & \delta_2 & 2\delta_2\bar{m}_2 \\ 1 & \delta_3 & 2\delta_3\bar{m}_3 \end{pmatrix} \begin{pmatrix} G_A^*(0) \\ F_A^*(0) \\ H_A^*(0) \end{pmatrix} = \begin{pmatrix} \bar{f}_1 \\ \bar{f}_2 \\ \bar{f}_3 \end{pmatrix}, \tag{2.10}$$

where

$$\delta \equiv (M_f^* - m_i); \quad \bar{m} \equiv \tfrac{1}{2}(M_f^* + m_i),$$

$$\bar{f}_i = \frac{C_\alpha K_{if\alpha}^*(0)}{\mu_\alpha^2} \frac{f_{if\alpha}}{\mathcal{C}_{\alpha if}}, \tag{2.11}$$

and the subscripts 1, 2, and 3 denote three choices of B_f^* and B_i. Solving (2.10) gives

$$H_A^*(0) = \varphi(123), \tag{2.12a}$$

$$F_A^*(0) = \frac{(\bar{f}_2 - \bar{f}_1)}{(\delta_2 - \delta_1)} - \frac{(\delta_2\bar{m}_2 - \delta_1\bar{m}_1)}{(\delta_2 - \delta_1)} \varphi(123), \tag{2.12b}$$

$$G_A^*(0) = \frac{(\delta_2\bar{f}_1 - \delta_1\bar{f}_2)}{(\delta_2 - \delta_1)} + \frac{[\delta_1\delta_2(\bar{m}_2 - 2\bar{m}_1) + \delta_1^2\bar{m}_1]}{(\delta_2 - \delta_1)} \varphi(123), \tag{2.12c}$$

where

$$\varphi(123) = \frac{1}{2}\left[\frac{(\bar{f}_2 - \bar{f}_1)}{(\delta_2 - \delta_1)} - \frac{(\bar{f}_3 - \bar{f}_2)}{(\delta_3 - \delta_2)}\right] \times \left[\frac{(\delta_2\bar{m}_2 - \delta_1\bar{m}_1)}{(\delta_2 - \delta_1)} - \frac{(\delta_3\bar{m}_3 - \delta_2\bar{m}_2)}{(\delta_3 - \delta_2)}\right]^{-1}. \tag{2.13}$$

Considering Eq. (2.12a) for all possible choices of B_i, B_f^* and P_α, we obtain seven sum rules of the form

$$\varphi(123) = \varphi(12j), \tag{2.14}$$

where $j = 4, 5, \cdots, 10$.

Noting (2.5), (2.6), and (2.11), it is seen that the sum rules (2.14) relate the PBB^* coupling constants to the PBB coupling constants and the physical B and B^* masses in broken SU_3, if the axial-vector renormalization constants $g_A^{(12\alpha)}$ are assumed to be known. The latter may be obtained from Cabibbo's theory of leptonic decays,[7] using the experimental value of the d/f ratio $(d/f)_A$ for the matrix elements of the axial-vector current between the spin-$\frac{1}{2}^+$ octet baryons.

Setting $\alpha = \pi$ in (2.4), taking (B_f^*, B_i) as (N^*, N), (Y_1^*, Σ), and (Y_1^*, Λ) in turn, and using the available experimental values of the decay widths for $N^* \to N\pi$, $Y_1^* \to \Lambda\pi$, and $Y_1^* \to \Sigma\pi$[12]:

$$\begin{aligned} \Gamma(N^* \to N\pi) &= 120 \text{ MeV}, \\ \Gamma(Y_1^* \to \Lambda\pi) &= 36.4 \text{ MeV}, \\ \Gamma(Y_1^* \to \Sigma\pi) &= 3.6 \text{ MeV}, \end{aligned} \tag{2.15}$$

we obtain[13]

$$\begin{aligned} G_A^*(0) &\approx 0.77, \\ F_A^*(0) &\approx 0.78, \\ H_A^*(0) &\approx -0.018. \end{aligned} \tag{2.16}$$

[11] See Ref. 1, Erratum. The data on leptonic decays used here were those of N. Brene, L. Veje, M. Roos, and C. Cronström, Phys. Rev. **149**, 1288 (1966). It was noted in Ref. 1 (Erratum) that instead of assuming $R=1$ and obtaining $\theta_V \neq \theta_A$ as in the work of Brene *et al.*, one may assume $\theta_V = \theta_A = 0.212$ and obtain $R = 1.27$.

[12] The masses and widths of the baryonic resonances used in this paper have been taken from the data compiled by A. H. Rosenfeld *et al.*, Tables from UCRL 8030 (revised), August 31, 1966 (unpublished). The $\frac{3}{2}^+ N^*$, Y_1^*, and Ξ^* partial decay widths are there quoted as $\Gamma(N^* \to N\pi) = 120 \pm 2$ MeV, $\Gamma(Y_1^* \to \Lambda\pi) = (36.4 \pm 0.8)$ MeV, $\Gamma(Y_1^* \to \Sigma\pi) = (3.6 \pm 0.8)$ MeV, and $\Gamma(\Xi^* \to \Xi\pi) = 7.3 \pm 1.7$ MeV.

[13] The signs of $G_A^*(0)$, etc. in (2.16) have been written with the convention that the reduced matrix element in the PBB^* coupling is positive.

TABLE I. Coupling constants of the decuplet $\frac{3}{2}^+$ resonances with the octet $\frac{1}{2}^+$ baryon and octet pseudoscalar meson.[a,b,c]

PBB^* coupling	Exact SU_3 Isoscalar factors	Exact SU_3 $f^2/4\pi$	Estimates for $f^2/4\pi$ Solution (i)	Estimates for $f^2/4\pi$ Solution (ii)	$f^2/4\pi$ from experimental widths	Freund and Nambu (Ref. 2)	Johnson and McCliment (Ref. 19)	Wali and Warnock (Ref. 20)	DDFS (Ref. 14)
πNN^*	$\frac{1}{2}\sqrt{2}$	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38
$\pi\Lambda Y_1^*$	$\frac{1}{2}$	0.19	0.14	0.14	0.14	0.15	0.16	0.12	0.14
$\pi\Sigma Y_1^*$	$1/\sqrt{6}$	0.127	0.11	0.11	0.11	0.043	0.036	0.06	0.039
$\pi\Xi\Xi^*$	$\frac{1}{2}$	0.19	0.1	0.1	0.076	0.08	0.08	0.08	0.059
$\eta\Sigma Y_1^*$	$-\frac{1}{2}$	0.19	$0.14r_\eta^{-1}$	$0.14r_\eta^{-1}$		0.064	0.035	0.06	0.01
$\eta\Xi\Xi^*$	$-\frac{1}{2}$	0.19	$0.1\ r_\eta^{-1}$	$0.1\ r_\eta^{-1}$		0.08	0.06	0.06	0.017
$\bar{K}NY_1^*$	$1/\sqrt{6}$	0.127	0.063	0.092		0.12	0.19	0.094	0.09
$K\Sigma N^*$	$-\frac{1}{2}\sqrt{2}$	0.38	0.03	0.043		0.13	0.008	0.15	0.039
$\bar{K}\Lambda\Xi^*$	$\frac{1}{2}$	0.19	0.076	0.11		0.14	0.26	0.11	0.091
$\bar{K}\Sigma\Xi^*$	$\frac{1}{2}$	0.19	0.058	0.084		0.06	0.11	0.075	0.029
$K\Xi Y_1^*$	$-1/\sqrt{6}$	0.127	0.011	0.016		0.06	0.008	0.04	0.013
$\bar{K}\Xi\Omega$	1	0.76	0.067	0.098		0.31	0.7	0.31	0.012

[a] Solution (i) corresponds to the choice (2.8) of $K_{\bar{K}N\Lambda}(0)$ and solution (ii) to the choice (2.9). r_η is defined by Eq. (2.19).
[b] In obtaining the solutions for $f^2/4\pi$, the πNN^*, $\pi\Lambda Y_1^*$, and $\pi\Sigma Y_1^*$ coupling constants, as deduced from the observed widths, have been used as input.
[c] See the note added at the end of the paper for a third solution.

