et al.[9] have estimated the contribution of the $I=0$ s-wave interaction to low-energy s-wave π-nucleon scattering. Their procedure effectively deduces one number, which is an integral over the $I=0$ s-wave $\pi\pi$ scattering cross section multiplied by a weighting factor which falls rapidly from threshold to an energy of about 900 MeV. The largest contribution to the integral comes from $\pi\pi$ energies below 600 MeV. They find a fit with a phase shift which rises rapidly at threshold to 30° and then levels off at that value. However, it would seem probable that other forms of variation of this phase shift would produce the same value of the integral they determine; in particular, a phase which is larger at higher masses, such as we propose, would then imply a much smaller scattering length at threshold. This smaller scattering length would be in better agreement with the conclusions drawn by Weinberg[10] from an analysis of K_{e4} decay. Chiu and Schechter[11] have remarked how the sum rule of Adler[12] can be completed if there is an s-wave $\pi\pi$ resonance with a mass of 390 MeV and a width of 90 MeV. They then show that this sum rule could, alternatively, be completed by a range of other s-wave resonances; all that is needed is that M/Γ^3 have a certain value. Therefore, we could complete this sum rule with an s-wave resonance with, for example, a mass of 700 MeV and a width of 520 MeV. The s-wave cross section due to such a resonance would be very similar to the conclusions which seem to be implied by these analyses of reaction (1). It can therefore be suggested that the sum rule of Adler[12] can probably be completed in this way.

[9] J. Hamilton, P. Menotti, G. C. Oades, and L. L. J. Vick, Phys. Rev. **128**, 1881 (1962).

[10] S. Weinberg, Phys. Rev. Letters **17**, 336 (1966).

[11] Y. T. Chiu and J. Schechter, Nuovo Cimento **46**, 548 (1966).

[12] S. L. Adler, Phys. Rev. **140**, B736 (1965).

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Field-Current Identities and Algebra of Fields*

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It is shown that the possible identity between the various hadronic current operators and the corresponding spin-1 meson field operators determines the general structure of the hadronic part of the total Lagrangian. In particular, the identity between the isovector electromagnetic current and the neutral ρ-meson field implies that the ρ dependence of the strong interaction must be the same as that in the Yang-Mills theory, except for the mass term of the ρ meson. The explicit form of the interaction Lagrangian makes possible a general study of the local equal-time commutators of the various hadronic current operators, including the effects of the electromagnetic interaction. Many of these electromagnetic correction terms depend only on the general requirement of gauge invariance, and are independent of whether the proposed field-current identities are valid or not. For example, the usual Schwinger term $\lambda(\partial/\partial r_j)\delta^3(r-r')$ in the commutator between the time component of any charged hadronic weak interaction current and the jth space component of its Hermitian conjugate should be replaced by $\lambda[(\partial/\partial r_j)+ieA_j]\delta^3(r-r')$, where A_j is the electromagnetic field operator. The contribution of such a correction term, i.e., $\lambda ieA_j\delta^3(r-r')$, remains present in the integrated form of the commutator. In the usual current algebra, λ is mathematically undefined. If field-current identities hold, then these current commutators are the same as the corresponding algebra of the field operators, and λ becomes a well-defined c number. Some speculative remarks concerning the possible extension of the algebra of fields to the lepton currents are presented.

1. INTRODUCTION

RECENTLY, it has been suggested[1] that the entire hadronic electromagnetic current operator is identical with a linear combination of the local field operators of the known neutral vector mesons, independent of whether the unrenormalized masses of these vector mesons are finite or infinite.[2] This identity is shown to be consistent with the requirement of gauge invariance; it gives a precise formulation of the idea of vector domi-

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[1] N. M. Kroll, T. D. Lee, and Bruno Zumino, Phys. Rev. **157**, 1376 (1967).

[2] There exists an alternative proposal in which the unrenormalized isovector part of the hadronic electromagnetic current is assumed to be the same operator as the unrenormalized current generating the neutral ρ-meson field. In such a case, the field-current identity holds only in the limit of an infinite unrenormalized meson mass. [See Refs. 1, 4, M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).] It is important to note that in this alternative possibility, the products of the current operators would in general, be different from the products of the corresponding field operators even in the limit of infinite unrenormalized masses; thus, the hadronic current operators entering in the electromagnetic and the weak interactions satisfy the usual current algebra instead of the algebra of fields.

nance[3] in the language of a local Lagrangian field theory, and it leads, among other consequences, to the conclusion that, in the absence of the leptons, the entire hadronic contribution to the electric charge renormalization is finite.[1,4] The generalization that such field-current identities hold also for other observed hadronic currents, such as the various weak-interaction vector and axial-vector currents, implies that these observed hadronic currents satisfy the algebra of fields,[5] which is simpler than the usual current algebra.[6] It is found that all successful applications previously obtained from the current algebra can be rederived by using the algebra of fields; in addition, the usually troublesome Schwinger term[7] becomes a well-defined c number in the algebra of fields.

The purpose of this paper is to point out that these field-current identities also determine the general structure of both the strong-interaction and the hadronic part of the electromagnetic interaction. The explicit form of these interaction Lagrangians enables one to study systematically the electromagnetic corrections of the equal-time commutator of the various hadron currents. That some of these commutators must be affected by the electromagnetic interaction follows from general considerations of gauge invariance. For example, suppose that, in the *absence* of the electromagnetic interaction, the commutator

$$[V_4{}^{\mathrm{wk}}(r,t)_{S=0}, V_k{}^{\mathrm{wk}}(r',t)^\dagger{}_{S=0}] \qquad (1.1)$$

is[8]

$$-2i\delta^3(r-r')J_k{}^\gamma(r',t)_{I=1}+\lambda(\partial/\partial r_k)\delta^3(r-r'), \qquad (1.2)$$

where $(V_\mu{}^{\mathrm{wk}})_{S=0}$ is the hadronic strangeness-conserving part of the weak-interaction vector current, $(J_\mu{}^\gamma)_{I=1}$ is the hadronic isovector part of the electromagnetic current and λ is the coefficient of the Schwinger term. The *inclusion* of the electromagnetic interaction requires that the same commutator should be given by, instead of (1.2),

$$-2i\delta^3(r-r')(J_k{}^\gamma)_{I=1}+\lambda[(\partial/\partial r_k)+ieA_k]\delta^3(r-r'), \qquad (1.3)$$

where A_μ is the electromagnetic field operator. We note that under the gauge transformation

$$A_\mu \to A_\mu - (\partial\Lambda/\partial x_\mu),$$
$$(V_\mu{}^{\mathrm{wk}})_{S=0} \to (V_\mu{}^{\mathrm{wk}})_{S=0}\exp(ie\Lambda),$$

the expression (1.1) acquires a multiplicative factor $\exp[ie\Lambda(r,t)-ie\Lambda(r',t)]$ while (1.3) acquires an additive term $-ie\lambda(\partial\Lambda/\partial r_k)\delta^3(r-r')$. It can be easily shown that these two changes are indeed identical by using the equality

$$\exp[ie\Lambda(r\ t)-ie\Lambda(r',t)](\partial/\partial r_j)\delta^3(r-r') = (\partial/\partial r_j)\delta^3(r-r')-ie(\partial\Lambda/\partial r_j)\delta^3(r-r'). \qquad (1.4)$$

The contribution of this new term $ie\lambda A_k\delta^3(r-r')$ in (1.3) remains present even after one integrates the commutator (1.1) over d^3r, or d^3r'. In the usual current algebra λ is mathematically undefined, therefore, so would be the corresponding *integrated* equal-time current commutator. In the algebra of fields, λ is a well-defined c number, related to the renormalized mass m_ρ and the renormalized coupling constant g_ρ of the neutral ρ meson by

$$\lambda = 2(m_\rho/g_\rho)^2. \qquad (1.5)$$

In Sec. 2, we begin with the identity that $(J_\mu{}^\gamma)_{I=1}$ is related to the neutral ρ-meson field by

$$[J_\mu{}^\gamma(x)]_{I=1} = -(m_0{}^2/g_0)\rho_\mu{}^0(x) = -(m_\rho{}^2/g_\rho)[\rho_\mu{}^0(x)]_{\mathrm{ren}}, \qquad (1.6)$$

where $(\rho_\mu{}^0)_{\mathrm{ren}}$ is the renormalized field operator, and m_0, g_0, $\rho_\mu{}^0(x)$ refer to the unrenormalized mass, the unrenormalized coupling constant, and the unrenormalized field operator of the neutral ρ meson.[9] Let $\boldsymbol{\varrho}_\mu(x)$ be the

[3] Y. Nambu, Phys. Rev. **106**, 1366 (1957); W. R. Frazer and J. R. Fulco, Phys. Rev. **117**, 1603 (1960); J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960); M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961); M. Gell-Mann, *ibid.* **125**, 1067 (1962); Y. Nambu and J. J. Sakurai, Phys. Rev. Letters **8**, 79 (1962); M. Gell-Mann, D. Sharp, and W. G. Wagner, *ibid.* **8**, 261 (1962); J. Schwinger, Phys. Rev. **140**, B158 (1965). For further discussions and applications of vector dominance ideas, see G. Feldman and P. Matthews, Phys. Rev. **132**, 823 (1963); S. Berman and S. Drell, *ibid.* **133**, B791 (1964); R. F. Dashen and D. H. Sharp, *ibid.* **133**, B1585 (1964); L. Stodolsky, *ibid.* **134**, B1099 (1964); G. Barton and B. G. Smith, Nuovo Cimento **36**, 436 (1965); R. Gatto, Ergeb. Exakt. Naturw. **39**, 106 (1965); M. Ross and L. Stodolsky, Phys. Rev. **149**, 1172 (1966); D. S. Beder, *ibid.* **149**, 1203 (1966); M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 51 (1965).

While the approximation that the matrix elements of the hadronic vector current operator may be replaced by the corresponding elements of the vector-meson field operator has been discussed in many of these papers, in the context of a Lagrangian field theory the difficulty has always been the apparent violation of gauge invariance. [See, e.g., G. Feldman and P. Matthews (Ref. 3).] This difficulty is revolved in Ref. 1.

[4] T. D. Lee and Bruno Zumino (to be published).

[5] T. D. Lee, S. Weinberg, and Bruno Zumino, Phys. Rev. Letters **18**, 1029 (1967).

[6] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

[7] T. Gotô and T. Imamura, Progr. Theoret. Phys. (Kyoto) **14**, 396 (1955); J. Schwinger, Phys. Rev. Letters **3**, 296 (1959). For an earlier discussion of a related term see R. Serber, Phys. Rev. **49**, 545 (1936).

[8] Throughout the paper, the subscript μ denotes the space-time index, $\mu=4$ is the time component, $x_4=it$, and $\mu=i$ (or j, or k) denotes the space component. The isospin index is represented by the superscript, e.g. a, which can be 0, 1, or 2, and $a=0$ denotes the neutral component. All boldface letters denote isovectors.

[9] We sketch the renormalization procedure for the ρ meson coupled to a conserved current. [For more details, see Ref. 1.] The renormalized and the unrenormalized propagators are related by $(D_{\mu\nu})_{\mathrm{unr}}=Z_\rho(D_{\mu\nu})_{\mathrm{ren}}$. At zero four-momentum $(D_{\mu\nu})_{\mathrm{unr}}=m_0{}^{-2}\delta_{\mu\nu}$, as a consequence of current conservation. By assumption $(D_{\mu\nu})_{\mathrm{ren}}$ is finite, and one can choose conveniently, $Z_\rho=(m_\rho/m_0)^2$, where the renormalized mass m_ρ of the ρ meson is defined as the zero of the real part of the inverse propagator. If the renormalized coupling constant g_ρ is defined at zero momentum transfer, one sees from the equation of motion of the ρ meson that it satisfies $g_\rho/g_0=m_\rho/m_0$. Since the current conservation holds more exactly for the neutral ρ meson, we will choose m_ρ and g_ρ to be those of the neutral ρ meson.

It is important to note that while the perturbation series of the interaction between a single neutral vector meson and a conserved current consisting of only spin-$\frac{1}{2}$ and spin-0 fields can be renormalized, this is not true for the perturbation series of the strong-interaction Lagrangian of the isovector ρ-meson system given by Eq. (2.6) following. Nevertheless, we shall *assume* in the same spirit as that used in the ξ-limiting process that the renormalized theory does exist [T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962); T. D. Lee, *ibid.* **128**, 899 (1962)]. The failure of the perturbation series method implies only that the renormalized theory does not have a power-series expansion in terms of the square of the renormalized coupling constant $g_\rho{}^2$. For example, it may contain terms such as $\ln g_\rho$, or $g_\rho{}^{1/2}$, etc.

unrenormalized isovector ρ-meson field whose 0th component[8] is the neutral ρ-meson field $\rho_\mu{}^0(x)$. The identity (1.6) implies that under the strong interaction, the ρ-meson field satisfies the equation of field conservation

$$\frac{\partial}{\partial x_\mu}\boldsymbol{\varrho}_\mu(x)=0. \qquad (1.7)$$

As we shall see, this field-conservation equation determines the general form of the ρ-meson part of the strong interaction. The resulting Lagrangian $\mathfrak{L}_{\rm st}$ is identical in form to that in the Yang-Mills theory,[10] except for a mass term $-\frac{1}{2}(m_0\boldsymbol{\varrho}_\mu)^2$. The presence of this mass term destroys the *local* isospin invariance which forms the starting point of the Yang-Mills theory. Such a local gauge invariance plays no role in the present paper; instead, we emphasize the importance of the field-conservation Eq. (1.7), which is a necessary consequence of the field-current identity (1.6).

