Analysis of the s-Wave $\pi\pi$ Interaction from the Reaction $\pi^- p \rightarrow \pi^- \pi^+ n$

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Some recent analyses which deduce s-wave $\pi\pi$ scattering amplitudes from observed peripheral production of s-wave dipions in $\pi^- p$ reactions are discussed; they assume that the ratio of s-wave production to s-wave scattering equals the ratio of p-wave production to p-wave scattering, and take p-wave scattering to be given by the Breit–Wigner amplitude describing the ρ meson. As a procedure involving fewer assumptions, it is suggested that one might only assume the phases of these two ratios to be the same, without making assumptions about their relative magnitudes. Applying this procedure to the compilation of experimental data made by Walker *et al.* provides a strong indication that the $I=0$ s-wave phase shift is ≈ 75–$80°$ at a mass of 750 MeV, and indicates that there is some uncertainty in the relative magnitudes of the two ratios mentioned above.

FOR some time it has been suggested[1] that the asymmetry in the angular distribution of the decay of the neutral ρ^0 meson indicates a strong, and possibly resonant, s-wave $\pi\pi$ interaction for $\pi\pi$ masses around 750 MeV. Recently, three groups[2–4] have attempted to deduce detailed s-wave $\pi\pi$ phase shifts from this asymmetry and from other measurements of the reaction

$$\pi^- + p \rightarrow \pi^- + \pi^+ + n. \quad (1)$$

These authors all assume that the ratio of the cross section for the reaction

$$\pi^- + \pi^+ \rightarrow \pi^- + \pi^+ \quad (2)$$

to the cross section for peripheral production of dipions of the same mass in reaction (1) scarcely varies with the $\pi\pi$ mass involved. The cross section for p-wave scattering in reaction (2) is then taken to be known, by assuming the ρ^0 meson to be an elastic resonance whose cross section is described by a Breit-Wigner formula, and then the s-wave scattering in reaction (2) can be deduced by taking the observed ratio of production of s-wave dipions to production of p-wave dipions in reaction (1) to be the same as the ratio of s-wave to p-wave scattering; essentially this seems to be a straightforward extension of a one-pion-exchange model. However, there is some uncertainty in making this extension as the polarization of the ρ^0 meson differs from that expected in the simplest one-pion-exchange model in which there are no corrections due to absorption. Both Gutay *et al.*[2] and Walker *et al.*[3] assume, in answer to this problem, that the ratio of s-wave to p-wave scattering amplitudes equals the ratio of the amplitude for production of an s-wave dipion in reaction (1) to that for production of the $m=0$ substate (defined with respect to the incident pion direction) of the p-wave dipion, without justifying this assumption. Gutay *et al.*[2] determine this latter ratio from a study of the density matrix deduced from the observed angular distribution of the $\pi\pi$ system, using the absorption model to guide them in deducing values of the density matrix elements corresponding to the lowest momentum transfers involved, while Walker *et al.*[3] correct for this depolarization of the ρ meson in a more empirical fashion. These two groups reach somewhat similar conclusions about the way the s-wave phase shifts vary with energy, but there are differences in detail between their results, presumably for three reasons: (i) Gutay *et al.*[2] assume p-wave phase shifts corresponding to a ρ meson of width 125 MeV, while Walker *et al.*[3] assume a width of 160 MeV. This should only make for appreciable differences in regions on the flanks of the ρ meson. (ii) Gutay *et al.*[2] neglect the effect of any s-wave $\pi\pi$ scattering in the $I=2$ state. It has been shown[3,5] how the probable amplitude for such scattering in the $I=2$ state ($\delta_2 \approx -20$ to $-30°$) interferes constructively with the $I=0$ amplitude in reaction (2): if one then analyses the cross sections for reaction (2) ignoring this $I=2$ amplitude, one will deduce an apparent $I=0$ phase shift δ_s ,which is larger than the true phase shift by approximately 10–15° if δ_0 is in the range from 60 to 120°. (iii) Gutay *et al.*[2] make a correction for the fact that the virtual pion is off the energy shell, assuming this correction to be solely the kinematic factor, $(q_{\rm off}/q)^l$. in the production amplitude, as introduced by Selleri,[6] while Walker *et al.*[3] make no such correction. This correction comes in as an extra factor relating the p-wave and s-wave production cross sections and so will make for a difference in the s-wave scattering cross sections deduced by these two groups. Taking an average of this correction factor over the range of momentum transfers involved, one finds that this would make for

