Lagrangian Method for Chiral Symmetries*

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The method of phenomenological nonlinear Lagrangians is described in detail for the case of chiral $SU(2)\times SU(2)$. The complete nonlinear Lagrangian is constructed for a theory of pions, nucleons, and vector and axial-vector mesons. The Lagrangian allows a study of the strong and weak interactions of the particles involved. Relations between masses and coupling constants emerge. A model containing only pions and nucleons is also described. Finally, the way to generalize the method to $SU(3)\times SU(3)$ is indicated.

1. INTRODUCTION

OVER the past few years a number of interesting and successful relations have been derived from the assumption of (broken) chiral $SU(2)\times SU(2)$ and of total or partial conservation of currents.[1] The techniques usually employed to derive these results are those of the algebra of currents,[2] supplemented by the idea that vector, axial-vector, or pseudoscalar states dominate the matrix elements of the currents.[3] In this paper we would like to describe a different method, that of the phenomenological Lagrangians with which it is possible to derive the consequences of the same physical ideas. Once one has acquired familarity with this method, one finds that it is perhaps simpler to use than the techniques of the algebra of currents and that a number of results can be obtained by means of rather elementary manipulations on classical field Lagrangians.

The use of Lagrangians in the study of broken chiral $SU(2)\times SU(2)$ is quite old and, as a matter of fact, the group was first introduced in particle physics in the context of a Lagrangian theory, the so-called σ model.[4] The σ model in its original form has the disadvantage of assigning the pion to the four-dimensional representation of $SU(2)\times SU(2)$ together with a scalar isoscalar meson, the σ meson, which appears not to exist in nature. Furthermore, as long as the symmetry is exact, the nucleon must have a vanishing mass. Of course one can argue that, since the symmetry is broken, the σ meson can acquire a very large mass and become highly unstable and that the nucleon can acquire a finite mass. However, if this is the case, it is natural to seek a formulation which does not require a scalar field in the theory at all. Theories of this kind have been discussed some time ago by Kramer, Rollnik, and Stech[5] and by Gürsey.[6] More recently, Weinberg,[7] starting from the σ model, eliminated from it the scalar field and obtained a Lagrangian in which only nucleons and pions occur and in which the nucleons have a manifestly nonvanishing mass. A transformation analogous to Weinberg's was performed some time ago for a somewhat simpler model by Gürsey and one of the authors (B.Z.). It resulted in the introduction of a nucleon field corresponding to the field N of Sec. 2 of this paper and having a manifestly nonvanishing mass (see the second paper of Ref. 6). The Lagrangians studied in these papers are highly nonlinear and, when expanded in terms of the coupling constant, generate many-particle vertices.

If one performs a transformation of $SU(2)\times SU(2)$, the fields entering the nonlinear Lagrangian undergo certain (in general, nonlinear) transformations which are realizations of the group.[8] The nonlinear Lagrangian consists of parts which are invariant under these transformations and, if the symmetry is broken, of parts which have simple transformation properties. In the present work we formulate the theory by assigning the fields to specified linear or nonlinear realizations of $SU(2)\times SU(2)$. The nonlinear Lagrangian is then essentially determined by requirements of invariance or covariance. The fact that one can limit oneself to a pion triplet and does not need to add a scalar meson is due to the existence of a three-dimensional nonlinear realization of $SU(2)\times SU(2)$, while one knows that there is no three-dimensional linear representation. The linear or nonlinear realizations of the group to which the various fields must be assigned are, of course, indicated by the agreement with empirical evidence. It is one of the advantages of the method of nonlinear phenomenological Lagrangians that this comparison with the experiments can be carried out very simply by expanding the Lagrangian in powers of the coupling constants and by using the various many-particle

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[1] We quote only a few of the relevant papers: S. L. Adler, Phys. Rev. Letters 14, 1051 (1965); W. I. Weisberger, *ibid.* 14, 1047 (1966); K. Kawarabayashi and M. Suzuki, *ibid.* 16, 255 (1966); S. Weinberg, *ibid.* 18, 507 (1967).
[2] M. Gell-Mann, Phys. Rev. 125, 1067 (1962); Physics 1, 63 (1964). More recently it has been suggested that the algebra of currents be replaced with an "algebra of fields": T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters 18, 1029 (1967).
[3] The dominance hypothesis has been mostly used in the form suggested by M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961). For a recent discussion based on Lagrangian field theory, see N. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967), where other references are given.
[4] J. Schwinger, Ann. Phys. (N. Y.) 2, 407 (1957); M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).
[5] G. Kramer, H. Rollnik, and B. Stech, Z. Physik 154, 564 (1959).
[6] F. Gürsey, Nuovo Cimento 16, 230 (1960); Ann. Phys. (N. Y.) 12, 91 (1961). These papers and the paper quoted in Ref. 5 contain many of the ideas which have been used in later work.
[7] S. Weinberg, Phys. Rev. Letters 18, 188 (1967).
[8] Following the mathematical usage we reserve the word representation for the linear realizations of a group.

vertices obtained in this way. One then adopts the rule that each process should be calculated by taking all graphs which can contribute to it and which have the topological structure of trees (no internal loops, no internal integrations). This procedure may perhaps be justified if the momentum transfers at the vertices are sufficiently small. This point of view is closely related to that of some recent work by Schwinger.

The main body of this paper describes a model which includes, besides nucleons and pions, an isovector vector meson (ρ meson) and an isovector axial-vector meson.[9] The requirements of invariance or covariance still leave certain essential parameters in the Lagrangian undetermined. The additional requirement to be imposed corresponds to that, which is usually made in other approaches, of vector dominance of certain matrix elements, or dominance by other meson states such as axial vectors or pseudoscalars. In phenomenological Lagrangian field theory this dominance requirement assumes an extremely simple form. It simply states that those interactions which can arise through the exchange of one particle should not also occur directly, i.e., be represented by their own vertex obtained by expanding the nonlinear Lagrangian. This requirement gives restrictions which determine some hitherto undetermined parameter (see Sec. 4).

The nonlinear realizations of the group by which the fields transform contain explicitly the coupling constant. As a result, if one expands the Lagrangian in powers of the coupling constant, terms of different order are transformed into each other by the group transformations and it is only the total nonlinear Lagrangian that has simple properties of invariance (or covariance). It is in this way that relations between processes of different order and with different numbers of particles arise, corresponding to the relations between processes with different numbers of soft pions which one obtains in the method of the algebra of currents.[10]

Knowledge of the Lagrangian permits the calculation of the vector and axial-vector currents which enter in the weak interactions. In nonlinear phenomenological field theory they appear as highly nonlinear functions of the various fields. From these expressions it is possible to read off the matrix elements of the currents between many-particle states and obtain relations between various weak-interaction parameters. Alternatively, one may assume that the weak interactions of hadrons are dominated by the vector and axial-vector mesons (see Sec. 6).