The extensions of this Lagrangian to include the electromagnetic and the weak interactions are discussed in Sec. 3. The resulting algebra of fields, valid to all orders in e, is given in Sec. 4. The generalization to an arbitrary broken symmetry is discussed in Sec. 5. As an illustration of the general formalism, we give in Sec. 6 the special example of the $SU_3\times SU_3$ field algebra which is valid to all orders in e.

In Sec. 7, some speculative remarks are made concerning the lepton currents. It is pointed out that at present, our experimental information is completely consistent with the possibility that the leptonic part of the electromagnetic current (or, the weak interaction current) may also satisfy the algebra of fields. The experimental consequences of such a possibility are discussed.

It may be emphasized that the distinction between fields and sources (or, currents) has its origin in the study of the electromagnetic and gravitational fields. Both fields, having a zero rest mass and satisfying Bose statistics, do approach their respective classical limits at large distances. Therefore, there is a clear physical distinction between such fields and their sources. The same physical distinction does not exist for a massive boson field. The difference between whether a set of observed hadron, or lepton, current operators are identical with a corresponding set of massive meson fields, or not, lies only in the algebraic relations that such operators satisfy. Although throughout the paper all derivations are based on the usual local Lagrangian field theory, it is hoped that the resulting equal-time commutation relations can be of a more general nature, not necessarily depending on the validity of the local Lagrangian field theory.

[10] C. N. Yang and F. Mills, Phys. Rev. **96**, 191 (1954). A discussion of the implications of the field conservation equation for the Yang-Mills theory has been given by V. I. Ogievetskij and I. V. Polubarinov, Ann. Phys. (N. Y.) **25**, 358 (1963); and by Bruno Zumino, Acta Phys. Austriaca, Suppl. II, 212 (1966).

2. STRONG INTERACTION

For clarity, we consider first only the system of the ρ-meson fields. The free Lagrangian is

$$\mathfrak{L}_{\rm free}(\rho)=-\tfrac{1}{4}\mathbf{G}_{\mu\nu}{}^2-\tfrac{1}{2}(m_0\boldsymbol{\varrho}_\nu)^2, \qquad (2.1)$$

where

$$\mathbf{G}_{\mu\nu}=\frac{\partial}{\partial x_\mu}\boldsymbol{\varrho}_\nu-\frac{\partial}{\partial x_\nu}\boldsymbol{\varrho}_\mu. \qquad (2.2)$$

In order that the field-current identity (1.6) holds, it follows from, e.g., Eq. (2.8) of Ref. 1 that, since the charged ρ meson does interact with the electromagnetic field, there must also exist a strong-interaction term between the neutral ρ meson and the charged ρ mesons. Such a term should be proportional to $(G_{\mu\nu}{}^1\rho_\nu{}^2-G_{\mu\nu}{}^2\rho_\nu{}^1)\rho_\mu{}^0$; from the isospin invariance, it becomes proportional to $(\mathbf{G}_{\mu\nu}\times\boldsymbol{\varrho}_\nu)\cdot(\boldsymbol{\varrho}_\mu)$. Thus, we assume the strong-interaction Lagrangian density of the ρ-meson system to be of the general form

$$\mathfrak{L}_{\rm int}(\rho)=\tfrac{1}{2}g_0\mathbf{G}_{\mu\nu}\cdot(\boldsymbol{\varrho}_\mu\times\boldsymbol{\varrho}_\nu)+F(\boldsymbol{\varrho}_\mu), \qquad (2.3)$$

where, for simplicity, F is assumed to depend *only* on $\boldsymbol{\varrho}_\mu$. This restriction is equivalent to the usual minimal principle in which the ρ-meson coupling is assumed to consist of only the minimal number of derivatives.

Theorem. If $\boldsymbol{\varrho}_\mu(x)$ satisfies the field conservation equation

$$\partial\boldsymbol{\varrho}_\mu(x)/\partial x_\mu=0, \qquad (2.4)$$

then the function F must be of the form

$$F=-\tfrac{1}{4}g_0{}^2(\boldsymbol{\varrho}_\mu\times\boldsymbol{\varrho}_\nu)^2. \qquad (2.5)$$

Therefore, the total Lagrangian density of the ρ-meson system is

$$\begin{aligned}\mathfrak{L}_\rho&=\mathfrak{L}_{\rm free}(\rho)+\mathfrak{L}_{\rm int}(\rho),\\ &=-\tfrac{1}{4}(\mathbf{f}_{\mu\nu})^2-\tfrac{1}{2}(m_0\boldsymbol{\varrho}_\mu)^2,\end{aligned} \qquad (2.6)$$

where

$$\mathbf{f}_{\mu\nu}=(\partial\boldsymbol{\varrho}_\nu/\partial x_\mu)-(\partial\boldsymbol{\varrho}_\mu/\partial x_\nu)-g_0(\boldsymbol{\varrho}_\mu\times\boldsymbol{\varrho}_\nu). \qquad (2.7)$$

Proof. Let $\mathbf{S}_\mu(\rho)$ be the current operator, whose components are defined by

$$\begin{aligned}S_\mu{}^a(\rho)&=-\epsilon^{abc}\left[\partial\mathfrak{L}(\rho)/\partial\left(\frac{\partial\rho_\nu{}^b}{\partial x_\mu}\right)\right]\rho_\nu{}^c,\\ &=\epsilon^{abc}f_{\mu\nu}{}^b\rho_\nu{}^c,\end{aligned} \qquad (2.8)$$

where the superscripts a, b, c denote the isospin indices and ϵ^{abc} is $+1$, -1, or 0 depending on whether abc is an even permutation of 012, an odd permutation, or otherwise. The isospin invariance of the strong interaction implies that in the absence of other fields

$$\frac{\partial}{\partial x_\mu}\mathbf{S}_\mu(\rho)=0. \qquad (2.9)$$

From (2.1) and (2.3), the equation of the ρ-meson field is

$$\frac{\partial}{\partial x_\mu}\mathbf{f}_{\mu\nu}-m_0{}^2\boldsymbol{\varrho}_\nu=g_0\mathbf{S}_\nu+g_0{}^2(\boldsymbol{\varrho}_\nu\times\boldsymbol{\varrho}_\mu)\times\boldsymbol{\varrho}_\mu-(\partial F/\partial\boldsymbol{\varrho}_\nu). \qquad (2.10)$$

Let us define A^{ab} by

$$A^{ab}\rho_\nu{}^b=[-(\partial F/\partial \boldsymbol{\rho}_\nu)+g_0{}^2(\boldsymbol{\rho}_\nu\times\boldsymbol{\rho}_\mu)\times\boldsymbol{\rho}_\mu]^a. \quad (2.11)$$

This is always possible since the right-hand side is an isovector function of $\boldsymbol{\rho}_\mu$, and it is also a space-time 4-vector. From Eqs. (2.4), (2.9), and (2.10), it follows that

$$\rho_\nu{}^b\frac{\partial}{\partial x_\nu}A^{ab}=0. \quad (2.12)$$

In this problem, the three spatial components $\boldsymbol{\rho}_i$ ($i=1$ 2, 3) and their first time derivatives $(\partial\boldsymbol{\rho}_i/\partial t)$ are independent variables. The time component $\boldsymbol{\rho}_4$ is, on account of (2.10), a function of $(\partial\boldsymbol{\rho}_i/\partial t)$ and $\boldsymbol{\rho}_i$. Thus (in the c-number theory), at any given time the values of $\boldsymbol{\rho}_i$ and $\boldsymbol{\rho}_4$ can be arbitrarily chosen. In order that (2.12) hold at all times, one must have $(\partial A^{ab}/\partial x_\nu)=0$. Thus, $A^{ab}=\kappa\delta^{ab}$ where κ is a constant. It follows from Eq. (2.11) that F is given by

$$F=-\tfrac{1}{2}\kappa(\boldsymbol{\rho}_\nu)^2-\tfrac{1}{4}g_0{}^2(\boldsymbol{\rho}_\mu\times\boldsymbol{\rho}_\nu)^2.$$

In this expression, the term $-\frac{1}{2}\kappa(\boldsymbol{\rho}_\nu)^2$ should be combined with the mass term $-\frac{1}{2}m_0(\boldsymbol{\rho}_\nu)^2$ in (2.1). The theorem is then proved.

We note that the first term $-\frac{1}{4}\mathbf{f}_{\mu\nu}{}^2$ in (2.6) is exactly the same as the Lagrangian in the Yang-Mills theory.[10] In contrast to the Yang-Mills theory, the second term $-\frac{1}{2}(m_0\boldsymbol{\rho}_\nu)^2$ destroys the *local* isospin gauge invariance. The fact that $m_0\neq 0$ makes it possible to connect the field conservation equation with the equation of motion.

The above considerations can be readily extended to include other fields. In the absence of the electromagnetic and the weak interactions, the total Lagrangian $\mathcal{L}$ is known to be invariant under isospin transformations. If we assume the minimal principle, the field-conservation equation (2.4) then requires the sum of the free and the strong-interaction Lagrangian densities to be of the form

$$\mathcal{L}_{\text{free}}+\mathcal{L}_{\text{st}}=\mathcal{L}_\rho+\mathcal{L}_m(\psi,D_\nu\psi),$$

where $\mathcal{L}_m$ is invariant under the isospin rotation, $\mathcal{L}_\rho$ is given by (2.6), ψ represents all other matter fields of either half-integer or integer spin, and $D_\nu\psi$ is related to the matrix representation $-i\mathbf{T}$ of the isospin generator on ψ by

$$D_\nu\psi=\frac{\partial}{\partial x_\nu}\psi+g_0(\mathbf{T}\cdot\boldsymbol{\rho}_\nu)\psi. \quad (2.13)$$

If ψ is an isospin $\frac{1}{2}$ field, then $\mathbf{T}=\frac{1}{2}i\boldsymbol{\tau}$, where $\boldsymbol{\tau}$ is the usual isospin Pauli matrix.

For the purpose of our subsequent discussions, the minimal principle is by no means necessary. The Lagrangian density can, then, assume a more general form

$$\mathcal{L}_{\text{free}}+\mathcal{L}_{\text{st}}=\mathcal{L}_\rho+\mathcal{L}_m(\psi,D_\nu\psi,\mathbf{f}_{\mu\nu}), \quad (2.14)$$

showing explicitly that $\mathcal{L}_m$ may also depend on $\mathbf{f}_{\mu\nu}$ which contains the derivatives of the ρ-meson field.

The ρ-meson equation becomes

$$\frac{\partial}{\partial x_\mu}\left(\mathbf{f}_{\mu\nu}-\frac{\partial\mathcal{L}_m}{\partial\mathbf{f}_{\mu\nu}}+\frac{\partial\mathcal{L}_m}{\partial\mathbf{f}_{\nu\mu}}\right)-m_0{}^2\boldsymbol{\rho}_\nu=g_0\mathbf{S}_\nu, \quad (2.15)$$

where the ath isospin component of $\mathbf{S}_\nu$ is given by

$$S_\nu{}^a=-\epsilon^{abc}\left(f_{\mu\nu}{}^b-\frac{\partial\mathcal{L}_m}{\partial f_{\mu\nu}{}^b}+\frac{\partial\mathcal{L}_m}{\partial f_{\nu\mu}{}^b}\right)\rho_\mu{}^c -\sum_\psi(\partial\mathcal{L}_m/\partial D_\nu\psi)T^a\psi. \quad (2.16)$$

Both the current $\mathbf{S}_\nu$ and the field $\boldsymbol{\rho}_\nu$ are conserved

$$\frac{\partial}{\partial x_\nu}\mathbf{S}_\nu=\frac{\partial}{\partial x_\nu}\boldsymbol{\rho}_\nu=0, \quad (2.17)$$

and the spatial integrals of both 4th components are proportional to the generators of the isospin group.