[1] See, typically, M. M. Islam and R. Piñon, Phys. Rev. Letters **12**, 310 (1964); S. H. Patil, *ibid.* **13**, 261 (1964).
[2] L. J. Gutay, P. B. Johnson, F. Loeffler, R. L. McIlwain, D. H. Miller, R. B. Williamson, and P. L. Csonka, Phys. Rev. Letters **18**, 142 (1967).
[3] W. D. Walker, J. Carroll, A. Garfinkel, and B. Y. Oh, Phys. Rev. Letters **18**, 630 (1967).
[4] L. W. Jones, D. O. Caldwell, B. Zacharov, D. Harting, E. Bleuler, W. C. Middelkoop, and B. Elsner, Phys. Letters **21**, 590 (1966).
[5] I. F. Corbett, C. J. S. Damerell, N. Middlemas, D. Newton, A. B. Clegg, W. S. C. Williams, and A. S. Carroll, Phys. Rev. **156**, 1451 (1967).
[6] F. Selleri, Phys. Letters **3**, 76 (1964).

a difference of a factor of about 1.16 between the two sets of s-wave cross sections deduced.

Jones *et al.*[4] proceed differently: They assume that the cross section for the reaction

$$\pi^- + p \to \pi^- + \pi^+ + N^0,$$

where N^0 here signifies all possible neutral baryon states, is determined by the simplest one-pion-exchange model without any corrections for form factors or absorption effects. They argue (see also Jones[7]) that all absorption effects are included when all possible neutral baryon states are included in this way. Another possible way to justify their procedures follows from the observation that the p-wave $\pi\pi$ cross section deduced by their procedure at the peak of the ρ meson, has the correct value, when one assumes the ρ meson to be an elastic resonance. Therefore, one can argue that what they are effectively doing is deducing the ratio of s-wave $\pi\pi$ scattering to p-wave $\pi\pi$ scattering, in particular to the ρ meson, in the same way as the other authors. This seems a useful way to justify the analysis of Jones *et al.*,[4] when one also notes that there is some uncertainty in the normalization of their cross sections due to uncertainties as to how to treat production from the bound protons in the carbon nuclei of their polyethylene target; such uncertainties disappear when one argues one is determining s-wave scattering from its ratio to p-wave scattering, which is known at the peak of the ρ meson. If one takes this viewpoint, one sees that Jones *et al.*[4] make no correction for any depolarization of the ρ meson from the alignment expected in the one-pion-exchange model, so their results should differ somewhat from the other analyses we have described. They also assume that all the s-wave scattering is taking place in the $I=0$ state, neglecting interference with scattering in the $I=2$ state, and they do make the Selleri[6] kinematic correction for the virtual pion being off the energy shell.

All these three analyses reach similar conclusions in that they suggest that the phase shift for s-wave $\pi\pi$ scattering is probably large and slowly varying for a broad range of $\pi\pi$ masses. Walker *et al.*[3] find one solution in which there is a rapid variation of the phase shift through 90°, suggesting a relatively narrow resonance, but this possibility can be ruled out by comparison with the peripheral $\pi\pi$ mass spectrum[5] from the reaction

$$\pi^- + p \to \pi^0 + \pi^0 + n. \qquad (3)$$

Therefore, the favored solution seems to be a slow rise in the s-wave phase shift, which may pass through 90° at some value of $\pi\pi$ mass: Walker *et al.*[3] suggest this happens at a mass near 1000 MeV, while Gutay *et al.*[2] suggest that it happens around 700 MeV (Gutay *et al.* also find a solution in which the s-wave phase shift does not rise above 60°). Thus, even though there seems to be some agreement that the $I=0$ s-wave scattering is strong over a broad range of $\pi\pi$ masses, there seems to be considerable uncertainty about the details of its behavior.

