Octet F/D Ratios and Spin–W-Spin Mixing*

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We consider the general problem of calculating the F/D ratios for the baryon+pseudoscalar-meson decays of particles assigned to representations of $U(6)\otimes U(6)$ and $SU(6)_S$, but decaying according to $SU(6)_W$. Unique predictions are found only for the spin-$\frac{5}{2}$ member of the **1134** and the spin-$\frac{3}{2}$ member of the **700**; the values are $-\frac{1}{3}$ and $\frac{2}{3}$, respectively. "Probable" values for the other cases are defined and calculated. Tables of the spin–W-spin mixing coefficients are also provided.

I. INTRODUCTION

THE success of unitary symmetry as a particle classification scheme has not, as yet, been matched by the higher symmetries which include particle spin in some fashion. Although the grouping of a large number of baryons and mesons into the **56** and **35**$\oplus$**1** of $SU(6)$ may be considered a substantial contribution to order among particle states, the question of how to classify the remaining known states is an open one. Two competing approaches of current interest are classification according to representations of $SU(6)\otimes O(3)$ and of $U(6)\otimes U(6)$. The first of these may logically be called the quark model, since it assumes the particle states have internal quantum numbers, including a spin which may be regarded as the sum of quark spins and which couples to orbital angular momentum to form the total spin. The general ideas of this scheme have been outlined by Dalitz,[1] and among recent work is the very interesting dynamical model of Mitra.[2]

The second approach has its roots in the relativistic generalizations of $SU(6)$. The particular form we will consider here[3–5] assigns each particle (at rest) to a representation of nonchiral $U(6)\otimes U(6)$ and to a representation of the $SU(6)_S$ subgroup. The particles are assumed to decay by means of an $SU(6)_W$-invariant interaction.[6] The most ambitious recent application of this scheme is the attempt by Coyne, Meshkov, and Yodh[7] to place the $Y_1^*(1765)$ and $Y_0^*(1520)$ in the **1134** representation of $SU(6)$. Horn, Lipkin, and Meshkov[8] have given plausible arguments to the effect that many members of high-dimensional representations will be difficult to observe. This means that the occurrence of such representations will not likely be signaled by the identification of most of the required states. It becomes necessary, instead, to rely on the ability of the theory to predict the decay parameters of the particles in a given representation (as in Ref. 7). For an octet member of a representation, the F/D ratio describing its coupling to the baryon and pseudoscalar meson octets is such a parameter. In this paper, we will be concerned with computing the F/D ratios for the octet members of those $SU(6)$ representations which can decay into baryon+psuedoscalar-meson, via an $SU(6)_W$-invariant interaction.

The baryons are assigned to the (**56**,**1**) of $U(6)\otimes U(6)$; and the mesons (pseudoscalar nonet+vector nonet), to the (**6**,**6***). Thus we are interested in (**56**,**1**)$\otimes$(**6**,**6***) = (**210**,**6***)$\oplus$(**126**,**6***). The (**210**,**6***) of $U(6)\otimes U(6)$ contains the **1134**, **70**, and **56** representations of the $SU(6)$ subgroups; the (**126**,**6***) contains the **700** and **56**. Spin–W-spin mixing is the chief technical complication in the calculation of the decay parameters. For example, a particle which is assigned to a given octet of the **1134** of $SU(6)_S$ will be a linear combination of octet states belonging to the **1134**, **70**, and **56** of $SU(6)_W$. The decay amplitude will thus be a linear combination of three invariant amplitudes. Finding the coefficients of this linear combination is the essential part of the task, and the next section describes a straightforward (if inelegant) technique for doing so.

II. SPIN–W-SPIN MIXING PARAMETERS

The procedure starts with a consideration of the states found in the (**56**,**1**) and (**6**,**6***) of $U(6)\otimes U(6)$. The spin and W-spin properties of all these states are known. A given state in one of the product representations may be explicitly constructed from these using the $SU(6)$ Clebsch–Gordan coefficients.[9] For example, a nonstrange member of the spin-$\frac{1}{2}$ octet of the **70** could be written

$$|8^2;70\rangle_S=\tfrac{1}{2}\sqrt{\tfrac{1}{2}}\,|N\pi\rangle+\tfrac{1}{4}\sqrt{\tfrac{1}{2}}\,|\Lambda K\rangle-\tfrac{1}{4}\sqrt{\tfrac{1}{2}}\,|\Sigma K\rangle+\cdots. \quad (1)$$

In this expression the symbols N, π, etc., represent the usual names given to the states of the (**56**,**1**) and (**6**,**6***).