3. ELECTROMAGNETIC AND WEAK INTERACTIONS

We discuss first the explicit form of the electromagnetic-interaction Lagrangian which yields the field-current identity (1.6). By following the general procedures given in Ref. 1, one can easily construct such a gauge-invariant electromagnetic interaction from any hadronic free and strong-interaction Lagrangian density $(\mathcal{L}_{\text{free}}+\mathcal{L}_{\text{st}})$ of the form (2.14). It is convenient to separate $(\mathcal{L}_{\text{free}}+\mathcal{L}_{\text{st}})$ into two parts:

$$\mathcal{L}_{\text{free}}+\mathcal{L}_{\text{st}}=-\tfrac{1}{2}(m_0\boldsymbol{\rho}_\mu)^2+\mathcal{L}_0, \quad (3.1)$$

where

$$\mathcal{L}_0=-\tfrac{1}{4}\mathbf{f}_{\mu\nu}{}^2+\mathcal{L}_m(\psi,D_\nu\psi,\mathbf{f}_{\mu\nu}). \quad (3.2)$$

The corresponding electromagnetic interaction can be generated by replacing $\boldsymbol{\rho}_\mu$ and $(\partial\boldsymbol{\rho}_\mu/\partial x_\nu)$ in $\mathcal{L}_0$ by $\hat{\rho}_\mu$ and $(\partial\hat{\rho}_\mu/\partial x_\nu)$, where the isospin components of $\hat{\rho}_\mu$ are related to those of $\boldsymbol{\rho}_\mu$ by

$$\hat{\rho}_\mu{}^1=\rho_\mu{}^1,\quad \hat{\rho}_\mu{}^2=\rho_\mu{}^2,$$

and

$$\hat{\rho}_\mu{}^0=\rho_\mu{}^0+(e_0/g_0)A_\mu. \quad (3.3)$$

A_μ is the unrenormalized electromagnetic field and e_0 the unrenormalized charge of the electron. The resulting Lagrangian density $\mathcal{L}=\mathcal{L}_{\text{free}}+\mathcal{L}_{\text{st}}+(\mathcal{L}_\gamma)_{I=1}$ is then given by

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}{}^2-\tfrac{1}{2}(m_0\boldsymbol{\rho}_\mu)^2+\mathcal{L}_0', \quad (3.4)$$

where

$$\mathcal{L}_0'=\mathcal{L}_0(\boldsymbol{\rho}_\mu\to\hat{\rho}_\mu)=-\tfrac{1}{4}\hat{f}_{\mu\nu}{}^2+\mathcal{L}_m(\psi,D_\nu'\psi,\hat{f}_{\mu\nu}), \quad (3.5)$$

$$\hat{f}_{\mu\nu}=(\partial\hat{\rho}_\nu/\partial x_\mu)-(\partial\hat{\rho}_\mu/\partial x_\nu)-g_0(\hat{\rho}_\mu\times\hat{\rho}_\nu), \quad (3.6)$$

$$D_\nu'\psi=(\partial\psi/\partial x_\nu)+g_0(\mathbf{T}\cdot\hat{\rho}_\nu)\psi, \quad (3.7)$$

and

$$F_{\mu\nu}=(\partial A_\nu/\partial x_\mu)-(\partial A_\mu/\partial x_\nu). \quad (3.8)$$

The Lagrangian density (3.4) is obviously gauge invariant. We note that from (3.4) both $\boldsymbol{\rho}_\nu$ and $(\partial F_{\mu\nu}/\partial x_\mu)$ are proportional to the variational derivative $(\delta\mathcal{L}_0'/\delta\hat{\rho}_\nu)$, which is defined to be $[(\partial\mathcal{L}_0'/\partial\hat{\rho}_\nu)-(\partial/\partial x_\mu)\partial\mathcal{L}_0'/\partial(\partial\hat{\rho}_\nu/\partial x_\mu)]$

$$m_0^2\boldsymbol{\rho}_\nu=\delta\mathcal{L}_0'/\delta\hat{\rho}_\nu, \quad (3.9)$$

and

$$\frac{\partial F_{\mu\nu}}{\partial x_\mu}=-\left(\frac{e_0}{g_0}\right)\frac{\delta\mathcal{L}_0'}{\delta\hat{\rho}_\nu^0}. \quad (3.10)$$

Thus, one has

$$\partial F_{\mu\nu}/\partial x_\mu=-(e_0m_0^2/g_0)\rho_\nu^0, \quad (3.11)$$

which gives the field-current identity (1.6).

For clarity, only the hadronic electromagnetic isovector current $[J_\mu^\gamma(x)]_{I=1}$ is included in the above Eqs. (3.4)–(3.11). It has been shown in Ref. 1 that the inclusion of the hadronic isoscalar electromagnetic current $[J_\mu^\gamma(x)]_{I=0}$ can be made in a similar way. In addition to the change $\boldsymbol{\rho}_\mu\to\hat{\rho}_\mu$ in the strong-interaction Lagrangian, one replaces also the unrenormalized field ϕ_μ^0 by $[\phi_\mu^0+\frac{1}{2}(e_0/g_Y^0)A_\mu]$ where ϕ_μ^0 is defined to be the field operator coupled to the hypercharge current with a coupling constant g_Y^0. As a result, $(J_\mu^\gamma)_{I=0}$ is proportional to ϕ_μ^0 which in turn is proportional to a linear combination of the renormalized fields $[\phi_\mu(x)]_{\rm ren}$ and $[\omega_\mu(x)]_{\rm ren}$ for the ϕ meson and the ω meson. We have, according to Eq. (1.6) of Ref. 1,

$$(J_\nu^\gamma)_{I=0}=-\tfrac{1}{2}g_Y^{-1}[\cos\theta_Y m_\phi^2(\phi_\mu)_{\rm ren}-\sin\theta_Y m_\omega^2(\omega_\mu)_{\rm ren}], \quad (3.12)$$

where g_Y is the renormalized coupling constant, θ_Y is one of the two mixing angles and m_ϕ, m_ω are the renormalized masses of the ϕ meson and the ω meson, respectively.

Next, we discuss the weak-interaction currents of the hadrons. The field-current identity (1.6) together with the isotriplet current hypothesis of Feynman and Gell-Mann[11] require that the strangeness conserving hadronic weak-interaction vector currents $(V_\mu^{\rm wk})_{S=0}$ be proportional to the appropriate charge component of $\boldsymbol{\rho}_\mu$.

$$(V_\mu^{\rm wk})_{S=0}=-(m_0^2/g_0)(\rho_\mu^1-i\rho_\mu^2) \equiv -\sqrt{2}(m_0^2/g_0)\rho_\mu^+, \quad (3.13)$$

where ρ_μ^+ is the field operator that annihilates (creates) the positively (negatively) charged ρ mesons. In terms of the renormalized mass m_ρ, the renormalized coupling constant g_ρ and the renormalized field $(\rho_\mu^+)_{\rm ren}$, (3.13) can be written as

$$(V_\mu^{\rm wk})_{S=0}=-\sqrt{2}(m_\rho^2/g_\rho)(\rho_\mu^+)_{\rm ren}. \quad (3.14)$$

[11] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

It is natural to extend the field-current identity also to other weak-interaction currents. For example, one would assume the strangeness-nonconserving hadronic weak-interaction current $(V_\mu^{\rm wk})_{S=1}$ to be proportional to the field operator of the $K^*(890)$ meson, and the strangeness-conserving and nonconserving parts of the hadronic weak axial-vector currents $(A_\mu^{\rm wk})_{S=0}$ and $(A_\mu^{\rm wk})_{S=1}$ to be, respectively, proportional to the field operators of a charged axial-vector meson such as $A1$ and its corresponding SU_3 multiplet member, e.g., K_A^*. Such a generalization and some of its consequences have already been discussed in Ref. 5. One has then

$$\begin{aligned}[V_\mu^{\rm wk}(x)]_{S=1}&=-\sqrt{2}(m_0^2/g_0)K_\mu^*(x)\\&=-\sqrt{2}(m_\rho^2/g_\rho)[K_\mu^*(x)]_{\rm ren},\\ [A_\mu^{\rm wk}(x)]_{S=0}&=-\sqrt{2}(m_0^2/g_0)A1_\mu(x)\\&=-\sqrt{2}(m_\rho^2/g_\rho)[A1_\mu(x)]_{\rm ren},\\ [A_\mu^{\rm wk}(x)]_{S=1}&=-\sqrt{2}(m_0^2/g_0)K_{A\mu}^*(x)\\&=-\sqrt{2}(m_\rho^2/g_\rho)[K_{A\mu}^*(x)]_{\rm ren},\end{aligned} \quad (3.15)$$

where $K_\mu^*(x)$, $A1_\mu(x)$, and $K_{A\mu}^*(x)$ are the unrenormalized field operators of K^*, $A1$. and K_A^* mesons. These are assumed to have the same unrenormalized coupling constant g_0 and the same unrenormalized mass m_0. [The general case that these mesons may have different unrenormalized masses is discussed in Sec. 5.] For convenience, we have chosen in (3.15) the same wave-function renormalization factor[9] $Z=Z_\rho=(m_\rho/m_0)^2$ for all these different meson fields, where m_ρ is the renormalized mass of the neutral ρ meson.

In the above expressions, all hadron currents are properly normalized, so that the semileptonic weak-interaction Lagrangian density is

$$2^{-1/2}GJ_\mu^{\rm wk}j_\mu^{\rm wk}+{\rm H.c.}, \quad (3.16)$$

where

$$J_\mu^{\rm wk}=\cos\theta_c(V_\mu^{\rm wk}+A_\mu^{\rm wk})_{S=0} +\sin\theta_c(V_\mu^{\rm wk}+A_\mu^{\rm wk})_{S=1}, \quad (3.17)$$

θ_c is the Cabibbo angle,[12] G is the Fermi constant for μ decay $\cong(10^{-5}/m_N^2)$, and i_μ^{wk} is the leptonic weak-interaction current.

4. SU_2 FIELD ALGEBRA (INCLUDING ELECTROMAGNETIC EFFECTS)

An important consequence of the field-current identities is that the observed hadronic electromagnetic and weak-interaction current operators should satisfy the same equal-time commutators as the corresponding fields. The details of these commutation relations which are called the field algebra have been analyzed in Ref. 5. The explicit Lagrangian density given in the previous section makes it possible to include in these algebraic relations also the necessary effects of the electromagnetic field.

[12] N. Cabibbo, Phys. Letters **10**, 513 (1963).

We begin with the Lagrangian density $\mathcal{L}$ given by (3.4) and, for convenience, adopt the Coulomb gauge. Let $A_j^{\rm tr}$ be the 3-vector which denotes the usual transverse electromagnetic vector potential; it satisfies

$$(\partial A_j^{\rm tr}/\partial r_j)=0. \tag{4.1}$$

In the Coulomb gauge, the 4-vector potential A_μ is given by

$$A_j=A_j^{\rm tr} \quad \text{and} \quad A_4=i\phi, \tag{4.2}$$

where, on account of (3.11), ϕ is the solution of the Laplace equation

$$\Delta\phi=i(e_0m_0^2/g_0)\rho_4^0.$$

Correspondingly, the electric field E_j can be written as $E_j=E_j^{\rm tr}+E_j^{\rm long}$, where

$$E_j^{\rm tr}=-(\partial A_j^{\rm tr}/\partial t) \quad \text{and} \quad E_j^{\rm long}=-(\partial\phi/\partial r_j). \tag{4.3}$$

To apply the canonical formalism, it is convenient to replace the $-\frac{1}{4}F_{\mu\nu}^2$ term in the Lagrangian density (3.4) by

$$\tfrac{1}{2}[(E_j^{\rm tr})^2+(E_j^{\rm long})^2-H_j^2], \tag{4.4}$$

where $H_j=\epsilon_{jkl}(\partial A_l^{\rm tr}/\partial r_k)$ denotes the magnetic field. It is clear that the spatial integral of (4.4) is the same as that of $-\frac{1}{4}F_{\mu\nu}^2$. The field algebra can then be most easily derived by choosing $\hat{\rho}_j^a$, $A_j^{\rm tr}$, and ψ as the generalized coordinates. By using (3.4), (3.5), and (4.4) one finds that their conjugate momenta are, respectively,

$$\hat{P}_j^a=i[\hat{f}_{4j}^a-(\partial\mathcal{L}_m/\partial\hat{f}_{4j}^a)+(\partial\mathcal{L}_m/\partial\hat{f}_{j4}^a)], \quad \Pi_j^{\rm tr}=-E_j^{\rm tr},$$

and (4.5)

$$P_\psi=-i(\partial\mathcal{L}_m/\partial D_4'\psi).$$

It is useful to introduce the renormalization-independent field operators

$$\rho_\mu'^a(r,t)=(m_0^2/g_0)\rho_\mu^a(r,t)=(m_\rho^2/g_\rho)[\rho_\mu^a(r,t)]_{\rm ren}. \tag{4.6}$$

From the canonical commutation relations it follows that the field operator $\rho_\mu'^a$ satisfies the following equal-time commutation relations:

$$[\rho_i'^a(r,t),\rho_j'^b(r',t)]=0; \tag{4.7}$$

$$[\rho_4'^a(r,t),\rho_j'^b(r',t)]=\epsilon^{abc}\delta^3(r-r')\rho_j'^c(r',t)+(m_\rho/g_\rho)^2\delta^{ab}(\partial/\partial r_j)\delta^3(r-r')+e\epsilon^{abc}(m_\rho/g_\rho)^2[A_j^{\rm tr}(r',t)]_{\rm ren}, \tag{4.8}$$

and

$$[\rho_4'^a(r,t),\rho_4'^b(r',t)]=\epsilon^{abc}\delta^3(r-r')\rho_4'^c(r',t). \tag{4.9}$$

In deriving these, we have made use of the fact that

$$\rho_4^a=m_0^{-2}[(\partial/\partial r_j)(i\hat{P}_j^a)-ig_0(-\epsilon^{abc}\hat{P}_j^b\hat{\rho}_j^c+P_\psi T^a\psi)],$$

and the renormalized electromagnetic field $(A_j^{\rm tr})_{\rm ren}$ and that the renormalized charge e are related to the unrenormalized quantities $A_j^{\rm tr}$ and e_0 by

$$e(A_j^{\rm rt})_{\rm ren}=e_0A_j^{\rm tr}. \tag{4.10}$$

Besides (4.7)–(4.9), there are other commutation relations between the electromagnetic fields and ρ_μ^a; these are given in Appendix A. Throughout the paper, e denotes the renormalized charge of the electron.

