Noninvariance Groups and Low-Energy Meson-Baryon Scattering*

N. G. DESHPANDE
Department of Physics, Northwestern University, Evanston, Illinois
(Received 7 June 1967)

The object of the paper is to show that consideration of noninvariance groups as well as the Cook–Goebel–Sakita strong-coupling solution may be used to derive low-energy sum rules for p-wave meson-baryon scattering. In particular we obtained two sum rules for pion-nucleon scattering which are in fair agreement with experiment. These are $p^{1,1}=4p^{1,3}=4p^{3,1}$. The method is generalized to $SU(3)$ to obtain twelve sum rules for the fourteen p-wave amplitudes. Throughout, a rather simple method of using crossing matrices of the groups $SU(2)_I\times SU(2)_J$ and $SU(3)\times SU(2)_J$ is employed. Several new results are obtained which were not obtained in the previous treatments.

I. INTRODUCTION

RECENTLY, Kuriyan and Sudarshan[1–3] have considered the consequences of an intermediate-coupling model. This model is based on the static Chew-Low model for p-wave scattering, as was the Cook, Goebel, and Sakita (CGS)[4,5] strong-coupling model. The model considered by the former authors differs from that of CGS in the fact that commutators of meson matrices are assumed to be nonvanishing, and are set equal to a certain linear combination of the generators of the invariance group. Such a model derives its validity from the fact that even in the limit of strong coupling, differences between infinite quantities may still remain finite. However, much of its predictive power comes from assuming that the coupling constants are in fact finite. For example, the noninvariance groups involved are compact, leading to finite dimensional representations for baryons. The authors further explore the consequences of their assumption on the spin-flip and spin-nonflip meson-baryon elastic scattering amplitudes.[2,3] Not all the sum rules are, however, exhausted by their method.

In this paper, we consider meson-baryon scattering within the framework of the intermediate-coupling model and rederive sum rules for p-wave scattering amplitudes. These sum rules are expected to be valid at low energies where the Born approximation has some validity. We consider the effect of the CGS mass-splitting term as well as the term arising from the commutators of meson matrices. The method used is the familiar one involving crossing matrices. Such a method is rather simple to use because the crossing matrices are well known for the groups involved here. The method also gives a clearer physical understanding of the different forces involved in the problem. In Sec. II, we consider the invariance group $SU(2)_I\times SU(2)_J$, and derive two sum rules that are fairly well satisfied by the p-wave scattering lengths. In Sec. III, we consider the group $SU(3)\times SU(2)_J$ and derive some of the consequences of this group. In Sec. IV, we summarize our conclusions.

II. MESON-BARYON SCATTERING IN $SU(2)_I\times SU(2)_J$

A. Intermediate-Coupling Model

We consider the p-wave pion-nucleon scattering assuming the invariance group $SU(2)_I\times SU(2)_J$. The Born approximation for the process

$$B+M_{i\alpha}\rightarrow B'+\bar{M}_{j\beta},$$

where i and α refer to spin and isospin, respectively, and B and M stand for baryons and mesons, is given as a matrix in the space of baryons B and B' by[4]

$$t_{i\alpha,\beta j}=-g^2\frac{[A_{i\alpha},A_{j\beta}]}{\omega}-\frac{1}{\omega^2}[A_{i\alpha},[\Delta,A_{\beta j}]]+O(1/g^2). \quad (1)$$

In this expression $A_{\alpha i}$ are meson matrices, and Δ is the mass operator. The intermediate-coupling model assumes that in the limit $g^2\rightarrow\infty$,

$$g^2[A_{i\alpha},A_{j\beta}]=i\theta[\epsilon_{\alpha\beta\gamma}I_\gamma\delta^{ij}+\epsilon_{ijk}J_k\delta^{\alpha\beta}]. \quad (2)$$

I_α and J_i are the generators of the invariance group $SU(2)_I$ and $SU(2)_J$, respectively. If $\theta\rightarrow 0$, we recover the results of the CGS strong-coupling model. The form of Eq. (2) though seemingly arbitrary, is the only one that allows N and N^* in the baryon spectrum as we shall show. An analogous assumption about meson commutators based on vector exchange has been suggested by Polkinghorne[6] in a slightly different context. Use has also been made of such commutators by Capps[7] in a model based on $SU(6)_W$ to construct a complete theory of baryon and meson bootstraps. Thus, it may be possible to obtain Eq. (2) based on a theory involving exchange of mesons, although we do not pursue that question here.