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TABLE I. Mixing coefficients for (210, 6*). Each row gives the coefficients for the expansion of the $SU(6)_S$ state in terms of the $SU(6)_W$ states or vice versa. All coefficients are to be divided by the number in the last column.

	1134							70		56	
	$8_A{}^2$	$8_B{}^2$	$8_C{}^2$	$8_A{}^4$	$8_B{}^4$	$8_C{}^4$	8^6	8^2	8^4	8^2	÷
1134:											
$8_A{}^2$	$13\sqrt{10}$	-10	5	$30\sqrt{2}$	$25\sqrt{2}$	$-5\sqrt{5}$	0	35	-15	$16\sqrt{10}$	$30\sqrt{10}$
$8_B{}^2$	-10	$-4\sqrt{10}$	$13\sqrt{10}$	$-20\sqrt{5}$	$20\sqrt{5}$	$-30\sqrt{2}$	0	$-5\sqrt{10}$	$-10\sqrt{10}$	0	$30\sqrt{10}$
$8_C{}^2$	10	$26\sqrt{10}$	$43\sqrt{10}$	$20\sqrt{5}$	$-30\sqrt{5}$	$-5\sqrt{2}$	0	$-15\sqrt{10}$	$5\sqrt{10}$	40	$60\sqrt{10}$
$8_A{}^4$	$3\sqrt{2}$	$-2\sqrt{5}$	$\sqrt{5}$	$\sqrt{10}$	0	0	0	$-\sqrt{5}$	0	$-4\sqrt{2}$	$3\sqrt{10}$
$8_B{}^4$	$25\sqrt{2}$	$20\sqrt{5}$	$-15\sqrt{5}$	0	$9\sqrt{10}$	5	$-6\sqrt{15}$	$-25\sqrt{5}$	$-5\sqrt{5}$	0	$30\sqrt{10}$
$8_C{}^4$	$-10\sqrt{5}$	$-60\sqrt{2}$	$-5\sqrt{2}$	0	10	$15\sqrt{10}$	$30\sqrt{6}$	$-75\sqrt{2}$	$25\sqrt{2}$	$40\sqrt{5}$	$60\sqrt{10}$
8^6	0	0	0	0	$-2\sqrt{15}$	$5\sqrt{6}$	$2\sqrt{10}$	0	$-5\sqrt{30}$	0	$10\sqrt{10}$
70:											
8^2	14	$-2\sqrt{10}$	$-3\sqrt{10}$	$-4\sqrt{5}$	$-10\sqrt{5}$	$-15\sqrt{2}$	0	$-\sqrt{10}$	$-\sqrt{10}$	8	$12\sqrt{10}$
8^4	-6	$-4\sqrt{10}$	$\sqrt{10}$	0	$-2\sqrt{5}$	$5\sqrt{2}$	$-6\sqrt{30}$	$-\sqrt{10}$	$-\sqrt{10}$	8	$12\sqrt{10}$
56:											
8^2	$8\sqrt{10}$	0	10	$-20\sqrt{2}$	0	$10\sqrt{5}$	0	10	10	$\sqrt{10}$	$15\sqrt{10}$

The subscript S on the left-hand side indicates that the states on the right have been combined to produce a state transforming under $SU(6)_S$. The corresponding $SU(6)_W$ state, with W-spin$=\frac{1}{2}$, has the form

$$|8^2;\mathbf{70}\rangle_W=\tfrac{1}{6}\sqrt{\tfrac{1}{2}}\,|N\eta\rangle-\tfrac{1}{3}\sqrt{\tfrac{1}{2}}\,|N\pi\rangle-\tfrac{1}{4}\sqrt{\tfrac{1}{2}}\,|\Lambda K\rangle-\tfrac{1}{12}\sqrt{\tfrac{1}{2}}\,|\Sigma K\rangle+\cdots, \quad (2)$$

which is clearly quite a different state. When expansions like (1) and (2) have been made for all the octets in all the product representations, the mixing coefficients may be determined by taking the scalar products of a given $SU(6)_S$ state with all the $SU(6)_W$ states.