From the field-current identities (1.6) and (3.13), it follows that

$$(J_\mu^\gamma)_{I=1}=-\rho_\mu'^0,$$

and

$$(V_\mu^{\rm wk})_{S=0}=-\sqrt{2}\rho_\mu'^+=-(\rho_\mu'^1-i\rho_\mu'^2).$$

These observed hadronic current operators, therefore, satisfy the following field algebra (valid to all orders in e):

$$[J_i^\gamma(r,t),J_j^\gamma(r',t)]=[J_4^\gamma(r,t),J_4^\gamma(r',t)]=0, \tag{4.11}$$

$$[J_4^\gamma(r,t),J_j^\gamma(r',t)]=(m_\rho/g_\rho)^2(\partial/\partial r_j)\times\delta^3(r-r'), \tag{4.12}$$

$$[V_i^{\rm wk}(r,t),J_j^\gamma(r',t)]=0, \tag{4.13}$$

$$[V_4^{\rm wk}(r,t),J_\mu^\gamma(r',t)]=-[J_4^\gamma(r,t),V_\mu^{\rm wk}(r',t)]=i\delta^3(r-r')V_\mu^{\rm wk}(r',t), \tag{4.14}$$

$$[V_i^{\rm wk}(r,t),V_j^{\rm wk}(r',t)]=[V_i^{\rm wk}(r,t),V_j^{\rm wk}(r',t)^\dagger]=0, \tag{4.15}$$

$$[V_4^{\rm wk}(r,t),V_\mu^{\rm wk}(r',t)]=0, \tag{4.16}$$

$$[V_4^{\rm wk}(r,t),V_4^{\rm wk}(r',t)^\dagger]=2i\delta^3(r-r')J_4^\gamma(r',t), \tag{4.17}$$

and

$$[V_4^{\rm wk}(r,t),V_j^{\rm wk}(r',t)^\dagger]=-2i\delta^3(r-r')J_j^\gamma(r',t)+2(m_\rho/g_\rho)^2\left(\frac{\partial}{\partial r_j}+ie[A_j^{\rm tr}(r',t)]_{\rm ren}\right)\delta^3(r-r'), \tag{4.18}$$

where the dagger denotes Hermitian conjugation. For clarity, we have omitted the subscripts $I=1$ and $S=0$; in the above Eqs. (4.11)–(4.18), J_μ^γ stands for $(J_\mu^\gamma)_{I=1}$ and $V_\mu^{\rm wk}$ for $(V_\mu^{\rm wk})_{S=0}$.

Although for convenience we have adopted the Coulomb gauge in our derivation, it can be readily shown that the above Eqs. (4.11)–(4.18) are valid in any gauge, provided that in (4.18) one replaces $(A_j^{\rm tr})_{\rm ren}$ by the appropriate $(A_j)_{\rm ren}$.

All above formulas, except (4.18), are formally unchanged with the inclusion of the electromagnetic interaction. They differ from those of current algebra by having the Schwinger term finite and the commutators of all spatial current components zero.[13] As already noted in the introduction, the presence of a term proportional to $eA_j\delta^3(r-r')$ in the commutator $[V_4^{\rm wk}(r,t),V_j^{\rm wk}(r',t)^\dagger]$ is a general consequence of gauge invariance.

[13] In principle, since the matrix elements of these current operators are measurable, the validity of these commutation relations can be tested. A convenient way is to use, e.g., the appropriate sum rules for the high-energy neutrino processes developed by S. L. Adler [Phys. Rev. **143**, 1144 (1965)]. At present, it is unclear whether the assumption of unsubtracted dispersion relations made in deriving these sum rules is justified or not.

The presence of the electromagnetic interaction destroys the isospin invariance. Nevertheless, one may still define the generator **I** of the isospin group by

$$\mathbf{I}=i\int \varrho_4' d^3r. \tag{4.19}$$

Through the field-current identities, its components I^a ($a=0, 1, 2$) are also given by

$$I^0=-i\int (J_4{}^\gamma)_{I=1}d^3r,$$
$$I^-=\tfrac{1}{2}(I^1-iI^2)=-i2^{-1/2}\int (V_4{}^{\rm wk})_{S=0}d^3r, \tag{4.20}$$

and

$$I^+=\tfrac{1}{2}(I^1+iI^2)=+i2^{-1/2}\int (V_4{}^{\rm wk})^\dagger{}_{S=0}d^3r.$$

The equal-time commutator of $I^a(t)$ is, on account of (4.9),

$$[I^a(t),I^b(t)]=\epsilon^{abc}I^c(t), \tag{4.21}$$

which is valid to all orders in e.

By using (4.7)–(4.9) and the additional canonical commutation relation

$$[\rho_4'^a(r,t),A_j{}^{\rm tr}(r',t)]=0, \tag{4.22}$$

one finds (valid to all orders in e)

$$[I^a(t),\rho_4{}^b(r,t)]=\epsilon^{abc}\rho_4{}^c(r,t), \tag{4.23}$$

but

$$[I^a(t),\hat{p}_j{}^b(r,t)]=\epsilon^{abc}\hat{p}_j{}^c(r,t). \tag{4.24}$$

Thus, under the isospin rotation, the components of ϱ_4, or the equivalent current operators

$$[V_4{}^{\rm wk}(r,t)]_{S=0},\quad [J_4{}^\gamma(r,t)]_{I=1},\quad \text{and}\quad [V_4{}^{\rm wk}(r,t)]^\dagger{}_{S=0}, \tag{4.25}$$

form an isotriplet, but for the spatial part it is those of $\hat{p}_j$, or

$$[V_j{}^{\rm wk}(r,t)]_{S=0},\quad [J_j{}^\gamma(r,t)]_{I=1}+e(m_\rho/g_\rho)^2[A_j{}^{\rm tr}(r,t)]_{\rm ren},\quad \text{and}\quad [V_j{}^{\rm wk}(r,t)]^\dagger{}_{S=0}, \tag{4.26}$$

that form an isotriplet.

5. BROKEN SYMMETRIES

As shown by Eq. (3.15), the generalization of the field-current identities to other weak-interaction currents requires the existence of both vector and axial-vector field operators. In place of the isospin symmetry, the appropriate symmetry group $\mathcal{G}$ becomes either

$$\mathcal{G}=SU_2\times SU_2,\quad \text{or}\quad SU_3,\quad \text{or}\quad SU_3\times SU_3, \tag{5.1}$$

or some other possibilities. All such symmetries are known to be broken or badly broken by the strong interaction. In the following, we assume the existence of N spin-1 fields $\phi_\mu{}^1$, $\phi_\mu{}^2$, $\cdots\phi_\mu{}^N$ consisting of both vector and axial-vector fields which include the three ρ-meson fields. The total number N of $\phi_\mu{}^a$ is the same as the number of generators of $\mathcal{G}$. For example, in the case $\mathcal{G}=SU_2\times SU_2$, one has $N=6$, and the set of spin-1 meson fields $\phi_\mu{}^a$ consists of three ρ-meson fields and three additional axial-vector meson fields. As we shall see, the existence of these spin-1 meson field operators makes possible a unified treatment of these broken symmetries.

It is useful to separate the total free and strong-interaction Lagrangian density of the hadrons into two parts

$$\mathcal{L}_{\rm free}+\mathcal{L}_{\rm st}=\mathcal{L}_\phi+\mathcal{L}_m, \tag{5.2}$$

where $\mathcal{L}_\phi$ depends *only* on the meson fields $\phi_\mu{}^a$, $\mathcal{L}_m$ depends on $\phi_\mu{}^a$ and also on other matter fields, represented by ψ. The infinitesimal transformations of the symmetry group $\mathcal{G}$ can be represented by

$$\phi_\mu{}^a\rightarrow\phi_\mu{}^a+C^{abc}(\delta\theta)^b\phi_\mu{}^c, \tag{5.3}$$

and

$$\psi\rightarrow\psi+T^a(\delta\theta)^a\psi, \tag{5.4}$$

where $\delta\theta^a$ is a set of infinitesimal numbers, C^{abc} is the totally antisymmetric structure constant of the symmetry group given by (5.1), and $-iT^a$ is the matrix representation of its Hermitian generators on ψ which satisfies

$$[T^a,T^b]=C^{abc}T^c. \tag{5.5}$$

In this section all superscripts a, b, or c, vary from 1 to N. [For SU_2 symmetry, $C^{abc}=-\epsilon^{abc}$, and T^a is related to the Pauli matrices τ^a by $T^a=\frac{1}{2}i\tau^a$ for the isospin-$\frac{1}{2}$ fields.]

We assume that the meson part $\mathcal{L}_\phi$ is *invariant* under the transformation (5.3), but $\mathcal{L}_m$ may violate the symmetry; i.e., under (5.3) and (5.4)

$$\mathcal{L}_\phi\rightarrow\mathcal{L}_\phi, \tag{5.6}$$

but

$$\mathcal{L}_m\rightarrow\mathcal{L}_m-P^a\delta\theta^a. \tag{5.7}$$

By assumption, the ρ-meson fields are included in the set $\phi_\mu{}^a$; therefore, the ρ-meson part of the Lagrangian, given by (2.6), is contained in $\mathcal{L}_\phi$. Thus, the invariance assumption of $\mathcal{L}_\phi$ under the larger group of transformations (5.3) requires that[14]

$$\mathcal{L}_\phi=-\tfrac{1}{4}(f_{\mu\nu}{}^a)^2-\tfrac{1}{2}m_0{}^2(\phi_\mu{}^a)^2, \tag{5.8}$$

where

$$f_{\mu\nu}{}^a=\frac{\partial}{\partial x_\mu}\phi_\nu{}^a-\frac{\partial}{\partial x_\nu}\phi_\mu{}^a+g_0C^{abc}\phi_\mu{}^b\phi_\nu{}^c. \tag{5.9}$$

In the case of broken symmetries, there is a great deal of arbitrariness in the symmetry-violating interaction

[14] Except for the mass term, $\mathcal{L}_\phi$ is the same as the Lagrangian of the generalized Yang-Mills theory discussed by R. Utiyama, Phys. Rev. **101**, 1597 (1956); M. Gell-Mann and S. Glashow, Ann. Phys. (N. Y.) **15**, 437 (1961).

$\mathcal{L}_m$. We will assume that, similar to (2.14), $\mathcal{L}_m$ is of the form

$$\mathcal{L}_m=\mathcal{L}_m(\psi,D_\nu\psi,f_{\mu\nu}{}^a); \quad (5.10)$$

i.e., $\mathcal{L}_m$ can be an arbitrary function of ψ, $f_{\mu\nu}{}^a$, and $D_\nu\psi$, where

$$D_\nu\psi=(\partial\psi/\partial x_\nu)+g_0T^a\phi_\nu{}^a\psi. \quad (5.11)$$

This symmetry-violating Lagrangian density (5.10) is quite general. Among others, it may contain symmetry-violating terms like

$$K^{ab}f_{\mu\nu}{}^af_{\mu\nu}{}^b,\quad M_{\mu\nu}{}^af_{\mu\nu}{}^a,\quad \text{etc.},$$

where K^{ab} can be an arbitrary set of constants, or it can be an arbitrary set of (space-time) scalar functions of ψ and $D_\nu\psi$, and $M_{\mu\nu}{}^a$ can be an arbitrary set of (space-time) tensor functions of ψ and $D_\nu\psi$.

It can be readily verified that, independent of the detailed structure of the symmetry-breaking form of $\mathcal{L}_m$, the canonical commutation rules imply that, in the absence of the electromagnetic interaction, $\phi_\mu{}^a$ satisfied the following algebra of fields[5]:

$$[\phi_i{}^a(r,t),\phi_j{}^b(r',t)]=0, \quad (5.12)$$

$$[\phi_4{}^a(r,t),\phi_4{}^b(r',t)] =-(g_0/m_0{}^2)C^{abc}\delta^3(r-r')\phi_4{}^c(r',t), \quad (5.13)$$

and

$$[\phi_4{}^a(r,t),\phi_j{}^b(r',t)]=-(g_0/m_0{}^2)C^{abc}\delta^3(r-r')\phi_j{}^c(r',t) +m_0{}^{-2}\delta^{ab}(\partial/\partial r_j)\delta^3(r-r'). \quad (5.14)$$

In deriving these, we have used the fact that the canonical momenta conjugate to $\phi_j{}^a$ and ψ, and are, respectively,

$$P_j{}^a=i[f_{4j}{}^a-(\partial\mathcal{L}_m/\partial f_{4j}{}^a)+(\partial\mathcal{L}_m/\partial f_{j4}{}^a)],$$

and

$$P_\psi=-i(\partial\mathcal{L}_m/\partial D_4\psi). \quad (5.15)$$

Through the equation of motion

$$\frac{\partial}{\partial x_\mu}\left[f_{\mu\nu}{}^a-\frac{\partial\mathcal{L}_m}{\partial f_{\mu\nu}{}^a}+\frac{\partial\mathcal{L}_m}{\partial f_{\nu\mu}{}^a}\right]-m_0{}^2\phi_\nu{}^a=g_0S_\nu{}^a, \quad (5.16)$$

where

$$S_\nu{}^a=C^{abc}[f_{\mu\nu}{}^b-(\partial\mathcal{L}_m/\partial f_{\mu\nu}{}^b)+(\partial\mathcal{L}_m/\partial f_{\nu\mu}{}^b)]\phi_\mu{}^c -\sum_\psi(\partial\mathcal{L}_m/\partial D_\nu\psi)T^a\psi, \quad (5.17)$$

the 4th component $\phi_4{}^a$ is given by

$$\phi_4{}^a=m_0{}^{-2}[(\partial/\partial r_j)(iP_j{}^a) -ig_0(C^{abc}P_j{}^b\phi_j{}^c-P_\psi T^a\psi)]. \quad (5.18)$$

The field-current identities (1.6), (3.14), and (3.15) imply that the various hadronic electromagnetic and weak-interaction current operators satisfy the same algebra of fields. From (5.16) and (5.17), it follows that

$$-(m_0{}^2/g_0)(\partial\phi_\nu{}^a/\partial x_\nu)=\partial S_\nu{}^a/\partial x_\nu=P^a, \quad (5.19)$$

where P^a is given by (5.7). The usual PCAC (partially conserved axial-vector current) approximation implies that for the divergence of the axial-vector isovector field $\phi_\mu{}^i$, the corresponding P^a can be approximated by a constant times the pion field; i.e.,

$$(\partial\phi_\mu{}^i/\partial x_\mu)\propto\pi^i(x),$$

where $\pi^i(x)$ denotes the pion fields and i represents the isospin index. [If one wishes, similar approximations can also be made for other pseudoscalar fields.]