* This work is supported by a NSF research grant.
[1] J. G. Kuriyan and E. C. G. Sudarshan, Phys. Letters 21, 106 (1966).
[2] J. G. Kuriyan and E. C. G. Sudarshan, Phys. Rev. Letters 16, 825 (1966).
[3] J. G. Kuriyan and E. C. G. Sudarshan, Phys. Rev. 162, 1650 (1967).
[4] T. Cook, C. J. Goebel and B. Sakita, Phys. Rev. Letters 15, 35 (1965).
[5] C. J. Goebel, Phys. Rev. Letters 16, 1130 (1966).
[6] J. C. Polkinghorne, Ann. Phys. (N. Y.) 34, 153 (1965).
[7] R. H. Capps, Phys. Rev. 161, 1538 (1967).

The dynamical postulate[3] made now is that the scattering amplitude in some range of energies is described by terms of the type in Eq. (1) multiplied by arbitrary functions of energy ω. Thus, we have

$$t_{i\alpha,j\beta}(\omega)=-\theta i[\epsilon_{\alpha\beta\gamma}I_{\gamma}\delta^{ij}+\epsilon_{ijk}J_k\delta^{\alpha\beta}]f_1(\omega)-[A_{i\alpha},[\Delta,A_{\beta j}]]f_2(\omega). \quad (3)$$

The functions $f_1(\omega)$ and $f_2(\omega)$ are, respectively, $1/\omega$ and $1/\omega^2$ at low energies, while they restore unitarity to the amplitude at high energies.

We now study the two terms on the right-hand side of Eq. (3). Using standard formulas involving Clebsch-Gordan coefficients[8] we obtain from Eq. (2), in the space of pion-nucleon elastic scattering, the 4×4 matrix equation

$$C\Gamma-\Gamma=\theta\Lambda, \quad (4)$$

where C is the crossing matrix, and Γ is the coupling constant vector. Λ is obtained by projecting the right-hand side of Eq. (2) in the four channels, i.e., (1,1), (1,3), (3,1), and (3,3) [notation is $(2I,2J)$], respectively. The vector Λ is[8] explicitly given by $\frac{1}{3}(8, 2, 2, -4)$ and C is

$$C=\frac{1}{9}\begin{bmatrix} 1 & -4 & -4 & 16 \\ -2 & -1 & 8 & 4 \\ -2 & +8 & -1 & 4 \\ 4 & 2 & 2 & 1 \end{bmatrix}. \quad (5)$$

With N and N^* exchange assumed, Γ is $(\gamma_{11},0,0,\gamma_{33})$ where $\gamma_{11}=3f^2=3(g^2/4\pi)(\mu_\pi/2M_N)^2\approx 0.25$. If $\theta=0$, we have the reciprocal bootstrap[9] solution $2\gamma_{33}=\gamma_{11}$. However, it is interesting to observe that any departure from this solution requires Λ to be of precisely the form assumed by Kuriyan and Sudarshan. Thus, their assumption is the only one compatible with N and N^* exchange.

The commutator $[A_{\alpha i},[\Delta,A_{\beta j}]]$ can be easily evaluated with N and N^* exchange *regardless of the form of the mass operator* Δ. Using methods similar to those used in getting Eq. (4), we have for the mass-splitting term in the space of the 4 channels, $M=(C+1)\Gamma'$ with $\Gamma'\equiv(0,0,0,\epsilon)$. Thus, M is $\epsilon(8,2,2,5)$ where ϵ is a measure of the N-N^* mass difference.

We finally have for the p-wave scattering amplitude

$$p^{2I,2J}(\omega)=-\theta f_1(\omega)\Lambda^{2I,2J}-f_2(\omega)M^{2I,2J}, \quad (6)$$

with

$$\Lambda=\tfrac{1}{3}(8,\ 2,\ 2,\ -4), \quad (7)$$

$$M=\epsilon(8,2,2,5). \quad (8)$$

We, therefore, have the following two sum rules:

$$p^{1,1}=4p^{1,3}=4p^{3,1}. \quad (9)$$

If we further assume that the N-N^* mass difference is small, we have from Eq. (7)

$$p^{1,3}-p^{1,1}=p^{3,3}-p^{3,1}. \quad (10)$$

This sum rule is, however, in poor agreement with experiment, because the N-N^* mass difference certainly cannot be ignored. The two sum rules in Eq. (9) are, however, in good agreement with p-wave scattering lengths, which are given by Hamilton and Woolcock[10] as

$$\begin{aligned} a^{11}&=-0.101\pm0.007, & a^{13}&=-0.029\pm0.005, \\ a^{31}&=-0.038\pm0.005, & a^{33}&=0.215\pm0.005. \end{aligned} \quad (11)$$

It is to be noted that the sum rules obtained in Eq. (9) are valid also for the CGS theory. We also observe that since we study Eq. (3) directly, we get predictions for elastic πN scattering. The method used by Kuriyan and Sudarshan[1,3] yields absolutely no predictions[11] for this case.