TABLE II. Mixing coefficients for (126, 6*).

	700		56	
	8^2	8^4	8^2	÷
700:				
8^2	$7\sqrt{15}$	$6\sqrt{5}$	$-10\sqrt{3}$	$9\sqrt{15}$
8^4	$2\sqrt{5}$	$\sqrt{15}$	10	$3\sqrt{15}$
56:				
8^2	$-10\sqrt{3}$	30	$-\sqrt{15}$	$9\sqrt{15}$

In fact, however, there is an additional complication. It should be noted that both the (**210**,**6***) and the (**126**,**6***) contain a **56**. Each is some linear combination of the **56**'s obtained in **56**⊗**35** and **56**⊗**1**. Fortunately, the correct linear combinations are fixed uniquely by the requirement that the $SU(6)_S$ states of the (**210**,**6***) be orthogonal to both the $SU(6)_S$ and $SU(6)_W$ states of the (**126**,**6***), and vice versa. That is, spin–W-spin mixing occurs only within a given $U(6)\otimes U(6)$ representation.

The mixing coefficients obtained as described above are given in Tables I and II. The notation for the states is that of Ref. 9. As an example of the use of the tables, we write

$$|8^6;\mathbf{1134}\rangle_S=-\tfrac{1}{5}\sqrt{\tfrac{3}{2}}\,|8_B{}^4;\mathbf{1134}\rangle_W+\tfrac{1}{2}\sqrt{\tfrac{3}{5}}\,|8_C{}^4;\mathbf{1134}\rangle_W+\tfrac{1}{5}|8^6;\mathbf{1134}\rangle_W-\tfrac{1}{2}\sqrt{3}\,|8^4;\mathbf{70}\rangle_W. \quad (3)$$

Expansions such as (3) enable us to find all the constraints on the decay parameters which are provided by the symmetry scheme. In the next section, we will apply the mixing coefficients to the particular task of calculating the octet F/D ratios for the baryon+pseudoscalar meson decays.

III. CALCULATION OF F/D RATIOS

The computation of the parameters for the baryon +pseudoscalar-meson decay modes begins with the expansion of the initial and final states in terms of states transforming under $SU(6)_W$. The pseudoscalar mesons are W-spin vector particles, due to the W-S flip phenomenon,[6] and so the final states are formed from W-spin-$\frac{1}{2}$ and W-spin-1 octets. The Clebsch-Gordan coefficients[9] provide us with the reduction of each octet in the expansion of the initial state into the symmetric and antisymmetric octets resulting from the $8^2\otimes8^3$ of final state. As an example, consider the spin-$\frac{5}{2}$ octet of the **1134**. Equation (3) gives the appropriate expansion. The amplitude for decay into baryon+pseudoscalar meson is

$$\langle BP|8^6;\mathbf{1134}\rangle_S=(\tfrac{1}{4}\sqrt{\tfrac{1}{6}})(A-D)(s\sqrt{5}+a). \quad (4)$$

In Eq. (4), s and a are the $SU(3)$ Clebsch-Gordan coefficients relevant to the reduction of $8\otimes8$ into the symmetric and antisymmetric octets, respectively. A and D are the invariant amplitudes for decays proceeding through channels with the quantum numbers of the **1134** and **70**, respectively. Equation (4) differs from the results of Ref. 7 only in the over-all sign, reflecting a slightly different choice of phases. Internal consistency

TABLE III. F/D ratios for (**210**, **6***). The first two rows give statistical values calculated from Eqs. (8) and (9) of the text. The last three rows give the values which follow from assuming that only the channel named in the first column contributes to the decay

	1134							**70**		**56**
	$8_A{}^2$	$8_B{}^2$	$8_C{}^2$	$8_A{}^4$	$8_B{}^4$	$8_C{}^4$	8^6	8^2	8^4	8^2
r_0	0.66	−0.88	1.15	0.78	3.22	0.79	−0.33	0.31	0.48	−4.01
Δr	0.03	0.02	0.47	0.18	0.22	0.15	0.00	0.08	0.55	5.32
1134	0.65	−0.88	1.73	0.90	3.45	1.22	···	0.30	0.35	−14.3
70	0.75	−1.0	0.87	1.67	3.00	0.87	···	−2.33	−2.33	−2.33
56	0.67	···	0.67	0.67	···	0.67	···	0.67	0.67	0.67

of phases has been checked in the present scheme by checking orthogonality of all the $SU(6)$ states.