We have already indicated some of the possible uncertainties in these analyses. In this note we raise the possibility of a further such uncertainty: the ratio of scattering to peripheral production may differ for different angular momentum states of the $\pi\pi$ system, which could happen as the virtual pions in reaction (1) are off the mass shell. One contribution to such an effect is the kinematic factor of Selleri,[6] which has already been mentioned; we are raising here the possibility of further contributions to such a difference, which could be due to form factor effects or to corresponding effects due to absorption which could differ for different angular momenta. The method of analysis we shall use to show this has been proposed previously[5] where it was applied to the data of West *et al.*[8] It is here applied to the much larger set of data compiled by Walker *et al.*[3]; this new data is of much greater statistical accuracy so that this new analysis reaches conclusions which were not possible before. In particular, the better statistics seem to indicate rather strongly that the s-wave phase in reaction (2) is close to 90° in the mass region 700–800 MeV.

We assume that three amplitudes are contributing to peripheral production in reaction (1):

A: production of p-wave dipion in $m=\pm1$ substates,
B: production of p-wave dipion in $m=0$ substate,
C: production of s-wave dipion.

We write a, b, c for the magnitudes of A, B, C, and assume that the relative phase of amplitudes B and C is $(\delta_p - \delta_s)$, where δ_p, δ_s are, respectively, the phases of the p-wave and s-wave scattering amplitudes in reaction (2). We are therefore assuming that the ratio of p-wave scattering amplitude to p-wave production amplitude has the same phase as the ratio of s-wave scattering amplitude to s-wave production amplitude, but are making no assumptions about the relative magnitudes of these ratios; this is therefore a less restrictive assumption than that made by the other authors[2–4] who assumed the magnitudes of these ratios are also the same. Then the $\pi\pi$ angular distribution in reaction (1), with respect to the incident pion direction, is

$$(a^2+c^2)+2bc\cos(\delta_p-\delta_s)\cos\theta+(b^2-a^2)\cos^2\theta. \qquad (4)$$

(We have integrated over azimuthal angles; this integration removes any need to know the phase of A with respect to those of B and C.) We have analyzed the data compiled by Walker *et al.*[3] into the form

$$B_0+B_1P_1(\cos\theta)+B_2P_2(\cos\theta). \qquad (5)$$

If then we neglect c^2 in Eq. (4) in comparison with b^2 (correction for this approximation would only make a small difference to our final results), we can deduce

[7] L. W. Jones, Phys. Rev. Letters **14**, 186 (1965).

[8] E. West, J. Boyd, A. R. Erwin, and W. D. Walker, Phys. Rev. **149**, 1089 (1966).

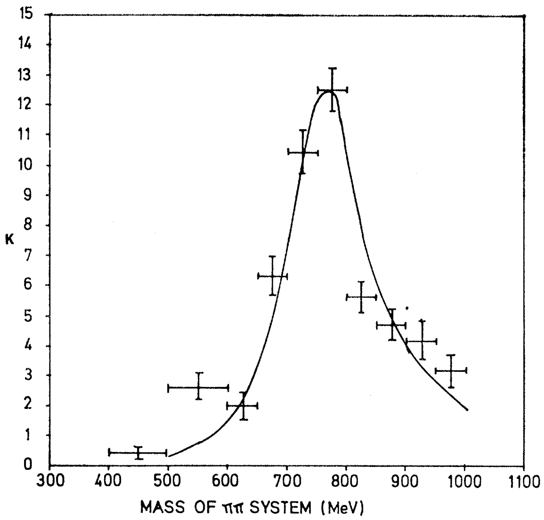

FIG. 1. Values of K, which is described in the text, as a function of $\pi\pi$ mass, as deduced from the compilation of data made by Walker *et al.*[3] The vertical scale is in arbitrary units. The full line is a p-wave Breit-Wigner intensity, with $M=760$ MeV and $\Gamma=160$ MeV, which has been normalized to go through the experimental points.

