Low-Energy Nucleon-Nucleon Scattering in a Three-Body Formalism*

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The $NN\pi$ system is studied using the Faddeev theory of three-particle interactions and the approximation scheme due to Lovelace, which preserves bound-state and three-particle unitarity and uses separable potentials to describe off-shell two-particle scattering amplitudes. One-dimensional coupled integral equations are obtained for elastic nucleon-nucleon scattering which decouple in the $I=0$, $S=0$ state. The theory, which contains no arbitrary parameters, predicts phase shifts which are in reasonable agreement with experiment for the higher partial waves.

1. INTRODUCTION

THE first mathematically correct theory of nonrelativistic three-particle scattering was given by Faddeev.[1,2] The difficulty with using the usual Lippmann-Schwinger equations to study three-particle interactions is that their kernels are not compact because of the presence of disconnected processes in which two of the particles interact while the third goes straight through. By summing over all such disconnected graphs, Faddeev succeeded in obtaining a set of linear equations, with compact kernels, for bound-state and unstable particle scattering involving off-the-energy-shell two-particle scattering amplitudes.

The three-body equations in their full generality are still too complicated to be solved without any approximation. Of the approximation schemes put forward,[3,4] which have been based either on the Faddeev equations or on a similar set of equations for n-particle scattering obtained by Weinberg,[5] we shall use the scheme put forward by Lovelace,[6,7] and use it to study the $NN\pi$ system of three particles.

The basis of the Lovelace theory is the fact that when each of the two-particle subsystems is dominated by a finite number of low-energy bound states and resonances, the Faddeev equations assume a relatively simple form.

In Sec. 2 we give the kinematics of the $NN\pi$ system. In Sec. 3, following Lovelace, we obtain the coupled integral equations for bound-state and resonance scatterings. In Sec. 4 we specialize to states of total isotopic spin zero in order to reduce the number of coupled channels in the integral equations, and finally solve the integral equations numerically for elastic nucleon-nucleon scattering in the $I=S=0$ state, the low-energy pion-nucleon interaction being approximated by a suitably chosen separable potential. There are no arbitrary parameters in the theory. The phase shifts are calculated for the $^1P_1$, $^1F_3$, and $^1H_5$ states. The results in the higher partial waves are found to be in reasonable agreement with experiment. We give our conclusions in Sec. 5.

2. KINEMATICS

A nonrelativistic system of three particles α, β, and γ, with momenta $\mathbf{k}_\alpha$, $\mathbf{k}_\beta$, and $\mathbf{k}_\gamma$ and masses m_α, m_β, and m_γ, can be specified in the total center-of-mass frame by the momenta $\mathbf{p}_\alpha$, $\mathbf{q}_\alpha$, where $\mathbf{p}_\alpha$ is the relative momentum in the (β,γ) subsystem and $\mathbf{q}_\alpha$ is the momentum of particle α relative to the (β,γ) subsystem, i.e.,

$$\mathbf{p}_\alpha=\frac{m_\gamma\mathbf{k}_\beta-m_\beta\mathbf{k}_\gamma}{m_\beta+m_\gamma},$$
$$\mathbf{q}_\alpha=\frac{(m_\beta+m_\gamma)\mathbf{k}_\alpha-m_\alpha(\mathbf{k}_\beta+\mathbf{k}_\gamma)}{m_\alpha+m_\beta+m_\gamma}, \tag{2.1}$$

with α, β, and γ cyclic. Only two of the six momenta that one can so define are actually independent and the description of the system in terms of any pair of them is equivalent to its description in terms of any other pair. The total kinetic energy in the center-of-mass system is

$$E=\mathbf{q}_\alpha^2/2M_\alpha+\mathbf{p}_\alpha^2/2\mu_\alpha,$$

where

$$M_\alpha=\frac{m_\alpha(m_\beta+m_\gamma)}{m_\alpha+m_\beta+m_\gamma}, \quad \mu_\alpha=\frac{m_\beta m_\gamma}{m_\beta+m_\gamma}. \tag{2.2}$$

For the case in which particles β and γ form a bound state n with binding energy $-\epsilon_{\alpha n}$, the total energy of the system is

$$E=\mathbf{q}_\alpha^2/2M_\alpha-\epsilon_{\alpha n}.$$

Since we shall be concerned with the $NN\pi$ system, we adopt the convention that particles 1 and 2 are the

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[1] L. D. Faddeev, Zh. Eksperim. i Teor. Fiz. **39**, 1459 (1960) [English transl.: Soviet Phys.—JETP **12**, 1014 (1961)]; Dokl. Akad. Nauk SSR **138**, 565 (1961); **145**, 301 (1962) [English transls.: Soviet Phys.—Doklady **6**, 384 (1961); **7**, 600 (1963)].

