As long as we are concerned only with the pion-nucleon system, the Lagrangian (A13) contains all the relevant information. We can use it to construct the vector and axial-vector currents associated with the generators of the group $SU(2)\times SU(2)$. The result for the vector current is

$$V_\mu=N_\mu-\frac{2}{1+a^2\phi^2}\left[\frac{f}{m_\pi}\phi\times N_\mu{}^5-a^2\phi\times(\phi\times N_\mu)\right]-\frac{\phi\times\partial_\mu\phi}{(1+a^2\phi^2)^2},\quad\text{(A16)}$$

and for the axial-vector current

$$\frac{am_\pi}{f}A_\mu=N_\mu{}^5-\frac{2a^2}{1+a^2\phi^2}\left[\frac{m_\pi}{f}\phi\times N_\mu-\phi\times(\phi\times N_\mu{}^5)\right]+\frac{m_\pi}{2f}\frac{\partial_\mu\phi}{1+a^2\phi^2}+\frac{m_\pi}{f}a^2\frac{\phi\times(\phi\times\partial_\mu\phi)}{(1+a^2\phi^2)^2}.\quad\text{(A17)}$$

Here we have used the abbreviations

$$N_\mu=\bar{N}\gamma_\mu\tfrac{1}{2}\tau N\quad\text{(A18)}$$

and

$$N_\mu{}^5=\bar{N}\gamma_\mu\gamma_5\tfrac{1}{2}\tau N.\quad\text{(A19)}$$

Since the Lagrangian is invariant under isospin transformations, the vector current is conserved. The axial-vector current satisfies a partial conservation equation, the exact form of which depends on the choice of the symmetry breaking term L_3. The first terms in the expansion of Eqs. (A16) and (A17) are, respectively,

$$V_\mu=\bar{N}\gamma_\mu\tfrac{1}{2}\tau N-\phi\times\partial_\mu\phi+\cdots\quad\text{(A20)}$$

and

$$A_\mu=(f/am_\pi)\bar{N}\gamma_\mu\gamma_5\tfrac{1}{2}\tau N+(1/2a)\partial_\mu\phi+\cdots.\quad\text{(A21)}$$

Comparing the coefficients of these two expressions with those of (A13) one obtains once more the Goldberger-Treiman relation and the Adler-Weisberger relation.

PHYSICAL REVIEW VOLUME 163, NUMBER 5 25 NOVEMBER 1967

Numerical Analysis of Hadron Total Cross Sections in the Quark Model*

MAURICE V. BARNHILL III
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California

(Received 14 March 1967)

The linear parametrization of the hadron-hadron total-scattering cross sections at high energy given by the additive quark model are compared with experiment by a least-squares analysis using several different kinematic assumptions. Expressions for the corrections due to shadowing and double scattering are obtained, and the corrections are shown to be large enough to be important. A nonlinear parametrization obtained from the expression for the shadowing correction is compared with experiment by a least-squares analysis. The agreement is good under two different kinematic assumptions. Agreement is also good using the linear parametrization with the kinematic assumptions of James and Watson. In all cases studied, the amplitude for scattering of the λ quark from a nonstrange quark is significantly lower than the amplitude for scattering of two nonstrange quarks. The amplitude for the scattering of the nonstrange quark and antiquark in an isosinglet state is both significantly larger and much more energy-dependent than any of the other amplitudes.

I. INTRODUCTION

SINCE Gell-Mann[1] and Zweig[2] introduced quarks as an explicit realization of the fundamental representation of $SU(3)$, many calculations of properties of hadrons have been done in the quark model. Among these calculations are relations among the high-energy total cross sections, using the additivity hypothesis first introduced by Levin and Frankfurt,[3] Anisovich,[4] and Lipkin and Scheck.[5] Many authors[6] have analyzed the total cross sections on this basis, and it has been possible to make statements about the amount of $SU(3)$ symmetry-breaking present in the amplitudes by examining the relative successes of the various sum rules.[7]

* Research sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under AFOSR Contract AF49(638)-1389.

[1] M. Gell-Mann, Phys. Letters **8**, 214 (1964).

[2] G. Zweig, CERN report TH. 412, 1964 (unpublished).

[3] E. M. Levin and L. I. Frankfurt, JETP Pisma v Redaktsiyu 2, 105 (1965) [English transl.: JETP Letters 2, 65 (1965)].

[4] V. V. Anisovich, JETP Pisma v Redaktsiyu 2, 439 (1965) [English transl.: JETP Letters **2**, 272 (1965)].

[5] H. J. Lipkin and F. Scheck, Phys. Rev. Letters **16**, 71 (1966).

[6] V. Barger and L. Durand III, Phys. Rev. **156**, 1525 (1967); C. H. Chan, *ibid.* **152**, 1244 (1966); Y. T. Chiu and J. Schechter, Nuovo Cimento **46A**, 548 (1966); M. Imachi and S. Sawada, Nagoya University report (unpublished); J. J. J. Kokkedee, Phys. Letters **22**, 88 (1966); J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento **42A**, 711 (1966); J. J. J. Kokkedee and L. Van Hove, Nucl. Phys. **B1**, 169 (1967); C. A. Levinson, H. S. Wall, and H. J. Lipkin, Phys. Rev. Letters **17**, 1122 (1966); H. J. Lipkin, *ibid.* **16**, 1015 (1966); L. Van Hove, in Proceedings of the Stony Brook Conference on High-Energy Two-Body Reactions (unpublished); L. Van Hove, CERN report TH. 676 (unpublished). There have also been a number of papers on inelastic processes.

[7] H. J. Lipkin, Ref. 6.

TABLE I. Evaluation of $A-B$. The quantity $\frac{1}{2}(A-B)$ is evaluated in two different ways, using Eqs. (2), for various laboratory momenta. The data are from Galbraith *et al.*[a] Momenta are in BeV/c; cross sections in mb.

Momentum	6	8	10	12	14	16	18	20
$(pp)-(pn)$	-2.0 ± 2.3	-1.8 ± 2.3	-1.6 ± 2.3	-1.0 ± 2.3	-1.1 ± 2.3	-1.5 ± 2.3	-1.5 ± 2.3	-0.3 ± 2.3
$(K^+p)-(K^+n)$	-0.5 ± 0.4	-0.3 ± 0.4	-0.2 ± 0.4	-0.3 ± 0.4	-0.1 ± 0.4	-0.4 ± 0.4	-0.5 ± 0.4	-0.2 ± 0.4

[a] Reference 10.

It is necessary to make certain kinematic assumptions in comparing the sum rules with experiment, and there are more than one set of assumptions in use. It has also been generally assumed that strict linearity[8] will be true at high energies; thus processes like Glauber shadowing have been ignored. It will be the purpose of this paper to evaluate the shadowing correction and to show that the agreement with experiment can be improved if the correction is included. At the same time we will evaluate the quark amplitudes numerically under several kinematic assumptions to see the effect of various ways of comparing the quark-model expressions with experiment.

