Current Algebra, s-Wave Meson-Baryon Scattering Lengths, and the Ademollo-Gatto Theorem

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s-wave meson-baryon scattering lengths to first order in symmetry breaking are investigated within the framework of the $SU(3)\times SU(3)$ chiral algebra. We find that there are no renormalization effects in these scattering lengths to first order in symmetry breaking under the assumption of current commutation relations, the hypothesis of partially conserved axial-vector current, and the Ademollo-Gatto theorem for the vector-baryon–baryon coupling. However, if we do not assume the Ademollo–Gatto theorem, we do observe symmetry-breaking effects in the scattering-length sum rules.

I. INTRODUCTION

IN recent years, the $SU(3)\times SU(3)$ chiral algebra[1] has found wide application[2] in the study of elementary-particle interactions. In spite of certain inherent difficulties[3] associated with these types of calculations, most of the results obtained are in remarkable agreement with experiments. Lately, Weinberg,[4] Tomozawa,[5] Raman and Sudarshan,[6] and others[6] have used the current commutation relations (CCR) and the hypothesis of partially conserved axial-vector current (PCAC) to calculate the low-energy matrix elements (s-wave scattering lengths) of meson-baryon scattering processes. We also recall a recent calculation by Bose and Hara,[7] in which they obtain sum rules between the meson-baryon coupling constants to first order in $SU(3)$ breaking by using the CCR and the generalized PCAC.[8] They conclude that the symmetry-breaking effects appear only when both π and K mesons occur in the coupling-constant sum rule. This result may be interpreted as an extension of the Ademollo-Gatto (AG) theorem[9] for vector-baryon–baryon coupling. A similar approach has been applied to vector- and tensor-meson decays by Biswas and Patil.[10]

In the present work we investigate the effect of $SU(3)$ symmetry breaking in the case of s-wave meson-baryon scattering lengths using the algebra of currents; we only consider up to first-order symmetry breaking, and further assume that the breaking transforms like S_8, the scalar charge density given in the quark model by

$$S_8(0)=\int d^3x\,\bar{q}(\bar{x},0)\lambda_8 q(\bar{x},0)\,. \qquad (1)$$

This work may be regarded as an extension of the ideas applied in Refs. 7 and 10 in the case of three-point functions to scattering processes at threshold. We include the S_8 symmetry breaking in the interaction Hamiltonian and follow the Weinberg technique.[4] *We find that if the AG theorem for vector-baryon–baryon coupling is assumed, then there is no symmetry-breaking effect in s-wave scattering lengths.* However, if we do not assume the AG theorem, all the s-wave scattering lengths can be expressed in terms of three parameters. This enables us to write a family of sum rules which have been summarized in Eqs. (10) and (11) and are discussed in the final section.

II. AG THEOREM AND NONRENORMALIZATION OF s-WAVE SCATTERING LENGTH

The off-mass-shell invariant pseudoscalar-meson–baryon scattering amplitude $\langle B_f,P_b|M|B_i,P_a\rangle$ is defined as

$$\int d^4x\, d^4y \langle B_f|T\{\partial_\mu A_b{}^\mu(x),\partial_\nu A_a{}^\nu(y)\}|B_i\rangle \times e^{-iq\cdot x+ik\cdot y}\equiv\frac{i(2\pi)^4\delta^4(p_f+q-p_i-k)c^2}{(q^2+m_b{}^2)(k^2+m_a{}^2)(2\pi)^3(2E_iE_f)^{1/2}} \times\langle B_f,P_b|M|B_i,P_a\rangle\,, \qquad (2)$$

where k_μ and q_μ are the momenta of the initial meson P_a and the final meson P_b, respectively, the indices a and b running over $1,\cdots,8$; B_i and B_f represent the ingoing and outgoing baryons, respectively, with p_i and p_f as their respective 4-momenta. The A's are the axial-vector