If the following combination of derivatives of $\mathcal{L}_m$ commutes with $\phi_j{}^b$ at equal time:

$$[\partial\mathcal{L}_m/\partial f_{4i}{}^a(r,t)-\partial\mathcal{L}_m/\partial f_{i4}{}^a(r,t),\ \phi_j{}^b(r',t)]=0, \quad (5.20)$$

then one has, in addition,

$$[(\partial/\partial t)\phi_j{}^a(r,t)-i(\partial/\partial r_j)\phi_4{}^a(r,t),\ \phi_k{}^b(r',t)] =-i\delta^{ab}\delta^3(r-r')\delta_{jk}-iC^{abc}(g_0/m_0{}^2)\phi_j{}^c(r,t) \times(\partial/\partial r_k)\delta^3(r-r')+i(g_0/m_0)^2C^{adc}C^{dbe}\delta^3(r-r') \times\phi_j{}^c(r,t)\phi_k{}^e(r',t). \quad (5.21)$$

As an example of (5.20), $\mathcal{L}_m$ can be of the form

$$M_{\mu\nu}{}^a(\psi)f_{\mu\nu}{}^a+\mathcal{L}_m'(\psi,D_\nu\psi), \quad (5.22)$$

where $M_{\mu\nu}{}^a=\partial\mathcal{L}_m/\partial f_{\mu\nu}{}^a$ depends only on ψ.

We note that according to (5.8), the unrenormalized masses of $\phi_\mu{}^a$ are all equal. A simple way to break the symmetry is to introduce different unrenormalized masses for different $\phi_\mu{}^a$. Such a possibility can be easily incorporated in our scheme by introducing a symmetry-breaking term $(K_{ab}f_{\mu\nu}{}^af_{\mu\nu}{}^b)$ in $\mathcal{L}_m$. The unrenormalized masses of the mesons $\phi_\mu{}^a$ become, then, the different eigenvalues of the $(N\times N)$ matrix $m_0{}^2(1+K)^{-1}$, where K=matrix (K_{ab}). In this case, the commutation relations (5.12)–(5.14) remain valid, but (5.21) should be modified.

If the symmetry-breaking interaction $\mathcal{L}_m$ is independent of $f_{\mu\nu}{}^a$, but otherwise can be an arbitrary function of ψ and $D_\nu\psi$, then one has the following additional equal-time commutation relations

$$[(\partial/\partial t)\phi_j{}^a(r,t)-i(\partial/\partial r_j)\phi_4{}^a(r,t),\ \phi_4{}^b(r',t)] =-(g_0/m_0{}^2)C^{abc}\delta^3(r-r')((\partial\phi_j{}^a/\partial t)-i(\partial\phi_4{}^a/\partial r_j)) +i(g_0/m_0{}^2)C^{abc}\phi_4{}^c(r,t)(\partial/\partial r_j)\delta^3(r-r'), \quad (5.23)$$

and

$$[f_{4j}{}^a(r,t),f_{4k}{}^b(r',t)]=0. \quad (5.24)$$

By adding these algebraic relations to those already derived, one can also determine the equal-time commutator between

$$(\partial/\partial t)\phi_j{}^a(r,t)-i(\partial/\partial r_j)\phi_4{}^a(r,t),$$

and

$$(\partial/\partial t)\phi_k{}^b(r',t)-i(\partial/\partial r_k{}')\phi_4{}^b(r',t),$$

though its explicit form is somewhat lengthy.

6. $SU_3\times SU_3$ FIELD ALGEBRA (INCLUDING ELECTROMAGNETIC EFFECTS)

As an illustration of how the above considerations can be extended to include the electromagnetic interaction, we discuss in some detail the special case of

$$\mathcal{G}=SU_3\times SU_3. \tag{6.1}$$

[The general case is discussed in Appendix A.] We assume the existence of eight vector fields v_μ^1, v_μ^2, $\cdots$, v_μ^8, and eight axial-vector fields a_μ^1, a_μ^2, $\cdots$, a_μ^8, where the vector fields v_μ^α are related to the ρ-meson fields $\boldsymbol{\rho}_\mu$ and the K^{*+}, K^{*0} meson fields by

$$v_\mu^1=\rho_\mu^1,\quad v_\mu^2=\rho_\mu^2,\quad v_\mu^3=\rho_\mu^0,$$
$$2^{-1/2}(v_\mu^4-iv_\mu^5)=K^{*+},\ \text{and}\ 2^{-1/2}(v_\mu^6-iv_\mu^7)=K^{*0}. \tag{6.2}$$

In this case, it is convenient to represent the field tensors (5.9) by $V_{\mu\nu}^\alpha$ and $A_{\mu\nu}^\alpha$:

$$V_{\mu\nu}^\alpha=\frac{\partial}{\partial x_\mu}v_\nu^\alpha-\frac{\partial}{\partial x_\nu}v_\mu^\alpha-g_0f^{\alpha\beta\gamma}(v_\mu^\beta v_\nu^\gamma+a_\mu^\beta a_\nu^\gamma), \tag{6.3}$$

and

$$A_{\mu\nu}^\alpha=\frac{\partial}{\partial x_\mu}a_\nu^\alpha-\frac{\partial}{\partial x_\nu}a_\mu^\alpha-g_0f^{\alpha\beta\gamma}(v_\mu^\beta a_\nu^\gamma+a_\mu^\beta v_\nu^\gamma), \tag{6.4}$$

where

$$-f^{\alpha\beta\gamma}=\text{totally antisymmetric structure constant of the } SU_3 \text{ group}, \tag{6.5}$$

and α, β, γ vary independently from 1 to 8. According to (5.8) and (5.10), the free and the strong-interaction Lagrangian density of hadrons can be written as

$$\mathcal{L}_{\text{free}}+\mathcal{L}_{\text{st}}=-\tfrac{1}{2}m_0^2(v_\mu^\alpha)^2-\tfrac{1}{2}m_0^2(a_\mu^\alpha)^2+\mathcal{L}_0, \tag{6.6}$$

and

$$\mathcal{L}_0=-\tfrac{1}{4}(V_{\mu\nu}^\alpha)^2-\tfrac{1}{4}(A_{\mu\nu}^\alpha)^2+\mathcal{L}_m(\psi,D_\nu\psi,V_{\mu\nu}^\alpha,A_{\mu\nu}^\alpha), \tag{6.7}$$

where$D_\nu\psi$ is defined by (5.11).

Following the arguments given in Sec. 3, one finds that, similar to (3.4), the Lagrangian density, $\mathcal{L}=\mathcal{L}_{\text{free}}+\mathcal{L}_{\text{st}}+\mathcal{L}_\gamma$, is

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}^2-\tfrac{1}{2}m_0^2(v_\mu^\alpha)^2-\tfrac{1}{2}m_0^2(a_\mu^\alpha)^2+\mathcal{L}_0'. \tag{6.8}$$

The function $\mathcal{L}_0'$ is the same as $\mathcal{L}_0$ except for the replacement of v_μ^α by $\hat{v}_\mu^\alpha$, and $\hat{v}_\mu^\alpha$ is given by

$$\hat{v}_\mu^3=v_\mu^3+(e_0/g_0)A_\mu,$$
$$\hat{v}_\mu^8=v_\mu^8+3^{-1/2}(e_0/g_0)A_\mu, \tag{6.9}$$

and

$$\hat{v}_\mu^\alpha=v_\mu^\alpha\quad\text{for}\quad 3\neq\alpha\neq 8.$$

This replacement changes $D_\nu\psi$, $V_{\mu\nu}^\alpha$, and $A_{\mu\nu}^\alpha$ to $D_\nu'\psi$, $\hat{V}_{\mu\nu}^\alpha$, and $\hat{A}_{\mu\nu}^\alpha$, respectively, where

$$\hat{V}_{\mu\nu}^\alpha=\frac{\partial}{\partial x_\mu}\hat{v}_\nu^\alpha-\frac{\partial}{\partial x_\nu}\hat{v}_\mu^\alpha-g_0f^{\alpha\beta\gamma}(\hat{v}_\mu^\beta\hat{v}_\nu^\gamma+a_\mu^\beta a_\nu^\gamma), \tag{6.10}$$

and

$$\hat{A}_{\mu\nu}^\alpha=\frac{\partial}{\partial x_\mu}a_\nu^\alpha-\frac{\partial}{\partial x_\nu}a_\mu^\alpha-g_0f^{\alpha\beta\gamma}(\hat{v}_\mu^\beta a_\nu^\gamma+a_\mu^\beta\hat{v}_\nu^\gamma). \tag{6.11}$$

Correspondingly,

$$\mathcal{L}_0'=\mathcal{L}_0(v_\mu^\alpha\to\hat{v}_\mu^\alpha)=-\tfrac{1}{4}(\hat{V}_{\mu\nu}^\alpha)^2-\tfrac{1}{4}(\hat{A}_{\mu\nu}^\alpha)^2+\mathcal{L}_m(\psi,D_\nu'\psi,\hat{V}_{\mu\nu}^\alpha,\hat{A}_{\mu\nu}^\alpha). \tag{6.12}$$

To derive the field algebra valid to all orders in e, we follow the steps outlined in Sec. 4. In the Coulomb gauge, A_μ is given by

$$A_j=A_j^{\text{tr}}\quad\text{and}\quad A_4=i\phi, \tag{6.13}$$

where A_j^{tr} satisfies (4.1) and ϕ is the solution of the Laplace equation

$$\Delta\phi=ie_0(m_0^2/g_0)(v_4^3+3^{-1/2}v_4^8). \tag{6.14}$$

It is convenient to regard $\hat{v}_j^\alpha$, a_j^α, A_j^{tr}, and ψ as independent variables. Similar to (4.5), their conjugate momenta are, respectively,

$$(\hat{P}_v)_j^\alpha=i[\hat{V}_{4j}^\alpha-(\partial\mathcal{L}_m/\partial\hat{V}_{4j}^\alpha)+(\partial\mathcal{L}_m/\partial\hat{V}_{j4}^\alpha)],$$
$$(\hat{P}_a)_j^\alpha=i[\hat{A}_{4j}^\alpha-(\partial\mathcal{L}_m/\partial\hat{A}_{4j}^\alpha)+(\partial\mathcal{L}_m/\partial\hat{A}_{j4}^\alpha)],$$
$$\Pi_j^{\text{tr}}=-E_j^{\text{tr}}=(\partial A_j^{\text{tr}}/\partial t), \tag{6.15}$$

and

$$P_\psi=-i(\partial\mathcal{L}_m/\partial D_4'\psi).$$

By using the canonical commutation relations, the algebra of fields (5.12)–(5.14) can be easily generalized to include the electromagnetic effects. In writing down these expressions, it is convenient to use the renormalization-independent field operators introduced in Ref. 5. We define

$$v_\mu'^\alpha=(m_0^2/g_0)v_\mu^\alpha=(m_\rho^2/g_\rho)(v_\mu^\alpha)_{\text{ren}},$$
$$a_\mu'^\alpha=(m_0^2/g_0)a_\mu^\alpha=(m_\rho^2/g_\rho)(a_\mu^\alpha)_{\text{ren}}, \tag{6.16}$$

and

$$\hat{v}_\mu'^\alpha=(m_0^2/g_0)\hat{v}_\mu^\alpha=(m_\rho^2/g_\rho)(\hat{v}_\mu^\alpha)_{\text{ren}},$$

where the subscript ren denotes the renormalized operator. For convenience, we have chosen the same wave-function renormalization for all v_μ^α and a_μ^α as that of the ρ meson; i.e.,

$$g_0v_\mu^\alpha=g_\rho(v_\mu^\alpha)_{\text{ren}},\quad g_0a_\mu^\alpha=g_\rho(a_\mu^\alpha)_{\text{ren}}, \tag{6.17}$$

where

$$(g_0/g_\rho)=(m_0/m_\rho).$$

Since, by definition, the renormalized electromagnetic field $(A_\mu)_{\text{ren}}$ and the renormalized charge e are related to the unrenormalized quantities A_μ and e_0 by

$$e(A_\mu)_{\text{ren}}=e_0A_\mu,$$

one finds from (6.9) that

$$(\hat{v}_\mu^3)_{\text{ren}}=(v_\mu^3)_{\text{ren}}+(e/g_\rho)(A_\mu)_{\text{ren}},$$
$$(\hat{v}_\mu^8)_{\text{ren}}=(v_\mu^8)_{\text{ren}}+3^{-1/2}(e/g_\rho)(A_\mu)_{\text{ren}}, \tag{6.18}$$

and

$$(\hat{v}_\mu{}^\alpha)_{\rm ren}=(v_\mu{}^\alpha)_{\rm ren} \quad \text{for} \quad 3\neq\alpha\neq 8.$$

The equal-time commutators (5.12)–(5.14) become (valid to all orders in e)

$$[v_i'^\alpha(r,t),v_j'^\beta(r',t)]=[a_i'^\alpha(r,t),a_j'^\beta(r',t)]=[v_i'^\alpha(r,t),a_j'^\beta(r',t)]=0\,, \quad (6.19)$$

$$[v_4'^\alpha(r,t),v_4'^\beta(r',t)]=[a_4'^\alpha(r,t),a_4'^\beta(r',t)]=f^{\alpha\beta\gamma}\delta^3(r-r')v_4'^\gamma(r',t)\,, \quad (6.20)$$

$$[v_4'^\alpha(r,t),a_4'^\beta(r',t)]=f^{\alpha\beta\gamma}\delta^3(r-r')a_4'^\gamma(r',t)\,, \quad (6.21)$$

$$[v_4'^\alpha(r,t),v_j'^\beta(r',t)]=[a_4'^\alpha(r,t),a_j'^\beta(r',t)]=f^{\alpha\beta\gamma}\delta^3(r-r')\hat{v}_j'^\gamma+(m_\rho/g_\rho)^2\times\delta^{\alpha\beta}(\partial/\partial r_j)\delta^3(r-r')\,, \quad (6.22)$$

and

$$[v_4'^\alpha(r,t),a_j'^\beta(r',t)]=[a_4'^\alpha(r,t),v_j'^\beta(r',t)]=f^{\alpha\beta\gamma}\delta^3(r-r')a_j'^\gamma(r',t)\,. \quad (6.23)$$

In the above expressions, the electromagnetic field enters explicitly only in (6.22) through $\hat{v}_j'^\gamma$. [Further commutation relations are given in Appendix A.]

