Strong Interactions as a Possible Mediator of Weak Interactions*

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Assuming current algebra and using Bjorken's methods, it is argued that inclusion of the strong interactions to all orders will almost certainly not remove the nonrenormalizability of the weak interactions.

I. INTRODUCTION

It has long been a tacit hope that the nonrenormalizability of the weak interactions[1] might go away if, at each order in the weak coupling, the strong interactions are introduced to all orders—that is, strong "form factors" may damp the singular weak forces. Such an idea is extremely attractive because, if all the higher-order weak corrections can be made finite in this way, one might have justification for neglecting them as small. Our purpose here is to argue that, if the (pure) strong interactions are described by current algebra, this hope is almost certainly unfounded.[2]

Our order of presentation is as follows. In Sec. II we use Bjorken's[3] methods to examine the exchange of two W-mesons between hadrons, to all orders in the strong interactions. The process is still quadratically divergent. In general, using a crude power-counting argument, the exchange of N W-mesons between hadrons (to all orders in the strong interactions) is divergent like Λ^{2N-2} (Λ an invariant cutoff mass); that is, the degree of divergence of these ladder graphs with all strong form factors is still just that of a ladder of bare nucleons. This is, in a sense, not surprising because, after all, the current algebra needed is true in a theory with free elementary nucleons. In Sec. III we study the sum of all graphs to fourth order in the weak coupling, i.e., vertex corrections and nucleon and W-meson self-energy corrections, in addition to the box graphs of Sec. II. Here we note that not all these graphs can be written in terms of currents with closed W loops, which keeps us from making universal statements: For some processes, such as elastic proton-proton scattering, it is clear that the quadratic divergence cannot cancel. For others, such as charge-exchange neutron-proton scattering, we cannot prove that the divergence does not cancel, although some simple arguments make a cancellation implausible. We do not attempt vertex function (etc.) analysis for higher than fourth order. At the end of this Section, we mention that the nonrenormalizability persists for semileptonic processes (e.g., higher-order correction to β decay, etc.), and purely leptonic processes.

II. LADDER GRAPHS WITH ALL STRONG INTERACTIONS

We begin with the expression for the exchange of two W-mesons between hadronic systems to all orders in the strong interactions (see Fig. 1):

$$M=\delta^{(4)}(p+q-p'-q')g^4\int d^4k\int d^4x\, e^{-ik\cdot x}$$
$$\times\langle p',c|T\{j_\mu(x),j_\nu^\dagger(0)\}|p,a\rangle\Delta_F^{\mu\mu'}(k)\Delta_F^{\nu\nu'}(k+p-p')$$
$$\times\int d^4x' e^{ik\cdot x'}\langle q',d|T\{j_{\mu'}^\dagger(x'),j_{\nu'}(0)\}|q,b\rangle, \quad (1)$$

where g is the weak coupling in the W theory, j_μ is the purely hadronic weak-interaction current,[4]

$$j_\mu=\cos\theta\, F_\mu{}^{1+i2}+\sin\theta\, F_\mu{}^{4+i5}, \quad F_\mu=V_\mu-A_\mu, \quad (2)$$

θ is Cabibbo's angle, and $\Delta_F^{\mu\mu'}(k)$ is the usual W-meson propagator. Note that hadronic system a is not necessarily the same as c, nor are they necessarily single hadron states. Assuming the absence of operator Schwinger terms, a time-ordered product between hadron states is covariant. This structure includes, of course, both direct and crossed box graphs whenever such are compatible with the external quantum numbers. It is important to realize that Eq. (1) contains many more graphs than, say, just the box graph for nucleons with a strong form factor at each vertex. Such a particular subset of graphs might well be convergent, but, in such an approximation, the time-ordered products would not satisfy the correct divergence

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[1] Representative references in the history of nonrenormalizable field theory can be found in M. B. Halpern, Phys. Rev. **140**, B1570 (1965); J. Math. Phys. **7**, 1226 (1966); Ann. Phys. (N. Y.) **39**, 351 (1966). See also, K. Bardakci and B. Schroer, J. Math. Phys. **7**, 10, 16 (1966); W. Guttinger and E. Pfaffelhuber, Nuovo Cimento (to be published); A. Jaffe, Phys. Rev. Letters **17**, 663 (1966); Phys. Rev. **158**, 1454 (1967).