In the quark-model parametrization of high-energy scattering, one assumes that the amplitude for hadron-hadron scattering is a sum of the amplitudes for scattering of the individual quarks. One normally assumes that isospin is a good symmetry for the quarks so that the quark-quark amplitudes are[9]

$$(\mathcal{P}\mathcal{P})=(\mathfrak{N}\mathfrak{N})=A\,,\quad (\bar{\mathcal{P}}\mathcal{P})=\tfrac{1}{2}(D+E)\,,$$
$$(\mathcal{P}\mathfrak{N})=\tfrac{1}{2}(A+B)\,,\quad (\bar{\mathcal{P}}\mathfrak{N})=(\bar{\mathfrak{N}}\mathcal{P})=E\,,\qquad(1)$$
$$(\lambda\mathcal{P})=(\lambda\mathfrak{N})=C\,,\quad (\bar{\lambda}\mathcal{P})=(\bar{\lambda}\mathfrak{N})=F\,,$$

where B and D are isoscalar amplitudes and A and E are isovector amplitudes. The hadron-hadron amplitudes are obtained by summing over all possible ways of selecting a pair of quarks with one coming from each hadron. The resulting parametrization of the experiments for which there are experimental data is

$$(pp)=7A+2B\,,\qquad (\pi^-p)=2A+B+D+2E\,,$$
$$(pn)=(13/2)A+\tfrac{5}{2}B\,,\qquad (K^+p)=\tfrac{5}{2}A+\tfrac{1}{2}B+3F\,,$$
$$(\bar{p}p)=\tfrac{5}{2}D+(13/2)E\,,\qquad (K^-p)=3C+D+2E\,,\quad(2)$$
$$(\bar{p}n)=2D+7E\,,\qquad (K^+n)=2A+B+3F\,,$$
$$(\pi^+p)=\tfrac{5}{2}A+\tfrac{1}{2}B+\tfrac{1}{2}D+\tfrac{5}{2}E\,,\qquad (K^-n)=3C+\tfrac{1}{2}D+\tfrac{5}{2}E.$$

Since there are ten experiments and only six parameters, there are already four sum rules available; more are possible if various amplitudes are set equal. Comparison with experiment is done with total-cross-section measurements, using the optical theorem. We will use the experimental data of Galbraith *et al.*[10]

II. NUMERICAL RESULTS IN THE LINEAR THEORY

To see why the results of the shadowing analysis might be interesting, we will present the results of a numerical analysis of the linear theory. It is possible to remove one of the arbitrary parameters by a relatively simple analysis of the data. We observe that it is possible to solve for A-B from Eq. (2) in two different ways:

$$\tfrac{1}{2}(A-B)=(pp)-(pn)=(K^+p)-(K^+n)\,.\qquad(3)$$

In Table I we present the results of using the experimental data of Galbraith *et al.*[10] to evaluate $A-B$. It is clear that the data are completely consistent with $A-B=0$. We will therefore, for the remainder of this paper, work in the restricted parametrization obtained by setting A equal to B.

If we assume for the moment that we should compare experiments at equal values of the total energy in the center of mass of the two hadrons, we may do a least-squares analysis to obtain the best possible fit of the quark-model parametrization to the experimental data. This has been done using the data of Galbraith *et al.*, interpolated as necessary, weighting the data by the inverse square of the quoted error. The amplitudes that are obtained by this procedure are plotted in Fig. 1(a),[11] and the fifth column of Table II gives the difference δ(lin) between a typical set of best-fit values and the corresponding experimental values.

As may be seen from the table, the fit is not especially impressive. In particular, the best-fit values are consistently low for the baryon-baryon data and even lower for the baryon-antibaryon data. The meson data are fairly well fit primarily because they are heavily weighted by the least-squares procedure.

At least two effects have been inadequately handled in the analysis above. First, it has been assumed that the amplitudes are strictly additive. This does not seem

[8] J. J. J. Kokkedee and L. Van Hove [Nucl. Phys. **B1**, 169 (1967)] have argued that the contribution of annihilation should not be included in the the $\bar{p}p$ and $\bar{p}n$ cross sections when comparing quark-model predictions with experiment. Unfortunately there are insufficient high-energy data to permit an extensive use of this suggestion.

[9] We use p and n for the proton and neutron and $\mathcal{P}$ and $\mathfrak{N}$ for the nonstrange quarks; λ is the strange quark. (AB) means either the amplitude for forward elastic scattering, assumed imaginary, for A incident on B in the laboratory, or, through the optical theorem, the total cross section for that process.

[10] W. Galbraith, E. W. Jenkins, F. F. Kycia, B. A. Leontic, R. H. Phillips, A. L. Read, and R. Rubinstein, Phys. Rev. **138**, B913 (1965).

[11] At this point it may be well to point out that the only amplitude with appreciable energy dependence is D, the $T=0$, nonstrange quark-antiquark amplitude. This particular characteristic of the dependence of the amplitudes on the energy will prove to be true for all the cases we will investigate. Why this amplitude should be distinguished is not understood at present.

likely even if the interaction is linear in the quarks, because of the possibility of the shadowing of one quark by the others in the system. This effect has been discussed by Glauber[12] for scattering processes involving the deuteron, where one nucleon shadows the other. In the next section we shall extend Glauber's treatment to cover the quark model and apply the results to the experimental data in Sec. V.

The other question that must be handled more carefully is the selection of the proper energies at which the various scattering systems should be compared. A criterion for the selection has been obtained recently by James and Watson[13]; in the fourth section we will rederive their results and point out the possibility of a second prescription being obtained from their analysis. The criterion obtained by James and Watson does not allow comparison of all of the ten available experiments simultaneously, but the kaon scattering experiments can be compared separately to the baryon experiments and to the pion experiments. This also will be done in Sec. V.

III. CORRECTIONS DUE TO SHADOWING

A. Black-Sphere Model

As was pointed out by Glauber in his original paper,[12] the essential results of the theory of shadowing may be obtained from the model in which the cross section for an interaction is estimated by assuming a black-sphere absorption. Then the absorption cross section is πa^2,

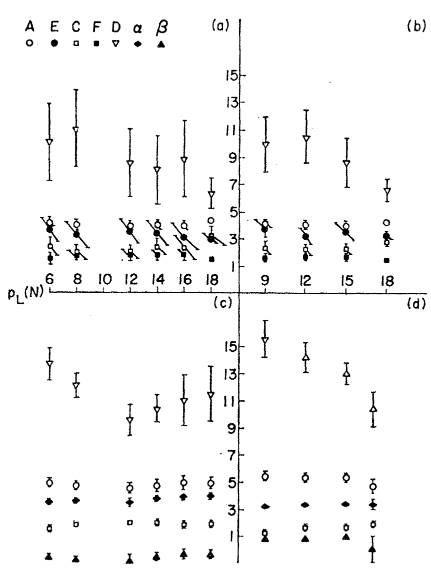

FIG. 1. Values obtained for the parameters in various analyses. The total cross sections A, C, D, E, and F are given in mb, the parameters α and β are given in mb^{-1}. The analyses are (a) holding s_{AB} constant in a linear fit, (b) holding the quark momentum constant in a linear fit, (c) holding s_{AB} constant including shadowing, (d) holding the quark momentum constant including shadowing. The results are plotted as a function of the laboratory momentum used in nucleon-nucleon scattering, in BeV/c, and the vertical scale is in mb for the cross sections and 10^{-2} mb^{-1} for α and β.