Here we have used the following relation between the widths and the coupling constants:

$$\Gamma(B^*\to BP)=\frac{f_{PBB^*}{}^2}{4\pi}\frac{(E+m)p^3}{3M^*}, \qquad (2.17)$$

where E and p are the energy and momentum of the final baryon B in the rest frame of the decaying resonance B^*, and $\Gamma(B^*\to B+P)$ is the total decay width of a given charge state of B^* (into all possible charge states of $B+P$).

To obtain the signs of the coupling constants derived from (2.15) [note that these signs are required in (2.4)], we have assumed that the relative signs of the coupling constants in broken SU_3 symmetry are the same as in exact SU_3 symmetry. Such an assumption is supported, for instance, by the self-consistent calculation of the coupling shifts in broken SU_3 carried out by Dashen, Dothan, Frautschi, and Sharp,[14] who find for the PBB and PBB^* couplings that the relative signs of the coupling constants are not altered even though some of the coupling shifts are quite large.

Using (2.16), we have evaluated all the remaining PBB^* coupling constants for both the choices (2.8) and (2.9) for the K-mesonic form factor. (These will be referred to as solutions (i) and (ii), respectively.) Among these, the $\pi\Xi\Xi^*$ coupling constant is the only one that is related to a decay width; this is predicted to be

$$\Gamma(\Xi^*\to\Xi+\pi)\approx 9.7\text{ MeV}. \qquad (2.18)$$

This may be compared with the experimental value of (7.3 ± 1.7) MeV.[12]

For both solutions (i) and (ii), it is found that the predicted coupling constants $f_{if\alpha}$ have the same signs as the coupling constants in exact SU_3. Thus our assumption, used in determining the signs of the input coupling constants, that the PBB^* coupling constants have the same sign as they would have in exact SU_3, appears to be consistent if we assume the values (2.15) for the widths.

The solution (2.16) is quite sensitive to the input values of the widths; thus, in order to obtain accurate values of the axial-vector renormalization constants, the input widths must be known accurately. The coupling constants predicted using (2.16) are, however, not very sensitive to the individual values of $G_A^*(0)$, $F_A^*(0)$, and $H_A^*(0)$, so that the predicted PBB^* coupling constants can be expected to give the correct orders of magnitude of the different coupling strengths and a reasonably good estimate of their relative magnitudes.

In the second column of Table I, we give the isoscalar factors for the various couplings,[15] which give the ratios of the coupling constants in exact SU_3.[16] In the third column we give the values of $f^2/4\pi$ in exact SU_3, assuming $f_{\pi NN^*}{}^2/4\pi=0.38$, which is obtained from the experimental value (2.15) of the N^* decay width.[12]

In the fourth column of Table I, we give the values of the coupling constants $f^2/4\pi$ obtained using the solution (2.16) and the value (2.8b) for the off-shell extrapolation factor for the K-meson vertex functions. In the fifth column of Table I are given the coupling constants obtained by using (2.16) and the value (2.9b) for the off-shell extrapolation factor.

In order to obtain the η coupling constants, one requires the value of $C_\eta/\mu_\eta{}^2$ and of the extrapolation factor $K_\eta^*(0)$ for the η-meson vertex functions (assum-

[14] R. Dashen, Y. Dothan, S. C. Frautschi, and D. Sharp, Phys. Rev. **143**, 1185 (1966) and Phys. Rev. **151**, 1127 (1966).

[15] The phases of the isoscalar factors used here are chosen as given by J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963). The second column of Table I is obtained on writing the couplings in the order $P+B\to B^*$.

[16] A discussion and review of coupling constants in unbroken SU_3 and their comparison with experiment have been given recently by M. Goldberg, J. Leitner, R. Musto, and L. O'Raifeartaigh, Nuovo Cimento **45**, 169 (1966).

ing this factor to be roughly the same for all the η-meson vertex functions). In the absence of any information about these, we have written the coupling constants f_η in terms of a factor r_η defined by

$$r_\eta=[C_\eta K_\eta^*(0)/\mu_\eta^2]/[C_\pi K_\pi^*(0)/\mu_\pi^2]. \quad (2.19)$$

We may make the following observations regarding the PBB^* coupling constants given in Table I:

(i) The pion coupling constants are of the same order of magnitude, $f_\pi^2/4\pi\approx 0.1$ to 0.14, except for the πNN^* coupling constant, which is considerably larger.

(ii) The η coupling constants f_η are of the order $0.1r_\eta^{-1}$ to $0.14r_\eta^{-1}$.

(iii) The K coupling constants in solution (i) are appreciably smaller than the π and η coupling constants; $f_K^2/4\pi\approx 0.01$ to 0.08. The values of the K coupling constants in solution (ii) are larger than those in solution (i) by a factor of about 1.2 and are closer to the values of the pion coupling constants, although still on the whole, smaller.

(iv) The deviations from exact SU_3 symmetry of the values of the K-meson coupling constants are seen to be considerable.

If, in solving for $G_A^*(0)$, $F_A^*(0)$, and $H_A^*(0)$ from Eqs. (2.10), we use as input the widths for $N^*\rightarrow N\pi$, $Y_1^*\rightarrow\Lambda\pi$, and $\Xi^*\rightarrow\Xi\pi$, assuming[12]

$$\Gamma(\Xi^*\rightarrow\Xi\pi)\approx 7.3\text{ MeV}, \quad (2.20)$$

and the first two widths in (2.15), we obtain[13]

$$G_A^*(0)\approx -0.48,\quad F_A^*(0)\approx 0.95,\quad H_A^*(0)\approx 0.01. \quad (2.21)$$

The coupling constants f obtained by using the values (2.21) for $G_A^*(0)$, $F_A^*(0)$, and $H_A^*(0)$ do not all have the same relative signs as they would have in exact SU_3. This is not consistent with the assumption made in determining the signs of the input coupling constants used in order to solve Eq. (2.10) for $G_A^*(0)$, $F_A^*(0)$, and $H_A^*(0)$. [This may indicate that the value (2.20) for the decay width $\Gamma(\Xi^*\rightarrow\Xi\pi)$ is in error.] We therefore prefer the solution for the coupling constants obtained from (2.16).

Earlier, various authors had obtained relations among the PBB^* coupling constants in broken SU_3 symmetry, starting with the assumption that, to the lowest order, the symmetry-breaking interaction transformed like a component of an octet tensor.[17] The PBB^* coupling constants may then all be expressed in terms of five parameters. For the PBB^* couplings, sufficiently reliable data are not available to evaluate these five parameters (and hence all the couplings). However, one may obtain seven sum rules between the 12 coupling constants listed in Table I; these are the following[17,18]:

$$2f(\pi\Xi\Xi^*)=-\sqrt{2}f(\pi NN^*)+3f(\pi\Lambda Y_1^*)+(\sqrt{\tfrac{3}{2}})f(\pi\Sigma Y_1^*), \quad (2.22a)$$

$$2f(\bar{K}\Sigma\Xi^*)=(\sqrt{6})f(\bar{K}NY_1^*)+(\sqrt{6})f(\pi\Sigma Y_1^*)-\sqrt{2}f(\pi NN^*), \quad (2.22b)$$

$$-2f(\bar{K}\Lambda\Xi^*)=-(\sqrt{6})f(\bar{K}NY_1^*)+\sqrt{2}f(\pi NN^*)-2f(\pi\Lambda Y_1^*), \quad (2.22c)$$

$$-\sqrt{2}f(K\Sigma N^*)=-(\sqrt{6})f(K\Xi Y_1^*)+2\sqrt{2}f(\pi NN^*)+2f(\pi\Xi\Xi^*)-6f(\pi\Lambda Y_1^*), \quad (2.22d)$$

$$f(\bar{K}\Xi\Omega)=(\sqrt{6})f(\bar{K}NY_1^*)+2f(\pi\Xi\Xi^*)-\sqrt{2}f(\pi NN^*), \quad (2.22e)$$

$$-2f(\eta\Sigma Y_1^*)=\tfrac{2}{3}(\sqrt{6})[-f(K\Xi Y_1^*)+f(\bar{K}NY_1^*)]+\tfrac{4}{3}[2f(\pi\Xi\Xi^*)+\sqrt{2}f(\pi NN^*)]-6f(\pi\Lambda Y_1^*), \quad (2.22f)$$

$$-2f(\eta\Xi\Xi^*)=(\tfrac{2}{3}\sqrt{6})[-f(K\Xi Y_1^*)+f(\bar{K}NY_1^*)]+\tfrac{1}{3}[2f(\pi\Xi\Xi^*)-2\sqrt{2}f(\pi NN^*)]. \quad (2.22g)$$

It is instructive to examine how well these first-order sum rules are satisfied by our estimates for the coupling constants. In Table II we list the numerical values of the two sides of the sum rules (2.22) for both our solutions.