[2] L. D. Faddeev, *Mathematical Problems of the Quantum Theory of Scattering for a 3-Particle System* [Steklov Mathematical Institute, Lenningrad (1963), No. 69], English transl. by J. B. Sykes (H. M. Stationary Office, Harwell, 1964), AERE Trans. 1002.

[3] R. D. Amado, Phys. Rev. **132**, 485 (1963).

[4] L. Rosenberg, Phys. Rev. **131**, 874 (1963); **135**, B715 (1964).

[5] S. Weinberg, Phys. Rev. **133**, B232 (1964).

[6] C. Lovelace, in *Strong Interactions and High-Energy Physics*, edited by R. G. Moorhouse (Oliver and Boyd, London, 1964).

[7] C. Lovelace, Phys. Rev. **135**, B1225 (1964), hereafter referred to as L.

nucleons and that particle 3 is the pion. We assume that the particles are always ordered cyclically, i.e. $\alpha=1$ corresponds to (2,3)+1, etc. We need not then distinguish between Bose and Fermi statistics. Because two of the particles are identical in the $NN\pi$ system, the relations for the momenta simplify to

$$\begin{aligned} \mathbf{p}_1&=\gamma\mathbf{q}_1+\mathbf{q}_2, & \mathbf{p}_2&=-\mathbf{q}_1-\gamma\mathbf{q}_2,\\ \mathbf{p}_3&=\tfrac{1}{2}(\mathbf{q}_1-\mathbf{q}_2), & \mathbf{q}_3&=-(\mathbf{q}_1+\mathbf{q}_2), \end{aligned} \tag{2.3}$$

where $\gamma=m_N/(m_N+m_\pi)$.

3. THE $NN\pi$ EQUATIONS

The Lovelace equations[8] for bound-state and resonance scattering are

$$X_{\alpha n,\beta m}(s) = -Z_{\alpha n,\beta m}(s)-\sum_{\gamma,r} X_{\alpha n,\gamma r}(s)\tau_{\gamma r}(s)Z_{\gamma r,\beta m}(s), \tag{3.1}$$

where $X_{\alpha n,\beta m}(s)$ is the amplitude for scattering, at the center-of-mass energy s, from an initial state of particle β and the mth composite system of the other two particles—the βm channel—to a final state consisting of the αn channel. $-Z_{\alpha n,\beta m}(s)$ is identified as the corresponding potential, while $-\tau_{\gamma r}(s)$ is the propagator for bound-state or resonance scattering. These equations can be used to study the $NN\pi$ system, because one can obtain a reasonably good low-energy description of the two-particle subsystems involved if one assumes that the two-nucleon interaction is dominated by the deuteron $D(I=0, J=1)$ and the singlet virtual state $S(I=1, J=0)$, and that the pion-nucleon interaction is dominated by a nucleon bound state $N(I=\frac{1}{2}, J=\frac{1}{2})$ and the N^* resonance $(I=\frac{3}{2}, J=\frac{3}{2})$.

A priori it would appear as if we would have an integral equation with four coupled channels to deal with, with additional couplings possible when a partial-wave analysis is carried out. However, we shall see that considerable simplification occurs when we make use of the identity of two of the three particles and the fact that the Lovelace potentials arise through rearrangement collisions, so that there can be no potentials for transitions involving two-nucleon composite systems in both the initial and final states.