[9] This model has also been studied by J. Schwinger, Phys. Letters **24B**, 473 (1967). Our own work, like Schwinger's, was directly stimulated by Weinberg's paper (Ref. 7). Most of the detailed results described in the present paper were obtained before we had knowledge of Schwinger's paper. A number of formulas, such as the expression for the full nonlinear Lagrangian, Eq. (50) below, appear here for the first time.

[10] For the current algebra method, see S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

The phenomenological Lagrangian which satisfies all the principles described above still contains a number of arbitrary parameters. The determination of their values requires principles which go beyond those embodied in the ideas of $SU(2)\times SU(2)$ or of vector and axial-vector dominance and thus in a very precise sense go beyond the present approach. We shall indicate in Sec. 5 how one can determine some parameter ratios if one is willing to make use of results of relativistic $SU(6)$ theory. This follows an idea of Schwinger, who makes use of his own formulation of relativistic $SU(6)$.[11]

The ideas and methods described in this paper can be generalized to $SU(3)\times SU(3)$. The detailed results will be described separately. However, many formulas of the present paper are written in such a way that the generalization to $SU(3)\times SU(3)$ is immediate.

For completeness we describe in the Appendix a model involving only nucleons and pions which has been already studied in detail by other authors, notably Weinberg and Schwinger. Our purpose here is to write it in a form which can be directly generalized to $SU(3)\times SU(3)$ and to give some additional results.

2. TRANSFORMATION PROPERTIES OF THE FIELDS

In the σ model the pion field π and the σ meson field σ belong to the four-dimensional representation of $SU(2)\times SU(2)$. By a chiral transformation they transform according to the law

$$\sigma+i\gamma_5\pi \rightarrow e^{-i\alpha\gamma_5}(\sigma+i\gamma_5\pi)e^{-i\alpha\gamma_5}, \tag{1}$$

where π and α are 2×2 Hermitian traceless matrices, $\pi=\boldsymbol{\pi}\cdot\boldsymbol{\tau}$ and $\alpha=\boldsymbol{\alpha}\cdot\boldsymbol{\tau}$. The corresponding transformation of the nucleon field is

$$\psi\rightarrow e^{i\alpha\gamma_5}\psi. \tag{2}$$

As a result, the expressions

$$R^2=\sigma^2+\pi^2 \tag{3}$$

and

$$\bar{\psi}(\sigma+i\gamma_5\pi)\psi \tag{4}$$

are invariant.

Following the papers quoted in Refs. 5–7, we replace the two fields σ and π by two new fields. One is the invariant R and the other is an isovector field $\boldsymbol{\xi}$. They are defined by

$$\sigma+i\gamma_5\pi=R(1-i\gamma_5\xi)/(1+i\gamma_5\xi). \tag{5}$$

Using the same notation as before, we denote with ξ the traceless 2×2 matrix $\xi=\boldsymbol{\xi}\cdot\boldsymbol{\tau}$. The physical interpretation of ξ is that it is, up to a multiplicative constant, the new field of the pion. Clearly ξ transforms so that

$$\frac{1-i\gamma_5\xi}{1+i\gamma_5\xi}\rightarrow e^{-i\alpha\gamma_5}\frac{1-i\gamma_5\xi}{1+i\gamma_5\xi}e^{-i\alpha\gamma_5}; \tag{6}$$

[11] J. Schwinger, Phys. Rev. **140**, B158 (1965); Phys. Rev. Letters **18**, 923 (1967).

when α is infinitesimal this can be written as

$$\delta\xi=\alpha+\xi\alpha\xi. \tag{7}$$

We also introduce a new nucleon field N defined so that the invariant (4) takes the simple form $R\bar{N}N$. Clearly

$$N=\left(\frac{1-i\gamma_5\xi}{1+i\gamma_5\xi}\right)^{1/2}\psi=\frac{1-i\gamma_5\xi}{(1+\xi^2)^{1/2}}\psi. \tag{8}$$

The transformation properties of N are determined by those of ψ and of ξ. The inverse of Eq. (8) is

$$\psi=[(1+i\gamma_5\xi)/(1+\xi^2)^{1/2}]N, \tag{9}$$

and one can easily see that

$$\delta N=iAN, \tag{10}$$

where

$$iA=[1/(1+\xi^2)^{1/2}][\xi\alpha(1+\xi^2)^{1/2}-\delta(1+\xi^2)^{1/2}]. \tag{11}$$

If we now make use of the special properties the matrix ξ has by virtue of its form $\xi=\boldsymbol{\xi}\cdot\boldsymbol{\tau}$, we see that Eq. (11) can also be written as

$$iA=\tfrac{1}{2}[\xi,\alpha]. \tag{12}$$

We remark here that Eqs. (6), (7), (10), and (11) are valid also for $SU(3)\times SU(3)$, in which case α and ξ are Hermitian 3×3 matrices (α is traceless while ξ isn't), and N is a three-component quark field.

We have indicated above how the transformation properties of the fields ξ and N can be obtained from those of the old fields σ, π, and ψ. However, as explained in the introduction, we shall now adopt a different attitude. Without referring any longer to the original σ model, we just postulate the transformation properties of the fields ξ and N and look for Lagrangians having suitable invariance or covariance properties. The invariant field R will never occur in the theory. The identification of the pion with ξ (suitably normalized) has the advantage that only the known pion triplet occurs in the theory. The identification of the nucleon with N has the advantage that the chiral transformation now allows the presence of a nucleon mass term in the Lagrangian. The matrix A is Hermitian and does not contain γ_5. From Eq. (10) and its adjoint

$$\delta\bar{N}=-i\bar{N}A, \tag{13}$$

it follows that $\bar{N}N$ is an invariant. It is obvious that the transformation properties of ξ and N under isospin transformations are

$$\delta N=i\beta N,\quad \delta\bar{N}=-i\bar{N}\beta, \tag{14}$$

$$\delta\xi=i[\beta,\xi], \tag{15}$$

where $\beta=\boldsymbol{\beta}\cdot\boldsymbol{\tau}$.