values of

$$K=\sigma_s \cos^2(\delta_p-\delta_s)=c^2 \cos^2(\delta_p-\delta_s)=\frac{B_1{}^2}{4(B_0+B_2)},$$

where σ_s denotes the cross section for production of s-wave dipions that would be observed in reaction (1) if the p-wave cross section were reduced to zero. The values of K deduced from the measured angular distributions are shown in Fig. 1. The curve drawn through the points, is, except for the normalization, the same Breit-Wigner formula for a p-wave resonance, with the same energy and width, as fits the ρ meson in reaction (1): ($M_\rho=760$ MeV, $\Gamma_\rho=160$ MeV). We see that this curve is an excellent fit to the experimental points. There would seem to be one explanation of the excellence of this fit which is most likely: σ_s and δ_s do not vary appreciably over this region of $\pi\pi$ mass, and $\delta_s\approx 90°$ so that $\cos^2(\delta_p-\delta_s)\approx\sin^2\delta_p$. Another possible interpretation could be that there is an s-wave resonance with the same mass and width as that of the ρ meson, but this would seem to be a rather unlikely coincidence; it would also disagree with the peripheral $\pi\pi$ mass spectrum observed[5] in reaction (3). Other explanations for the peaking in K could conceivably be invented, but they would involve even more remarkable coincidences; they would also be difficult to reconcile with the comparison[5] made between the values of K (deduced solely from the data at $p=2.03$ GeV/c) and the peripheral $\pi\pi$ mass spectrum from reaction (3) at the same energy. [In making this comparison, we assumed that the ratio of s-wave scattering to s-wave peripheral production is the same in both these reactions (1) and (3) at the same incident pion energy, an assumption which seems reasonable.]

Thus this analysis gives a very direct indication that the phase of the s-wave scattering amplitude is probably very close to 90° at a $\pi\pi$ mass value close to 750 MeV and is varying slowly in this mass region; the latter of these two conclusions agrees with the other analyses we have described, while the former helps considerably to pin down the value of the phase shift at one value of the $\pi\pi$ mass. This s-wave scattering amplitude is the result of interference between $I=0$ and $I=2$ scattering, so that this phase shift δ_s is not the $I=0$ phase shift δ_0. It has been shown[3,5] how the $I=2$ s-wave phase shift in this region is probably somewhere in the range -20 to $-30°$; if this is so, and the $I=0$ s-wave phase shift δ_0 is near 90°, then δ_0 is approximately 10–15° less than δ_s.

Our conclusion then implies that $K=\sigma_s$ at the peak. If this is so, the ratio of s-wave to p-wave scattering cross sections at 750 MeV should be related to the ratio of K to the p-wave production cross section at that mass. Taking the s-wave scattering to be the result of an interference between scattering in $I=0$ and $I=2$ states as we have described, we find a ratio of s-wave to p-wave scattering cross sections of 0.161. To estimate the corresponding ratio of production cross sections we should probably include a correction for the Selleri[6] kinematic factor which increases the p-wave production cross section and so reduces the expected ratio of production cross sections to 0.139. In deducing the ratio of production cross sections from the experimental measurements we have the problem of which polarization of the ρ meson we should consider. We can take either

(i) the ratio of K to production of $m=0$ substate of ρ meson $[\frac{1}{3}(B_0+B_2)]$ which gives a ratio of 0.166 ± 0.010,

or

(ii) the ratio of K to total production of ρ meson ($=B_0$) which gives a ratio of 0.109 ± 0.007. We see that the expected ratio of 0.139 falls between these two possible experimental values, indicating something of the uncertainty in taking ratios of s-wave to p-wave production cross sections.

We conclude that this analysis, which has the merit of making fewer assumptions than others (in particular it only makes assumptions about relative phases of amplitudes and not about their relative magnitudes), gives a strong indication that the phase of the s-wave $\pi\pi$ scattering amplitude in reaction (2) is very close to 90° at a mass of 750 MeV. Relating this to other evidence implying $\delta_2\approx-20$ to $-30°$, this implies $\delta_0=75$–$80°$ at this mass. We also obtain, in agreement with other analyses, an independent indication that the s-wave phase shift is varying slowly with $\pi\pi$ mass; that any $I=0$ s-wave resonance must be very broad. This procedure also makes it possible to test assumptions about ratios of magnitudes of amplitudes which were made in other analyses.