TABLE II. Quality of fit for two parametrizations with s_{AB} held constant. The second column gives p_L, the laboratory momentum of the projectile, in BeV/c; the third column gives the interpolated experimental value of the total cross section in mb; the fourth column gives Δ, the experimental error; the fifth column gives the difference δ(lin) between the theoretical values in a linear fit and the experimental measurements; the sixth column gives δ(shad), the same quantity for a parametrization including the shadowing correction. The data are from Galbraith *et al.*[a] interpolated as necessary. Both cases have five adjustable parameters.

System	p_L	Total cross section (meas.)	Δ	δ(lin)	δ(shad)
(pp)	16	38.7	0.6	−3.5	0.0
(pn)	16	40.2	1.7	−5.0	−1.5
$(\bar{p}p)$	16	49.2	0.8	−6.7	0.4
$(\bar{p}n)$	16	52.7	3.7	−13.1	−5.1
(π^+p)	16.5	23.4	0.2	0.6	0.1
(π^-p)	16.5	25.1	0.3	1.7	−0.2
(K^+p)	16.4	17.0	0.1	0.0	0.0
(K^-p)	16.4	21.2	0.5	0.7	0.2
(K^+n)	16.4	17.4	0.4	−0.4	−0.4
(K^-n)	16.4	20.3	0.7	−1.3	−1.0

[a] Reference 10.

where a is the radius of the sphere, and

$$\sigma^{\text{tot}}=2\sigma^{\text{abs}}=2\pi a^2. \qquad (4)$$

Some of the projectiles that would be expected to interact with a given particle in the target will be removed from the incident beam by having first interacted with one of the other components of the target. The probability of a projectile having been removed from the beam before reaching the second of two particles in the target is just the solid angle subtended by the second black sphere at the first particle, divided by 4π, or

$$p=\langle \pi a^2(4\pi R^2)^{-1}\rangle=(8\pi)^{-1}\sigma^{\text{tot}}\langle R^{-2}\rangle, \qquad (5)$$

where $\langle R^{-2}\rangle$ is the expectation value of the inverse square of the distance between the two particles in the target. To get the total correction we must sum over all ways of choosing quarks in the projectile *and in the target*. If we denote the total cross section for interaction of the jth quark in the projectile with the ith quark in the target by σ_{ij}, we get a correction $\delta\sigma$ to the total cross section[14] of

$$\delta\sigma_1+\delta\sigma_2=-\sum_{\substack{i,i',j\\ i\neq i'}}(8\pi)^{-1}\sigma_{ij}\sigma_{i'j}\langle R^{-2}\rangle_A -\sum_{\substack{i,j',j\\ j\neq j'}}(8\pi)^{-1}\sigma_{ij}\sigma_{ij'}\langle R^{-2}\rangle_B, \qquad (6)$$

where we have called the projectile A and the target

[12] R. J. Glauber, Phys. Rev. **100**, 242 (1955).

[13] P. B. James and H. D. D. Watson, Phys. Rev. Letters **18**, 179 (1967).

[14] The flux factors necessary to obtain the cross section from the probability of interaction are the same on both sides of the equation.

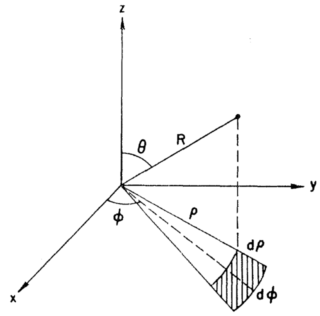

FIG. 2. Geometrical relations for discussion of double-counting corrections. $\rho = R\sin\theta$ is the projection of R in the scattering plane xy. See text.

B.[15] The quantity $\langle R^{-2}\rangle_A$ is the expectation value of the inverse square of the interquark distance in A.

So far we have included only terms involving one quark from one system and two quarks from the other; other terms involving the interaction of two pairs of quarks will be discussed in a moment. First we wish to estimate the size of the correction. If we take each of the σ_{ij} to be about 4 mb, and estimate $\langle R^{-2}\rangle$ from the mean-square radius of the proton of about 0.8 F^2 determined from electron scattering, we have a correction of the order of 3 mb for baryon-baryon scattering. The correction will be higher if the radius is smaller, but in any case it is large enough to be seen.

There is a second set of correction terms which are not present in the scattering of an elementary particle from the deuteron and which we have not yet considered. We must subtract off the probability that two different pairs of quarks from the two systems interact. Two simultaneous interactions are each counted separately by the linear theory, but they make only a single contribution to the total cross section. A correction for this double-counting error may also be estimated in the black-sphere model. We require the probability that, given an interaction between one pair of quarks, another pair of quarks will pass within a distance a of each other and thus interact. If in Fig. 2, θ and φ are considered to be the angles of the radius vector between two quarks (taken to be at a fixed distance from each other) in spherical coordinates, all spherical angles are equally probable and we have

$$P(\theta,\varphi)=(4\pi)^{-1}d\Omega=(4\pi)^{-1}\sin\theta\delta\theta\delta\varphi. \qquad (7)$$

Converting to a function of ρ and φ, still taking fixed R, we have for the probability that the second member of a pair will be in the shaded region of Fig. 2

$$P(\rho,\varphi)=\rho\delta\rho\delta\varphi/[4\pi R(R^2-\rho^2)^{1/2}]. \qquad (8)$$

Finally, we require the probability that the second members of the two pairs strike the scattering plane (plane xy in Fig. 2) within a distance a of each other. Allowing for the four choices of θ_1 and θ_2 independently less than or greater than $\frac{1}{2}\pi$, we have a total probability of

$$P=4\int\rho d\rho d\phi\,[4\pi R_1(R_1^2-\rho^2)^{1/2}]^{-1}\int\rho' d\rho' d\phi' \times[4\pi R_2(R_2^2-\rho'^2)^{1/2}]^{-1}\theta[a^2-(\boldsymbol{\varrho}-\boldsymbol{\varrho}')^2], \qquad (9)$$

for fixed R_1 and R_2, where $\theta(x)$ is 1 when x is positive and 0 otherwise. We will take the appropriate averages over R_1 and R_2 in a moment. If we substitute $\mathbf{y}=\boldsymbol{\varrho}'-\boldsymbol{\varrho}$ and assume that the range of the interaction is small compared to the size of the composite particle so that $y<a\ll R_2$, $\rho'\leq R_2$, and $\rho\leq R_1$, then to lowest approximation there is no dependence on φ or φ', and we may do those integrations and the integration over dy to get

$$P=a^2(2R_1R_2)^{-1}\int_0^M\rho d\rho\,(R_1^2-\rho^2)^{-1/2}(R_2^2-\rho^2)^{-1/2}, \qquad (10)$$

where M is the smaller of R_1 and R_2. The remaining integral can be done and yields

$$P=a^2(4R_1R_2)^{-1}\ln|(R_1+R_2)(R_1-R_2)^{-1}|, \qquad (11)$$

regardless of the relative sizes of R_1 and R_2.