The physically interesting amplitudes in the $NN\pi$ system are those with either a nucleon, as a composite $N\pi$ system, or a deuteron present in the initial state. In order to simplify the Lovelace equations, we define the amplitudes

$$\begin{aligned} X_{\alpha a,\alpha N}(s)&=X_{aN}{}^d(s),\\ X_{\alpha a,\beta N}(s)&=X_{aN}{}^n(s), \end{aligned} \tag{3.2}$$

$$\begin{aligned} X_{\alpha a,3D}(s)&=X_{aD}(s),\\ X_{3b,\alpha N}(s)&=X_{bN}(s),\\ X_{3b,3D}(s)&=X_{bD}(s), \end{aligned} \tag{3.3}$$

where α, $\beta=1$, 2 with $\alpha\neq\beta$; while $a\equiv N$ or N^* and $b\equiv S$ or D. If, further, we define

$$\begin{aligned} X_{aN}(s)&=X_{aN}{}^d(s)+X_{aN}{}^n(s),\\ Y_{aN}(s)&=X_{aN}{}^d(s)-X_{aN}{}^n(s), \end{aligned} \tag{3.4}$$

we find that the equations for $X_{aN}(s)$ and $Y_{aN}(s)$ decouple. Then, essentially using the same arguments as in Sec. 3c of L, we can show that the $Y_{aN}(s)$ do not correspond to anything observable and hence need never be calculated. The $X_{aN}(s)$ correspond to the observed scattering amplitude $N+N\to N+(N \text{ or } N^*)$, because we cannot distinguish between the two pairings of the particles corresponding to $\alpha=1$, 2 but must sum over them.

Let us consider first the case when we have a nucleon bound state in the initial state, and a composite two-nucleon system in the final state. Equation (3.1) can then be written as

$$\begin{pmatrix} X_{SN}\\ X_{DN}\end{pmatrix}=-\begin{pmatrix} Z_{SN}\\ Z_{DN}\end{pmatrix} -\begin{pmatrix} Z_{SN}\tau_N & Z_{SN^*}\tau_{N^*}\\ Z_{DN}\tau_N & Z_{DN^*}\tau_{N^*}\end{pmatrix}\begin{pmatrix} X_{NN}\\ X_{N^*N}\end{pmatrix}, \tag{3.5}$$

where all the amplitudes, potentials and propagators are functions of the same total energy s. Notice that there is no term like

$$\begin{pmatrix} Z_{SS}\tau_S & Z_{SD}\tau_D\\ Z_{DS}\tau_S & Z_{DD}\tau_D\end{pmatrix}\begin{pmatrix} X_{SN}\\ X_{DN}\end{pmatrix}.$$

This is because composite states of the two-nucleon subsystem can only occur in the channel (1,2)+3; while by definition,[9] the potentials

$$Z_{3n,3m}=0. \tag{3.6}$$

If we now define

$$V_{ab}(s)=Z_{ab}(s)-2\sum_{c=S,D} Z_{ac}(s)\tau_c(s)Z_{cb}(s) \tag{3.7}$$

for a, $b=N$ or N^*, we obtain a set of two coupled equations for the amplitudes X_{NN} and X_{N^*N}:

$$\begin{pmatrix} X_{NN}\\ X_{N^*N}\end{pmatrix}=-\begin{pmatrix} V_{NN}\\ V_{N^*N}\end{pmatrix} -\begin{pmatrix} V_{NN}\tau_N & V_{NN^*}\tau_{N^*}\\ V_{N^*N}\tau_N & V_{N^*N^*}\tau_{N^*}\end{pmatrix}\begin{pmatrix} X_{NN}\\ X_{N^*N}\end{pmatrix}. \tag{3.8}$$

In order to clarify the significance of Eqs. (3.5), (3.7), and (3.8) we have represented them graphically in Fig. 1. The equation for the bound-state disintegration process $N+N\to N+N+\pi$ can easily be written in

[8] Equation (3.29) of L.

[9] Equation (3.18) of L.

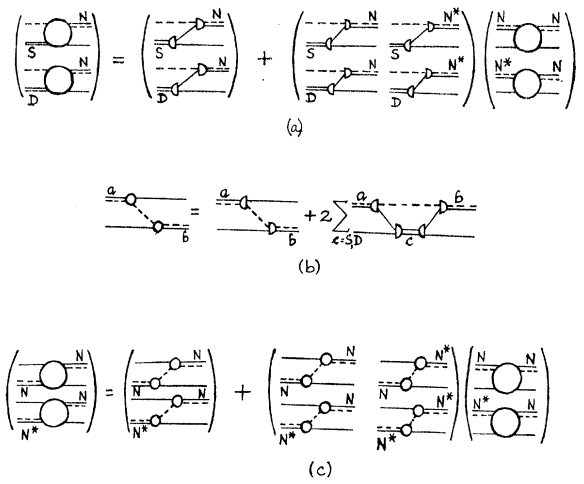

FIG. 1. Parts (a), (b), and (c) show Eqs. (3.5), (3.7), and (3.8), respectively, in graphical form. The firm lines refer to nucleons, the broken ones to pions, and the double lines to two-particle composite states.

terms of the amplitudes X_{aN} $(a=N, N^*, S, D)$ by using Eq. (3.28) of L.