B. Relations between Spin-Flip and Spin-Nonflip Amplitudes

The scattering amplitude can be separated into spin-flip and spin-nonflip amplitudes as

$$T=f+g\boldsymbol{\sigma}\cdot\mathbf{n}, \quad (12)$$

where f is the spin-nonflip and g the spin-flip amplitude. If we assume that *only the first term in Eq. (3) is important*, we get the result that only $\epsilon_{ijk}J_k\delta^{\alpha\beta}$ contributes to g and only $\epsilon_{\alpha\beta\gamma}I_\gamma\delta^{ij}$ contributes to f. It then follows that for spin-flip amplitudes we have the following relation between isospin amplitudes A^1 and A^3:

$$A^1=A^3, \quad (13)$$

and for spin-nonflip the relation

$$A^1=-2A^3. \quad (14)$$

Using the well-known formulas[12]

$$f(\theta)=\sum_l [(l+1)f_{l+}+lf_{l-}]P_l(\cos\theta), \quad (15)$$

$$g(\theta)=\sum_l (f_{l+}-f_{l-})P_l^1(\cos\theta), \quad (16)$$

we obtain two sum rules

$$p^{3,3}-p^{3,1}=p^{1,3}-p^{1,1}, \quad (17)$$

and

$$2p^{1,3}+p^{1,1}=-2[2p^{3,3}+p^{3,1}]. \quad (18)$$

These sum rules are already contained in the three sum rules in Eqs. (9) and (10). These are, however, in poor agreement with experiment again the reason being

[8] V. Singh and B. Udgaonkar, Phys. Rev. **149**, 1164 (1966). Note that any function $\langle N|f_{\alpha i,\beta j}|N\rangle=\sum_n P_nF^n$, where P^n are projection operators for the different channels. F is the matrix representation of $f_{\alpha j,\beta j}$ in the space of meson-baryon scattering. Thus $\langle N|i\epsilon_{\alpha\beta\gamma}I_\gamma\delta^{ij}+i\epsilon_{ijk}J_k\delta^{\alpha\beta}|N\rangle=\sum_n P_n\Lambda^n$ defines Λ.

[9] G. F. Chew, Phys. Rev. Letters **9**, 233 (1962).

[10] J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. **35**, 737 (1963).

[11] E. C. G. Sudarshan (private communication). In Ref. 2 note that Y should be defined as $Y(B_1M_1,B_2M_2)=g(B_1+M_1\rightarrow B_2+M_2)+g(\bar{M}_2+B_1\rightarrow\bar{M}_1+B_2)$.

[12] S. C. Frautschi and J. D. Walecka, Phys. Rev. **120**, 1486 (1960).

that $p^{3,3}$ is very sensitive to location $N^*(33)$ resonance. Thus the assumption of ignoring the CGS mass-splitting term is in poor agreement with the experiment, at least at low energies. At energies much higher than the N^*-N mass difference these relations however could play an important role.

III. MESON-BARYON SCATTERING IN $SU(3)\times SU(2)_J$

A. Intermediate-Coupling Model with Mass Splitting

We now consider the elastic scattering of a p-wave octet of pseudoscalar mesons off an octet of baryons. The scattering amplitude is the same as in Eq. (1), where α now refers to $SU(3)$ index. The appropriate intermediate-coupling hypothesis for this group is

$$g^2[A_{\alpha i},A_{\beta j}] = i\theta[f_{\alpha\beta\gamma}F_\gamma\delta_{ij}+2\epsilon_{ijk}(J_k\delta_{\alpha\beta}/3+d_{\alpha\beta\gamma}A_{k\gamma})]. \quad (19)$$