To finish the expression, we multiply by the cross section for the interaction of the first pair of quarks, substitute for a^2 in terms of the cross section for interaction of the second pair of quarks, multiply by the radial wave functions in the two composites and integrate over all values of the radii, and finally sum over all distinct ways of choosing the quarks from the two systems, to get a correction $\delta\sigma_3$ to the cross section of

$$\delta\sigma_3=-\sum_{iji'j'}{}''\,\tfrac{1}{2}(8\pi)^{-1}\sigma_{ij}\sigma_{i'j'}I_{AB}, \qquad (12)$$

$$I_{AB}=(4\pi)^2\int dR_1dR_2\,R_1|\varphi_A(R_1)|^2R_2|\varphi_B(R_2)|^2 \times\ln|(R_1+R_2)(R_1-R_2)^{-1}|, \qquad (13)$$

where $\varphi_A(R_1)$ is the radial wave function of system A and the double prime on the summation denotes omitting terms with either $i=i'$ or $j=j'$. We have implicitly assumed that both composites have over-all S-state wave functions.

If we make the substitution $x=\frac{1}{2}(R_1+R_2)$ and $y=\frac{1}{2}(R_1-R_2)$, the integral becomes

$$I_{AB}=2(4\pi)^2\int_0^\infty dx\int_{-x}^{x}dy\,|\varphi_A(x+y)|^2 \times|\varphi_B(x-y)|^2(x^2-y^2)\ln|xy^{-1}|. \qquad (14)$$

If the wave functions are slowly varying functions at $y=0$, the integral over y is dominated by the divergence in the logarithm; the wave functions can then be removed from the integral over y and the remaining in-

[15] We will always use i and i' to label quarks in A and j and j' to label quarks in B.

tegral evaluated to give

$$I_{AB}=(32/9)(4\pi)^2\int_0^\infty dx\, x^3|\varphi_A(x)|^2|\varphi_B(x)|^2. \quad (15)$$

No further reduction is possible without specific knowledge of the form of the wave functions.

Altogether we have a final expression for the total cross section of hadron A incident on hadron B of

$$\sigma^{\text{tot}}(A,B)=\sum_{i,j}\sigma_{ij}+\delta\sigma_1+\delta\sigma_2+\delta\sigma_3, \quad (16)$$

where $\delta\sigma_1+\delta\sigma_2$ is given in Eq. (6) and $\delta\sigma_3$ is given in Eqs. (12) and (15).

It is necessary to discuss our approximations a little more carefully. The expression for P in Eq. (11) cannot be correct as R_1 approaches R_2, since a probability greater than 1 results. The problem is that the approximations made in going from Eq. (9) to Eq. (10) are not valid for $R_1-R_2\lesssim a$. The divergence in P must be cut off somewhere in this region; the cutoff will be late enough that the peak in P, although finite, will still be present, so that Eq. (13) survives. It is in fact possible to obtain an expression for P valid in the restricted region $R_1-R_2\ll R_1+R_2$ (where the trouble arises) which does not suffer from the incorrect approximations. In the region $R_1-R_2\ll R_1+R_2$ there is an additional term P' in P given by

$$P'=-\frac{\xi a^2}{2\pi R_1R_2}\left[\frac{R_1-R_2}{a}\ln\left|\frac{R_1-R_2+a}{R_1-R_2-a}\right| +\ln\left|\frac{(R_1-R_2)^2-a^2}{R_1^2-R_2^2}\right|-2\right],$$

where ξ is of order unity and is a function of R_1-R_2 which, to lowest order, is the same in the limits $R_1\to R_2$ and $R_1-R_2\to\pm\infty$.

If we assume that $|\varphi_A(x\pm a)|^2\simeq|\varphi_A(x)|^2$, the derivation of Eq. (15) is changed only by the addition of an integral of P' over y from $-x$ to x. This integral yields terms that are of order a^4. Since we have already neglected terms of this order in ignoring triple-scattering corrections, we may safely ignore P'.

We are not able to obtain explicit results for these higher-order terms in the eikonal-approximation discussion in the next section without assuming an explicit expression for the quark-quark interaction. The derivation of P' depends on doing the angular integrals by using the θ function in Eq. (9), and the point of the eikonal-approximation derivation is to avoid using an explicit expression for the amplitudes. If the θ-function form of the interaction were to be assumed, the discussion of the preceding paragraphs would apply.

B. Derivation of the Correction in Eikonal Approximation

The forms we have obtained for the nonlinear corrections to the additive quark model due to shadowing may be obtained somewhat more generally by making use of the eikonal approximation. One must assume that the interaction region is small compared to the radius of the composite (an assumption we have already made). If the forward amplitudes are purely imaginary, we recover our previous results. The dependence of the correction on the real part of the amplitude is also obtained.

The derivation parallels the work of Glauber.[12] We start from the eikonal representation of the scattering amplitude[16]

$$f(\mathbf{k}',\mathbf{k})=ik(2\pi)^{-1}\int_s \Gamma(\mathbf{b})\exp[i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{b}]d^2b,$$
$$\Gamma(\mathbf{b})=1-\exp[-i\chi(\mathbf{b})], \quad (17)$$

where $\mathbf{b}$ is perpendicular to the direction of incidence, $\mathbf{k}$ is the incident momentum, and $\mathbf{k}'$ is the final momentum. The optical theorem is

$$\operatorname{Im}f(\mathbf{k},\mathbf{k})\equiv\operatorname{Im}f(0)=k(4\pi)^{-1}\sigma^{\text{tot}}(k).$$