The equations for the scattering processes in which we have a deuteron in the initial state,

$$\pi+D\rightarrow\pi+(S \text{ or } D),$$
$$\pi+D\rightarrow N+(N \text{ or } N^*),$$

are, respectively, given by

$$\begin{pmatrix}X_{SD}\\X_{DD}\end{pmatrix}=-2\begin{pmatrix}Z_{SN}\tau_N & Z_{SN^*}\tau_{N^*}\\Z_{DN}\tau_N & Z_{DN^*}\tau_{N^*}\end{pmatrix}\begin{pmatrix}X_{ND}\\X_{N^*D}\end{pmatrix}, \quad (3.9)$$

$$\begin{pmatrix}X_{ND}\\X_{N^*D}\end{pmatrix}=-\begin{pmatrix}Z_{ND}\\Z_{N^*D}\end{pmatrix}-\begin{pmatrix}V_{NN}\tau_N & V_{NN^*}\tau_{N^*}\\V_{N^*N}\tau_N & V_{N^*N^*}\tau_{N^*}\end{pmatrix}\begin{pmatrix}X_{ND}\\X_{N^*D}\end{pmatrix}, \quad (3.10)$$

where the quantities V_{ab} occurring in (3.10) are the same as those defined by Eq. (3.7). The equations for deuteron breakup, $\pi+D\rightarrow\pi+N+N$, can again be written in terms of the amplitudes X_{aD} $(a=N, N^*, S, D)$ by using Eq. (3.28) of L.

At this stage we have succeeded in reducing the number of coupled channels in the integral equations for bound state and resonance scatterings from four to two.

4. FURTHER SIMPLIFICATIONS

A. The $I=0$ Equations

For the case when the total isotopic spin $I=0$, the only two possible bound-state and free-particle configurations that we can have in the $NN\pi$ system are $(N+N)$ and $(\pi+S)$. Hence, if we are willing to restrict ourselves to scattering processes with $I=0$, we shall obtain single-channel integral equations for $X_{SN}(s)$ and $X_{NN}(s)$. We have from Eqs. (3.5), (3.7), and (3.8)

$$X_{SN}=-Z_{SN}-Z_{SN}\tau_N X_{NN}, \quad (4.1)$$

$$X_{NN}=-V_{NN}-V_{NN}\tau_N X_{NN}, \quad (4.2)$$

where

$$V_{NN}=+Z_{NN}-2Z_{NS}\tau_S Z_{SN}. \quad (4.3)$$

The amplitude for $N+N\rightarrow N+N+\pi$ in the $I=0$ state would now be given in terms of $X_{NN}(s)$ and $X_{SN}(s)$.

In order to obtain integral equations in only one variable from Eqs. (4.1)–(4.3), which involve integrations over angles and energies, we must carry out a partial-wave analysis. Consider a transition with total angular momentum J, from an initial state of total spin S and relative orbital angular momentum l in the total center-of-mass frame, to a final state with quantum numbers S' and l'. The scattering amplitude can then be written in terms of the angular operators[10] $\mathcal{L}_{JS'l'Sl}$ as

$$X(\mathbf{q}',\mathbf{q};s)=\sum_{JS'l'Sl}X^{JS'l'Sl}(q',q,s)\mathcal{L}_{JS'l'Sl}(\hat{q}',\hat{q}). \quad (4.4)$$