TABLE II. Comparison of $\frac{3}{2}^+$ decuplet coupling constants with the first-order sum rules (2.22).

	Solution (i) [for $K_{\bar{K}N\Lambda}(0)\approx 1.4$]		Solution (ii) [for $K_{\bar{K}N\Lambda}(0)\approx 1.15$]	
Sum rule in (2.22)	Left-hand side of sum rule	Right-hand side of sum rule	Left-hand side of sum rule	Right-hand side of sum rule
(a)	2.34	2.22	2.34	2.22
(b)	1.7	1.4	2.0	1.85
(c)	−1.96	−1.85	−2.36	−2.30
(d)	−0.86	−0.70	−1.0	−0.89
(e)	0.92	1.42	1.11	1.87
(f)	$2.32r_\eta^{-1}$	0.99	$2.32r_\eta^{-1}$	1.42
(g)	$2.34r_\eta^{-1}$	0.78	$2.34r_\eta^{-1}$	1.2

As a whole, it is seen that the coupling constants given by solution (ii) agree better with the first-order sum rules than those given by solution (i). This was to be expected, because the choice (2.9) of the K-mesonic form factor which gives solution (ii) was obtained by assuming $G_{\bar{K}N\Lambda}^2/4\pi\approx 7.4$, which is closer to the value of $G_{\bar{K}N\Lambda}^2/4\pi$ in exact SU_3 (with a d/f ratio of 1.5 to 2 and with $G_{\pi NN}^2/4\pi\approx 14.6$) than the value of 4.8 which led to (2.8) [which was used in obtaining solution (i)].

For the first four sum rules, the deviation (with our values of the coupling constants) from the equality required by the sum rules is comparatively small,

[17] C. Dullemond, A. J. Macfarlane, and E. C. G. Sudarshan, Phys. Rev. Letters **10**, 423 (1963); E. C. G. Sudarshan, in *Proceedings of the Athens Topical Conference on Recently Discovered Resonant Particles, Athens, Ohio, 1963*, edited by B. A. Munir and L. J. Gallaker (University of Ohio Press, Athens, Ohio, 1963); V. Gupta and V. Singh, Phys. Rev. **135**, B1442 (1964); C. Becchi, E. Eberle, and G. Morpurgo, *ibid.* **136**, B808 (1964); M. Konuma and Y. Tomozawa, Phys. Letters **10**, 347 (1964); M. Suzuki, Progr. Theoret. Phys. (Kyoto) **32**, 279 (1964).

[18] The sum rules (2.22) are written in the form given by Gupta and Singh (Ref. 17), with a redefinition of the phases.

though not negligible. The deviation is smaller for solution (ii). For the fifth sum rule, involving $f(\bar{K}\Xi\Omega)$, the deviation is quite large; the reason for this is not clear. For the sixth and seventh sum rules, the deviation is large if we assume $r_\eta \approx 1$ [see Eq. (2.19)]. If one assumes that for the η couplings, which do not involve a change in the strangeness, the deviation from first-order symmetry breaking should be small, one is led to expect that

$$r_\eta \approx 1.5 \quad \text{to} \quad 1.7, \tag{2.23}$$

from solution (ii). Noting that the mass of the η is large and of the same order as that of the K meson, it is not unreasonable to expect an off-shell extrapolation factor $K_\eta^*(0)$ of the same order as that for the K-meson vertex functions [for which we used the estimates (2.8) and (2.9)]. This would account for a part of the deviation of r_η from unity.

We shall finally compare our estimates of the coupling constants with those made by other authors using different methods. These are shown in the seventh, eighth, ninth, and tenth columns of Table I. For convenience in comparing the results, all the coupling constants have been normalized so that $f_{\pi NN^*}{}^2/4\pi = 0.38$, corresponding to a width of 120 MeV for the $(\frac{3}{2},\frac{3}{2})$ N^*.[12]

The seventh column gives the predictions of Freund and Nambu.[2] Although our starting point is the same as theirs, their detailed assumptions and results are different. Our predictions for the π and η couplings are of the same order of magnitude as those of Freund and Nambu, except for the $\pi\Sigma Y_1^*$ coupling, for which their value is considerably smaller. The $K\Sigma N^*$, $K\Xi Y_1^*$, and $\bar{K}\Xi\Omega$ coupling constants predicted by us are considerably smaller than those of Freund and Nambu.

The results obtained by Johnson and McCliment[19] are given in the eighth column of Table I. Their value for the $\pi\Sigma Y_1^*$ coupling constant $f^2/4\pi$ is about a third as large as ours. Their predictions for the $\bar{K}NY_1^*$, $\bar{K}\Xi\Omega$, and $\bar{K}\Lambda\Xi^*$ couplings are considerably larger than ours, while their values for the $K\Sigma N^*$ and $\eta\Sigma Y_1^*$ coupling constants are appreciably smaller than ours, even if we assume $r_\eta \approx 1.5$ in obtaining our value of $f(\eta\Sigma Y_1^*)$.

In the ninth column of Table I are given the values of the coupling constants obtained by Wali and Warnock[20] from an N/D calculation; we have taken the solution for which their symmetry-breaking parameter x has the value unity, which gives the physical masses. If we choose r_η as in (2.23), then their values for the η coupling constants are roughly of the same order as ours. Their predictions for the $K\Sigma N^*$, $K\Xi Y_1^*$, and $\bar{K}\Xi\Omega$ coupling constants are considerably larger than ours. We note that their value for $f(\pi\Sigma Y_1^*)$, although smaller than our input value, is closer to the experimental value than those of the other authors discussed here.

The tenth column in Table I gives the coupling constants obtained by Dashen, Dothan, Frautschi, and Sharp[14] by a self-consistent calculation. Their K coupling constants are roughly of the same order of magnitude as ours except for $\bar{K}\Xi\Omega$ and $\bar{K}\Sigma\Xi^*$, which are considerably smaller. The major difference is that their η couplings are much smaller than ours, even when we use (2.23) for r_η. The $\pi\Sigma Y_1^*$ coupling constant obtained by these authors is also considerably smaller than our input value.

Table I thus shows how the predictions obtained for the coupling constants by other authors compare with ours. We note that our estimates of the coupling constants are fitted to the observed value of $f(\pi\Sigma Y_1^*)$ which now experimentally appears to be considerably larger than was expected earlier. In Sec. III we apply our method to the couplings of the $\frac{3}{2}^-$ baryonic resonances.

III. COUPLINGS OF $\frac{3}{2}^-$ OCTET BARYON RESONANCES

In this section we discuss the coupling of a spin-$\frac{3}{2}^-$ octet baryon resonance B^* to a $\frac{1}{2}^+$ octet baryon B and a 0^- meson P. We shall group into an octet the N^* at 1512 MeV, the Y_1^* at 1660 MeV, and the Ξ^* at 1815 MeV.[12] The first-order Gell-Mann–Okubo mass formula[21] then suggests the existence of a Y_0^* at about 1670 MeV. Some evidence for such a state has been obtained from the analysis of K^-p reactions.[22] Also, the dynamical calculation of Martin suggests that the Y_0^* to be included in this octet should have a mass of (1660 ± 60) MeV.[23] We shall therefore assume a Y_0^* at about 1670 MeV and examine the predictions for its couplings.

The expressions analogous to (2.2) and (2.3) are given by

$$\begin{aligned}\langle B_f^*(p_f)|\mathcal{A}_\alpha{}^\lambda(0)|B_i(p_i)\rangle = {} & [m_i M_f^*/E_i E_f]^{1/2}\bar{\psi}_\rho(p_f) \\ & \times(-i)\{d_{\alpha if}[\mathcal{G}_d(q^2)g^{\lambda\rho}\gamma_5+\mathcal{F}_d(q^2)\gamma^\lambda q^\rho\gamma_5 \\ & +\mathcal{H}_d(q^2)P^\lambda q^\rho\gamma_5+\mathcal{L}_d(q^2)q^\lambda q^\rho\gamma_5] \\ & +f_{\alpha if}[\mathcal{G}_f(q^2)g^{\lambda\rho}\gamma_5+\mathcal{F}_f(q^2)\gamma^\lambda q^\rho\gamma_5 \\ & +\mathcal{H}_f(q^2)P^\lambda q^\rho\gamma_5+\mathcal{L}_f(q^2)q^\lambda q^\rho\gamma_5]\}u(p_i),\end{aligned} \tag{3.1}$$

[19] E. Johnson and E. R. McCliment, Phys. Rev. **139**, B951 (1965).