We shall now allow the group parameters α and β to be coordinate-dependent. The transformation formulas given above are still valid but, in addition to the fields already introduced, we need now as gauge fields a vector field

$$\rho_\mu=\boldsymbol{\rho}_\mu\cdot\boldsymbol{\tau} \tag{16}$$

and an axial-vector field

$$a_\mu=\mathbf{a}_\mu\cdot\boldsymbol{\tau}. \tag{17}$$

Coordinate-dependent $SU(2)\times SU(2)$ transformations for such fields have been considered by a number of authors.[12] An infinitesimal isospin gauge transformation can be written as

$$\delta\rho_\mu=i[\beta,\rho_\mu]+(2/g)\partial_\mu\beta, \tag{18}$$

$$\delta a_\mu=i[\beta,a_\mu], \tag{19}$$

while an infinitesimal chiral gauge transformation has the form

$$\delta\rho_\mu=i[\alpha,a_\mu], \tag{20}$$

$$\delta a_\mu=i[\alpha,\rho_\mu]+(2/g)\partial_\mu\alpha. \tag{21}$$

We do not intend the Lagrangian to be invariant under the coordinate-dependent $SU(2)\times SU(2)$ gauge group. Rather we have in mind a Lagrangian which consists of a part L_1 invariant under the gauge group, a part L_2 which breaks the gauge invariance but preserves invariance under the $SU(2)\times SU(2)$ group with constant parameters, and finally a part L_3 which reduces the symmetry to ordinary isospin invariance. L_2 will be chosen of a particularly simple form

$$L_2=\tfrac{1}{2}m^2(\rho_\mu{}^2+a_\mu{}^2). \tag{22}$$

This has an interesting consequence which follows immediately by performing on the total Lagrangian

$$L=L_1+L_2+L_3, \tag{23}$$

an infinitesimal gauge transformation. One can see easily in this way that the equations of motion will have as a consequence the two relations

$$\partial_\mu\boldsymbol{\rho}_\mu=0 \tag{24}$$

and

$$\partial_\mu\mathbf{a}_\mu=-(g/2m^2)\mathbf{P}. \tag{25}$$

Here $\mathbf{P}$ is defined in terms of the change of L_3 under an infinitesimal chiral gauge transformation

$$\delta L_3=\mathbf{P}\cdot\boldsymbol{\alpha}. \tag{26}$$

In general, $\mathbf{P}$ will be a function of the fields occurring in L_3. For a particular form of L_3, P will be proportional to the pion field. Although this choice may be particularly appealing, we must point out that only comparison with the empirical evidence can justify a particular choice of the symmetry-breaking term L_3. In Sec. 4 we shall discuss the form of L_3 in more detail.

3. CONSTRUCTION OF INVARIANTS

Our main task is the construction of the gauge-invariant part L_1 of the Lagrangian as a function of the fields ξ, N, ρ_μ and a_μ. Four-dimensional covariant curls can be easily constructed for the fields ρ_μ and a_μ. They

[12] C. N. Yang and F. Mills, Phys. Rev. **96**, 191 (1954); R. Utiyama, *ibid.* **101**, 1597 (1956); M. Gell-Mann and S. Glashow, Ann. Phys. (N. Y.) **15**, 437 (1961).

are, respectively,

$$\rho_{\mu\nu}=\partial_\mu\rho_\nu-\partial_\nu\rho_\mu-\tfrac{1}{2}ig[\rho_\mu,\rho_\nu]-\tfrac{1}{2}ig[a_\mu,a_\nu] \quad (27)$$

and

$$a_{\mu\nu}=\partial_\mu a_\nu-\partial_\nu a_\mu-\tfrac{1}{2}ig[\rho_\mu,a_\nu]+\tfrac{1}{2}ig[\rho_\nu,a_\mu]$$
$$=D_\mu a_\nu-D_\nu a_\mu\,, \quad (28)$$

where

$$D_\mu=\partial_\mu-\tfrac{1}{2}ig[\rho_\mu,\ \ ]. \quad (29)$$

The expression $\rho_{\mu\nu}^2+a_{\mu\nu}^2$ is invariant under the full $SU(2)\times SU(2)$ gauge group. Observe that Eqs. (27) to (29) can be taken over directly for $SU(3)\times SU(3)$.

The construction of invariants containing also the fields ξ and N is greatly facilitated by the introduction of two auxiliary fields ρ_μ' and a_μ' which can be defined as the transforms of ρ_μ and a_μ by the same ξ-dependent finite chiral gauge transformation which transforms ψ into N, according to Eq. (8). They satisfy, therefore, the identity

$$\bar\psi\gamma^\mu(i\partial_\mu+\tfrac{1}{2}g\rho_\mu+\tfrac{1}{2}g\gamma_5 a_\mu)\psi$$
$$=\bar N\gamma^\mu(i\partial_\mu+\tfrac{1}{2}g\rho_\mu'+\tfrac{1}{2}g\gamma_5 a_\mu')N. \quad (30)$$

In view of Eq. (9), this can be written as

$$\rho_\mu'+\gamma_5 a_\mu'=\frac{1-i\gamma_5\xi}{(1+\xi^2)^{1/2}}\left(\rho_\mu+\gamma_5 a_\mu+\frac{2i}{g}\partial_\mu\right)\frac{1+i\gamma_5\xi}{(1+\xi^2)^{1/2}}, \quad (31)$$

or, more explicitly, in the form

$$\rho_\mu'=\frac{1}{1+\xi^2}\left(\rho_\mu+\xi\rho_\mu\xi-i[\xi,a_\mu]+\frac{i}{g}[\xi,\partial_\mu\xi]\right), \quad (32)$$

$$a_\mu'=\frac{1}{1+\xi^2}\left(a_\mu+\xi a_\mu\xi-i[\xi,\rho_\mu]-\frac{2}{g}\partial_\mu\xi\right). \quad (33)$$

In deriving these last two equations we have made use of the particular properties of the matrix $\xi=\boldsymbol{\xi}\cdot\boldsymbol{\tau}$. Equations (30) and (31), however, are valid for $SU(3)\times SU(3)$ as well, and equations analogous to (32) and (33) can also be obtained.

Let us observe that the left-hand side of Eq. (30) is invariant under a chiral gauge transformation. Then the right-hand side of the equation must also be invariant. Since we know the transformation law of the field N, which is given by Eq. (10), we can immediately infer the transformation properties of the fields ρ_μ' and a_μ'. We need only observe that A does not contain γ_5 and therefore Eq. (10) has the form of an *isospin* gauge transformation, except for the fact that the ξ-dependent matrix A rather than β occurs. It follows immediately that a chiral gauge transformation changes ρ_μ' and a_μ' according to

$$\delta\rho_\mu'=i[A,\rho_\mu']+(2/g)\partial_\mu A\,, \quad (34)$$

$$\delta a_\mu'=i[A,a_\mu']. \quad (35)$$

These equations are valid for $SU(3)\times SU(3)$ as well. For the case of $SU(2)\times SU(2)$, the matrix A is given by Eq. (12).

The transformation of the fields ρ_μ' and a_μ' under an isospin gauge transformation can be obtained in exactly the same way. They are

$$\delta\rho_\mu'=i[\beta,\rho_\mu']+(2/g)\partial_\mu\beta\,, \quad (36)$$

$$\delta a_\mu'=i[\beta,a_\mu']. \quad (37)$$

We see that these equations have the same form as for a chiral gauge transformation, except that the matrix β instead of the matrix A is to be used. Equations (34) to (37) can also be verified directly from explicit forms of the fields ρ_μ' and a_μ', such as those given in Eqs. (32) and (33).