If $\mathbf{r}_i$ is the radius vector from the center of mass of A to the position of the ith quark in A, and if the projection of $\mathbf{r}_i$ in the scattering plane is $\boldsymbol{\rho}_i$, and if $\mathbf{s}_j$ and $\boldsymbol{\varrho}_j$ are the corresponding quantities for the jth quark in B,[15] and if $\mathbf{b}$ is the vector from the projection in the scattering plane of the center of mass of A to the projection of the center of mass of B, we write the total scattering amplitude in terms of χ_{tot} which is assumed to be

$$\chi_{\text{tot}}(\mathbf{b},\boldsymbol{\rho}_1\cdots\boldsymbol{\rho}_A,\boldsymbol{\sigma}_1\cdots\boldsymbol{\sigma}_B)=\sum_{i,j}\chi_{ij}(\mathbf{b}-\boldsymbol{\rho}_i+\boldsymbol{\sigma}_j), \quad (18)$$

when there are A quarks in A and B quarks in B. Γ is now to be thought of as an operator, inducing various transitions, so to get the full amplitude for elastic scattering we must write

$$F(\mathbf{k}',\mathbf{k})=ik(2\pi)^{-1}\int\exp[i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{b}]d^2b \times\int|\psi_A(r_1\cdots r_A)|^2 \times|\psi_B(s_1\cdots s_B)|^2\Gamma_{\text{tot}}(\cdots)d\tau_Ad\tau_B, \quad (19)$$

where $d\tau_A$ and $d\tau_B$ represent all independent coordinates of A and B, respectively, not including the center-of-mass coordinates, and where ψ_A and ψ_B are the full time-independent wave functions of A and B, respectively.

[16] Equation (17) is valid only for small-angle scattering. See L. I. Schiff [Phys. Rev. 103, 443 (1956)] where an expression which is also valid for large angles is derived.

We can now write Γ_{tot} in terms of the Γ_{ij},

$$\Gamma_{\text{tot}}(\mathbf{b},\boldsymbol{\varrho}_1\cdots\boldsymbol{\varrho}_A,\boldsymbol{\sigma}_1\cdots\boldsymbol{\sigma}_B)=1-\prod_{i,j}[1-\Gamma_{ij}(\mathbf{b}-\boldsymbol{\varrho}_i+\boldsymbol{\sigma}_j)],\quad(20)$$

where the product contains factors labeled by all i and j, with i and j chosen independently. If we expand the product, neglecting products of more than two Γ_{ij}'s and substitute in Eq. (19), taking $k=k'$, we get

$$F(0)=ik(2\pi)^{-1}\int|\psi_A(r_1\cdots r_A)|^2|\psi_B(s_1\cdots s_B)|^2d\tau_Ad\tau_B\times\int d^2b\,[\sum_{i,j}\Gamma_{ij}(\mathbf{b}-\boldsymbol{\varrho}_i+\boldsymbol{\sigma}_j)-\tfrac{1}{2}\sum_{ii'jj'}{}''\Gamma_{ij}(\mathbf{b}-\boldsymbol{\varrho}_i+\boldsymbol{\sigma}_j)\Gamma_{i'j'}(\mathbf{b}-\boldsymbol{\varrho}_{i'}+\boldsymbol{\sigma}_{j'})],\quad(21)$$

where the double prime on the sum means to omit terms for which both $i=i'$ and $j=j'$. If we substitute $\mathbf{b}'=\mathbf{b}-\boldsymbol{\varrho}_i+\boldsymbol{\sigma}_j$ in the first term, all the integrals in that term may be done, yielding simply $f_{ij}(0)$. Then, applying the optical theorem, we have

$$\sigma_{AB}=\sum_{i,j}\sigma_{ij}+\delta\sigma,\quad(22)$$

$$\delta\sigma=-\text{Re}\int|\psi_A|^2|\psi_B|^2d\tau_Ad\tau_B\int d^2b\times\sum_{ii'jj'}{}''\Gamma_{ij}(\mathbf{b}-\boldsymbol{\varrho}_i+\boldsymbol{\sigma}_j)\Gamma_{i'j'}(\mathbf{b}-\boldsymbol{\varrho}_{i'}+\boldsymbol{\sigma}_{j'}).\quad(23)$$

At this point it is most convenient to take three special cases, corresponding to the three corrections to the cross section in the black-sphere model: (1) $i=i'$, (2) $j=j'$, and (3) $i\neq i'$ and $j\neq j'$. The first two cases correspond to the shadowing correction and the last case corresponds to the correction for the double counting of the simultaneous interaction of two pairs of quarks.

In case (1) we have

$$\delta\sigma_1=-\text{Re}\sum_{ijj'}{}'\int|\psi_A|^2|\psi_B|^2d\tau_Ad\tau_B\times\int d^2b\,\Gamma_{ij}(\mathbf{b}-\boldsymbol{\varrho}_i+\boldsymbol{\sigma}_j)\Gamma_{ij'}(\mathbf{b}-\boldsymbol{\varrho}_i+\boldsymbol{\sigma}_{j'}),\quad(24)$$

where the prime on the sum means that j is to be taken unequal to j'. With the substitution of the new variables

$$\mathbf{s}=\tfrac{1}{2}(\mathbf{s}_j+\mathbf{s}_{j'}),\quad\boldsymbol{\sigma}=\tfrac{1}{2}(\boldsymbol{\sigma}_j+\boldsymbol{\sigma}_{j'}),$$
$$\mathbf{t}=(\mathbf{s}_j-\mathbf{s}_{j'}),\quad\boldsymbol{\tau}=(\boldsymbol{\sigma}_j-\boldsymbol{\sigma}_{j'}),$$

and $\mathbf{b}'=\mathbf{b}-\boldsymbol{\varrho}_i+\boldsymbol{\sigma}$, we have no dependence of the integrals on $\boldsymbol{\varrho}_i$ other than through the wave functions; thus we may write $\delta\sigma_1$ in terms of the function defined by

$$d\tau_B=d\tau_B'd^3t,\qquad\int|\psi_B(\mathbf{s}_1\cdots\mathbf{s}_B)|^2d\tau_B'=|\varphi_B(\mathbf{t})|^2.\quad(25)$$

The resulting expression for $\delta\sigma_1$ is

$$\delta\sigma_1=-\text{Re}\sum_{ijj'}{}'\int|\varphi_B(\mathbf{t})|^2\times\Gamma_{ij}(\mathbf{b}-\tfrac{1}{2}\boldsymbol{\tau})\Gamma_{ij'}(\mathbf{b}+\tfrac{1}{2}\boldsymbol{\tau})d^3td^2b.\quad(26)$$

Now if ψ_B is an over-all S-wave function, $|\varphi_B(\mathbf{t})|^2$ is independent of the angles of $\mathbf{t}$ and it is advantageous to change variables to

$$\mathbf{t}=\boldsymbol{\tau}+\mathbf{z},\quad d^3t=d^2\tau dz=tdtd^2\tau(t^2-\tau^2)^{-1/2}.$$