Here, F_α is the generator of $SU(3)$, and $f_{\alpha\beta\gamma}$ and $d_{\alpha\beta\gamma}$ are the usual antisymmetric and symmetric tensors of $SU(3)$. The scattering amplitude is again given by the combination of two terms in Eq. (3). Specializing to Eq. (19) in the space of 14 channels of $SU(3)\times SU(2)$, we have

$$C\Gamma-\Gamma=-\theta\Lambda\,, \quad (20)$$

where C is a 14×14 crossing matrix obtained by finding the direct product of the $SU(3)$ crossing matrix[13] and the $SU(2)_J$ crossing matrix. We label the channels, in an obvious generalization of the notation used earlier, by $(S,2J)$ where S takes the values 1, 8_{as}, 8_{aa}, 8_{ss}, 10*, 10, and 27, while $J=\frac{1}{2}$ and $\frac{3}{2}$. The nucleon contribution to Γ in the channels $(8_{as},1)$, $(8_{aa},1)$ and $(8_{ss},1)$ depends on the D/F ratio, and is $(2\sqrt{5})\alpha(1-\alpha)\gamma_8$, $3(1-\alpha^2)\gamma_8$, $5\alpha^2\gamma_8$, respectively, where $D/F=\alpha/(1-\alpha)$. The contribution of the decuplet of spin-$\frac{3}{2}$ resonances is γ_{10} in the channel (10,3). The projection of Λ, unlike the previous case, depends on the D/F ratio, which in turn depends on the choice of Γ. For example, if Γ has contributions from nucleon and (1,3) only, the coupling is pure D, and Λ takes the form $\Lambda\equiv\theta(-2, -26, 0, 0, -3, -12, -19, -4, -12, 6, -12, 6, 6, 6)$. The channels are in the order (1,1); (1,3); $(8_{as},1)$; $(8_{as},3)$; $(8_{aa},1)$; $(8_{aa},3)$; $(8_{ss},1)$; $(8_{ss},3)$; $(10^+,1)$; $(10^*,3)$; (27,1); (27,3). With nucleon and (10,3) exchange, the value of Λ is found to be

$$\Lambda=\theta(-42, -6, -8\sqrt{5}, 4\sqrt{5}, -23, -2, -7, -10, 12, -6, -4, 2, -2, 10). \quad (21)$$

If this term dominates the low-energy scattering, we would have 13 independent sum rules, which can be obtained from Eq. (21) in an obvious way.

The CGS mass-splitting term may be calculated as before, and is $(C+1)\Gamma'$, where Γ' has only a (10,3) contribution[14] for the **56** representation of baryons in $SU(6)$. Thus

$$M\equiv\epsilon(-60, -15, 24\sqrt{5}, 6\sqrt{5}, 0, 0, 24, 6, 12, 3, 12, 39, 4, 1). \quad (22)$$

It is clear that by combining the two terms Λ and M we get 12 independent sum rules. These include the two sum rules that we have obtained in Sec. II. It may be possible to check these sum rules as data on other meson-baryon scattering becomes available. However, it should be emphasized that $SU(3)$ mass splitting has not been taken into account, so that the agreement is not expected to be too good.

B. Relations between Spin-flip and Spin-nonflip Amplitudes

If we again assume that the meson commutator term (19) dominates the scattering, we have the result that only $f_{\alpha\beta\gamma}F_\gamma\delta^{ij}$ contributes to spin-nonflip amplitude f; and only $\epsilon_{ijk}(J_k\delta^{\alpha\beta}/3+d_{\alpha\beta\gamma}A_{k\gamma})$ contributes to g. The spin-nonflip amplitude is independent of the choice of baryon representation and by projecting it into $SU(3)$ invariant amplitudes, we have the result

$$A^{10}=A^{10*}=A^{8as}=0\,, \quad (23)$$

$$A^1=2A^{8aa}=2A^{8ss}=-3A^{27}. \quad (24)$$

These equations give essentially all the results for spin-nonflip amplitudes obtained in Ref. 3. Since only the nonflip amplitude survives in the forward direction, these relations are valid for the complete amplitude in the forward direction. They also imply the Johnson-Treiman relations as is pointed out in Ref. 3. They yield six independent sum rules for p-wave scattering amplitudes, and these can be obtained from Eq. (15). Spin-flip results assuming (10,3) resonances are given by

$$A^1=3A^{27}=6A^{10}=-2A^{10*}=-12A^{8ss} =(12/7)A^{8aa}=(3/2\sqrt{5})V^{8as}. \quad (25)$$

These equations give six more sum rules, which can be obtained using Eq. (16). All the twelve sum rules are contained in the thirteen that follow from Λ in Eq. (21). However, the twelve sum rules that are obtained by including the CGS term are probably in better agreement with experiment.