Using the above formulas it is easy to construct all functions of the fields ξ, N, ρ_μ, and a_μ which are invariant under $SU(2)\times SU(2)$ gauge transformations. They are those functions of the fields N, ρ_μ', and a_μ' which are invariant under the isospin gauge transformations (14), (36), and (37), since these functions will be automatically invariant also under the chiral gauge transformations (10), (13), (34), and (35). Having constructed a function of N, ρ_μ', and a_μ', invariant under isospin gauge transformations, one needs only to substitute the expressions (32) and (33) to solve the problem. For isospin gauge transformations it is well known how to construct invariants by use of "covariant" derivatives like that defined in Eq. (29).

4. THE PHENOMENOLOGICAL LAGRANGIAN

In the following we find it convenient to use vector notation for isospin vectors, rather than the 2×2 matrix notation we have used until now.[13] Furthermore, for the sake of simplicity, we use ordinary type (not bold face) for the isospin vectors. A form for the Lagrangian, which is general enough to contain most desired interaction terms, in the following:

$$L=-\tfrac{1}{2}(\bar m^2-m^2)(a_\mu')^2-\tfrac{1}{4}(\rho_{\mu\nu}^2+a_{\mu\nu}^2)+i\bar N(\gamma_\mu\partial_\mu+M)N$$
$$+\tfrac{1}{2}g\bar N\gamma_\mu\tau N\cdot\rho_\mu'+\tfrac{1}{2}\lambda\bar N\gamma_\mu\gamma_5\tau N\cdot a_\mu'+\kappa g(a_\mu'\times a_\nu')\cdot\rho_{\mu\nu}'$$
$$+\mu\bar N\sigma_{\mu\nu}\tau N\cdot\rho_{\mu\nu}'-\tfrac{1}{2}m^2(\rho_\mu^2+a_\mu^2)+L_3\,, \quad (38)$$

where[14]

$$\rho_{\mu\nu}'=\partial_\mu\rho_\nu'-\partial_\nu\rho_\mu'+g\rho_\mu'\times\rho_\nu'+ga_\mu'\times a_\nu'$$
$$=\rho_{\mu\nu}+[2/(1+\xi^2)]\xi\times(a_{\mu\nu}+\xi\times\rho_{\mu\nu}). \quad (39)$$

[13] The correspondence between the two notations is well known. If $a=\mathbf{a}\cdot\boldsymbol{\tau}$, $b=\mathbf{b}\cdot\boldsymbol{\tau}$, then the commutator and the anticommutator of a and b are given by $[a,b]=2i(\mathbf{a}\times\mathbf{b})\cdot\tau$ and $\{a,b\}=2\mathbf{a}\cdot\mathbf{b}$.

[14] There is a corresponding equation

$$a_{\mu\nu}'=\partial_\mu a_\nu'-\partial_\nu a_\mu'+g\rho_\mu'\times a_\nu'-g\rho_\nu'\times a_\mu'$$
$$=a_{\mu\nu}+[2/(1+\xi^2)]\xi\times(\rho_{\mu\nu}+\xi\times a_{\mu\nu}).$$

To see that this last expression, as well as the last form of Eq. (39), are correct, one repeats the argument that led to Eq. (31). Since $\rho_{\mu\nu}'$ and $a_{\mu\nu}'$ have a gauge covariant form, one has here the simpler relation

$$\rho_{\mu\nu}'+\gamma_5 a_{\mu\nu}'=\frac{1-i\gamma_5\xi}{(1+\xi^2)^{1/2}}(\rho_{\mu\nu}+\gamma_5 a_{\mu\nu})\frac{1+i\gamma_5\xi}{(1+\xi^2)^{1/2}}.$$

The relative simplicity of the expression for $\rho_{\mu\nu}'$ in terms of the unprimed fields is exactly the reason why we chose to use it in the Lagrangian, instead of $\partial_\mu\rho_\nu'-\partial_\nu\rho_\mu'+g\rho_\mu'\times\rho_\nu'$ which transforms in the same way as $\rho_{\mu\nu}'$.

Except for the last two terms, this Lagrangian is invariant under $SU(2)\times SU(2)$ gauge transformations. We have written the coefficient of the $(a_\mu')^2$ term, without loss of generality, in a form which will turn out to be convenient a little later. Let us make use of Eq. (33), which can also be written as

$$a_\mu' = a_\mu - \frac{1}{1+\xi^2}\left[\frac{2}{g}D_\mu\xi - 2\xi\times(\xi\times a_\mu)\right], \tag{40}$$

with

$$D_\mu = \partial_\mu + g\rho_\mu\times. \tag{41}$$

We see that the term $(a_\mu')^2$ contains, in addition to higher nonlinearities, a bilinear interaction proportional to $a_\mu\cdot\partial_\mu\xi$. Such an interaction must be eliminated by going over to a new axial-vector field $\hat{a}_\mu$ which is an appropriate linear combination of a_μ and $\partial_\mu\xi$. It is easy to see that a suitable choice is

$$\hat{a}_\mu = a_\mu - \frac{2}{g}\frac{\bar{m}^2-m^2}{\bar{m}^2}\frac{1}{1+\xi^2}D_\mu\xi. \tag{42}$$

In this expression only the coefficient $(2/g)(\bar{m}^2-m^2)/\bar{m}^2$ of $\partial_\mu\xi$ is determined by the requirement that the bilinear interaction be eliminated. However, the particular choice of nonlinear terms given in Eq. (42) has the advantage of simplifying certain terms in the expression for the nonlinear Lagrangian as a function of $\hat{a}_\mu$ and the other fields and is therefore very convenient. We now have

$$\begin{aligned}\tfrac{1}{2}(\bar{m}^2-m^2)(a_\mu')^2+\tfrac{1}{2}m^2a_\mu^2 &= \tfrac{1}{2}\bar{m}^2\hat{a}_\mu^2+\frac{2}{g}\frac{\bar{m}^2-m^2}{\bar{m}^2}\frac{1}{(1+\xi^2)^2}\left\{\frac{m^2}{g}(D_\mu\xi)^2\right.\\ &\quad +2(2m^2-\bar{m}^2+\bar{m}^2\xi^2)\frac{(\xi\times\hat{a}_\mu)\cdot(\xi\times D_\mu\xi)}{1+\xi^2}-\bar{m}^2g(\xi\times\hat{a}_\mu)^2\\ &\quad \left.+\frac{4}{g}\frac{\bar{m}^2-m^2}{\bar{m}^2}(m^2+\bar{m}^2\xi^2)\frac{(\xi\times D_\mu\xi)^2}{(1+\xi^2)^2}\right\}. \end{aligned}\tag{43}$$

At the same time ρ_μ', which is given by Eq. (32), can be written in terms of $\hat{a}_\mu$ as

$$\rho_\mu' = \rho_\mu + \frac{2}{1+\xi^2}\xi \times\left\{\hat{a}_\mu - \frac{1}{g}\frac{1}{\bar{m}^2}(2m^2-\bar{m}^2+\bar{m}^2\xi^2)\frac{1}{1+\xi^2}D_\mu\xi\right\}. \tag{44}$$