In terms of $v_\mu'^\alpha$ and $a_\mu'^\alpha$, the observed hadronic electromagnetic current operators and the weak-interaction current operators become

$$(J_\mu{}^\gamma)_{I=1}=-v_\mu'^3\,,\quad (J_\mu{}^\gamma)_{I=0}=-3^{-1/2}v_\mu'^8\,,$$
$$(V_\mu{}^{\rm wk})_{S=0}=-(v_\mu'^1-iv_\mu'^2)\,,$$
$$(V_\mu{}^{\rm wk})_{S=1}=-(v_\mu'^4-iv_\mu'^5)\,, \quad (6.24)$$
$$(A_\mu{}^{\rm wk})_{S=0}=-(a_\mu'^1-i\alpha_\mu'^2)\,,$$

and

$$(A_\mu{}^{\rm wk})_{S=1}=-(a_\mu'^4-ia_\mu'^5)\,.$$

These field-current identities imply that these hadron current operators satisfy the same equal-time commutation relations as those of the fields. Most of the commutators of these hadron currents do not explicitly depend on the electromagnetic field. Those which are modified by the presence of the electromagnetic fields are

$$[V_4{}^{\rm wk}(r,t)_{S=0},V_j{}^{\rm wk}(r',t)^\dagger{}_{S=0}]=[A_4{}^{\rm wk}(r,t)_{S=0},A_j{}^{\rm wk}(r',t)^\dagger{}_{S=0}]=-2i\delta^3(r-r')(J_j{}^\gamma)_{I=1}+2(m_\rho/g_\rho)^2\left[\frac{\partial}{\partial r_j}+ie(A_j)_{\rm ren}\right]\delta^3(r-r')\,, \quad (6.25)$$

and

$$[V_4{}^{\rm wk}(r,t)_{S=-1},V_j{}^{\rm wk}(r',t)^\dagger{}_{S=-1}]=[A_4{}^{\rm wk}(r,t)_{S=-1},A_j{}^{\rm wk}(r',t)^\dagger{}_{S=-1}]=-i\delta^3(r-r')[(J_j{}^\gamma)_{I=1}+3(J_j{}^\gamma)_{I=0}]+2(m_\rho/g_\rho)^2\left[\frac{\partial}{\partial r_j}+ie(A_j)_{\rm ren}\right]\delta^3(r-r')\,. \quad (6.26)$$

In both expressions the Schwinger term has a finite constant coefficient and the covariant derivative $[\partial/\partial r_j+ie(A_j)_{\rm ren}]$ occurs.

7. LEPTON CURRENTS

In this section, we will discuss some speculations concerning the lepton currents. Consider first the electromagnetic interaction. The hadronic part of the electromagnetic current $J_\nu{}^\gamma$ and the leptonic part of the electromagnetic current $j_\nu{}^\gamma$ are, by definition, related to the electromagnetic field $F_{\mu\nu}$ by

$$\partial F_{\mu\nu}/\partial x_\mu=e_0(J_\nu{}^\gamma+j_\nu{}^\gamma)\,. \quad (7.1)$$

Through the current field identities (1.6) and (3.12), the hadron currents satisfy the following field algebra:

$$[J_j{}^\gamma(r,t),J_k{}^\gamma(r',t)]=[J_4{}^\gamma(r,t),J_4{}^\gamma(r',t)]=0\,, \quad (7.2)$$

and

$$[J_4{}^\gamma(r,t),J_k{}^\gamma(r',t)]=\lambda_h(\partial/\partial r_j)\delta^3(r-r')\,, \quad (7.3)$$

where λ_h is a finite constant. [In the SU_3 field algebra, one has $\lambda_h=\frac{4}{3}(m_\rho/g_\rho)^2$.] Equations (7.2) and (7.3) reduce to (4.11) and (4.12), respectively, if only the isovector part is included in $J_\mu{}^\gamma$.

A natural question to ask is whether the lepton current operator $j_\mu{}^\gamma$ can satisfy a similar set of algebraic relations. As we shall see, our present experimental information is completely consistent with the proposal that the leptonic part of the electromagnetic current $j_\lambda{}^\gamma$ also satisfies the algebra of fields; i.e.,

$$[j_i{}^\gamma(r,t),j_k{}^\gamma(r',t)]=[j_4{}^\gamma(r,t),j_4{}^\gamma(r',t)]=0\,, \quad (7.4)$$

and

$$[j_4{}^\gamma(r,t),j_k{}^\gamma(r',t)]=\lambda_l(\partial/\partial r_k)\delta^3(r-r')\,, \quad (7.5)$$

where λ_l is a finite constant. To be sure, in the usual quantum electrodynamics, the operator $j_\lambda{}^\gamma$ is assumed to be equal to

$$s_\lambda=i\psi_e{}^\dagger\gamma_4\gamma_\lambda\psi_e+i\psi_\mu{}^\dagger\gamma_4\gamma_\lambda\psi_\mu\,, \quad (7.6)$$

where ψ_e and ψ_μ are the field operators of the electron and the μ meson; therefore, it cannot possibly satisfy the algebra of fields.

The feasibility of (7.4) and (7.5) can be demonstrated by considering a particular model in which one assumes the existence of a (hypothetical) neutral spin-1 boson field $B_\mu{}^0(x)$. There is a direct interaction between $B_\mu{}^0(x)$ and the charged lepton. This interaction can be represented by the Lagrangian density

$$\mathcal{L}_{\rm free}+\mathcal{L}_{\rm int}=-\tfrac{1}{2}(m_B{}^0B_\mu{}^0)^2-\frac{1}{4}\left(\frac{\partial}{\partial x_\mu}B_\nu{}^0-\frac{\partial}{\partial x_\nu}B_\mu{}^0\right)^2-\sum_l\psi_l{}^\dagger\gamma_4\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu}-if_0B_\mu{}^0\right)+m_l{}^0\right]\psi_l\,, \quad (7.7)$$

where f_0 is the unrenormalized coupling constant, $m_B{}^0$, $m_l{}^0$ are, respectively, the unrenormalized masses of B^0

and the charged lepton l, $l=e$ or μ, and ψ_l is the field operator of the lepton.

Just as in (3.1), we may write

$$\mathcal{L}_{\text{free}}+\mathcal{L}_{\text{int}}=-\tfrac{1}{2}(m_B{}^0B_\mu{}^0)^2+\mathcal{L}_0. \tag{7.8}$$

The electromagnetic interaction Lagrangian $\mathcal{L}_\gamma$ is then generated by replacing in $\mathcal{L}_0$ the operator $B_\mu{}^0(x)$ by

$$\hat{B}_\mu{}^0=B_\mu{}^0+(e_0/f_0)A_\mu, \tag{7.9}$$

and $(\partial B_\mu{}^0/\partial x_\nu)$ by $(\partial\hat{B}_\mu{}^0/\partial x_\nu)$; i.e., in the absence of hadrons, one has

$$\mathcal{L}_{\text{free}}+\mathcal{L}_{\text{int}}+\mathcal{L}_\gamma=-\tfrac{1}{4}F_{\mu\nu}{}^2-\tfrac{1}{2}(m_B{}^0B_\mu{}^0)^2+\mathcal{L}_0', \tag{7.10}$$

where

$$\mathcal{L}_0'=\mathcal{L}_0(B_\mu{}^0\rightarrow\hat{B}_\mu{}^0). \tag{7.11}$$

It can be readily verified that in this model, if one includes also the hadron current $J_\nu{}^\gamma$, the Maxwell equation is

$$\frac{\partial}{\partial x_\mu}F_{\mu\nu}=-(e_0/f_0)(m_B{}^0)^2B_\nu{}^0+e_0J_\nu{}^\gamma. \tag{7.12}$$

Comparison between (7.12) and (7.1) leads to the identity

$$\begin{aligned} j_\nu{}^\gamma(x)&=-[(m_B{}^0)^2/f_0]B_\nu{}^0(x)\\ &=-(m_B{}^2/f)[B_\nu{}^0(x)]_{\text{ren}}, \end{aligned} \tag{7.13}$$

where $[B_\nu{}^0(x)]_{\text{ren}}$ is the renormalized field, related to the renormalized coupling constant f and the renormalized mass m_B by

$$(B_\nu{}^0)_{\text{ren}}=(m_B{}^0/m_B)B_\nu{}^0 \text{ and } (m_B{}^0/m_B)=(f_0/f). \tag{7.14}$$

Consequently $j_\nu{}^\gamma(x)$ satisfies the field algebra (7.4) and (7.5), and the constant λ_l is given by

$$\lambda_l=(m_B{}^0/f_0)^2=(m_B/f)^2. \tag{7.15}$$

In (7.10), there is a direct B^0-photon coupling through the replacement $B_\mu{}^0\rightarrow\hat{B}_\mu{}^0$ in

$$-\tfrac{1}{4}[(\partial B_\nu{}^0/\partial x_\mu)-(\partial B_\mu{}^0/\partial x_\nu)]^2.$$

As will be shown in Appendix B, this direct B^0-photon coupling can be removed by a canonical transformation. Furthermore, the physical consequences of the B^0 meson become particularly simple if one assumes that the unrenormalized theory is divergent; i.e., the unrenormalized mass $m_B{}^0\rightarrow\infty$. In this case, as a result of the canonical transformation, the B^0 meson becomes coupled *only* to $e^\pm$ and $\mu^\pm$ with the same renormalized coupling constant f given by (7.14). By using the results derived in Appendix B, one sees that the existence of the B^0 meson has essentially only the following two experimental consequences:

(i) Scattering between charged leptons (e.g., $e^\pm$ on $e^\pm$) can occur, besides through the usual photon exchanges, also through a virtual exchange of the B^0 meson. The lowest-order perturbation formulas for all photon-exchange amplitudes are the *same* as the usual ones. Since quantum electrodynamics has been tested by the colliding beam experiment[15] for 4-momentum transfer $|q^2|^{1/2}$ up to $\sim(\frac{1}{2}$ BeV), one should require, in the same range of q^2, the matrix element due to B^0 exchange $(f^2/4\pi)(q^2+m_B{}^2)^{-1}$ to be much smaller than that due to photon exchange (α/q^2). Thus, one expects

$$m_B{}^2\gg\alpha^{-1}(f^2/4\pi)(\tfrac{1}{2}\text{ BeV})^2\sim(f^2/4\pi)(5\text{ BeV})^2, \tag{7.16}$$

which implies that $m_B\gg5$ BeV if $(f^2/4\pi)$ is $\sim O(1)$ and[15a] $m_B\gg\frac{1}{2}$ BeV if $(f^2/4\pi)\sim O(\alpha)$.

If $(f/m_B)^2$ is of the same order of magnitude as the weak-interaction coupling constant G [i.e., $(f^2/4\pi)\sim10^{-6}(m_B/m_N)^2$], it would be natural to identify B^0 as the neutral component of the (hypothetical) intermediate boson $W^\pm$ of the weak interaction. One may then expect m_B to have a comparable lower limit ~2.5 BeV to that of $W^\pm$. [See remark (4) below.]

(ii) The B^0 meson can be produced through inelastic processes by scattering $\mu^\pm$ or $e^\pm$ on hadrons; e.g.,

$$\mu^\pm+p\rightarrow\mu^\pm+p+B^0, \tag{7.17}$$

and

$$e^\pm+p\rightarrow e^\pm+p+B^0. \tag{7.18}$$

Subsequently, the B^0 meson would decay into lepton pairs

$$B^0\rightarrow(e^++e^-),\quad\text{or}\quad(\mu^++\mu^-). \tag{7.19}$$

The B^0-meson production cross section $\sigma(l^\pm+p\rightarrow l^\pm+p+B^0)$ at an energy high above the threshold can be roughly estimated to be $\sim\alpha^{-1}(f^2/4\pi)$ times the cross section $\sigma(l^\pm+p\rightarrow l^\pm+p+\gamma)$ for a photon-emission process at the same incoming lepton energy and with a comparable 4-momentum transfer to the proton. The rates for the B^0 decay can be estimated by using the lowest-order perturbation formula. One finds, upon neglecting the lepton mass,

$$\begin{aligned}\text{Rate}(B^0\rightarrow\mu^++\mu^-)&=\text{Rate}(B^0\rightarrow e^++e^-)\\ &=\tfrac{1}{3}(f^2/4\pi)m_B.\end{aligned} \tag{7.20}$$

From (i), one expects m_B to be $>(\sim1$ BeV). Thus, the effective way to investigate reactions (7.17) and (7.18) is to use either the high-energy electrons from,

[15] W. C. Barber, B. Gittelman, G. K. O'Neill, and B. Richter, Phys. Rev. Letters **16**, 1127 (1966).