Finally we will make some comments about the relation of these results with other information that has been deduced about s-wave $\pi\pi$ scattering. Hamilton *et al.*[9] have estimated the contribution of the $I=0$ s-wave interaction to low-energy s-wave π-nucleon scattering. Their procedure effectively deduces one number, which is an integral over the $I=0$ s-wave $\pi\pi$ scattering cross section multiplied by a weighting factor which falls rapidly from threshold to an energy of about 900 MeV. The largest contribution to the integral comes from $\pi\pi$ energies below 600 MeV. They find a fit with a phase shift which rises rapidly at threshold to 30° and then levels off at that value. However, it would seem probable that other forms of variation of this phase shift would produce the same value of the integral they determine; in particular, a phase which is larger at higher masses, such as we propose, would then imply a much smaller scattering length at threshold. This smaller scattering length would be in better agreement with the conclusions drawn by Weinberg[10] from an analysis of K_{e4} decay. Chiu and Schechter[11] have remarked how the sum rule of Adler[12] can be completed if there is an s-wave $\pi\pi$ resonance with a mass of 390 MeV and a width of 90 MeV. They then show that this sum rule could, alternatively, be completed by a range of other s-wave resonances; all that is needed is that M/Γ^3 have a certain value. Therefore, we could complete this sum rule with an s-wave resonance with, for example, a mass of 700 MeV and a width of 520 MeV. The s-wave cross section due to such a resonance would be very similar to the conclusions which seem to be implied by these analyses of reaction (1). It can therefore be suggested that the sum rule of Adler[12] can probably be completed in this way.

[9] J. Hamilton, P. Menotti, G. C. Oades, and L. L. J. Vick, Phys. Rev. **128**, 1881 (1962).

[10] S. Weinberg, Phys. Rev. Letters **17**, 336 (1966).

[11] Y. T. Chiu and J. Schechter, Nuovo Cimento **46**, 548 (1966).

[12] S. L. Adler, Phys. Rev. **140**, B736 (1965).

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Field-Current Identities and Algebra of Fields*

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It is shown that the possible identity between the various hadronic current operators and the corresponding spin-1 meson field operators determines the general structure of the hadronic part of the total Lagrangian. In particular, the identity between the isovector electromagnetic current and the neutral ρ-meson field implies that the ρ dependence of the strong interaction must be the same as that in the Yang-Mills theory, except for the mass term of the ρ meson. The explicit form of the interaction Lagrangian makes possible a general study of the local equal-time commutators of the various hadronic current operators, including the effects of the electromagnetic interaction. Many of these electromagnetic correction terms depend only on the general requirement of gauge invariance, and are independent of whether the proposed field-current identities are valid or not. For example, the usual Schwinger term $\lambda(\partial/\partial r_j)\delta^3(r-r')$ in the commutator between the time component of any charged hadronic weak interaction current and the jth space component of its Hermitian conjugate should be replaced by $\lambda[(\partial/\partial r_j)+ieA_j]\delta^3(r-r')$, where A_j is the electromagnetic field operator. The contribution of such a correction term, i.e., $\lambda ieA_j\delta^3(r-r')$, remains present in the integrated form of the commutator. In the usual current algebra, λ is mathematically undefined. If field-current identities hold, then these current commutators are the same as the corresponding algebra of the field operators, and λ becomes a well-defined c number. Some speculative remarks concerning the possible extension of the algebra of fields to the lepton currents are presented.

1. INTRODUCTION

RECENTLY, it has been suggested[1] that the entire hadronic electromagnetic current operator is identical with a linear combination of the local field operators of the known neutral vector mesons, independent of whether the unrenormalized masses of these vector mesons are finite or infinite.[2] This identity is shown to be consistent with the requirement of gauge invariance; it gives a precise formulation of the idea of vector domi-

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[1] N. M. Kroll, T. D. Lee, and Bruno Zumino, Phys. Rev. **157**, 1376 (1967).

[2] There exists an alternative proposal in which the unrenormalized isovector part of the hadronic electromagnetic current is assumed to be the same operator as the unrenormalized current generating the neutral ρ-meson field. In such a case, the field-current identity holds only in the limit of an infinite unrenormalized meson mass. [See Refs. 1, 4, M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).] It is important to note that in this alternative possibility, the products of the current operators would in general, be different from the products of the corresponding field operators even in the limit of infinite unrenormalized masses; thus, the hadronic current operators entering in the electromagnetic and the weak interactions satisfy the usual current algebra instead of the algebra of fields.