Using the expressions (43) and (44) in the Eq. (38) and expanding the nonlinear denominators, we see that among others, the following direct interactions are present in the Lagrangian:

$$(\xi\times\hat{a}_\mu)\cdot(\xi\times D_\mu\xi), \tag{45}$$

$$(\xi\times D_\mu\xi)^2, \tag{46}$$

and

$$\bar{N}\gamma_\mu\tau N\cdot(\xi\times D_\mu\xi). \tag{47}$$

On the other hand, the Lagrangian (38) would give rise to these interactions also through the exchange of a ρ meson. In accordance with the formulation of vector dominance given in the introduction, we must require that the direct interactions be absent or that

$$\bar{m}^2 = 2m^2. \tag{48}$$

That this relation ensures the absence of the interactions (45) and (47) is obvious. To see that the interaction (46) will also be absent one must combine in Eq. (43) the term proportional to $\xi^2(D_\mu\xi)^2$ with the term proportional to $(\xi\times D_\mu\xi)^2$. From Eq. (43) swe see that $\bar{m}$ is the mass of the axial vector which is correctly described by the field $\hat{a}_\mu$. Therefore Eq. (48) is exactly Weinberg's relation[15] between the mass of the axial-vector particle and the mass m of the ρ meson. In view of this relation it is natural to identify the axial vector with the A_1 resonance. Making use of Eq. (48) we see from Eq. (43) that the coefficient of $(D_\mu\xi)^2$, to lowest order in the nonlinear terms, is equal to m^2/g^2. This shows that the pion field ξ is not normalized. We introduce therefore the normalized pion field

$$\phi = (m\sqrt{2}/g)\xi. \tag{49}$$

We can now write the final expression for the phenomenological Lagrangian (38). Since the detailed expression is rather lengthy we exhibit explicitly only certain terms and leave some of the substitutions undone. The result is

$$\begin{aligned}L = &-\tfrac{1}{4}(\rho_{\mu\nu}^2+a_{\mu\nu}^2)+i\bar{N}(\gamma_\mu\partial_\mu+M)N-\tfrac{1}{2}m^2\rho_\mu^2-\tfrac{1}{2}\bar{m}^2\hat{a}_\mu^2\\ &-\frac{1}{2}\frac{1}{(1+\xi^2)^2}(D_\mu\phi)^2+\frac{g^2}{(1+\xi^2)^2}\left\{\phi\times\left(\hat{a}_\mu\right.\right.\\ &\left.\left.-\frac{g^2}{2m^3\sqrt{2}}\frac{\phi^2}{1+\xi^2}D_\mu\phi\right)\right\}^2-\frac{g^2}{2m^2}\frac{1}{(1+\xi^2)^2}(\phi\times D_\mu\phi)^2\\ &+\kappa g(a_\mu'\times a_\nu')\cdot\rho_{\mu\nu}'+\tfrac{1}{2}g\bar{N}\gamma_\mu\tau N\cdot\rho_\mu'\\ &+\tfrac{1}{2}\lambda\bar{N}\gamma_\mu\gamma_5\tau N\cdot a_\mu'+\mu\bar{N}\cdot\sigma_{\mu\nu}\tau N\cdot\rho_{\mu\nu}'+L_3. \end{aligned}\tag{50}$$

Here

$$\begin{aligned}\rho_{\mu\nu} = &\ \partial_\mu\rho_\nu-\partial_\nu\rho_\mu+g\rho_\mu\times\rho_\nu+g\hat{a}_\mu\times\hat{a}_\nu\\ &+\frac{g}{m\sqrt{2}}\frac{1}{1+\xi^2}(D_\mu\phi\times\hat{a}_\nu-D_\nu\phi\times\hat{a}_\mu)\\ &+\frac{g}{2m^2}\frac{1}{(1+\xi^2)^2}D_\mu\phi\times D_\nu\phi, \end{aligned}\tag{51}$$

[15] See Weinberg's paper quoted in Ref. 1.

$$a_{\mu\nu}=D_\mu\hat{a}_\nu-D_\nu\hat{a}_\mu-\frac{g}{m\sqrt{2}}\frac{1}{1+\xi^2}\phi\times(\partial_\mu\rho_\nu-\partial_\nu\rho_\mu+g\rho_\mu\times\rho_\nu)+\frac{g^2}{m^3\sqrt{2}}\frac{1}{(1+\xi^2)^2}\phi\times(D_\mu\phi\times D_\nu\phi), \quad (52)$$

$$\rho_\mu'=\rho_\mu+\frac{g\sqrt{2}}{m}\frac{1}{1+\xi^2}\phi\times\left(\hat{a}_\mu-\frac{g^2}{2m^3\sqrt{2}}\frac{\phi^2}{1+\xi^2}D_\mu\phi\right), \quad (53)$$

$$a_\mu'=\hat{a}_\mu+\frac{g^2}{m^2}\frac{1}{1+\xi^2}\left(\phi\times(\phi\times\hat{a}_\mu)+\frac{1}{m\sqrt{2}}\frac{\phi}{1+\xi^2}\phi\cdot D_\mu\phi\right)-\frac{1}{m\sqrt{2}}\frac{1}{(1+\xi^2)^2}\left(1+\frac{3}{2}\frac{g^2}{m^2}\phi^2\right)D_\mu\phi, \quad (54)$$

and $\rho_{\mu\nu}'$ can be taken from Eq. (39).

The transformation laws of the various fields have been given in Secs. 2 and 3 in terms of the field ξ. One may observe that, if one expresses them in terms of the normalized field ϕ, they will contain the coupling constant g. It will also be natural to rescale the group parameter and use $\alpha'=(m\sqrt{2}/g)\alpha$ instead of α.

We can now discuss the form of the symmetry-breaking term L_3. The simplest assumption is that L_3 is a function of the pion field alone. This function is not completely arbitrary since, when it is expanded in powers of the pion field, its quadratic term must reproduce the pion mass term

$$L_3(\phi^2)=-\tfrac{1}{2}m_\pi^2\phi^2+\cdots. \quad (55)$$

From Eqs. (25), (26), and (55) we see that Eq. (25) has the general form

$$\partial_\mu a_\mu=(m_\pi^2/\sqrt{2}m)\phi+\cdots. \quad (56)$$

Two choices for L_3 appear particularly simple. One is just

$$L_3=-\tfrac{1}{2}m_\pi^2\phi^2. \quad (57)$$

The other is a function chosen so that the higher-order terms indicated by the dots in Eq. (56) are not present. It is easy to see that this choice is

$$L_3=-m_\pi^2\frac{m^2}{g^2}\ln\left(1+\frac{g^2}{2m^2}\phi^2\right). \quad (58)$$

The different choices for L_3 give rise by expanding in ϕ^2 to different many-pion interactions. These effects can, in principle, be compared with the experiments.

5. SOME EXPERIMENTAL CONSEQUENCES

Expanding the Lagrangian (50) in powers of the coupling constant g, one can generate a number of interaction terms. We make a few simple observations on these interactions and establish contact with previous work.