On making a similar expansion for the kernel as well as the inhomogeneous term in Eqs. (4.1) and (4.2), and using the orthogonality properties of the angular operators to carry out the integration over the angles, we obtain

$$X_{SN}{}^{J\nu'\nu}(q',q;s)=-Z_{SN}{}^{J\nu'\nu}(q',q;s)-\int dq''q''^2\sum_{\nu''}Z_{SN}{}^{J\nu'\nu''}(q',q'';s)\times\tau_N(s)X_{NN}{}^{J\nu''\nu}(q'',q;s), \quad (4.5)$$

$$X_{NN}{}^{J\nu'\nu}(q',q;s)=-V_{NN}{}^{J\nu'\nu}(q',q;s)-\int dq''q''^2\sum_{\nu''}V_{NN}{}^{J\nu'\nu''}(q',q'';s)\times\tau_N(s)X_{NN}{}^{J\nu''\nu}(q'',q;s), \quad (4.6)$$

where

$$V_{NN}{}^{J\nu'\nu}(q',q;s)=-Z_{NN}{}^{J\nu'\nu}(q',q;s)-2\int dq''q''^2\sum_{\nu''}Z_{NS}{}^{J\nu'\nu''}(q',q'';s)\times\tau_S(s)Z_{SN}{}^{J\nu''\nu}(q'',q;s), \quad (4.7)$$

with $\nu\equiv l, S$.

In the process $N+N\rightarrow\pi+S$, parity conservation forbids any transitions when the two nucleons are in a state of total spin $S=0$. When the nucleons are in a state $S=1$, the allowed transitions are for $J=l\pm1=l'$.

[10] V. S. Varma, Nuovo Cimento 42, 87 (1965). The angular operators in this reference must be multiplied by appropriate recoupling coefficients for three-particle scattering.

In the process $N+N \to N+N$, for $S=0$ the allowed transitions are $J=l=l'$; and for $S=1$ we have the four independent amplitudes corresponding to $J=l=l'$, $J=l+1=l'+1$, $J=l-1=l'-1$, and $J=l\pm1=l'\mp1$. Therefore, the second term in Eq. (4.7) will be nonzero only when $S=1$ and $J\neq l$.

Hence for $I=S=0$ nucleon-nucleon scattering we obtain the one-dimensional single-channel integral equation in the Jth partial wave as

$$X_{NN}{}^{J}(q',q;s)=-Z_{NN}{}^{J}(q',q;s) - \int_0^\infty dq''q''^2 Z_{NN}{}^{J}(q',q'';s) \times \tau_N(s) X_{NN}{}^{J}(q'',q;s). \quad (4.8)$$

Notice that this involves only the original Lovelace potential and that there are no contribution from $Z_{SN}(s)$. There is a similar equation for the $I=0$, $S=1$, $J=l=l'$ amplitude, while there are coupled two-channel equations for the remaining amplitudes in $I=0$, $S=1$ scattering.

B. The $N\pi$ Interaction

The potentials $Z_{NN}{}^{J}(q',q;s)$ occurring in Eq. (4.8) require a knowledge of the two-particle p-wave $N\pi$ scattering in the $I=\frac{1}{2}$, $J=\frac{1}{2}$ state. For this we use the separable potentials (forgetting isospin for the moment)

$$\langle \mathbf{p}'|V|\mathbf{p}\rangle=\lambda_N \sum_{s_z=-1/2}^{+1/2} g_{Ns_z}{}^{\dagger}(\mathbf{p}')g_{Ns_z}(\mathbf{p}), \quad (4.9)$$

with

$$g_{Ns_z}(\mathbf{p})=g_N(p)\sum_{\mu} C_{1/2,s_z}{}^{1/2,\mu;1/2,s_z-\mu} \times Y_{1,s_z-\mu}{}^{*}(\hat{p})T_{\mu}{}^{1/2\dagger}, \quad (4.10)$$

where we have decomposed $J=\frac{1}{2}$ into the spin $\frac{1}{2}$ of the nucleon and the orbital angular momentum 1 of the pion. For the radial part of the form factor we choose the form

$$g_N(p)=pA_N/(p^2+\beta_N{}^2), \quad (4.11)$$

where A_N is a normalization constant. Then $g_N(p)$ satisfies the following conditions[11] that are required of it: (i) $g_N{}^2(p)$ is a positive-definite function of p^2, vanishing as p^{2l} as $p^2 \to 0$, (ii) $g_N(p)$ has a branch cut from $p^2=-\beta_N{}^2$ to $-\infty$ ($\beta_N{}^{-1}$ is proportional to the range of the corresponding Yukawa potential), (iii) $g_N(p)$ goes to zero as $p^2 \to \infty$.