[20] K. C. Wali and R. Warnock, Phys. Rev. **135**, B1358 (1964); F. Ernst, R. Warnock, and K. C. Wali, *ibid.* **141**, 1354 (1966).

[21] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).

[22] Evidence suggesting the existence of a $\frac{3}{2}^-$ Y_0^* was obtained by J. D. Davies *et al.* and by R. Armenteros *et al.* [see the reports presented by these authors and the review by M. Ferro-Luzzi, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967), p. 183], who found that K^-p reactions of energies near 1700 MeV could be better fitted by assuming a Y_0^* in the $\frac{3}{2}^-$ partial wave in this region, in addition to the $\frac{3}{2}^-$ Y_1^* at 1660 MeV. We are obliged to Professor J. Leitner for a discussion on this.

[23] A dynamical calculation by A. W. Martin, using as input the properties of the $\frac{3}{2}^-$ N^* and Y_1^*, predicts the Y_0^* mass to be (1660 ± 60) MeV and the Ξ^* mass to be (1800 ± 80) MeV. See A. W. Martin, Nuovo Cimento **32**, 1645 (1964).

TABLE III. Solutions of Eq. (3.3) for the $\frac{3}{2}^-$ resonances.

	Solution (i) [for $K_{\bar{K}N\Lambda}(0)\approx 1.4$]	Solution (ii) [for $K_{\bar{K}N\Lambda}(0)\approx 1.15$]
$\mathcal{G}_d(0)$	0.135	0.68
$\mathcal{F}_d(0)$	0.075	0.056
$\mathcal{H}_d(0)$	−0.017	−0.019
$\mathcal{G}_f(0)$	1.14	0.89
$\mathcal{F}_f(0)$	−0.025	−0.034
$\mathcal{H}_f(0)$	−0.016	−0.01

and

$$\langle B_f^*(p_f)|j_\alpha(0)|B_i(p_i)\rangle = [m_i M_f^*/E_i E_f]^{1/2} g_{if\alpha} \times \mathcal{K}_{if\alpha}^*(q^2)\bar{\psi}_\rho(p_f) q^\rho \gamma_5 u(p_i). \quad (3.2)$$

Here $\psi_\rho(p_f)$ is a Rarita-Schwinger spinor for a spin-$\frac{3}{2}^-$ particle, and $d_{\alpha if}$ and $f_{\alpha if}$ are the symmetric and antisymmetric SU_3 coupling coefficients of Gell-Mann.[21]

Taking matrix elements of (2.1) between B^* and B states leads, at $q^2=0$, to the following relation:

$$[d_{\alpha if}\mathcal{G}_d(0)+f_{\alpha if}\mathcal{G}_f(0)] + (M_f^*+m_i)[d_{\alpha if}\mathcal{F}_d(0)+f_{\alpha if}\mathcal{F}_f(0)] + (M_f^{*2}-m_i^2)[d_{\alpha if}\mathcal{H}_d(0)+f_{\alpha if}\mathcal{H}_f(0)] = d_\alpha g_{if\alpha}\mathcal{K}_{if\alpha}^*(0). \quad (3.3)$$

We again assume that for given α

$$\mathcal{K}_{if\alpha}^*(0)\approx K_{12\alpha}(0), \quad (3.4)$$

where $K_{12\alpha}(0)$ is the form factor of the PBB vertex (at zero square of momentum transfer). The factor multiplying $g_{if\alpha}$ on the right-hand side of (3.3) is then known for $\alpha=\pi$ and $\alpha=K$. We shall again consider the two choices (2.8) and (2.9) for the K-mesonic form factor.

By considering Eq. (3.3) for seven different choices of i, f, and α, one may derive sum rules similar to (2.14); there are 11 such sum rules.

Data for the decay widths are at present available for six decay modes; these are as follows[12]:

$$\Gamma(N^*\to N\pi)\approx 40\text{ MeV}, \quad \Gamma(Y_1^*\to\Sigma\pi)\approx 15\text{ MeV},$$
$$\Gamma(Y_1^*\to\bar{K}N)\approx 7.5\text{ MeV}, \quad \Gamma(Y_1^*\to\Lambda\pi)\approx 2.5\text{ MeV},$$
$$\Gamma(\Xi^*\to\Xi\pi)\approx 1.6\text{ MeV}, \quad \Gamma(\Xi^*\to\Lambda\bar{K})\approx 10.4\text{ MeV}. \quad (3.5)$$

Using these and the relation

$$\Gamma=\frac{g^2}{4\pi}\frac{p^3(E-m)}{3M^*}, \quad (3.6)$$

with a notation similar to (2.17), we may obtain the coupling constants $g_{if\alpha}^2/4\pi$. To obtain the relative signs of the coupling constants $g_{if\alpha}$, we write the expressions for the coupling constants corresponding to the vertices in (3.5) which would hold in exact symmetry, i.e., as a sum of D and F couplings with appropriate coefficients, and examine what choice of the relative signs of $g_{if\alpha}$ would allow the best approximate solution for the D and F reduced matrix elements. We assume that this gives the correct relative signs of the $g_{if\alpha}$.

Writing the six equations of the form (3.3) corresponding to the six couplings in (3.5), we may solve for the six unknowns $\mathcal{G}_d(0)$, etc. Because there are two K-meson coupling constants among the input couplings (3.5), the solution will depend on the value assumed for the K-meson form factors. The solutions corresponding to the assumptions (2.8) and (2.9) for this form factor will be denoted as solutions (i) and (ii), respectively; they are shown in Table III.[24] Using these values of $\mathcal{G}_d(0)$, etc., we have evaluated the other coupling constants.

In the second column of Table IV we list, for comparison, the forms of the couplings in exact SU_3. In the third column we give the input values of the coupling constants, obtained from the observed widths (3.5), used in solving Eqs. (3.3). The predicted values of the coupling constants for solutions (i) and (ii) are given in the fourth and fifth columns, respectively, for g, and in the sixth and seventh columns for $g^2/4\pi$. The predicted values of the partial widths for the decay modes $Y_0^*\to\Sigma\pi$, $Y_0\to N\bar{K}$, and $\Xi^*\to\Sigma\bar{K}$ are given in the eighth and ninth columns of Table IV.

The η couplings listed in Table IV correspond to the assumption $r_\eta\approx 1.5$ [see Eq. (2.23)]. For $r_\eta\approx 1$, the predicted coupling constants g would be larger by a factor $\frac{3}{2}$.

Because the input decay widths (3.5) are not known accurately at present, our predictions for the coupling constants can be regarded as giving only rough values of the strengths of the various couplings. Although the solutions for the individual values of $\mathcal{G}_d(0)$, etc., are quite sensitive to the input widths, the values of the predicted coupling constants are much less sensitive to these. A measurement of the decay width for $\Xi^*\to\Sigma\bar{K}$ and of the partial decay widths of the Y_0^*, if the existence of a $\frac{3}{2}^-$ Y_0^* at about 1670 MeV is confirmed, would provide tests of our predictions.

We shall first compare our results with those of Martin,[23] who made a simple dynamical analysis of the $\frac{3}{2}^-$ octet baryon resonances. He assumed an N^* at 1512 MeV and a Y_1^* at 1660 MeV with the partial widths for $N^*\to N\pi$, $Y_1^*\to\Sigma\pi$, $Y_1^*\to\Lambda\pi$, and

[24] For comparison, we note that if the $\frac{3}{2}^-$ octet baryons are assigned to the **70** representation of SU_6 [e.g., see I. Gyuk and S. F. Tuan, Phys. Rev. **140**, B164 (1965)], and if the $\frac{1}{2}^+$ octet baryons and the axial-vector current are assigned to the **56** and **35**, respectively, the D/F ratio of the matrix element of the axial-vector current between the $\frac{1}{2}^+$ and $\frac{3}{2}^-$ octet baryons is predicted to be 3. If the $\frac{3}{2}^-$ octet is assigned to the **700** of SU_6, this ratio is −0.6. In the **1134** representation, there are three possible assignments for a $\frac{3}{2}^-$ octet; for two of these the matrix element of the axial-vector current (between a $\frac{1}{2}^+$ octet baryon in the **56** and the $\frac{3}{2}^-$ octet baryon) is pure D or pure F, while for the third assignment, the D/F ratio is 0.6. [The SU_6 Clebsch-Gordan coefficients used in obtaining these were taken from J. C. Carter, J. J. Coyne, and S. Meshkov, Phys. Rev. Letters **14**, 523 (1965).]