[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).
[2] See, e.g., the works listed in K. C. Gupta, Phys. Rev. **155**, 1758 (1967).
[3] Namely, the difficulties associated with (i) the appearance of Schwinger terms in the equal-time commutator, and (ii) the corrections arising from extrapolating the off-mass-shell amplitudes to physical amplitudes.
[4] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).
[5] Y. Tomozawa, Nuovo Cimento **46**, 707 (1966).
[6] K. Raman and E. C. G. Sudarshan, Phys. Rev. **154**, 1499 (1967); A. P. Balachandran, M. Gundzik, and F. Nicodemi (to be published).
[7] S. K. Bose and Y. Hara, Phys. Rev. Letters **17**, 409 (1966).
[8] Of course, one knows that the PCAC is not as good for the K meson as for the π meson. However, following Ref. 7, we use PCAC for both π and K mesons. See also M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).
[9] M. Ademollo and R. Gatto, Phys. Rev. Letters **13**, 264 (1964).
[10] S. N. Biswas and S. H. Patil, Phys. Rev. **155**, 1599 (1967).

currents coming from the generalized PCAC condition

$$\partial_\mu A_b{}^\mu(x)=c\phi_b(x)\,, \qquad (3)$$

where $\phi_b(x)$ is the renormalized b-meson field. (Henceforth whenever we use the word "meson" we shall mean "pseudoscalar meson.") In order to calculate the first-order symmetry-breaking effects we introduce S_8 in the Hamiltonian, and consequently the complete amplitude M is given by Eq. (2) with the following term added on the left-hand side:

$$\lambda\int d^4x\, d^4y \langle B_f|T\{\partial_\mu A_b{}^\mu(x),\; \times\partial_\nu A_a{}^\nu(y),S_8(0)\}|B_i\rangle e^{-iq\cdot x+ik\cdot y}, \qquad (4)$$

where λ is the first-order symmetry-breaking interaction-strength parameter. Now both terms on the left-hand side of the new equation for the amplitude M have to be evaluated in exact $SU(3)$ limit, in the spirit of first-order perturbation theory. Consequently, we take the baryon and meson masses as degenerate. The relation between the S matrix and the invariant amplitude M is the same as before [see Eq. (3) in Ref. 4].

As far as the first term (without S_8) is concerned, the analysis is the same as that carried out by Weinberg[4] for πN scattering, except that now we use the generalized PCAC condition, Eq. (3). Ignoring the poles (as is the case, in general, for the s-wave part of the scattering amplitude), we see that the s-wave scattering lengths for the various processes are given in terms of a single parameter A_E. Strictly speaking, A_E is not a parameter, as it is related to g_v in terms of known factors.[11]

Next we try to estimate the contribution from the first-order symmetry-breaking term written above as Eq. (4). The expansion of the T product contains the usual eight terms [see, e.g., Eq. (4) in Ref. 12]. One of these vanishes identically because of the conserved vector current (CVC) hypothesis. Further, since we are interested in getting the s-wave part of the amplitude, we have to retain only such terms as contain $p\cdot k$ as a factor (in the limit as $k,q\to 0$, $p_i\to p_f\to\infty$, $p\cdot k$ remaining finite). Neglecting the terms not containing $p\cdot k$, namely, the double commutators, Eq. (4) becomes equal to

$$\lambda\int d^4x d^4y\, e^{-iq\cdot x+ik\cdot y}\bigg[\langle B_f|\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}T\{A_b{}^\mu(x),A_a{}^\nu(y),S_8(0)\}|B_i\rangle$$
$$-\langle B_f|\delta(x^0-y^0)T\{[A_a{}^0(y),\partial_\mu A_b{}^\mu(x)],S_8(0)\}|B_i\rangle-\frac{1}{2}\left\langle B_f\left|\left(\frac{\partial}{\partial y^\nu}-\frac{\partial}{\partial x^\nu}\right)\delta(x^0-y^0)T\{[A_b{}^0(x),A_a{}^\nu(y)],S_8(0)\}\right|B_i\right\rangle$$
$$-\langle B_f|\delta(y^0)T\{[A_a{}^0(y),S_8(0)],\partial_\mu A_b{}^\mu(x)\}|B_i\rangle-\langle B_f|\delta(x^0)T\{[A_b{}^0(x),S_8(0)],\partial_\nu A_a{}^\nu(y)\}|B_i\rangle\bigg]. \qquad (5)$$

The first term in Eq. (5) represents the scattering of a baryon with an external axial-vector current with S_8-type symmetry breaking; it may have poles. Explicit calculation shows that in the prescribed limit they do not contain terms proportional to $p\cdot k$. Further, using the equal-time commutator,

$$\delta(x^0-y^0)[A_a{}^0(y),\partial_\mu A_b{}^\mu(x)]=ig_v\sigma_{ab}(x)\delta^4(x-y)\,. \qquad (6)$$