The pseudovector interaction of the nucleon to lowest order is

$$\tfrac{1}{2}\lambda\bar{N}\gamma_\mu\gamma_5\tau N\cdot(\hat{a}_\mu-(1/\bar{m})\partial_\mu\phi). \quad (59)$$

The coupling to the axial vector and that to the pion are related to each other, but they are independent of the vector coupling to the ρ meson which is, also to lowest order,

$$\tfrac{1}{2}g\bar{N}\gamma_\mu\tau N\cdot\rho_\mu. \quad (60)$$

The constant λ is related to the usual pseudovector nucleon-pion coupling constant f by

$$\lambda/2\bar{m}=f/m_\pi. \quad (61)$$

One knows that

$$f^2/4\pi\cong0.08. \quad (62)$$

As pointed out by Sakurai,[16] the coupling constant g can be estimated from nucleon-pion scattering, since the exchange of a ρ meson gives rise to the interaction

$$-(g^2/2m^2)\bar{N}\gamma_\mu\tau N\cdot(\phi\times\partial_\mu\phi). \quad (63)$$

This interaction is responsible for most of the low-energy nucleon-pion s-wave scattering. The value for the coupling constant obtained in this way is

$$g^2/4\pi\cong2.8. \quad (64)$$

In the Lagrangian (50) the meson-meson interactions depend upon two parameters, g and κ. A particularly simple (in some sense minimal) form would obtain for $\kappa=0$ and it is interesting to see if this choice is in agreement with the available experimental information. The interaction terms in the Lagrangian (50) which are responsible for ρ-meson decay into two pions are

$$L_{\rho\pi\pi}=-g\rho_\mu\cdot(\phi\times\partial_\mu\phi)-(g/4m^2)(1-2\kappa)(\partial_\mu\rho_\nu-\partial_\nu\rho_\mu)\cdot(\partial_\mu\phi\times\partial_\nu\phi). \quad (65)$$

Performing an integration by parts on the second term, we have the equivalent form

$$L_{\rho\pi\pi}=-g(\tfrac{3}{4}+\tfrac{1}{2}\kappa)\rho_\mu\cdot(\phi\times\partial_\mu\phi). \quad (66)$$

On the other hand, the terms responsible for the decay of the axial vector into a ρ meson and a pion are

$$L_{A_1\rho\pi}=-\frac{1}{2\sqrt{2}}\frac{g}{m}(1+2\kappa)(\partial_\mu\rho_\nu-\partial_\nu\rho_\mu)\cdot(\partial_\mu\phi\times\hat{a}_\nu-\partial_\nu\phi\times\hat{a}_\mu)-\frac{1}{2\sqrt{2}}\frac{g}{m}(\partial_\mu\rho_\nu-\partial_\nu\rho_\mu)\cdot[\phi\times(\partial_\mu\hat{a}_\nu-\partial_\nu\hat{a}_\mu)]. \quad (67)$$

This expression can be transformed into

$$L_{A_1\rho\pi}=(1/\sqrt{2})gm(1+2\kappa)\rho_\mu\cdot(\phi\times\hat{a}_\mu)+\frac{1}{\sqrt{2}}\kappa\frac{g}{m}(\partial_\mu\rho_\nu-\partial_\nu\rho_\mu)\cdot[\phi\times(\partial_\mu\hat{a}_\nu-\partial_\nu\hat{a}_\mu)]. \quad (68)$$

[16] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960); Phys. Rev. Letters **17**, 1021 (1966).

In the rest frame of the axial-vector meson, it is easy to see that the second term in Eq. (68) can be written approximately in the same form as the first term. Using Eq. (48) for the mass of the axial vector we have

$$L_{A_1\rho\pi}\cong(1/\sqrt{2})gm[1-2\kappa(\sqrt{2}-1)]\rho_\mu\cdot(\phi\times\hat{a}_\mu). \quad (69)$$

Equations (66) and (69) can be compared with the experiments. Taking both the width of the ρ meson and that of the A_1 resonance to be 125 MeV, we obtain $\kappa\cong\frac{1}{4}$ and

$$g^2/4\pi\cong3.2, \quad (70)$$

in reasonably good agreement with the other determination of this coupling constant, Eq. (64). However, this value of κ should not be taken too seriously in view of the large uncertainties in the experimental data. For instance, $\kappa=0$ cannot be excluded and even $\kappa=\frac{1}{2}$ may still be possible. The corresponding values of g are still very close to that given by Eq. (70).

It is interesting to observe that the coupling constants g and f satisfy rather well the $SU(6)$ relation[17]

$$\frac{f}{m_\pi}=\frac{5}{3}\frac{g}{2m}, \quad (71)$$

where m is the mass of the ρ meson. Using Eq. (61), this relation can be written as

$$\frac{\lambda}{\bar{m}}=\frac{5}{3}\frac{g}{m}. \quad (72)$$

$SU(6)$ considerations can also be used to determine the constants κ and μ. It is well known that one obtains

$$\mu=(5/3)(g/2m). \quad (73)$$

The $SU(6)$ value of κ depends somewhat on the particular form of relativistic $SU(6)$ one is willing to accept. A simple version[18] would give $\kappa=\frac{1}{2}$.

6. WEAK INTERACTIONS

There are two ways of introducing the weak interactions. The first is to construct, by means of a well-known procedure,[19] the hadronic weak-interaction currents from the Lagrangian (50). The currents obtained in this way are highly nonlinear functions of the fields ϕ, N, ρ_μ, and a_μ. The second way is to postulate[20] that the hadronic vector and axial-vector currents of the weak interactions are proportional to the fields ρ_μ and a_μ [more precisely, that they are equal, respectively, to $(m^2/g)\rho_\mu$ and $(m^2/g)a_\mu$] and that the weak interactions of all hadrons are determined by the strong coupling to these two fields. We shall follow here this second approach, which is more convenient when fields such as ρ_μ and a_μ are present in the theory. We give an example of the first approach in the Appendix, where we discuss a model with only pions and nucleons.