Since the nucleon is being considered as a bound state of mass (m_N+m_π) in the p-wave $J=\frac{1}{2}$ channel with binding energy equal to $-m_\pi=-\alpha_N{}^2/2m_r$ (m_r being the $N\pi$ reduced mass, $\alpha_N=181$ MeV), $g_N(p)$ can be related to the radial part of the nucleon wave-function (see L), which when normalized to unity yields

$$A_N{}^2=(\alpha_N+\beta_N)^3/\pi m_r{}^2. \quad (4.12)$$

The condition that the bound state occurs at the energy $-\alpha^2/2m_r$ gives

$$\lambda_N=-2m_r/(\alpha_N+\beta_N)(2\alpha_N+\beta_N). \quad (4.13)$$

The effective range formula for the P_{11} state can then be written as

$$k^3\cot\delta_{11}=-\frac{1}{2(\alpha_N+\beta_N)^2}[\alpha_N{}^2\beta_N{}^3 +\beta_N(3\alpha_N{}^2+2\alpha_N\beta_N+\beta_N{}^2)k^2+(2\alpha_N+\beta_N)k^4]. \quad (4.14)$$

For the scattering length

$$a_{11}{}^{-1}=-\alpha_N{}^2\beta_N{}^3/2(\alpha_N+\beta_N)^2,$$

we use the value -0.107 ± 0.007 ($\hbar=m_\pi=c=1$) obtained by Hamilton and Woolcock.[12,13] The cubic equation for β_N has only one real root, and we get $\beta_N=1874$ MeV.

It must be pointed out that in a nonrelativistic formalism, as mass has nothing to do with binding energy, the nucleon when regarded as a pion-nucleon bound state has total mass equal to (m_N+m_π). By demanding that the binding energy of the nucleon bound state be equal to $-m_\pi$, we make sure, at least from a relativistic standpoint, that in its restframe, this bound state has the same energy as an ordinary nucleon of mass m_N. This asymmetry between the masses of the bound and ordinary nucleons is, however, necessary in order to maintain the Galilean invariance of the theory.

If we consider, for example, the potential represented by the graph in Fig. 1(b), Galilean invariance requires the conservation of mass at each vertex. Thus, at an $N \to N+\pi$ vertex, where the initial bound nucleon dissociates into an ordinary nucleon and a pion as the first part of the bound-state rearrangement process which gives rise to the potentials in the Lovelace scheme, Galilean invariance is maintained because the initial nucleon which is bound has mass (m_N+m_π), while the ordinary nucleon has mass m_N, i.e., exactly because of the asymmetry between the masses of the bound and ordinary nucleons.

The P_{11} phase shifts (4.14) predicted by such a separable potential are plotted in Fig. 2. Notice that this is roughly consistent with experiments in the

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[12] J. Hamilton and W. S. Woolcock, Phys. Rev. **118**, 291 (1960); Rev. Mod. Phys. **35**, 737 (1963).

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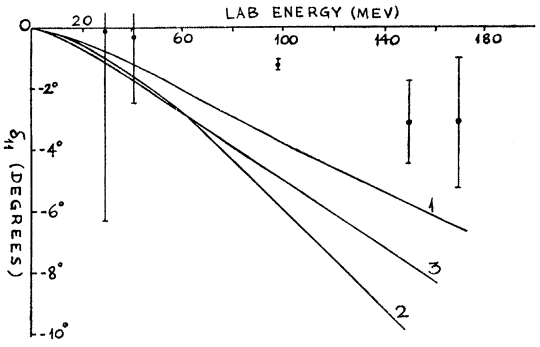

FIG. 2. The P_{11} phase shifts for $N\pi$ scattering. Curve 1 is for $g_N(p)=A_N p/(p^2+\beta_N^2)$; curve 2 is for $g_N(p)=A_N' p/(p^2+\beta_N'^2)^2$; curve 3 is that due to Balázs (Ref. 19). The experimental points are from Refs. 14–16.

energy range[14–16] 0–150 MeV but the high-energy behavior is completely wrong. The experimental phase shifts are known to cross over at about 200 MeV,[17] this behavior being probably due to a P_{11} object[18] at about 400 MeV. It is, however, impossible to make the phase shifts cross over with just a single separable potential.[11] For example, a different choice of the form factor

$$g_N(p)=A_N' p/(p^2+\beta_N'^2)^2$$

is found to give an even poorer fit. We have therefore used the form (4.11) throughout the rest of the calculations.