In general, each decay amplitude will have the form $\alpha s+\beta a$, with s and a as defined above. The conventional F/D ratio is defined by

$$r=-(\beta\sqrt{5})/3\alpha. \tag{5}$$

Applying this to Eq. (4) we find that the coupling of the spin $\frac{5}{2}$ octet of the **1134** has F/D ratio $r=-\frac{1}{3}$.

Unfortunately, only the 8^6, **1134** and the 8^4, **700** have uniquely determined F/D ratios in this theory; for the latter we find $r=\frac{2}{3}$. The reason for this is easy to see: α and β are linear combinations of the various invariant amplitudes corresponding to the available decay channels. Generally, these linear combinations are not simply multiples of one another and since the symmetry scheme imposes no restrictions on the invariant amplitudes, r will be a function of them. An illuminating way of discussing the problem is to rewrite Eq. (5) in the form

$$r=\mathbf{g}\cdot\mathbf{b}/\mathbf{g}\cdot\mathbf{a}. \tag{6}$$

In (6), the vector $\mathbf{g}$ has the invariant amplitudes as its components, and $\mathbf{b}$ and $\mathbf{a}$ are vectors formed from the coefficients of these amplitudes found in the numerator and denominator of (5). The vectors $\mathbf{b}$ and $\mathbf{a}$ are all that the theory provides; $\mathbf{g}$ contains the dynamics. It is clear that Eq. (6) gives a unique value of r only if $\mathbf{a}$ and $\mathbf{b}$ are exactly parallel; otherwise r can vary from zero (for $\mathbf{g}$ perpendicular to $\mathbf{b}$) to infinity (for $\mathbf{g}$ perpendicular to $\mathbf{a}$). Intuitively, however, we feel that when $\mathbf{a}$ and $\mathbf{b}$ are nearly parallel, the theory has made a stronger statement than otherwise. This feeling can be made quantitative as follows. First, we can take both $\mathbf{g}$ and $\mathbf{a}$ to be unit vectors in (6) without loss of generality. Now let us assume that all directions of $\mathbf{g}$ are equally likely. We find that the probability that the F/D ratio lies between r and $r+dr$ is $P(r)dr$, where

$$P(r)=\frac{1}{\pi}\frac{|\mathbf{a}\times\mathbf{b}|}{[r-(\mathbf{a}\cdot\mathbf{b})]^2+(\mathbf{a}\times\mathbf{b})^2}. \tag{7}$$

This distribution is of Breit-Wigner type with its maximum at

$$r_0=(\mathbf{a}\cdot\mathbf{b}), \tag{8}$$

and half-width at half-maximum of

$$\Delta r=|\mathbf{a}\times\mathbf{b}|. \tag{9}$$

These numbers give us a quantitative measure of how well the theory determines the F/D ratio. Values of r_0 and Δr are given in Tables III and IV for all the octets of interest here.

TABLE IV. F/D ratios for (**126**, **6***).

	700		**56**
	8^2	8^4	8^2
r_0	0.54	0.67	−2.79
Δr	0.05	0.00	3.73
700	0.52	···	−8.34
56	0.67	···	0.67

It should be emphasized that the statistical treatment given above is not intended as an argument that all directions of $\mathbf{g}$ are indeed equally likely. It is to be hoped that nature has not depended upon chance to fix her basic laws. The assumption merely reflects the fact that only some additional dynamical assumptions can reduce our ignorance. In Tables III and IV we have given for comparison the F/D ratios which follow from the crude dynamical assumption of single-channel dominance: i.e., that only one component of $\mathbf{g}$ is significant.