If at the same time we assume that the range of the interaction is small compared to the average separation of the quarks in a system so that we may neglect τ with respect to t in the factor $(t^2-\tau^2)^{-1/2}$, we get

$$\delta\sigma_1=-2\,\text{Re}\sum_{ijj'}{}'\int_0^\infty dt\,|\varphi_B(t)|^2\times\int d^2\tau\,d^2b\Gamma_{ij}(\mathbf{b}-\tfrac{1}{2}\boldsymbol{\tau})\Gamma_{ij'}(\mathbf{b}+\tfrac{1}{2}\boldsymbol{\tau})=4\pi(2k^2)^{-1}\sum_{ijj'}{}'\{\langle r_{jj'}^{-2}\rangle_B\,\text{Re}[f_{ij}(0)f_{ij'}(0)]\}.\quad(27)$$

We have used Eq. (17) to obtain the $f_{ij}(0)$ and the relation

$$\langle r_{jj'}^{-2}\rangle_B\equiv\langle t^{-2}\rangle=\int d^3t\,t^{-2}|\varphi_B(\mathbf{t})|^2,\quad(28)$$

to obtain $\langle r_{jj'}^{-2}\rangle_B$.

We can handle case (2) in a completely analogous manner, obtaining

$$\delta\sigma_2=4\pi(2k^2)^{-1}\sum_{ii'j}{}'\{\langle r_{ii'}^{-2}\rangle_A\,\text{Re}[f_{ij}(0)f_{i'j}(0)]\}.\quad(29)$$

Now if we assume that the f_{ij} are purely imaginary and apply the optical theorem, we reproduce Eq. (6), which was obtained previously under the somewhat more restrictive assumptions of the black-sphere model.

The correction in case (3) is

$$\delta\sigma_3=-\text{Re}\int|\psi_A|^2|\psi_B|^2d\tau_Ad\tau_B\int d^2b\times\sum_{ii'jj'}{}''\Gamma_{ij}(\mathbf{b}-\boldsymbol{\varrho}_i+\boldsymbol{\sigma}_j)\Gamma_{i'j'}(\mathbf{b}-\boldsymbol{\varrho}_{i'}+\boldsymbol{\sigma}_{j'}),\quad(30)$$

where the double prime on the sum now denotes the restriction $i\neq i'$ and $j\neq j'$. The reduction in this case is somewhat complicated by having to integrate over the coordinates of two quarks instead of just one. The necessary set of substitutions is

$$\mathbf{r}=\tfrac{1}{2}(\mathbf{r}_i+\mathbf{r}_{i'}),\quad \boldsymbol{\rho}=\tfrac{1}{2}(\boldsymbol{\rho}_i+\boldsymbol{\rho}_{i'}),\quad d\tau_A=d\tau_A'd^3u,$$
$$\mathbf{u}=(\mathbf{r}_i-\mathbf{r}_{i'}),\quad \boldsymbol{\mu}=(\boldsymbol{\rho}_i-\boldsymbol{\rho}_{i'}),\quad d\tau_B=d\tau_B'd^3t,$$
$$\mathbf{s}=\tfrac{1}{2}(\mathbf{s}_j+\mathbf{s}_{j'}),\quad \boldsymbol{\sigma}=\tfrac{1}{2}(\boldsymbol{\sigma}_j+\boldsymbol{\sigma}_{j'}),\quad |\varphi_A(\mathbf{u})|^2=\int|\psi_A|^2d\tau_A',$$
$$\mathbf{t}=(\mathbf{s}_j-\mathbf{s}_{j'}),\quad \boldsymbol{\tau}=(\boldsymbol{\sigma}_j-\boldsymbol{\sigma}_{j'}),\quad |\varphi_B(\mathbf{t})|^2=\int|\psi_B|^2d\tau_B',$$
$$\mathbf{b}'=\mathbf{b}-\boldsymbol{\rho}+\boldsymbol{\sigma}. \tag{31}$$

After making the substitutions we have the expression

$$\delta\sigma_3=-\mathrm{Re}\sum_{ii'jj'}''\int|\varphi_A(\mathbf{u})|^2|\varphi_B(\mathbf{t})|^2d^3ud^3t \times\int d^2b'\,\Gamma_{ij}[\mathbf{b}'-\tfrac{1}{2}(\boldsymbol{\mu}-\boldsymbol{\tau})]\Gamma_{i'j'}[\mathbf{b}'+\tfrac{1}{2}(\boldsymbol{\mu}-\boldsymbol{\tau})]. \tag{32}$$

We may again exploit the assumed lack of angular dependence of $|\varphi_A|^2$ and $|\varphi_B|^2$ by making the replacements $d^3t=tdtd^2\tau\ (t^2-\tau^2)^{-1/2}$ and

$$d^3u=udud^2\mu\ (u^2-\mu^2)^{-1/2},$$

to get

$$\delta\sigma_3=-4\ \mathrm{Re}\sum_{ii'jj'}''\int dtdu\ d^2\mu d^2\tau d^2b \times\left[\frac{ut|\varphi_A(u)|^2|\varphi_B(t)|^2}{(t^2-\tau^2)^{1/2}(u^2-\mu^2)^{1/2}}\right]\Gamma_{ij}[\mathbf{b}-\tfrac{1}{2}(\boldsymbol{\mu}-\boldsymbol{\tau})] \times\Gamma_{i'j'}[\mathbf{b}+\tfrac{1}{2}(\boldsymbol{\mu}-\boldsymbol{\tau})], \tag{33}$$

where the integration regions are given by $0\le u,\ t\le\infty$, $\mu<u$, and $\tau<t$. We may now neglect the variation of the factor in brackets over the interaction region. This region is given by the requirement that both $|\mathbf{b}-\frac{1}{2}(\boldsymbol{\mu}-\boldsymbol{\tau})|<a$ and $|\mathbf{b}+\frac{1}{2}(\boldsymbol{\mu}-\boldsymbol{\tau})|<a$, which together imply that $|\frac{1}{2}(\boldsymbol{\mu}-\boldsymbol{\tau})|<a$. Thus if we write $(t^2-\tau^2)=[t^2-\{(\boldsymbol{\tau}-\boldsymbol{\mu})+\boldsymbol{\mu}\}^2]$ we see that neglecting $\tau-\mu$ with respect to μ gives $t^2-\tau^2=t^2-\mu^2$. Defining $\mathbf{v}=\boldsymbol{\mu}-\boldsymbol{\tau}$, we have

$$\delta\sigma_3=-4\ \mathrm{Re}\sum_{ii'jj'}''\int dtdud^2\mu\left[\frac{ut|\varphi_A(u)|^2|\varphi_B(t)|^2}{(t^2-\mu^2)^{1/2}(u^2-\mu^2)^{1/2}}\right] \times\int d^2bd^2\nu\ \Gamma_{ij}(\mathbf{b}-\tfrac{1}{2}\mathbf{v})\Gamma_{i'j'}(\mathbf{b}+\tfrac{1}{2}\mathbf{v}). \tag{34}$$

The integrals over d^2b and $d^2\nu$ give the f_{ij} factors, and the integral over $d^2\mu$ is the same integral that appears in Eq. (10), so we have

$$\delta\sigma_3=2\pi(2k^2)^{-1}\sum_{ii'jj'}''\mathrm{Re}[f_{ij}(0)f_{i'j'}(0)]\int_0^\infty 4\pi udu \times\int_0^\infty 4\pi tdt|\varphi_A(u)|^2|\varphi_B(t)|^2\ln|(u+t)(u-t)^{-1}|. \tag{35}$$

If we again assume that the f_{ij} are pure imaginary and apply the optical theorem, we reproduce the black-sphere results of Eq. (12).