It is interesting to note that the phase shifts predicted by (4.11) are moderately better than those obtained by Balázs,[19] who also used a nucleon pole, but in an N/D calculation. More recent N/D calculations,[20,21] which use exchanges of the ρ and of the s-wave dipion resonance, are able to get better agreement with experiment at high energies.

C. The $I=0$, $S=0$ Equations

We are now in a position to write down the Lovelace bound-state scattering potential for $I=0$ singlet nucleon-nucleon scattering from an initial 1, N-channel state to a final 2, N-channel state. We can decompose the initial-spin-zero state into

$$|0,0\rangle=\sum_{s_1,s_2} C_{0,0}{}^{1/2,-s_1;1/2,s_1} C_{1/2,-s_1}{}^{1/2,s_2;1,-s_1-s_2} \times Y_{1,-s_1-s_2}(\hat{p}) T_{s_1}{}^{1/2} T_{s_2}{}^{1/2} = \frac{1}{\sqrt{3}} \sum_m T_m {}^1Y_{1,m}(\hat{p}). \quad (4.15)$$

Although the product of two spin-$\frac{1}{2}$ functions can give rise to either a scalar or a vector, only the latter can match the orbital angular momentum of the pion and give rise to a total-spin-zero state. The potential for singlet scattering consists of a product of two terms like

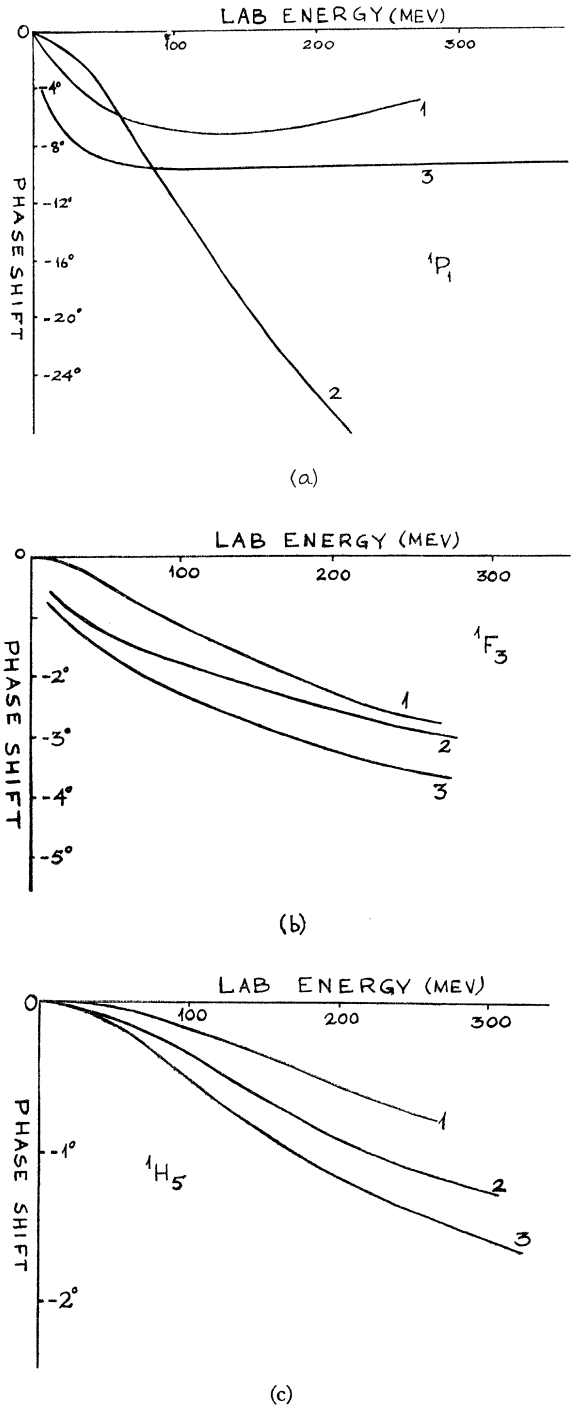

FIG. 3. The phase shifts for (a) the $^1P_1$, (b) the $^1F_3$, and (c) the $^1H_5$ state in elastic nucleon-nucleon scattering for $I=S=0$. Curve 1 corresponds to the present calculations, curve 2 is from the phase-shift analysis of Arndt and MacGregor (Ref. 22), and curve 3 is from the one-pion-exchange model.