TABLE IV. Couplings of the octet $\frac{3}{2}^-$ resonances with a $\frac{1}{2}^+$ octet baryon and an octet 0^- meson.[a,b,c]

PBB^* couplings	Couplings g in exact SU_3	Coupling constants g from the observed widths	Estimates for g Solution (i) [for $K_{\bar{K}N\Lambda}(0)\approx1.4$]	Estimates for g Solution (ii) [for $K_{\bar{K}N\Lambda}(0)\approx1.15$]	Estimates for $g^2/4\pi$ Solution (i)	Estimates for $g^2/4\pi$ Solution (ii)	Predicted decay widths (MeV) Solution (i)	Predicted decay widths (MeV) Solution (ii)	Martin's estimates (Ref. 23) Decay widths (MeV)	Martin's estimates (Ref. 23) Values of $g^2/4\pi$
πNN^*	$\sqrt{3}(D-F)$	2.16			(0.37)	(0.37)			80	0.74
$\pi\Lambda Y_1^*$	$-\frac{2}{3}\sqrt{3}D$	−0.66			(0.035)	(0.035)			11	0.15
$\pi\Sigma Y_1^*$	$-2\sqrt{2}F$	2.36			(0.44)	(0.44)			13	0.375
$\pi\Sigma Y_0^*$	$2D$		1.72	1.74	0.24	0.24	9	9	6	0.16
$\pi\Xi\Xi^*$	$-\sqrt{3}(D+F)$	0.71			(0.04)	(0.04)			1	0.025
ηNN^*	$\frac{1}{3}\sqrt{3}(D+3F)$		−0.83	−0.69	0.054	0.038				
$\eta\Lambda Y_0^*$	$\frac{2}{3}\sqrt{3}D$		0.42	0.41	0.014	0.014				
$\eta\Sigma Y_1^*$	$-\frac{2}{3}\sqrt{3}D$		−0.71	−0.72	0.04	0.04				
$\eta\Xi\Xi^*$	$\frac{1}{3}\sqrt{3}(D-3F)$		1.78	1.64	0.25	0.21				
$\bar{K}NY_1^*$	$\sqrt{2}(D+F)$	−1.31			(0.14)	(0.14)			3	0.056
$\bar{K}NY_0^*$	$(2/\sqrt{6})(D-3F)$		2.1	1.96	0.35	0.3	21.5	18.4	46	0.76
$K\Sigma N^*$	$-\sqrt{3}(D+F)$		−1.25	−1.41	0.12	0.16				
$K\Lambda N^*$	$\frac{1}{3}\sqrt{3}(D-3F)$		0.72	1.06	0.04	0.09				
$K\Xi Y_1^*$	$\sqrt{2}(D-F)$		1.51	2.05	0.18	0.33				
$K\Xi Y_0^*$	$(-2/\sqrt{6})(D+3F)$		−0.02	0.23	3×10^{-5}	4×10^{-3}				
$\bar{K}\Lambda\Xi^*$	$\frac{1}{3}\sqrt{3}(D+3F)$	−1.85			(0.27)	(0.27)			5	0.13
$\bar{K}\Sigma\Xi^*$	$\sqrt{3}(D-F)$		1.85	1.78	0.27	0.25	2.9	2.7	6	0.56

[a] In the sixth and seventh columns, the numbers in parentheses are the values of $g^2/4\pi$ obtained from the observed widths (3.5), corresponding to the values of g in the third column.
[b] The η couplings here have been given for $r_\eta\approx1.5$.
[c] See the note added in proof at the end of the paper for a third solution.

$Y_1^*\rightarrow N\bar{K}$ as given in the tenth column of Table IV, and estimated that the location of the Ξ^* and Y_0^* should be the following:

$$M(\Xi^*)=1800\pm80 \text{ MeV},\quad M(Y_0^*)=1660\pm60 \text{ MeV}. \tag{3.7}$$

His estimates for the partial widths of the Ξ^* and Y_0^* are also given in the tenth column of Table IV.

In the last column of Table IV we give the coupling constants $g^2/4\pi$ corresponding to the widths quoted by Martin. We note that the πNN^*, $\pi\Lambda Y_1^*$, and $\bar{K}NY_1^*$ coupling constants assumed by Martin differ considerably from our input values, while the $\pi\Sigma Y_1^*$ coupling constant is roughly the same. His estimates for the $\pi\Sigma Y_0^*$ and $\pi\Xi\Xi^*$ couplings are of the same order as ours, while his estimated values of the $\bar{K}NY_0^*$ and $\bar{K}\Sigma\Xi^*$ coupling strengths differ considerably from our values.

We finally note the following features of our estimates for the coupling constants given in Table IV:

(i) In contrast to the PBB couplings and PBB^* couplings for the decuplet $\frac{3}{2}^+$ resonances, it is no longer true that the K couplings are, on the whole, smaller than the π couplings. This is already so for the (input) values of the couplings obtained from the observed decay widths. Thus the $\bar{K}NY_1^*$ and $\bar{K}\Lambda\Xi^*$ coupling constants are of roughly the same order of magnitude as the πNN^* and $\pi\Sigma Y_1^*$ coupling constants, while the $\pi\Lambda Y_1^*$ and $\pi\Xi\Xi^*$ coupling constants are considerably smaller.

(ii) Solution (i), corresponding to the choice (2.8) for the K-mesonic form factor, differs from solution (ii), obtained with the choice (2.9), mainly in the values of the $K\Lambda N^*$, $K\Xi Y_1^*$, and the ηNN^* coupling constants. The $K\Xi Y_0^*$ coupling constants given by the two solutions differ in sign; however, because its magnitude is very small, this difference in sign is not significant. The other coupling constants given by the two solutions do not differ significantly.

(iii) The values of the coupling constants listed in Table IV enable estimates of the relative weights of different pole contributions to reactions in which $\frac{3}{2}^-$ resonances are produced, such as $\pi+N\rightarrow Y_1^*+K$, $K+N\rightarrow N^*+K$, $\bar{K}+N\rightarrow\Xi^*+K$, etc. For instance, the small magnitude of the $K\Xi Y_0^*$ coupling constant would predict that the contribution of the $\frac{3}{2}^-$ Y_0^* resonance at 1670 MeV to the reaction $\bar{K}+N\rightarrow K+\Xi$ would be much suppressed (relative to the contribution of the Y_1^* resonance at 1660 MeV), in contrast to the reaction $\bar{K}+N\rightarrow\bar{K}+N$, in which the Y_0^* would give an important contribution which may enable it to be detected.

(iv) We have examined how well the coupling constants obtained by us satisfy the sum rules that would follow from a first-order symmetry breaking that transformed like an SU_3 octet.[25] These sum rules are

[25] V. Gupta and V. Singh, Phys. Rev. **136**, B782 (1965). For the analogous sum rules for the PBB couplings, see M. Muraskin and S. L. Glashow, *ibid.* **132**, 482 (1963).