IV. CHOICE OF ENERGY FOR THE COMPARISON

We are still faced with the serious question of the choice of the energies of the various experiments to be used in applying the quark model. This question is complicated by the possible dependence of the amplitudes on the quark masses, which may or may not be renormalized by the symmetry-breaking interaction, and by the possible direct dependence of the amplitudes on the strength of the binding; but these possible effects must be ignored, since there is at present no way of estimating them.

James and Watson[13] have argued that the most reasonable choice is to take constant $s_{ij}=(p_i^\mu+p_j^\mu)^2$, where i and j are quark labels. This quantity is rather easily evaluated in terms of s_{AB}, the equivalent quantity for the composite hadrons. If we assume that we may neglect the momentum of the quarks relative to the center of mass of the hadron, we find, in the hadron rest frame,[15]

$$p_i^\mu=(m_i,0)=(m_i/m_A)p_A^\mu,\qquad p_j^\mu=(m_j,0)=(m_j/m_B)p_B^\mu. \tag{36}$$

Thus we have

$$s_{ij}=(m_im_A^{-1}p_A^\mu+m_jm_B^{-1}p_B^\mu)^2,\qquad (s_{ij}-m_i^2-m_j^2)/(m_im_j)=(s_{AB}-m_A^2-m_B^2)/(m_Am_B). \tag{37}$$

If hadron A is incident on hadron B in the laboratory with momentum p_L, and if we neglect the hadron masses compared to momenta and energies, we have

$$p_L\approx\frac{s_{AB}}{2m_B}\approx\frac{m_Am_B}{m_im_j}\left(\frac{s_{ij}-m_i^2-m_j^2}{2m_B}\right). \tag{38}$$

Now we have two choices. James and Watson choose to assume that the symmetry-breaking interaction does not renormalize the quark masses. In this case we hold $s_{ij}-m_i^2-m_j^2$ constant, and for A and A' incident on B we have

$$p_L(A)/P_L(A')=m_A/m_{A'},\quad \text{no renormalization}. \tag{39}$$

If, however, we assume that the masses are renormalized from system to system, then we take $m_i=\frac{1}{2}m_A$ if A is a meson (and $\frac{1}{3}m_A$ if A is a baryon). We may further assume that all m_i may be neglected with respect to momenta and energies, and the equivalent of Eq. (39) is

$$p_L(A)/P_L(A')=A/A',\quad \text{quark masses renormalized}, \tag{40}$$

where A is the number of quarks in hadron A. This pre-

TABLE III. Quality of fit with and without shadowing correction with momentum in the quark-quark c.m. frame held constant. See Table II for explanation of symbols. There are five adjustable parameters.

System	Lab. mom.	Total cross section (meas.)	Δ	δ(lin)	δ(shad)
(pp)	15.2	38.9	0.6	−2.6	0.0
(pn)	15.2	40.2	1.7	−3.9	−1.3
$(\bar{p}p)$	15.2	49.8	0.8	−5.1	0.3
$(\bar{p}n)$	15.2	53.0	3.7	−10.8	−4.8
(π^+p)	9.9	24.8	0.2	0.5	0.1
(π^-p)	9.9	26.6	0.3	1.2	−0.2
(K^+p)	10	17.3	0.1	0.0	0.0
(K^-p)	10	22.5	0.2	0.1	0.1
(K^+n)	10	17.5	0.4	−0.2	−0.2
(K^-n)	10	20.6	0.4	−0.5	−0.5

scription has been extensively used by Kokkedee and Van Hove.[17]

Because of the structure of the sum rules, James and Watson could only apply their prescription to the relations

$$(\bar{p}p)+(pn)=2(\pi^-p)+(\pi^+p),\qquad (\bar{p}n)+(pp)=(\pi^-p)+2(\pi^+p),\tag{41}$$

for which there is no overlapping data; they were able to compare the left-hand sides only with extrapolations of the right-hand sides, obtaining satisfactory agreement. However, the kaon-scattering data can be compared, using their prescription [Eq. (39)], with both pion-scattering experiments and baryon-scattering experiments, if a numerical analysis is used. We shall make this comparison in the next section, and we will find that adequate agreement is obtained. Somewhat better agreement is obtained by using Eq. (40) in conjunction with a parametrization including the shadowing correction. Unfortunately, the parametrization obtained by including the shadowing correction is rather flexible, so it is not clear that the rather good fit to the data is really significant.

V. NUMERICAL ANALYSIS

In this section we will present the results of a least-squares fitting of the experimental data under various kinematic assumptions, including the shadowing correction whenever the amount of usable data permits. The results of such an analysis for a linear parametrization holding s_{AB} constant from experiment to experiment has already been presented [see Table II and Fig. 1(a)].

In the first analysis to be presented in this section we will hold the momentum in the quark-quark center-of-momentum frame constant. This prescription is almost the same as that contained in Eq. (40); it differs from that prescription by terms of the same order as those that have been neglected in the derivation of Eq. (40), and the quality of the fit is not affected by the difference between the two prescriptions. The results for a linear parametrization, Eq. (2) with $A=B$, are given in Table III and Fig. 1(b). The baryon-baryon scattering data are still not very well reproduced, although the fit is somewhat better than that obtained in Sec. II.

[17] See Ref. 6.

Before we can include the shadowing correction in an effort to improve the agreement, we must reduce the number of parameters. We have five amplitudes; Eq. (6) contains two additional parameters, $\langle R^{-2}\rangle$ for the meson and for the baryon; and the double-counting correction in Eq. (12) has three parameters, I_{AB} with A and B both baryons, both mesons, or one a baryon and one a meson. Thus there are ten parameters for ten experiments and no predictions are obtained. We intend to ignore the double-counting terms, Eq. (12). The size of these terms cannot be estimated without knowing details of the internal quark wave functions; their neglect can be justified only in terms of the success of the resulting parametrization. We also intend to apply the Pomeranchuk theorem to the quark amplitudes to justify taking $A=E$ and $C=F$; it may be seen from Fig. 1 that these relations are consistent with the results of the linear analyses.