[15a] *Note added in proof.* In the case $f^2/4\pi\sim O(\alpha)$, a stronger lower limit for m_B can be obtained by using the recent result for the anomalous magnetic moment of the negative μ-meson. Farley *et al.* (at the 1967 Stanford International Symposium on Electron-Photon Interactions) reported the experimental value

$$(g-2)_{\text{exp}}=(23332\pm10)\times10^{-7}.$$

The usual quantum electrodynamics prediction for the same quantity is

$$(g-2)_{\text{th}}=23312\times10^{-7}.$$

Assuming that the discrepancy is due to the existence of the B^0 meson, and using the relation

$$\delta g=(f^2/4\pi)(1/3\pi)(m_\mu/m_B)^2,$$

one finds $m_B\gtrsim1.6$ BeV, provided $(f^2/4\pi)\cong\alpha$. We wish to thank Professor L. Lederman for pointing this out to us.

e.g., SLAC, or the high-energy muons from, e.g., the AGS and CERN, provided $(f^2/4\pi)$ is not much smaller than α. At present, we are unaware of any such experiments.[16]

In the model, the existence of this hypothetical B^0 meson implies that j_ν^γ satisfies the algebra of fields. At present, the converse, which is perhaps the more interesting question, whether the algebraic relations (7.4) and (7.5) also imply the existence of a vector meson, like B^0, remains an open one. It is hoped that these algebraic relations and the generalized field-current identities may have a wider domain of applicability than the special local-field-theoretical model. In this connection, the following remarks, though based on the special model, may clarify some of the implications of these field-current identities.

(1) The Lagrangians (7.7) and (7.10) are both renormalizable in the usual sense. The mathematical problem is identical with the one discussed in Ref. 4 The unrenormalized mass $m_B{}^0$ cannot be zero; it can be infinite, if the unrenormalized theory is divergent. In the limit of an infinite $m_B{}^0$, the matrix element of s_λ between any two states $|a\rangle$ and $|b\rangle$ becomes identical with that of $j_\nu^\gamma(x) = -(m_B{}^2/f)[B_\nu{}^0(x)]_{\text{ren}}$; i.e.,

$$\lim_{m_B{}^0=\infty} \langle b|s_\lambda|a\rangle = \langle b|j_\nu^\gamma(x)|a\rangle = -(m_B{}^2/f)\langle b|(B_\nu{}^0)_{\text{ren}}|a\rangle, \quad (7.21)$$

where s_λ is given by (7.6). Nevertheless, j_ν^γ satisfies the algebra of fields, but s_ν does not. [Similar conclusions also apply to the hadrons.[1,4]]

(2) By using the same arguments given in Refs. 1 and 4, one finds that, independently of whether the unrenormalized mass $m_B{}^0$ is finite or infinite, the unrenormalized photon propagator must be finite. As a result, the ratio between the unrenormalized charge e_0 and the renormalized charge e is also finite. We note that the Maxwell equation for the renormalized electromagnetic field tensor

$$(F_{\mu\nu})_{\text{ren}} = (e_0/e)(F_{\mu\nu}) \quad (7.22)$$

can be decomposed into a leptonic part

$$\left[\frac{\partial}{\partial x_\mu}(F_{\mu\nu})_{\text{ren}}\right]_{\text{lep}} = -(e_0/e)^2 e(m_B{}^2/f)(B_\nu{}^0)_{\text{ren}}, \quad (7.23)$$

a hadronic $I=1$ part

$$\left[\frac{\partial}{\partial x_\mu}(F_{\mu\nu})_{\text{ren}}\right]_{I=1} = -(e_0/e)^2 e(m_\rho{}^2/g)(\rho_\nu{}^0)_{\text{ren}}, \quad (7.24)$$

and a hadronic $I=0$ part

$$\left[\frac{\partial}{\partial x_\mu}(F_{\mu\nu})_{\text{ren}}\right]_{I=0} = -\tfrac{1}{2}(e_0/e)^2(e/g_Y) \times[\cos\theta_Y m_\phi{}^2(\phi_\mu)_{\text{ren}} - \sin\theta_Y m_\omega{}^2(\omega_\mu)_{\text{ren}}]. \quad (7.25)$$

Except for $(e_0/e)^2$, only renormalized quantities occur in these expressions. The finiteness of $(e_0/e)^2$ and $(e_0/e)^2>1$ make possible the general expectation that the matrix elements of the renormalized field operators $(F_{\mu\nu})_{\text{ren}}$, $(\rho_\mu{}^0)_{\text{ren}}$, $\cdots$ between any two physical states should be finite.

The hadronic contribution to the photon propagator has already been discussed in Refs. 1 and 4. For the special case that

$$(f^2/4\pi) = O(\alpha^{1/2}) \ll 1, \quad (7.26)$$

and, therefore,

$$\alpha(f^2/4\pi)^{-1} = O(\alpha^{1/2}) \ll 1, \quad (7.27)$$

where $\alpha = (e^2/4\pi) \cong (137)^{-1}$, the leptonic contribution can be explicitly calculated.

From a straightforward perturbation calculation, one finds that, to first order in $(f^2/4\pi)$, the *renormalized* B^0-meson propagator is

$$(q^2+m_B{}^2)^{-1}(\delta_{\mu\nu}+m_B{}^{-2}q_\mu q_\nu) \times[1+(q^2+m_B{}^2)^{-1}(f^2/4\pi)q^2(q^2\delta_{\mu\nu}-q_\mu q_\nu)F]. \quad (7.28)$$

The function F can be most conveniently expressed in terms of the usual spectral representation

$$F(q^2) = F_e(q^2)+F_\mu(q^2), \quad (7.29)$$

and

$$F_l(q^2) = -\frac{1}{3\pi}\int_0^1 \frac{(3-2x)x^2(1-2x)dx}{m_l{}^2+q^2x(1-x)}, \quad (7.30)$$

where $l=e$ or μ, and m_l is the renormalized lepton mass. The unrenormalized photon propagator $(D_{\mu\nu}{}^\gamma)^0$ can be obtained by using Eq. (55) of Ref. 4. In the limit that the unrenormalized mass $m_B{}^0$ is infinite, one has, upon neglecting the hadronic contribution and higher-order terms in $\alpha(f^2/4\pi)^{-1}$, $(f^2/4\pi)$, and α,

$$(D_{\mu\nu}{}^\gamma)^0 = q^{-2}(\delta_{\mu\nu}-q^{-2}q_\mu q_\nu)[1-\alpha(f^2/4\pi)^{-1} \times(q^2+m_B{}^2)^{-1}m_B{}^2+\alpha(q^2+m_B{}^2)^{-2}m_B{}^4q^2F]. \quad (7.31)$$

Thus to the same order, the leptonic contribution to $(e_0/e)^2$ is

$$(e_0/e)^2 = 1+\alpha(f^2/4\pi)^{-1}. \quad (7.32)$$

From (7.31), one sees that the usual vacuum polarization term[17] $\alpha(15\pi)^{-1}q^2[m_e{}^{-2}+m_\mu{}^{-2}]$ is now changed to

$$\alpha q^2[(15\pi)^{-1}(m_e{}^{-2}+m_\mu{}^{-2})+(f^2/4\pi)^{-1}m_B{}^{-2}]. \quad (7.33)$$

(3) The possible presence of the B^0 meson leaves a

[16] We wish to thank Dr. R. Cool and Dr. W. Panofsky for discussions of these experimental problems.

[17] E. A. Uehling, Phys. Rev. 48, 55 (1935); R. Serber, *ibid.* 48, 49 (1935).

certain arbitrariness in the definition of the electromagnetic field A_μ. For example, one may define

$$A_\mu' = [1+(e_0/f_0)^2]^{1/2} A_\mu + (e_0/f_0)[1+(e_0/f_0)^2]^{-1/2} B_\mu^0. \quad (7.34)$$

Thus, by using the Lagrangian (7.10), one sees that, in the absence of hadrons, A_μ' satisfies

$$\frac{\partial}{\partial x_\mu} F_{\mu\nu}' = -e_1 s_\nu, \quad (7.35)$$

where s_ν is given by (7.6)

$$e_1 = [1+(e_0/f_0)^2]^{-1/2} e_0, \quad (7.36)$$

and

$$F_{\mu\nu}' = (\partial A_\nu'/\partial x_\mu) - (\partial A_\mu'/\partial x_\nu).$$

If the hadron current J_ν^γ is included, then A_μ' satisfies

$$\frac{\partial}{\partial x_\mu} F_{\mu\nu}' = e_1(-s_\nu + J_\nu^\gamma). \quad (7.37)$$

[Further discussions will be given in Appendix B.] From (7.14) and the finiteness of $(e_0/e)^2$, it follows that in the limit of the unrenormalized mass $m_B{}^0 = \infty$, (7.21) holds and $F_{\mu\nu}' = F_{\mu\nu}$.

(4) If the renormalized coupling constant $(f^2/4\pi)$ is of the same order as the semiweak coupling constant,

$$(f^2/4\pi) \sim 10^{-6}(m_B/m_N)^2, \quad (7.38)$$

where m_N is the nucleon mass, then it becomes natural to identify the B^0 meson as the neutral component of the usual charged weak-interaction intermediate boson $W^\pm$. One replaces the lepton current $j_\mu{}^{\mathrm{wk}}$ in (3.16) by the $W^\pm$-meson field operator W_μ. The weak-interaction Lagrangian density (3.16) becomes, then,

$$f J_\mu{}^{\mathrm{wk}} W_\mu + \mathrm{H.c.}, \quad (7.39)$$

where $J_\mu{}^{\mathrm{wk}}$ is given by (3.17). In addition, there is a direct W-lepton coupling, in complete analogy with the direct B^0-lepton coupling in (7.7),

$$f s_\mu{}^{\mathrm{wk}} W_\mu + \mathrm{H.c.}, \quad (7.40)$$

where

$$s_\mu{}^{\mathrm{wk}} = i \sum_l \psi_l{}^\dagger \gamma_4 \gamma_\mu (1+\gamma_5) \psi_{\nu_l}, \quad (7.41)$$

and ψ_{ν_l} is the field operator of the neutrino ν_l.

A full investigation of these interesting, but hypothetical, possibilities clearly lies outside both the scope and the spirit of the present paper.

APPENDIX A

In this Appendix, we discuss the complete set of equal-time commutators of the local fields for the general case discussed in Sec. 5, but extended to include also the electromagnetic field. On account of (5.2) and (5.8), the strong-interaction Lagrangian density and the hadronic part of the free Lagrangian density can be written as

$$-\tfrac{1}{2} m_0{}^2 (\phi_\nu{}^a)^2 + \mathcal{L}_0(\psi, D_\nu\psi, f_{\mu\nu}{}^a), \quad (\mathrm{A.1})$$

where $\mathcal{L}_0$ is related to $\mathcal{L}_m$ of (5.10) by

$$\mathcal{L}_0 = -\tfrac{1}{4}(f_{\mu\nu}{}^a)^2 + \mathcal{L}_m. \quad (\mathrm{A.2})$$

The function $\mathcal{L}_0$ is invariant under isospin rotations; otherwise, it can be an arbitrary function of ψ, $D_\nu\psi$, and $f_{\mu\nu}{}^a$.

The total Lagrangian density $\mathcal{L}_{\mathrm{free}} + \mathcal{L}_{\mathrm{st}} + \mathcal{L}_\gamma$ can be derived from (A.1) by replacing in $\mathcal{L}_0$

$$\phi_\mu{}^a \to \hat{\phi}_\mu{}^a = \phi_\mu{}^a + (e_0/g_0)\xi^a A_\mu, \quad (\mathrm{A.3})$$

where ξ^a depends on the group $\mathcal{G}$. If $\mathcal{G}$ is the isospin SU_2 group, or the usual $SU_2 \times SU_2$ group, then

$$\xi^a = 1 \quad \text{for} \quad \phi_\mu{}^a = \rho_\mu{}^0, \qquad = 0 \quad \text{otherwise}. \quad (\mathrm{A.4})$$

If $\mathcal{G}$ is the usual SU_3, or $SU_3 \times SU_3$ group, then

$$\begin{aligned} \xi^a &= 1 \quad &&\text{for} \quad \phi_\mu{}^a = v_\mu{}^3 = \rho_\mu{}^0, \\ &= 3^{-1/2} &&\text{for} \quad \phi_\mu{}^a = v_\mu{}^8, \\ &= 0 &&\text{otherwise}. \end{aligned} \quad (\mathrm{A.5})$$

Through the replacement (A.3), the total Lagrangian density becomes

$$\mathcal{L}_{\mathrm{free}} + \mathcal{L}_{\mathrm{st}} + \mathcal{L}_\gamma = -\tfrac{1}{4}F_{\mu\nu}{}^2 - \tfrac{1}{2}m_0{}^2(\phi_\nu{}^a)^2 - \tfrac{1}{4}(\hat{f}_{\mu\nu}{}^a)^2 + \mathcal{L}_m(\psi, D_\nu'\psi, \hat{f}_{\mu\nu}{}^a), \quad (\mathrm{A.6})$$

where

$$D_\nu'\psi = (\partial\psi/\partial x_\nu) + g_0 T^a \hat{\phi}_\nu{}^a \psi, \quad (\mathrm{A.7})$$

and

$$\hat{f}_{\mu\nu}{}^a = \frac{\partial}{\partial x_\mu}\hat{\phi}_\nu{}^a - \frac{\partial}{\partial x_\nu}\hat{\phi}_\mu{}^a + g_0 C^{abc}\hat{\phi}_\mu{}^b \hat{\phi}_\nu{}^c. \quad (\mathrm{A.8})$$

From the Lagrangian density (A.6), one sees that the Maxwell equation takes the form

$$\frac{\partial}{\partial x_\mu} F_{\mu\nu} = -(e_0 m_0{}^2/g_0)\xi^a \phi_\nu{}^a. \quad (\mathrm{A.9})$$