the following[26]:

$$(\sqrt{6})g(\bar{K}NY_1^*)+(\sqrt{\tfrac{3}{2}})g(\pi\Sigma Y_1^*)-2g(\pi\Xi\Xi^*)+3g(\pi\Lambda Y_1^*)-3g(\bar{K}\Lambda\Xi^*)=g(\bar{K}\Sigma\Xi^*)\,, \tag{3.8a}$$

$$(-\sqrt{6})g(\bar{K}NY_1^*)-(\sqrt{\tfrac{3}{2}})g(\pi\Sigma Y_1^*)+g(\pi NN^*)=-2g(K\Sigma N^*)+3g(\eta\Sigma Y_1^*)-3g(\eta NN^*)\,, \tag{3.8b}$$

$$(\sqrt{\tfrac{3}{2}})g(\pi\Sigma Y_1^*)-g(\pi\Xi\Xi^*)=2g(K\Sigma N^*)+\sqrt{3}g(\pi\Sigma Y_0^*)+(\sqrt{\tfrac{3}{2}})g(K\Xi Y_1^*)-\tfrac{3}{2}\sqrt{2}g(K\Xi Y_0^*)\,, \tag{3.8c}$$

$$-2g(\pi NN^*)+(\sqrt{\tfrac{3}{2}})g(\pi\Sigma Y_1^*)-3g(\pi\Lambda Y_1^*)=(\sqrt{6})g(K\Xi Y_1^*)+g(K\Sigma N^*)-3g(K\Lambda N^*)\,, \tag{3.8d}$$

$$(\sqrt{\tfrac{3}{2}})g(\pi\Sigma Y_1^*)-g(\pi\Xi\Xi^*)=(\sqrt{6})g(K\Xi Y_1^*)+2g(\bar{K}\Sigma\Xi^*)+3g(\eta\Sigma Y_1^*)-3g(\eta\Xi\Xi^*)\,, \tag{3.8e}$$

$$2g(\pi NN^*)-(\sqrt{\tfrac{3}{2}})[g(\pi\Sigma Y_1^*)-g(\bar{K}NY_1^*)]=-2g(\bar{K}\Sigma\Xi^*)+\sqrt{3}g(\pi\Sigma Y_0^*)+\tfrac{3}{2}\sqrt{2}g(\bar{K}NY_0^*)\,, \tag{3.8f}$$

$$-\sqrt{3}g(\pi\Lambda Y_1^*)-\sqrt{2}g(\bar{K}NY_1^*)=-\sqrt{3}[g(\eta\Lambda Y_0^*)-g(\eta\Sigma Y_1^*)]+\sqrt{2}g(K\Xi Y_1^*)+\tfrac{1}{3}g(\pi\Sigma Y_0^*)+(\sqrt{\tfrac{2}{3}})[g(\bar{K}NY_0^*)-g(K\Xi Y_0^*)]\,. \tag{3.8g}$$

In Table V we have given the values of the left- and right-hand sides of each of the Eqs. (3.8), using our values of the coupling constants [for each of our solutions (i) and (ii)]. It is seen that the deviations, with our values of the coupling constants, from the predictions of first-order symmetry breaking are, for most of the sum rules, smaller for solution (ii) than for solution (i), as for the $\frac{3}{2}^+$ decuplet couplings.[27,28] As suggested earlier for the $\frac{3}{2}^+$ couplings, this may be expected, because the value (2.9) for $K_{\bar{K}N\Lambda}(0)$, which was used in obtaining solution (ii), was in turn obtained by assuming $G_{\bar{K}N\Lambda}^2/4\pi\approx 7.4$, which is closer to the SU_3 symmetric value than the value $G_{\bar{K}N\Lambda}^2/4\pi\approx 4.8$ used in obtaining (2.8). However, for the $\frac{3}{2}^-$ couplings we have used two K-meson coupling constants as input constants in the basic equation (3.3) (in contrast to the $\frac{3}{2}^+$ couplings, where only pion couplings were used as input). The result that an input value of the K-mesonic form factor $K_{\bar{K}N\Lambda}(0)$ which was obtained from a value of $G_{\bar{K}N\Lambda}$ that is closer to exact SU_3 leads, after solving the equations (3.3) and using the solutions to predict the coupling constants, to output values of the coupling constants which are again closer to exact SU_3 (as they deviate less from the first-order sum rules), may be regarded as a check of the consistency of our procedure and of the qualitative nature of our results.

For the sum rules (3.8e) and (3.8g), which involve the η couplings, the deviation from the first-order sum rules is much less, and more strikingly so for solution (ii), when one uses $r_\eta\approx 1.5$ rather than $r_\eta\approx 1$. For $r_\eta\approx 1.5$, this deviation is of the same order as the deviation from the other sum rules in (3.8). [For the sum rule (3.8b), the deviation is roughly the same for $r_\eta\approx 1.5$ and for $r_\eta\approx 1$, for solution (ii).] This is in agreement with the corresponding results for the first-order sum rules for the $\frac{3}{2}^+$ couplings, where the deviation from the sum rules (2.22f) and (2.22g) is much less when one uses $r_\eta\approx 1.5$ rather than $r_\eta\approx 1$ (which is what suggested the estimate $r_\eta\approx 1.5$).

IV. COUPLINGS OF SPIN-$\frac{5}{2}$ RESONANCES

Describing a $\frac{5}{2}^+$ baryon by a Rarita-Schwinger spinor $\psi_{\rho\sigma}$, we may write the expressions analogous to (2.2) and (2.3) for the couplings of a $\frac{5}{2}^+$ octet baryon to a $\frac{1}{2}^+$ octet baryon and a pseudoscalar meson as

$$\begin{aligned}\langle B_f^*(p_f)|\,\mathfrak{A}_\alpha^\lambda(0)\,|B_i(p_i)\rangle =[m_iM_f^*/E_iE_f]^{1/2}\bar{\psi}_{\rho\sigma}(p_f)(-i)\{d_{\alpha if}[(\mathfrak{F}_d(q^2)\gamma^\lambda\gamma_5+\mathfrak{H}_d(q^2)P^\lambda\gamma_5+\mathfrak{L}_d(q^2)q^\lambda\gamma_5)P_\rho P_\sigma-\tfrac{1}{2}\mathfrak{G}_d(q^2)(g^{\rho\lambda}P^\sigma\gamma_5+g^{\sigma\lambda}P^\rho\gamma_5)]\\ +f_{\alpha if}[(\mathfrak{F}_f(q^2)\gamma^\lambda\gamma_5+\mathfrak{H}_f(q^2)P^\lambda\gamma_5+\mathfrak{L}_f(q^2)q^\lambda\gamma^5)P_\rho P_\sigma-\tfrac{1}{2}\mathfrak{G}_f(q^2)(g^{\rho\lambda}P^\sigma\gamma_5+g^{\sigma\lambda}P^\rho\gamma_5)]\}u(p_i)\,,\end{aligned} \tag{4.1}$$

and

$$\langle B_f^*(p_f)|\,j_\alpha(0)\,|B_i(p_i)\rangle=[m_iM_f^*/E_iE_f]^{1/2}g(q^2)\bar{\psi}_{\rho\sigma}(p_f)P^\rho P^\sigma\gamma_5u(p_i)\,. \tag{4.2}$$

The couplings for a $\frac{5}{2}^-$ baryon would differ from (4.1) and (4.2) in not having the factor γ_5 between the spinors.

Taking matrix elements of (4.1) between B^* and B states gives, at $q^2=0$, a relation identical in form to (3.3), where the quantities $\mathfrak{G}(0)$, $\mathfrak{F}(0)$, and $\mathfrak{H}(0)$ now refer to the coupling of the $\frac{5}{2}^+$ resonance to a $\frac{1}{2}^+$ baryon.

Present data suggest the existence of the following $\frac{5}{2}^+$ resonances: (i) an N^* with $I=\frac{1}{2}$ at about 1688 MeV, (ii) a Y_0^* at about 1815 MeV, (iii) a Y_1^* at about 1915 MeV, and (iv) a Ξ^* (with $I=\frac{1}{2}$) at about 1933 MeV.[12] If the N^* is regarded as a Regge recurrence of the

[26] The sum rules (3.8) were obtained from those given by Gupta and Singh (Ref. 25), with a redefinition of the phases. Note that we write the couplings in the order PBB^*.

[27] For the sum rule (3.8a), the deviation is smaller for solution (i) than for solution (ii); however, for both solutions the deviation is small.

[28] Note that five of the six coupling constants occurring in the first sum rule, (3.8a), are input coupling constants, obtained from the available experimental values of the widths. Using the experimental values of the coupling constants, the value of $f_{\bar{K}\Sigma\Xi^*}{}^2/4\pi$ predicted by the sum rule (3.8a) is 0.29. The values given by us are 0.27 for solution (i) and 0.25 for solution (ii).