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[15] H. Y. Chiu and E. L. Lomon, Ann. Phys. (N.Y.) 6, 50 (1959).
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[17] A. Donnachie, J. Hamilton, and A. T. Lea, Phys. Rev. **135**, B515 (1964).
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[20] J. S. Ball and D. Y. Wong, Phys. Rev. **133**, B179 (1964).
[21] S. Rai Choudhury, Aditya Kumar, and R. P. Saxena, Phys. Rev. **143**, 1159 (1966).

(4.15), one each for the initial and final states. This product can again be decomposed into spin tensors of rank 0, 1, and 2. However, it is known that singlet nucleon-nucleon scattering involves only scalar interactions; hence the angular part of the Lovelace potential for singlet scattering will be just

$$(1/4\pi)P_1(\hat{p}\cdot\hat{p}')T^0.$$

The isospin dependence Λ^I of the potential will be given in terms of a 6-j symbol (see L), and for $I=0$, $\Lambda^0=1$. The potential for nucleon-nucleon elastic scattering in the Jth partial wave for $I=S=0$ is

$$Z_{NN}{}^J(q_2',q_1;s)=\tfrac{1}{2}\int_{-1}^{+1}dz\,P_J(z)(\hat{p}_2'\cdot\hat{p}_1)\times\frac{g_N(p_2')g_N(p_1)}{p_1^2/2\mu_1+q_1^2/2M_1-s}. \quad (4.16)$$

The integration can be carried out after expressing $\mathbf{p}_2'$ and $\mathbf{p}_1'$ in terms of q_2', q_1 and $z=\hat{q}_2'\cdot\hat{q}_1$ by using (2.3). Following L, the bound-state propagator is given by

$$-\tau_N(s)=-t_N(s-q_1^2/2M_1)=\frac{2M_1}{q_1^2-q_0^2-i\epsilon}\frac{(R+\beta_N)^2(R+\alpha_N)}{(\alpha_N+\beta_N)\{\beta_N(R+\alpha_N)+2\alpha_N R\}}, \quad (4.17)$$

where $R=[(\mu_1/M_1)q_1^2-2\mu_1 s]^{1/2}$ and $q_0^2/2M_1-m_\pi=s$.

Instead of solving Eq. (4.8) for the off-shell T matrix, its analog for the corresponding off-shell K-matrix was solved. The advantage of this procedure is that the bound-state scattering unitarity relation is preserved whatever numerical approximation scheme is adopted; and also, below the three-particle threshold, the K matrix is always real so the equations are easier to solve than those for the T matrix, which is complex.

The integral equation for the K matrix was cast into a 50×50 matrix equation and solved by inversion using standard matrix routines. Below the three-particle threshold, this size of the matrix was sufficient to produce stable solutions.

The equations were solved for the ${}^1P_1$, ${}^1F_3$, and ${}^1H_5$ states and the phase shifts predicted for these partial waves are given in Fig. 3.

The agreement with the experimental phase-shift analysis of Arndt and MacGregor[22] is poor in the ${}^1P_1$ state and is reasonably good in the other two partial waves. Our phase shifts are consistently lower than those predicted by the one-pion-exchange model. This may be related to the fact that whereas the one-pion-exchange model is equivalent to taking the Born term, we solve an integral equation, because we find that our solutions are always smaller than their corresponding inhomogeneous terms.

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5. CONCLUSIONS

We have seen how to obtain single-channel one-dimensional integral equations for nucleon-nucleon scattering in the $I=S=0$ state starting from the Lovelace-Faddeev theory for the $NN\pi$ system of three particles. We have approximated the $N\pi$ system in the P_{11} state (the only two-particle system which contributes to the scattering in our model) by a single separable potential, and we find that the phase shifts for $I=S=0$ nucleon-nucleon elastic scattering agree moderately well with experiments in the higher partial waves.

The disagreement is almost certainly due to the fact that our separable approximation does not provide a good description for the P_{11} state at high energies. One method of improving our results would be to approximate the P_{11} $N\pi$ state by a sum of two separable potentials.[23] This would make the two-body scattering amplitude a 2×2 matrix and certainly make the calculations more complicated. The alternative of introducing a phenomenological three-body force, as other similar calculations in the three-nucleon system[24] have had to do, is not very satisfying. An additional parameter is introduced in the theory, which may still be worthwhile if good agreement is obtained with a large number of experimental quantities.

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