We are left with five parameters. If we evaluate the correction terms from Eq. (6) and add them to the linear terms from Eq. (2) we have

$$\begin{aligned}
(pp)&=(pn)=9A-36\beta A^2,\\
(p\bar{p})&=(13/2)A+\tfrac{5}{2}D-18\beta A^2-16\beta AD-2\beta D^2,\\
(\bar{p}n)&=7A+2D-21\beta A^2-14\beta AD-\beta D^2,\\
(\pi^+p)&=(11/2)A+\tfrac{1}{2}D-(5\alpha+10\beta)A^2-(\alpha+2\beta)AD,\\
(\pi^-p)&=5A+D-[4\alpha+(17/2)\beta]A^2\\
&\qquad-(2\alpha+3\beta)AD-\tfrac{1}{2}\beta D^2,\\
(K^+p)&=(K^+n)=3A+3C-6\beta A^2-6\alpha AC-6\beta C^2,\\
(K^-p)&=2A+3C+D-\tfrac{5}{2}\beta A^2-4\alpha AC\\
&\qquad-3\beta AD-6\beta C-2\alpha CD-\tfrac{1}{2}\beta D^2,\\
(K^-n)&=\tfrac{5}{2}A+3C+\tfrac{1}{2}D-4\beta A^2-5\alpha AC\\
&\qquad-2\beta AD-6\beta C^2-\alpha CD,\\
\alpha&\equiv\langle(8\pi R^2)^{-1}\rangle_M,\\
\beta&\equiv\langle(8\pi R^2)^{-1}\rangle_B.
\end{aligned}\tag{42}$$

The results of the least-square analysis with the parametrization in Eq. (42) are given in Fig. 1(d), and a typical set of best-fit values is compared with experiment and with the linear fit in Table III. As may be seen, the fit is quite good. Moreover, there is no appreciable energy dependence in α or β, which is as it should be. If we take the values for α and β seriously (which may well be dangerous) we get values of 8.5 F^{-2} and 2 F^{-2} for the expectation values of R^{-2} for the meson and for the baryon, respectively. Hence the magnitudes of α and β are not unreasonable.

Since the parametrization including the shadowing correction is nonlinear, it is not clear exactly how flexible the parametrization is. Thus the significance of the rather good fit we have obtained is doubtful. This is especially true considering the large number of adjustable parameters we have. To check on this situation, we have done the analysis keeping s_{AB} constant from experiment to experiment, giving a somewhat different set of experimental numbers to be fit. A typical set of best-fit values are given in Table II, and the resulting parameters are given in Fig. 1(c). The fit is embarrassingly good; the only feature of this fit that is less than ideal is a slight energy dependence of α and β. The problem is that the parametrization of the meson-baryon data depends almost entirely on the linear combination $\alpha+2\beta$, leaving β as a free parameter to achieve agreement of the baryon-baryon data. Since the meson data are already well fit by a linear analysis, it is not surprising that a good fit is obtained when shadowing is included. We conclude that it is not possible to decide whether or not shadowing is present by examining the results of the numerical analysis.

Finally, we present the results of an analysis using the prescription for choice of energy given by James and Watson [our Eq. (39)]. Since with this criterion we cannot use all of the experiments in any one analysis, a three-parameter linear analysis has been used (independent parameters A, C, and D). For reasonably high energies, only three distinct sets of experimental data are available. The results of the analysis for all three are given in Table IV. It may be seen that the fit is good, although not quite as good as the fit with the shadowing correction and holding quark-quark momentum constant.[18]

TABLE IV. Parameters and quality of fit using James-Watson prescription of $p_L(A')/p_L(A)=m_{A'}/m_A$. The first ten lines are arranged similarly to Tables II and III; the last six lines give the laboratory momenta used (in BeV/c) and the results for the parameters (in mb). The column labeled Δ gives the experimental error; the column labeled δ gives the difference between the experimental measurement and the best-fit value. Data are from Galbraith *et al.*[a] with slight interpolation and extrapolation.

System	Δ	δ	Δ	δ	Δ	δ
(pp)	0.6	−1.0	0.6	−0.8	Not available	
(pn)	1.7	−2.2	1.7	−2.1	Not available	
$(\bar{p}p)$	0.8	2.3	0.9	2.7	Not available	
$(\bar{p}n)$	3.7	−2.8	3.7	−3.3	Not available	
(π^+p)		Not available			0.2	0.0
(π^-p)		Not available			0.3	0.0
(K^+p)	0.1	0.2	0.1	0.1	0.1	0.0
(K^-p)	0.3	−0.4	0.2	−0.4	4.6	−0.2
(K^+n)	0.4	−0.3	0.4	−0.2	0.4	−0.2
(K^-n)	0.4	−1.5	0.4	0.6	1.6	−0.5
p_L kaon		6		8		20
p_L nucleon		11.3		15		...
p_L pion		...		...		5.6
A		4.29±0.14		4.23±0.11		4.01±0.03
C		1.43±0.15		1.56±0.12		1.83±0.03
D		10.8 ±0.6		10.1 ±0.4		8.66±0.02

[a] Reference 10.

VI. CONCLUSIONS

We have seen that, under a wide variety of kinematic assumptions, the meson-baryon total cross sections are well represented by a quark-model parametrization while the baryon-baryon data may or may not be well represented. In all cases, the cross sections involving one strange quark are smaller than the cross sections involving nonstrange quarks; thus $SU(3)$ is broken. In addition, the cross section for the $T=0$, state of the nonstrange quark and nonstrange antiquark is appreciably larger and more energy-dependent than the other cross sections.

If we use the kinematic prescription of equal s_{ij} with unrenormalized masses given by James and Watson[13] we can obtain a fairly good agreement of the quark-model parametrization with experiment using three parameters in a simple, linear relation. If we assume that the quark masses are renormalized from system to system, we must use a parametrization based on including a correction for the shadowing of one quark by others in the composite, but if we use this parametrization we get impressive agreement with the data. The agreement is marred by the apparent flexibility of the parametrization. Neglecting the double-counting terms of Eq. (12) we can obtain estimates of hadron sizes which are entirely reasonable ($\langle R^{-2}\rangle \approx 2$ F^{-2} for a nucleon). It appears that the quark model, with physically reasonable assumptions, is quite flexible enough to avoid contradiction with experiments of the present accuracy.

ACKNOWLEDGMENT

The author would like to thank Professor L. I. Schiff for his continuing interest and encouragement and for many helpful comments.

[18] We have deliberately avoided using χ^2 with so few degrees of freedom and the nonlinear parametrization. However, for whatever it may be worth, the average value of χ^2 divided by the degrees of freedom in the fit with momentum constant was 1.5; for the present analysis the values are 6.4 for a kaon momentum of 6 BeV/c, 4 for 8 BeV/c, and 0.1 for 20 BeV/c.