TABLE V. Comparison of the estimates for the couplings of the $\frac{3}{2}^-$ octet with the first-order sum rules (3.8).[a]

	Solution (i) [for $K_{\bar{K}N\Lambda}(0)\approx 1.4$]		Solution (ii) [for $K_{\bar{K}N\Lambda}(0)\approx 1.15$]	
Sum rule in (3.8)	Left-hand side of sum rule	Right-hand side of sum rule	Left-hand side of sum rule	Right-hand side of sum rule
(a)	1.9	1.85	1.9	1.77
(b)	2.48	2.86(3.04)	2.48	2.71(2.67)
(c)	1.47	2.37	1.47	2.22
(d)	0.50	0.28	0.50	0.42
(e)	1.47	−0.08(−3.8)	1.47	1.48(−1.86)
(f)	3.0	3.73	3.0	3.6
(g)	3.0	2.48(1.51)	3.0	2.9(2.0)

[a] For the sum rules involving the η couplings, the first number on the right-hand side is for $r_\eta \approx 1.5$, while the number in parentheses is for $r_\eta \approx 1$.

nucleon, this would suggest it should be assigned to an octet. If one assumes that the Y_0^* and Y_1^* at 1815 and 1915 MeV should also be assigned to this octet, the Gell-Mann–Okubo first-order mass formula[21] predicts a Ξ^* at about 1990 MeV, which is not too different from the mass of 1933 MeV for the observed Ξ^*. However, if one starts by assuming that the Y_0^* and the Ξ^* belonging to the same octet as the N^* (at 1688 MeV) have masses of 1815 and 1933 MeV, the mass formula predicts that the mass of the Y_1^* in the octet should be about 1815 MeV. This relatively large discrepancy makes it doubtful whether it is valid to assign the Y_0^*, Y_1^*, and Ξ^* referred to above into an octet together with the N^* at 1688 MeV. However, it may be useful to examine whether such an assignment, used in conjunction with our procedure for determining the coupling constants, can lead to a consistent solution for the latter. This may provide a test of the validity of the assignment of these resonances to an octet.

The following estimates are available for the widths of some of the decay modes of these resonances[12]:

$$\Gamma(N^*\to N\pi)\approx 72.5\text{ MeV},\quad \Gamma(Y_0^*\to \bar{K}N)\approx 37\text{ MeV},$$
$$\Gamma(Y_0^*\to \Sigma\pi)\approx 4.5\text{ MeV},\quad \Gamma(Y_0^*\to \Lambda\eta)\approx 0.5\text{ MeV},$$
$$\Gamma(Y_1^*\to \bar{K}N)\approx 6.5\text{ MeV},\quad \Gamma(Y_1^*\to \Lambda\pi)\approx 7.8\text{ MeV}. \tag{4.3}$$

Using the relation

$$\Gamma=(g^2/30\pi)p^5(E-m)/M^* \tag{4.4}$$

between the decay width and the coupling constant, in a notation similar to (2.17), the magnitudes of the coupling constants corresponding to (4.3) may be worked out.[29]

The equations for the $\frac{5}{2}^+$ resonances analogous to (3.3) for the $\pi\Lambda Y_1^*$, $\pi\Sigma Y_0^*$, and $\eta\Lambda Y_0^*$ couplings involve only the D-type couplings. If we assume that the coupling constants for these vertices have the same relative signs as they would have in unbroken SU_3 symmetry, then we may solve for $\mathcal{G}_d(0)$, $\mathcal{F}_d(0)$, and $\mathcal{H}_d(0)$. Using these in the other three equations, we may attempt to deduce the signs of the coupling constants for the other three couplings in (4.3) by the same argument as was used for the $\frac{3}{2}^-$ couplings.

When this is done, we find that these simple arguments do not lead to any consistent solutions for the signs of the coupling constants to be used as input on the right-hand sides of the equations analogous to (3.3).

The simplest conclusion would be that the N^*, Y_0^*, Y_1^*, and Ξ^* considered here should not be together assigned to an SU_3 octet. No relations between the couplings can then be obtained.

An alternative possibility would be that the PBB^* coupling constants of the $\frac{5}{2}^+$ resonances in broken SU_3 do not have the same relative signs as they would have in exact SU_3. In the absence of any information regarding these signs, the signs of the right-hand sides of the equations analogous to (3.3) are undetermined, and no solutions can be obtained.

When sufficient information is available about the $\frac{5}{2}^+$ baryon resonances considered here and other possible $\frac{5}{2}^+$ resonances to enable a reliable SU_3 classification, our method may be applied to them to obtain information about their coupling strengths and axial-vector renormalization constants.

V. CONCLUSIONS

Using the PCAC relation and assuming that the axial-vector current remains an SU_3 octet to a good approximation in the presence of symmetry breaking, we have in this paper evaluated, in broken SU_3, the coupling constants of $\frac{3}{2}^+$ and $\frac{3}{2}^-$ baryonic resonances with the normal $\frac{1}{2}^+$ baryons and the pseudoscalar mesons. A sufficient number of observed values of the decay widths were used as input in order to solve the basic equations for the unknown parameters. For the $\frac{3}{2}^-$ resonances, additional values of the decay widths are not available to serve as a check of the solutions. For the $\frac{3}{2}^+$ decuplet couplings, the predicted value of the $\Xi^*\to\Xi\pi$ decay width is a little larger than the observed width; however, more accurate data for this width and for the input coupling constants are required before one can make a quantitative comparison.

Our procedure gives also values for the various $(B^*\to B)$ axial-vector renormalization constants.[30] However, these are quite sensitive to the values of the input decay widths, and more accurate values of the latter are needed in order to obtain good estimates of the axial-vector renormalization constants.

We have compared our results for the coupling constants of the $\frac{3}{2}^+$ and $\frac{3}{2}^-$ resonances with the results obtained from dynamical calculations and with the sum

[29] The couplings of the $\frac{5}{2}^+$ resonances in exact SU_3 have been considered by D. M. Brudnoy, Phys. Rev. Letters 14, 273 (1965).

[30] An estimate of the axial-vector renormalization constants has been made recently by Horn by considering matrix elements of the PCAC relation between baryon states at infinite momentum; see D. Horn, Phys. Rev. Letters 17, 778 (1966).

rules predicted by first-order SU_3 breaking. It is found that the deviation of the coupling constants from the first-order sum rules involving π and K couplings is on the whole smaller if we choose the off-shell extrapolation factor for the K couplings as in (2.9), corresponding to the larger value of $G_{\bar{K}N\Lambda}{}^2$, namely, $G_{\bar{K}N\Lambda}{}^2 \approx 7.4$. (For the $\bar{K}\Xi\Omega$ coupling, the deviation is still large; the reason for this is not clear.) Further, for the sum rules involving the η couplings, the deviation of our results for the coupling constants from the sum rules is found to be smaller if we choose r_η [as defined by (2.19)] to be of the order of 1.5.

The deviation of our estimates of the coupling constants from the first-order sum rules, although relatively small for most of the sum rules with a suitable choice of r_η and $K_{\bar{K}N\Lambda}(0)$, is still appreciable. A more precise statement can be made about these deviations when sufficient data on the couplings are available to provide accurate values of the input coupling constants and to enable our predictions to be tested. If it turns out that the data agree better with our estimates than with the first-order sum rules, this may be taken as suggesting that even when the symmetry breaking in the couplings of the baryon resonances is too large to be well approximated as a first-order perturbation, one may still obtain a useful description of it in terms of the nonlinear relations between the shifts in the masses and the coupling constants provided by Goldberger-Treiman relations such as those we have used in this paper.

The main limitations of our work are probably the following:

(1) Because of the uncertainties in many of the experimental values of the decay widths we have used as input, we cannot expect our estimates for the coupling constants to be accurate ones. If the data we have taken for the input widths are not much in error, we can expect our results to give the correct orders of magnitude of the coupling constants and an estimate of their relative strengths. When the widths used as input are measured accurately, good estimates may be obtained of the coupling strengths as well as of the $(B^* \to B)$ axial-vector renormalization constants.

(2) Because few reliable estimates have been made of coupling strengths of baryons with a η meson, any assumptions about the η couplings cannot be adequately checked at present.

However, if one requires that for couplings involving no change in strangeness, the SU_3 symmetry breaking should be relatively small and hence described fairly well by a first-order perturbation transforming as an octet, then one obtains a rough estimate of the unknown factor r_η entering the equations for the η couplings: $r_\eta \approx 1.5$.

For the π and K couplings, the off-mass-shell factor entering the basic equations has been eliminated by, in effect, taking the ratios of equations for PBB and PBB^* couplings, assuming that the off-shell extrapolation factor is roughly the same for both these latter couplings, and using the πNN and $\bar{K}N\Lambda$ coupling constants as input. We believe that this procedure for eliminating the off-mass-shell factor is a good approximation and will not introduce much error.