IV. DISCUSSION

In this paper, we have investigated the ability of one currently interesting symmetry scheme to make predictions about the decay modes of the particles which it is supposed to encompass. This ability is particularly important for the theory in question, since experimental observation of the decay rates is expected to play a predominant role in the assignment of particles to multiplets. In this respect, the lack of uniqueness of the F/D ratios is disappointing, but it should be noted that the constraints imposed on these parameters are nonetheless considerable. For the (**210**,**6***), all decays are dependent on but three invariant amplitudes; the (**126**,**6***) requires only two. Thus, it should be possible to eliminate or reduce the ambiguities if the F/D ratios of more than one octet in a representation can be measured. The statistical values of r given in Sec. III may prove of some value in choosing a hypothesis when only one ratio is known. F/D ratios which differ greatly from the statistical values are also interesting, because in the present context they provide rather strong statements about the dynamics; i.e., about the relative strength of the invariant amplitudes.

As a case in point, consider the $N^*(1570)$ resonance with $J^P=\frac{1}{2}^-$. There is evidence that this state has a coupling to $N\eta$ at least comparable to its coupling to $N\pi$.[10] This means that r is unlikely to lie between 0 and 1. Tables III and IV reveal that the $8_B{}^2$ and $8_C{}^2$ of **1134**, and the 8^2 of either **56** are the only likely assignments, using the statistical values. The former possibilities fit in well with the work of Ref. 7. On the other hand, this state has previously been mentioned as a candidate for the **70**$^-$ by Gyuk and Tuan.[11] If their hypothesis is accepted, then the observed coupling gives a fairly strong constraint on the dynamics; for example, **70** dominance is obviously favored.

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Some Consequences from Superconvergence for πN Scattering

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Sum rules of the superconvergent type are obtained for the πN helicity-flip and helicity-nonflip amplitudes. The sum rules for the helicity-nonflip amplitudes are shown to be consistent with the Regge-pole-dominance model. Investigation of the sum rule for $B^{(-)}(\nu)$ leads us to speculate as to the existence of resonances on N_β, Δ_γ, and N_δ baryon trajectories.

I. INTRODUCTION

Recently, an exact sum rule for the $\pi^- p$ helicity-nonflip forward-scattering amplitude with charge exchange has been proposed in order to investigate singularities in the complex J plane.[1,2] Assuming that there are no other singularities in the complex J plane except the ρ Regge pole above $J=-1$ at $t=0$, we separated the helicity-nonflip amplitude $f^{(-)}(\nu)$ into the ρ-pole term $f_\rho(\nu)$ and the remaining term $f^{(-)\prime}(\nu)$, which vanishes faster than ν^{-1} at infinity. Since the $f^{(-)\prime}(\nu)$ is odd under crossing symmetry, satisfies an unsubtracted dispersion relation, and vanishes faster than ν^{-1} for $\nu\to\infty$, we were immediately led to the following sum rule of the superconvergent type:

$$4\pi f^2-\frac{1}{2\pi}\int_\mu^\infty d\nu\{(\nu^2-\mu^2)^{1/2}[\sigma_{\pi^-p}(\nu)-\sigma_{\pi^+p}(\nu)]-4\pi\beta_\rho P_{\alpha_\rho}(\nu/\mu)\}=0\,, \quad (1)$$

with[3]

$$f^2=\frac{g_r{}^2}{4\pi}\left(\frac{\mu}{2m}\right)^2=0.081\pm0.002\,. \quad (2)$$

An experimental check of the above sum rule has suggested to us that Eq. (1) holds[4] within the present accuracy of the total cross-section measurements. Therefore, we have concluded that the experiments support the ρ Regge-pole-dominance model at high energy, even though we cannot rule out the possibility of the existence of other singularities (including a ρ' pole or a cut) if the pole residue or discontinuities are reasonably small.

This ρ-pole-dominance model has also been strongly favored[5] by the remarkable diffraction shrinkage at high energy for the reaction $\pi^-+p\to\pi^0+n$, and the dip phenomena[6] observed in the above and other reactions. The single-ρ-exchange model, however, predicted no polarization for the above reaction $\pi^-+p\to\pi^0+n$, which was not consistent with the observed nonzero polarization.

Recently, a possible model to overcome this difficulty was proposed by Desai, Gregorich, and Ramachandran.[7] They pointed out that if baryon trajectories continue to rise for quite large energies, then, as a consequence of assuming the total amplitude to be given by the single ρ Regge-pole term and the direct-channel contribution from baryon trajectories, it is possible to explain the

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