IV. CONCLUSIONS

We have derived sum rules for low-energy meson-baryon scattering using the intermediate-coupling theory, which is based on the static Chew-Low model. Such a theory, although approximate, may have a certain validity for low-energy phenomena as borne out by the success of static $SU(6)$. Though higher

[13] See for example, S. Gasiorowicz, *Elementary Particle Physics* (John Wiley & Sons, Inc., New York, 1966).

[14] The CGS solution for $\theta\to 0$ contains **10*** and **27** exchange also; however, now θ may be taken as nonzero to obtain the value of the CGS term consistent with only **10** exchange.

invariance groups like $SU(4)$ or $SU(6)$ have not been explicitly assumed, the meson commutator is chosen to yield baryons belonging to representations of these groups. Many of the results are independent of the choice of any specific representation and are valid even if baryons belong to representations of noncompact groups. The sum rule $p^{1,1}=4p^{1,3}=4p^{3,1}$ is an example of such a result. A large number of sum rules are obtained from the group $SU(3)\times SU(2)_J$ which may be checked against experiment as results become available. The method used throughout is rather simple and involves only the knowledge of the relevant crossing matrices. Under certain simplifying assumptions results are obtained also for spin-flip and spin-nonflip amplitudes which generalize the work of earlier authors. The nonflip results are in agreement with the Johnson-Treiman relations in the forward direction as was shown in previous work.

ACKNOWLEDGMENTS

The author has benefited from many helpful discussions with Professor P. R. Auvil and a correspondence with Professor E. C. G. Sudarshan.

PHYSICAL REVIEW VOLUME 163, NUMBER 5 25 NOVEMBER 1967

Analysis of Pion Photoproduction in the Second Resonance Region

Y. C. Chau*
Glasgow University, Glasgow, Scotland

AND

Norman Dombey
University of Sussex, Brighton, England

AND

R. G. Moorhouse†
Rutherford Laboratory, Didcot, England

(Received 6 April 1967)

A phenomenological analysis is made of the pion photoproduction data in the energy range $535<E_\gamma<850$ MeV. The analysis is based on the pion-nucleon phase-shift analyses using a generalized isobar model. The Watson theorem for the elastic pion-nucleon partial waves is imposed. η photoproduction data in the same energy range is also analyzed using an effective range expansion of the K matrix. The nucleon-isobar electromagnetic couplings are determined.

I. INTRODUCTION

In this paper we study the photoproduction of pions from protons in the region $535<E_\gamma<850$ MeV where E_γ is the photon laboratory energy. Analysis of pion photoproduction in the lower-energy region from threshold has been extensive[1] and has been theoretically approachable in terms of dispersion relations following Chew, Goldberger, Low, and Nambu[2] for two basic reasons. First, this region is dominated by the nucleon isobar $N_{3/2}^*(1238)$, $J^P=\frac{3}{2}^+$. Second, this region (or at least the lower end of it) is elastic, and so elastic unitarity and the Watson theorem apply. This means that the γN amplitude is related to the πN amplitude and that the dispersion integrals can be reasonably evaluated.

The center-of-mass (c.m.) energy W of the photoproduced pion-nucleon system in the second resonance region is in the range 1375–1575 MeV, which contains three N^* resonances. These are the $N_{1/2}^*(1525)$ $J^P=\frac{3}{2}^-$ (d_{13} wave in the pion-nucleon system), the $N_{1/2}^*(1570)$ $J^P=\frac{1}{2}^-$ (the $N\eta$ resonance, s_{11} wave in the pion-nucleon system), and the $N_{1/2}^*(1400)$ $J^P=\frac{1}{2}^+$ (p_{11} wave in the pion-nucleon system).[3] Furthermore, all three resonances are appreciably inelastic and so we cannot expect to be able to make the same type of theoretical analysis as in the lower-energy region. The details of the photoproduction of these resonances are of importance in considerations of symmetry theories. Besides this resonance question it has also become evident recently that a knowledge of the multipole amplitudes for photoproduction in this energy region is necessary for the more exact evaluation of some current commutator sum rules.

We have made a phenomenological analysis of the photoproduction processes

$$\gamma+p\to\pi^0+p,\quad \gamma+p\to\pi^++n$$

* Based in part on a thesis submitted to the University of Sussex in 1966, in fulfillment of the requirements for the degree of Doctor of Philosophy.

† Present address: Glasgow University, Glasgow, Scotland.

[1] A. Donnachie and G. Shaw, Ann. Phys. **37**, 333 (1966). This paper has a fairly complete list of references.

[2] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345, (1957).

[3] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).