Neglecting higher nonlinear terms, the matrix element for the vector part of the hadronic weak-interaction current is approximately

$$[m^2/(k^2+m^2)](\bar{N}\gamma_\mu\tfrac{1}{2}\tau N-\phi\times\partial_\mu\phi+\cdots), \quad (74)$$

where the dots refer to other fields. For the axial-vector current, let us make use of the relation

$$a_\mu=\hat{a}_\mu+(1/m\sqrt{2})[1/(1+\xi^2)]D_\mu\phi. \quad (75)$$

Neglecting higher nonlinear terms we have

$$(m^2/g)a_\mu=(m^2/g)\hat{a}_\mu+(m/g\sqrt{2})\partial_\mu\phi. \quad (76)$$

The coefficient of the $\partial_\mu\phi$ term can be immediately related to the pion-decay constant

$$F_\pi=(m/g)\sqrt{2}. \quad (77)$$

Remembering the interaction (59) we obtain for the matrix element of the axial-vector current between nucleon states the expression $A_{\mu\nu}\bar{N}\gamma_\nu\gamma_5\tau N$, where

$$A_{\mu\nu}=\frac{\lambda}{4g}\left(g_{\mu\nu}+\frac{k_\mu k_\nu}{\bar{m}^2}\right)\frac{\bar{m}^2}{k^2+\bar{m}^2}-\frac{\lambda}{4g}\frac{k_\mu k_\nu}{k^2+m_\pi^2}. \quad (78)$$

This is strongly reminiscent of an expression given by Nambu,[21] who did not, however, consider the axial-vector contribution to the form factor. In the limit of zero-momentum transfer we obtain the result

$$A_{\mu\nu}=(\lambda/4g)g_{\mu\nu} \quad (79)$$

which, by comparison with Eq. (74), gives

$$-\frac{G_A}{G_V}=\frac{\lambda}{2g}=\frac{f}{g}\frac{\bar{m}}{m_\pi}=\frac{f}{g}\frac{m\sqrt{2}}{m_\pi}. \quad (80)$$

This equation is equivalent to a relation of Kawarabayashi and Suzuki.[22] On the other hand, through Eqs. (59), (61), and (63), Eq. (80) relates G_A/G_V to pion-nucleon scattering parameters. It can therefore be interpreted as a form of the Adler-Weisberger relation.[23] Finally, combining Eq. (80) and Eq. (77),

[17] F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters 8, 299 (1964). Actually, in the $SU(6)$ formula instead of the mass of the ρ meson there appears an average mass of the multiplet. The particular form of Eq. (71) with the ρ-meson mass occurs in Schwinger's version of the $SU(6)$ theory, described in Ref. 11.

[18] See, for instance, B. Sakita and K. C. Wali, Phys. Rev. 139, B1355 (1965).

[19] See N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Interscience Publishers, Inc., New York, 1959), pp. 19–26.

[20] See, for instance, the paper by Lee, Weinberg, and Zumino quoted in Ref. 2.

[21] Y. Nambu, Phys. Rev. Letters 4, 380 (1960).

[22] These authors use the algebra of currents. See their paper quoted in Ref. 1.

[23] See the papers by Adler and Weisberger quoted in Ref. 1. The interpretation in terms of pion-nucleon s-wave scattering lengths was given by Y. Tomozawa, Nuovo Cimento **46A**, 707 (1966); S. Weinberg, Phys. Rev. Letters **17**, 616 (1966); J. J. Sakurai, *ibid.* **17**, 552 (1966).

we have

$$-G_A/G_V=(f/m_\pi)F_\pi\,, \tag{81}$$

which is the Goldberger-Treiman relation,[24] expressed in terms of the pseudovector nucleon-pion coupling constant. If one combines Eqs. (71) and (80) one obtains[11]

$$-G_A/G_V=5/3\sqrt{2}\,, \tag{82}$$

a relation which agrees with experiment better than (71).

Note added in proof. The problem of the widths of the ρ and of the A_1 has been discussed recently by H. J. Schnitzer and S. Weinberg [Phys. Rev. (to be published)] using current commutators and a meson-dominance assumption. They obtain effective vertices which are essentially equivalent to our interactions (66) and (67) and concur with our determination $\kappa\cong\frac{1}{4}$ (to identify our results with theirs use Eq. (77) and $\delta=-2\kappa$). It may be worth pointing out that, if one uses slightly more realistic values for the ρ width ($\Gamma_\rho=128$ MeV) and for the A_1 width ($\Gamma_{A_1}=90$ MeV), one obtains $\kappa\cong\frac{1}{3}$ and $g^2/4\pi\cong2.9$, a result which improves the agreement with Eq. (64).

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APPENDIX

In this Appendix we describe briefly a model involving only pions and nucleons. It is not difficult to construct an invariant Lagrangian involving only the fields ξ and N. Since there are now no gauge fields, we restrict ourselves here to the group with constant parameters. The transformations of the fields ξ and N are the same as in Sec. 2, but α and β are now coordinate independent. For constant parameters the expression $\bar\psi\gamma_\mu\partial_\mu\psi$ is an invariant. In terms of the fields ξ and N it takes the form

$$\bar\psi\gamma_\mu\partial_\mu\psi=\bar N\gamma_\mu\partial_\mu N+\bar N\gamma_\mu\frac{1-i\gamma_5\xi}{(1+\xi^2)^{1/2}}\left(\partial_\mu\frac{1+i\gamma_5\xi}{(1+\xi^2)^{1/2}}\right)N\,. \tag{A1}$$

Since we know the transformation property of N under a chiral transformation [given by Eq. (10)], we can immediately obtain that of the anti-Hermitian matrix

$$X_\mu=\frac{1-i\gamma_5\xi}{(1+\xi^2)^{1/2}}\partial_\mu\frac{1+i\gamma_5\xi}{(1+\xi^2)^{1/2}}\,. \tag{A2}$$

It is

$$\delta X_\mu=i[A,X_\mu]-i\partial_\mu A\,, \tag{A3}$$

where A is given by Eq. (11) [or Eq. (12)]. Similarly, if we introduce the matrix

$$Y_\mu=\frac{1+i\gamma_5\xi}{(1+\xi^2)^{1/2}}\partial_\mu\frac{1-i\gamma_5\xi}{(1+\xi^2)^{1/2}}\,, \tag{A4}$$

we see that it also transforms as

$$\delta Y_\mu=i[A,Y_\mu]-i\partial_\mu A\,. \tag{A5}$$

(One can see it most simply by considering the substitution $\gamma_5\to-\gamma_5$.) It is convenient to introduce the two Hermitian matrices

$$v_\mu=-(1/2i)(X_\mu+Y_\mu)\,, \tag{A6}$$

and

$$p_\mu=+(1/2i)(X_\mu-Y_\mu)\,, \tag{A7}$$

which transform as

$$\delta v_\mu=i[A,v_\mu]+\partial_\mu A\,, \tag{A8}$$

and

$$\delta p_\mu=i[A,p_\mu]. \tag{A9}$$

These equations have the same form as Eqs. (34) and (35), except for a normalization factor. All equations given so far in this Appendix are valid also for $SU(3)\times SU(3)$. They permit, quite simply, to construct invariants.