We see that the second term in Eq. (5) does not contribute to the s-wave scattering length since it depends only on p^2, $p\cdot(q-k)$, and $(q-k)^2$. Here we would like to point out that throughout our analysis we neglect the Schwinger terms[3] appearing in the CCR. It is easy to see that the last two terms in Eq. (5), with the neglect of surface terms, and the use of CCR, can again be reduced to vertex functions. These terms, however, do not give rise to additional first-order–dependent terms which contribute to the s-wave scattering length. The only surviving term left in Eq. (5) can be written as

$$\lambda(\tfrac{1}{2}i)(k+q)_\nu\int d^4x\, e^{-i(k-q)\cdot x}(-2ig_v) \times\begin{pmatrix}8 & 8 & \vline & 8_a\\ a & b & \vline & c\end{pmatrix}\langle B_f|T\{V_c{}^\nu(x),S_8(0)\}|B_i\rangle, \qquad (7)$$

where we have used the CCR

$$\delta(x^0-y^0)[A_b{}^0(x),A_a{}^\nu(y)]=2ig_v \times\begin{pmatrix}8 & 8 & \vline & 8_a\\ a & b & \vline & c\end{pmatrix}V_c{}^\nu(x)\delta^4(x-y)\,. \qquad (8)$$

It is evident from the structure of the matrix element in Eq. (7) that the first-order symmetry-breaking effects in s-wave meson-baryon scattering lengths appear only as due to first-order symmetry-breaking effects in baryon-baryon vector coupling. But if the AG theorem[9] is assumed, then there are no BBV-coupling renormalization effects to first order in symmetry breaking. *We therefore conclude that there are no renormalization effects in s-wave meson-baryon scattering lengths to first order in symmetry breaking under the assumption of CCR, PCAC, and the AG theorem.*

In the next section, we discard the hypothesis of the AG theorem; we obtain the scattering lengths in terms of three parameters and derive a set of sum rules which are discussed at the end.

[11] See, e.g., Ref. 4.

[12] S. Weinberg, Phys. Rev. Letters **17**, 336 (1966).

III. SUM RULES AND DISCUSSION

With the definition

$$M_c^{\nu}=\int d^4x\, e^{i(k-q)\cdot x}\langle B_f|T\{V_c^{\nu}(x),S_8(0)\}|B_i\rangle, \quad (9)$$

we can interpret M_c^{ν} as the matrix element for the scattering of baryons and massless vector and scalar mesons. With the assumption of pure f-type BBV coupling[13] and the "baryon-octet dominance" it is easy to simplify Eq. (7) in terms of two parameters α and A_B, where α is the F/D ratio for BBS coupling and A_B is the product of BBV and BBS couplings. Thus, in this model with first-order symmetry-breaking effects, all s-wave meson-baryon scattering lengths have been expressed in terms of three parameters. This enables us to obtain a set of sum rules which are summarized below.

$$\begin{aligned}
&a_{1/2}(K\Sigma)+2a_{3/2}(K\Sigma)=0,\\
&a_0(K\Xi)-3a_1(K\Xi)=0,\\
&a_{1/2}(\pi N)+2a_{3/2}(\pi N)=0,\\
&a_0(\pi\Sigma)=2a_1(\pi\Sigma)=-2a_2(\pi\Sigma),\\
&a_{1/2}(\pi\Xi)+2a_{3/2}(\pi\Xi)=0,\\
&a_0(\bar{K}N)-3a_1(\bar{K}N)=0,\\
&a_{1/2}(\bar{K}\Sigma)+2a_{3/2}(\bar{K}\Sigma)=0,\\
&a_0(KN)=a_{1/2}(K\Lambda)=a_1(\pi\Lambda)\\
&\qquad=a_{1/2}(\bar{K}\Lambda)=a_0(\bar{K}\Xi)=0;
\end{aligned} \quad (10a)$$

$$\begin{aligned}
a_1(KN)&=-a_{1/2}(\pi N)=-\tfrac{2}{3}a_0(\bar{K}N),\\
a_{1/2}(K\Sigma)&=\tfrac{1}{2}a_0(\pi\Sigma)=a_{1/2}(\bar{K}\Sigma),\\
a_0(K\Xi)&=\tfrac{3}{2}a_{1/2}(\pi\Xi)=-\tfrac{3}{2}a_1(\bar{K}\Xi).
\end{aligned} \quad (10b)$$