Just as in Secs. 4 and 6, we will, for convenience, adopt the Coulomb gauge, and regard $\hat{\phi}_j{}^a$, $A_j{}^{\mathrm{tr}}$, and ψ as the generalized coordinates. Their conjugate momenta are, respectively,

$$\begin{aligned} \hat{P}_j{}^a &= i[\hat{f}_{4j}{}^a - (\partial\mathcal{L}_m/\partial\hat{f}_{4j}{}^a) + (\partial\mathcal{L}_m/\partial\hat{f}_{j4}{}^a)], \\ \Pi_j{}^{\mathrm{tr}} &= -E_j{}^{\mathrm{tr}} = (\partial A_j{}^{\mathrm{tr}}/\partial t), \end{aligned} \quad (\mathrm{A.10})$$

and

$$P_\psi = -i(\partial\mathcal{L}_m/\partial D_4'\psi).$$

For $\mathcal{G} = SU_2$, (A.10) reduces to (4.5), and for $\mathcal{G} = SU_3 \times SU_3$, (A.10) reduces to (6.15). From the equations of motion, it follows that

$$\phi_4{}^a = m_0{}^{-2}[(\partial/\partial r_j)(i\hat{P}_j{}^a) - ig_0(C^{abc}\hat{P}_j{}^b\hat{\phi}_j{}^c - P_\psi T^a \psi)]. \quad (\mathrm{A.11})$$

The totality of the canonical commutation relations between these generalized coordinates and their conjugate momenta determines the complete set of field algebra. The following equal-time commutators for $\phi_\mu{}^a(r,t)$ are valid to all orders in e^2:

$$[\phi_i{}^a(r,t),\phi_j{}^b(r',t)]=0\,, \qquad (A.12)$$

$$[\phi_4{}^a(r,t),\phi_4{}^b(r',t)] = -(g_0/m_0{}^2)C^{abc}\phi_4{}^c(r',t)\delta^3(r-r')\,, \qquad (A.13)$$

and

$$[\phi_4{}^a(r,t),\phi_j{}^b(r',t)] = -(g_0/m_0{}^2)C^{abc}\hat{\phi}_j{}^c(r',t)\delta^3(r-r') + m_0{}^{-2}\delta^{ab}(\partial/\partial r_j)\delta^3(r-r')\,. \qquad (A.14)$$

We note that (A.12) and (A.13) are not affected by the presence of the electromagnetic field; they are identical with (5.12) and (5.13), respectively. Equation (A.14) differs from (5.14) by the presence of $\hat{\phi}_j{}^c$, instead of $\phi_j{}^c$, on the right-hand side. The above Eqs. (A.12)–(A.14) become (4.7)–(4.9) for $\mathcal{G}=SU_2$, and (6.19)–(6.23) for $\mathcal{G}=SU_3\times SU_3$.

In the Coulomb gauge, the transverse part of the vector potential $A_j{}^{\rm tr}$ and the corresponding transverse part of the electric field $E_j{}^{\rm tr}$ are canonical variables. They satisfy

$$[A_j{}^{\rm tr}(r,t),E_k{}^{\rm tr}(r',t)] = -i(\delta_{jk}-\partial_j\partial_k\nabla^{-2})\delta^3(r-r')\,, \qquad (A.15)$$

$$[\phi_\mu{}^a(r,t),A_k{}^{\rm tr}(r',t)]=0\,, \qquad (A.16)$$

$$[\phi_4{}^a(r,t),E_k{}^{\rm tr}(r',t)]=0\,, \qquad (A.17)$$

$$[\phi_j{}^a(r,t),E_k{}^{\rm tr}(r',t)] = [\partial_t\phi_j{}^a(r,t)-i\partial_j\phi_4{}^a(r,t),A_k{}^{\rm tr}(r',t)] = i(e_0/g_0)\xi^a(\delta_{jk}-\partial_j\partial_k\nabla^{-2})\delta^3(r-r')\,, \qquad (A.18)$$

and

$$[\partial_t\phi_j{}^a(r,t)-i\partial_j\phi_4{}^a(r,t),E_k{}^{\rm tr}(r',t)]=0\,, \qquad (A.19)$$

where $\partial_t=(\partial/\partial t)$, $\partial_j=(\partial/\partial r_j)$, $\nabla^2=\partial_j{}^2$ and

$$\nabla^{-2}\delta^3(r-r')=-(4\pi)^{-1}[\sum_j(r_j-r_j')^2]^{-1/2}\,. \qquad (A.20)$$

The magnetic field H_j is related to $A_j{}^{\rm tr}$ by

$$H_j=\epsilon_{jkl}\partial_k A_l{}^{\rm tr}\,. \qquad (A.21)$$

The electric field E_j consists of two parts

$$E_j=E_j{}^{\rm tr}+E_j{}^{\rm long}\,, \qquad (A.22)$$

in which $E_j{}^{\rm long}$ is not an independent variable, but is determined by $\phi_4{}^a$. One has, on account of (A.9),

$$\partial_j E_j{}^{\rm long}=-i(e_0m_0{}^2/g_0)\xi^a\phi_4{}^a\,. \qquad (A.23)$$

From (A.21)–(A.23) and the above commutation relations, it can be readily verified that

$$[\phi_\mu{}^a(r,t),H_k(r',t)]=0\,, \qquad (A.24)$$

$$[\phi_4{}^a(r,t),E_k(r',t)] = -ie_0C^{abc}\xi^b\phi_4{}^c\partial_k\nabla^{-2}\delta^3(r-r')\,, \qquad (A.25)$$

and

$$[\phi_j{}^a(r,t),E_k(r',t)] = i(e_0/g_0)(\xi^a\delta_{jk}-g_0C^{abc}\xi^b\phi_j{}^c\partial_k\nabla^{-2})\delta^3(r-r')\,. \qquad (A.26)$$

All these commutation relations are valid independently of the detailed form of $\mathcal{L}_m$, and to all orders in e. It has been shown in Sec. 5 that, in the absence of the electromagnetic interaction, one has the additional commutation relations (5.21), if $\mathcal{L}_m$ satisfies (5.20), and also (5.23), if $\mathcal{L}_m$ is independent of $f_{\mu\nu}{}^a$. The generalization of (5.21) and (5.23) to include the electromagnetic effects can be derived in a straightforward, though somewhat tedious, way by using the above commutation relations together with the canonical ones;

$$[\hat{P}_j{}^a(r,t),\hat{\phi}_k{}^b(r',t)]=-i\delta_{jk}\delta^{ab}\delta^3(r-r')\,,$$
$$[\hat{P}_j{}^a(r,t),\hat{P}_k{}^b(r',t)]=0\,,$$

and the definitions of $\hat{P}_\nu{}^a$ and $\hat{\phi}_k{}^b$.

APPENDIX B

In this Appendix, we discuss some consequences of the (hypothetical) B^0 meson. For clarity, we consider a model containing the B^0 meson, the electromagnetic field, the electron and the muon, but in which the hadrons are represented only by the proton and the ρ^0 meson. The total Lagrangian density is

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}{}^2-\tfrac{1}{2}(m_B{}^0B_\mu{}^0)^2-\tfrac{1}{2}(m_\rho{}^0\rho_\mu{}^0)^2 -\frac{1}{4}\left[\frac{\partial\hat{B}_\nu{}^0}{\partial x_\mu}-\frac{\partial\hat{B}_\mu{}^0}{\partial x_\nu}\right]^2-\frac{1}{4}\left[\frac{\partial\hat{\rho}_\nu{}^0}{\partial x_\mu}-\frac{\partial\hat{\rho}_\mu{}^0}{\partial x_\nu}\right]^2 -\sum_l\psi_l{}^\dagger\gamma_4\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu}-if_0\hat{B}_\mu{}^0\right)+m_l{}^0\right]\psi_l -\psi_p{}^\dagger\gamma_4\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu}+ig_0\hat{\rho}_\mu{}^0\right)+m_p{}^0\right]\psi_p\,, \qquad (B.1)$$

where

$$\hat{\rho}_\mu{}^0=\rho_\mu{}^0+(e_0/g_0)A_\mu\,, \qquad (B.2)$$

and

$$\hat{B}_\mu{}^0=B_\mu{}^0+(e_0/f_0)A_\mu\,. \qquad (B.3)$$

We find that (B.1) becomes the same as (7.10) if one neglects the hadrons p and ρ^0. From (B.1), the electromagnetic field satisfies

$$\frac{\partial}{\partial x_\mu}F_{\mu\nu}=-e_0[f_0{}^{-1}(m_B{}^0)^2B_\nu{}^0+g_0{}^{-1}(m_\rho{}^0)^2\rho_\nu{}^0]\,. \qquad (B.4)$$

It is convenient to define

$$s_\nu=\sum_l i\psi_l{}^\dagger\gamma_4\gamma_\nu\psi_l\,, \qquad (B.5)$$

and

$$S_\nu = i\psi_p^\dagger \gamma_4 \gamma_\nu \psi_p. \tag{B.6}$$

One can readily verify that both s_ν and S_ν are conserved. Similarly, both ρ_ν^0 and B_ν^0 fields satisfy the field-conservation equation. There exist in (B.1), besides the $(\rho_\nu^0 S_\nu)$ and the $(B_\nu^0 s_\nu)$ interactions, also a ρ_ν^0-photon interaction and a B_ν^0-photon interaction with coupling constants (e_0/g_0) and (e_0/f_0). These photon-meson direct couplings can be removed by the transformation

$$A_\mu \to A_\mu',$$

where A_μ' is defined by

$$A_\mu' = N^{-1}A_\mu + N[(e_0/g_0)\rho_\mu^0 + (e_0/f_0)B_\mu^0], \tag{B.7}$$

and

$$N = [1+(e_0/f_0)^2 + (e_0/g_0)^2]^{-1/2}. \tag{B.8}$$

In terms of A_μ', the Lagrangian density (B.1) becomes

$$\begin{aligned}\mathcal{L} = -\tfrac{1}{4}[F_{\mu\nu}'^2 + \tilde{\mathcal{G}}_{\mu\nu}^0 K_0 \mathcal{G}_{\mu\nu}^0] - \tfrac{1}{2}\tilde{\phi}_\mu^0 M_0^2 \phi_\mu^0 \\ + e_1(s_\mu - S_\mu)A_\mu' + \tilde{\phi}_\mu^0 G_0 \mathcal{J}_\mu \\ - \sum_l \psi_l^\dagger \gamma_4 \left(\gamma_\mu \frac{\partial}{\partial x_\mu} + m_l^0\right)\psi_l \\ - \psi_p^\dagger \gamma_4 \left(\gamma_\mu \frac{\partial}{\partial x_\mu} + m_p^0\right)\psi_p,\end{aligned} \tag{B.9}$$

where $\sim$ denotes the transpose of a matrix,

$$\begin{aligned} e_1 &= Ne_0, \\ F_{\mu\nu}' &= \frac{\partial}{\partial x_\mu}A_\nu' - \frac{\partial}{\partial x_\nu}A_\mu', \\ \phi_\mu^0 &= \begin{pmatrix}\rho_\mu^0 \\ B_\mu^0\end{pmatrix}, \quad \mathcal{J}_\mu = \begin{pmatrix} S_\mu \\ s_\mu \end{pmatrix}, \\ \mathcal{G}_{\mu\nu}^0 &= \frac{\partial}{\partial x_\mu}\phi_\nu^0 - \frac{\partial}{\partial x_\nu}\phi_\mu^0, \end{aligned} \tag{B.10}$$

K_0, M_0, and G_0 are all (2×2) matrices,

$$K_0 = N^2 \begin{pmatrix} 1+(e_0/f_0)^2 & -e_0^2/(f_0 g_0) \\ -e_0^2/(f_0 g_0) & 1+(e_0/g_0)^2 \end{pmatrix}, \tag{B.11}$$

$$M_0^2 = \begin{pmatrix} (m_\rho^0)^2 & 0 \\ 0 & (m_B^0)^2 \end{pmatrix}, \tag{B.12}$$

and

$$G_0 = N^2 \begin{pmatrix} -g_0[1+(e_0/f_0)^2] & -e_0^2/g_0 \\ e_0^2/f_0 & f_0[1+(e_0/g_0)^2] \end{pmatrix}. \tag{B.13}$$

One sees that the transformed electromagnetic field A_μ' satisfies

$$\frac{\partial}{\partial x_\mu} F_{\mu\nu}' = e_1(S_\nu - s_\nu). \tag{B.14}$$

The mathematical problem of the two coupled fields ρ_μ^0 and B_μ^0 is identical with the ϕ-ω mixing problem discussed in Ref. 1. Here we will only discuss the case that the unrenormalized theory is divergent. In this case, the unrenormalized masses m_B^0 and m_ρ^0 tend to infinity; i.e.,

$$m_B^0 \to \infty \quad \text{and} \quad m_\rho^0 \to \infty. \tag{B.15}$$

The renormalized coupling constants f and g are related to the unrenormalized ones by

$$(f/f_0) = (m_B/m_B^0) \quad \text{and} \quad (g/g_0) = (m_\rho/m_\rho^0). \tag{B.16}$$

In the limit (B.15), $(e_0/e)^2$ remains finite, but f_0 and g_0 both become infinity; therefore,

$$K_0 \to \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{B.17}$$

and

$$G_0 \to \begin{pmatrix} -g_0 & 0 \\ 0 & f_0 \end{pmatrix}. \tag{B.18}$$

By substituting these limiting forms into (B.9), one finds that in the same limit the ρ^0 meson is coupled *only* to the hadron, while the B^0 meson is coupled *only* to the leptons. Equation (B.14) is *exactly the same* as the Maxwell equation in the usual electrodynamics without the B^0 meson. The experimental consequences of the possible existence of such a B^0 meson have already been discussed in Sec. 7.