If we now specialize our results to the case of $SU(2)\times SU(2)$ and $\xi=\boldsymbol{\xi}\cdot\boldsymbol{\tau}$ we can transform our expressions and obtain

$$v_\mu=\tfrac{1}{2}i([\xi,\partial_\mu\xi]/(1+\xi^2))\,, \tag{A10}$$

and

$$p_\mu=\gamma_5[\partial_\mu\xi/(1+\xi^2)]. \tag{A11}$$

An invariant Lagrangian, containing two arbitrary parameters a and b, can be immediately written.[25]

$$L=-(1/2a^2)p_\mu{}^2+i\bar N(\gamma_\mu\partial_\mu+M)N+\bar N\gamma_\mu v_\mu N+b\bar N\gamma_\mu p_\mu N\,. \tag{A12}$$

The first term indicates that the normalized pion field is

$$\phi=(1/a)\xi\,.$$

We can rewrite Eq. (A12) in terms of ϕ. Adding a symmetry-breaking term L_3 (as discussed in Sec. 4), we obtain finally in vector notation

$$L=-\tfrac{1}{2}(1+a^2\phi^2)^{-2}(\partial_\mu\phi)^2+i\bar N(\gamma_\mu\partial_\mu+M)N-(1+a^2\phi^2)^{-1}\{(f/m_\pi)\bar N\gamma_\mu\gamma_5\tau N\cdot\partial_\mu\phi+a^2\bar N\gamma_\mu\tau N\cdot(\phi\times\partial_\mu\phi)\}+L_3\,. \tag{A13}$$

Here we have identified

$$-ab=f/m_\pi\,. \tag{A14}$$

Comparison with Eq. (63) shows that we must also identify

$$a^2=g^2/2m_\rho{}^2\,. \tag{A15}$$

[24] M. Goldberger and S. Treiman, Phys. Rev. **110**, 1178 (1958); see also the paper by Gell-Mann and Lévy quoted in Ref. 4.

[25] For the case of $SU(3)\times SU(3)$ the analog of the first term in Eq. (A12) can be written in the form

$$-(1/2a^2)\mathrm{Tr}p_\mu{}^2=-(1/8a^2)\mathrm{Tr}(\partial_\mu U^\dagger\partial_\mu U),$$

where $U=(1-i\gamma_5\xi)/(1+i\gamma_5\xi)$.

As long as we are concerned only with the pion-nucleon system, the Lagrangian (A13) contains all the relevant information. We can use it to construct the vector and axial-vector currents associated with the generators of the group $SU(2)\times SU(2)$. The result for the vector current is

$$V_\mu=N_\mu-\frac{2}{1+a^2\phi^2}\left[\frac{f}{m_\pi}\phi\times N_\mu{}^5-a^2\phi\times(\phi\times N_\mu)\right]-\frac{\phi\times\partial_\mu\phi}{(1+a^2\phi^2)^2}, \quad (A16)$$

and for the axial-vector current

$$\frac{am_\pi}{f}A_\mu=N_\mu{}^5-\frac{2a^2}{1+a^2\phi^2}\left[\frac{m_\pi}{f}\phi\times N_\mu-\phi\times(\phi\times N_\mu{}^5)\right]+\frac{m_\pi}{2f}\frac{\partial_\mu\phi}{1+a^2\phi^2}+\frac{m_\pi}{f}a^2\frac{\phi\times(\phi\times\partial_\mu\phi)}{(1+a^2\phi^2)^2}. \quad (A17)$$

Here we have used the abbreviations

$$N_\mu=\bar{N}\gamma_\mu\tfrac{1}{2}\tau N \quad (A18)$$

and

$$N_\mu{}^5=\bar{N}\gamma_\mu\gamma_5\tfrac{1}{2}\tau N. \quad (A19)$$

Since the Lagrangian is invariant under isospin transformations, the vector current is conserved. The axial-vector current satisfies a partial conservation equation, the exact form of which depends on the choice of the symmetry breaking term L_3. The first terms in the expansion of Eqs. (A16) and (A17) are, respectively,

$$V_\mu=\bar{N}\gamma_\mu\tfrac{1}{2}\tau N-\phi\times\partial_\mu\phi+\cdots \quad (A20)$$

and

$$A_\mu=(f/am_\pi)\bar{N}\gamma_\mu\gamma_5\tfrac{1}{2}\tau N+(1/2a)\partial_\mu\phi+\cdots. \quad (A21)$$

Comparing the coefficients of these two expressions with those of (A13) one obtains once more the Goldberger-Treiman relation and the Adler-Weisberger relation.

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Numerical Analysis of Hadron Total Cross Sections in the Quark Model*

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The linear parametrization of the hadron-hadron total-scattering cross sections at high energy given by the additive quark model are compared with experiment by a least-squares analysis using several different kinematic assumptions. Expressions for the corrections due to shadowing and double scattering are obtained, and the corrections are shown to be large enough to be important. A nonlinear parametrization obtained from the expression for the shadowing correction is compared with experiment by a least-squares analysis. The agreement is good under two different kinematic assumptions. Agreement is also good using the linear parametrization with the kinematic assumptions of James and Watson. In all cases studied, the amplitude for scattering of the λ quark from a nonstrange quark is significantly lower than the amplitude for scattering of two nonstrange quarks. The amplitude for the scattering of the nonstrange quark and antiquark in an isosinglet state is both significantly larger and much more energy-dependent than any of the other amplitudes.

I. INTRODUCTION

SINCE Gell-Mann[1] and Zweig[2] introduced quarks as an explicit realization of the fundamental representation of $SU(3)$, many calculations of properties of hadrons have been done in the quark model. Among these calculations are relations among the high-energy total cross sections, using the additivity hypothesis first introduced by Levin and Frankfurt,[3] Anisovich,[4] and Lipkin and Scheck.[5] Many authors[6] have analyzed the total cross sections on this basis, and it has been possible to make statements about the amount of $SU(3)$ symmetry-breaking present in the amplitudes by examining the relative successes of the various sum rules.[7]

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[1] M. Gell-Mann, Phys. Letters **8**, 214 (1964).

[2] G. Zweig, CERN report TH. 412, 1964 (unpublished).

[3] E. M. Levin and L. I. Frankfurt, JETP Pisma v Redaktsiyu 2, 105 (1965) [English transl.: JETP Letters 2, 65 (1965)].

[4] V. V. Anisovich, JETP Pisma v Redaktsiyu 2, 439 (1965) [English transl.: JETP Letters **2**, 272 (1965)].

[5] H. J. Lipkin and F. Scheck, Phys. Rev. Letters **16**, 71 (1966).

[6] V. Barger and L. Durand III, Phys. Rev. **156**, 1525 (1967); C. H. Chan, *ibid.* **152**, 1244 (1966); Y. T. Chiu and J. Schechter, Nuovo Cimento **46A**, 548 (1966); M. Imachi and S. Sawada, Nagoya University report (unpublished); J. J. J. Kokkedee, Phys. Letters **22**, 88 (1966); J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento **42A**, 711 (1966); J. J. J. Kokkedee and L. Van Hove, Nucl. Phys. **B1**, 169 (1967); C. A. Levinson, H. S. Wall, and H. J. Lipkin, Phys. Rev. Letters **17**, 1122 (1966); H. J. Lipkin, *ibid.* **16**, 1015 (1966); L. Van Hove, in Proceedings of the Stony Brook Conference on High-Energy Two-Body Reactions (unpublished); L. Van Hove, CERN report TH. 676 (unpublished). There have also been a number of papers on inelastic processes.

[7] H. J. Lipkin, Ref. 6.