$$\begin{aligned}
&a_{3/2}(\pi N\to K\Sigma)=2a_{1/2}(\pi N\to K\Sigma)\\
&\qquad=a_1(\pi\Sigma\to\bar{K}N)=(\sqrt{\tfrac{2}{3}})a_0(\pi\Sigma\to\bar{K}N),\\
&-a_1(\pi\Sigma\to K\Xi)=-(\sqrt{\tfrac{2}{3}})a_0(\pi\Sigma\to K\Xi)\\
&\qquad=a_{3/2}(\pi\Xi\to\bar{K}\Sigma)=2a_{1/2}(\pi\Xi\to\bar{K}\Sigma),
\end{aligned} \quad (10c)$$

$$\begin{aligned}
&(\sqrt{6})a_1(\pi\Lambda\to\bar{K}N)+2a_{1/2}(\pi N\to K\Lambda)=0,\\
&(\sqrt{6})a_1(\pi\Lambda\to K\Xi)-2a_{1/2}(\pi\Xi\to\bar{K}\Lambda)=0,\\
&a_0(\bar{K}N\to K\Xi)=a_1(\bar{K}N\to K\Xi)=0;\\
&a_1(\pi\Sigma\to\pi\Lambda)=a_{1/2}(K\Sigma\to K\Lambda)\\
&\qquad=a_{1/2}(\bar{K}\Sigma\to\bar{K}\Lambda)=0.
\end{aligned} \quad (10d)$$

$$\left.\begin{aligned}
&4a_{3/2}(\pi N)-a_0(\pi\Sigma)-[(8\sqrt{6})/3]a_0(\pi\Sigma\to\bar{K}N)=0\\
&4a_0(K\Xi)+3a_0(\pi\Sigma)-(8\sqrt{6})a_0(\pi\Sigma\to\bar{K}N)=0;
\end{aligned}\right\}, \quad (11a)$$

$$\begin{aligned}
&a_0(\pi\Sigma\to K\Xi)+a_0(\pi\Sigma\to\bar{K}N)+a_1(\pi\Lambda\to K\Xi)\\
&\qquad-a_1(\pi\Lambda\to\bar{K}N)=0,\\
&6a_0(\pi\Sigma)=(3\sqrt{6})[a_1(\pi\Lambda\to K\Xi)+a_1(\pi\Lambda\to\bar{K}N)]\\
&\qquad+(7\sqrt{6})[a_0(\pi\Sigma\to K\Xi)-a_0(\pi\Sigma\to\bar{K}N)].
\end{aligned} \quad (11b)$$

[13] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960).

The sum rules obtained above fall under four categories:

(i) Sum rules between s-wave scattering lengths of elastic processes (having the same mesons and baryons on either side of the S matrix) in different isotopic spin channels. These are covered by Eqs. (10a) and (10b) and have no symmetry-breaking effects.

(ii) Sum rules between s-wave scattering lengths of inelastic processes (having different mesons and baryons in the ingoing and outgoing states) in different isospin channels. These are contained in Eq. (10c) and have no renormalization effects.

(iii) Sum rules between s-wave scattering lengths of inelastic processes having the same meson but different baryons on the two sides of the S matrix. These are given in Eq. (10d) and these scattering lengths vanish identically in this model.

(iv) New sum rules, which embody the renormalization effects due to first-order symmetry breaking in the strong-interaction Hamiltonian. These are contained in in Eqs. (11a) and (11b).

Some of the sum rules in Eqs. (10a) and (10b) have also been obtained by Tomozawa.[5] In particular, he finds (without the S_8 breaking) that the sum rules

$$\begin{gathered}
a_{1/2}(\pi N)+2a_{3/2}(\pi N)=0,\\
a_0(KN)=0,\quad a_1(KN)\simeq 2\frac{m_K}{m_\pi}a_{3/2}(\pi N)
\end{gathered} \quad (12)$$

show qualitative agreement with the experiments. Since, at present, we have little experimental information for other scattering lengths,[14] we cannot test the rest of the sum rules.

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[14] We would like to mention here that in comparing our results for inelastic processes, only the real parts of the respective experimental scattering lengths have to be taken into account.