The method used here may be used for estimating the coupling strengths of the higher baryon resonances as well, when sufficient input data become available. Similar work on the couplings of mesonic resonances will be discussed in a separate paper.

Note added in proof. (i) Recently, evidence has been obtained confirming the existence of a Y_0^* near 1700 MeV; the spin-parity assignment $\frac{3}{2}^-$ is favored. [See R. Armenteros *et al.*, Phys. Letters **24B**, 198 (1967).] This supports our assumption that there exists a $D_{3/2}Y_0^*$ near 1670 MeV which should be assigned to an octet along with the N^* at 1512 MeV, the Y_1^* at 1660 MeV, and the Ξ^* at 1815 MeV. (See Sec. III of the paper.) The mass and width of the Y_0^* are quoted by the authors (see R. Armenteros *et al.*) as (1682 ± 2) and (55 ± 4) MeV, respectively; the branching ratio for decay into the $\Sigma\pi$ channel is quoted as being about 50%.

When more measurements are made on this Y_0^*, enabling an accurate determination of its parameters, a comparison may be made with our estimates.

(ii) Recently, a detailed analysis of KN scattering has been made by J. K. Kim [see report on Strong Interactions by G. Goldhaber, in Proceedings of the International Theoretical Physics Conference on Particles and Fields, Rochester, 1967 (to be published)] who obtains $(1/4\pi)G_{\bar{K}N\Lambda}{}^2 \simeq 16.0$, $(1/4\pi)G_{\bar{K}N\Sigma}{}^2 \simeq 0.3$.

This value of $G_{\bar{K}N\Lambda}{}^2$ is close to the value which would be obtained in exact SU_3 symmetry for the PBB couplings (with a d/f ratio of about 1.5). With this value of $G_{\bar{K}N\Lambda}{}^2$, the estimates of d_K/d_π, etc., would be the following:

$$d_K K_K(0)/d_\pi K_\pi(0) \simeq 0.97\,, \quad K_{\bar{K}N\Lambda}(0)/K_{\pi NN}(0) \simeq 0.77\,.$$

Thus, $K_{\bar{K}N\Lambda}(0) \simeq 0.77$ if $K_{\pi NN}(0) \simeq 1$. Note that $\bar{K}_{KN\Lambda}(0)$ is now less than unity; also, the deviations of d_K/d_π and $K_{\bar{K}N\Lambda}(0)/K_{\pi NN}(0)$ from unity seem to largely compensate each other.

If Kim's value of $G_{\bar{K}N\Lambda}$ is used as input, one obtains for the coupling constants $(1/4\pi)f^2$ for the decuplet couplings $\bar{K}NY_1^*$, $K\Sigma N^*$, $\bar{K}\Lambda\Xi^*$, $\bar{K}\Sigma\Xi^*$, $K\Xi Y_1^*$, and $\bar{K}\Xi\Omega$, the values 0.21, 0.1, 0.253, 0.193, 0.037, and 0.22, respectively. (For the π and η couplings, the results are the same as in solutions (i) and (ii), see Table I.)

Similarly, for the $\frac{3}{2}^-$ octet baryons, the values of $g^2/4\pi$ for the couplings $\pi\Sigma Y_0^*$, ηNN^*, $\eta\Lambda Y_0^*$, $\eta\Sigma Y_1^*$, $\eta\Xi\Xi^*$, $\bar{K}NY_0^*$, $K\Sigma N^*$, $K\Lambda N^*$, $K\Xi Y_1^*$, $K\Xi Y_0^*$, and $\bar{K}\Sigma\Xi^*$ are found to be 0.33, 0.023, 0.025, 0.13, 0.33, 0.058, 0.58, 0.46, 1.56, 0.056, and 0.048, respectively.

These results will be referred to as solution (iii). The relative signs of the coupling constants are the same as those in solutions (i) and (ii).

For the $\frac{3}{2}^+$ decuplet couplings, the couplings in solution (iii) seem to be as a whole closer to those in exact SU_3 [with $(1/4\pi)f_{\pi NN^*}{}^2 \simeq 0.38$]. However, there is still a significant symmetry breaking shown by the K-couplings, particularly the $K\Sigma N^*$, $K\Xi Y_1^*$ and $\bar{K}\Xi\Omega$ couplings. On substituting solution (iii) for f into the first-order sum rules (2.22), the left- and right-hand sides of (2.22b)–(2.22g) are found to be 3.1, 3.8; -3.57, -3.54; -1.58, -1.83; 1.67, 2.84; $2.32r_\eta^{-1}$, 2.52; and $2.34r_\eta^{-1}$, 2.35, respectively. [Equation (2.22a) is the same as before.] It is now seen that the deviation from the first-order sum rules (2.22f) and (2.22g) will be small if r_η is of the order of 0.9 to 1. For the other sum rules, the deviation is roughly of the same order as in solutions (i) and (ii).

For the $\frac{3}{2}^-$ octet couplings, the $Y_0^*\Sigma\pi$ coupling in solution (iii) is not much different from that in solution (i) or (ii). On the other hand, the K couplings in solution (iii), and some of the η couplings, are considerably different from those in solutions (i) and (ii). The $K\Xi Y_0^*$ coupling appears to be the one most sensitive to the input value of $G_{\bar{K}N\Lambda}$. The decay widths for $Y_0^* \to \pi\Sigma$, $Y_0^* \to \bar{K}N$ and $\Xi^* \to \bar{K}\Sigma$ given by solution (iii) are about 12.4, 3.6, and 0.5 MeV, respectively. Here, the decay $\Xi^* \to \Sigma\bar{K}$ seems to be relatively suppressed, while the $\pi\Sigma$ decay mode of the Y_0^* is predicted to be the dominant decay mode, in contrast to solutions (i) and (ii).

On substituting solution (iii) for the couplings of the $\frac{3}{2}^-$ octet into the first-order sum rules (3.8), the right-hand sides of the sum rules (3.8a)–(3.8g) are found to be 0.8, 2.9, 1.8, 0.98, 1.43, 3.7, and 3.4, respectively when we take $r_\eta \simeq 0.9$. It is found that there is better agreement with the sum rules for $r_\eta \simeq 0.9$ (i.e., better than for a larger value of r_η). For the sum rules (3.8a) and (3.8d), particularly the former, there is a larger discrepancy than for solution (ii). A part of this may be due to errors in the input values of the widths; a more accurate knowledge of the latter will enable a better assessment of our results.

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$\pi^- + p \to \eta + n$ Reaction near Threshold and Resonant States of the $\pi^- + p$ System*

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We calculate the differential cross sections for the reaction $\pi^- + p \to \eta + n$ in the $T_\pi = 593$ MeV to $T_\pi = 704$ MeV energy range using field-theoretic techniques, and taking into account three resonant states of the $\pi^- + p$ system. We find interference effects among these resonances to be of importance, and that locating the resonances at 1430, 1512, and 1567 MeV gives the best fit to the experimental data.

RECENTLY partial differential cross sections for the production of η mesons in the reaction $\pi^- + p \to \eta + n$ have been obtained experimentally.[1] Several authors[2,3] have analyzed the data in terms of strong interactions in an S_{11} resonant state. One author[4] also took into account the D_{13} resonant state. The purpose of this paper is to calculate the angular distribution of the produced η mesons near threshold from a field-theoretic viewpoint and compare these results with the experimental data. It was found from these calculations that three resonant states must be taken into account in order to fit the experimental data well.

Calculations were made by taking into account the Feynman diagrams shown in Fig. 1. The resonant states are labeled following Rosenfeld *et al.*,[5] with $N(1400)$ assumed to be a P_{11} or S_{11} resonance, $N(1518)$ a D_{13} resonance, and $N(1570)$ an S_{11} or P_{11} resonance. The nonresonant or crossing diagrams were neglected in comparison with the diagrams (c).

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[1] W. B. Richards *et al.*, Phys. Rev. Letters **16**, 1221 (1966).

[2] J. S. Ball, Phys. Rev. **149**, 1191 (1966).

[3] F. Uchiyama-Campbell and R. K. Logan, Phys. Rev. **149**, 1220 (1966).

[4] S. Minami, Phys. Rev. **147**, 1123 (1966).

[5] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).