[2] A start in this direction was made by M. B. Halpern and G. Segrè, Phys. Rev. Letters **19**, 611 (1967), in which it is noted that, even with strong corrections, all second-order weak nonleptonic decays are quadratically divergent in the W theory.

[3] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

[4] We are assuming the usual theory with only charged W mesons. In this theory, of course, the $\Delta I=\frac{1}{2}$ rule emerges dynamically through the current algebra. There exist theories with neutral W's, such as the "schizon" theory of T. D. Lee and C. N. Yang [Phys. Rev. **119**, 1410 (1960)]. Although there are more graphs in such a theory, all our qualitative conclusions about degree of divergence remain unchanged. It should also be noted that, if the Fermi theory can be considered as the infinite-mass limit of the W theory, our conclusions apply to it as well.

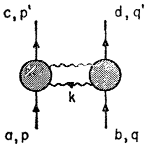

FIG. 1. Two W-meson exchange between hadrons.

condition, etc.—i.e., the current algebra is not represented by such a subset of graphs.

The loop in Fig. 1 is really only a function of one 4-vector k, so we can use Bjorken's methods directly to isolate the most divergent term. In a frame with $\mathbf{k}=0$, each of the time-ordered products go, for large k_0, as $(k_0)^{-1}$; explicitly for the left-most, we have

$$-i\int d^4x\, e^{-ik\cdot x}\langle p',c|T\{j_\mu(x),j_\nu{}^\dagger(0)\}|p,a\rangle \rightarrow \frac{1}{k_0}\int d\mathbf{x}\langle p',c|[j_\mu(\mathbf{x},0),j_\nu{}^\dagger(0)]|p,a\rangle. \quad (3)$$

As a model, we work out Eq. (3) using the recently proposed gauge-field algebra of currents[5] in the case of strangeness conservation. Any other reasonable model[6] gives the same degree of divergence with different coefficients. Doing the commutation, and going to the covariant form, we obtain

$$-i\int d^4x\, e^{-ik\cdot x}\langle p',c|T\{j_\mu(x),j_\nu{}^\dagger(0)\}|p,a\rangle \rightarrow \frac{1}{k^2}\langle p',c|\left[k_\mu\mathfrak{F}_\nu(0)-k_\nu\mathfrak{F}_\mu(0)+\frac{k_\mu k_\nu}{k^2}k\cdot\mathfrak{F}(0)\right]|p,a\rangle, \quad (4)$$

where

$$\mathfrak{F}_\mu=2\cos^2\theta F_\mu{}^3+\sin^2\theta[F_\mu{}^3+(1/\sqrt{3})F_\mu{}^8]. \quad (5)$$

With this in hand, we can isolate the most singular part of Eq. (1), keeping only the $k_\mu k_\nu$ term in the W propagators,

$$M_{\text{singular}}=\frac{g^4}{M^4}\int\frac{d^4k}{(k^2-M^2)}\times\frac{\langle p',c|k\cdot\mathfrak{F}|p,a\rangle\langle q',d|k\cdot\mathfrak{F}|q,b\rangle}{(k+p-p')^2-M^2} \quad (6)$$

where M is the W-meson mass, and we have suppressed the energy-momentum conserving δ function. We see the two-meson exchange is quadratically divergent. For example, if all the hadrons are identical and spinless, this reduces, with all the hadrons at rest, to

$$M_{\text{singular}}=\frac{g^4\mu^2}{M^4}[Q(2\cos^2\theta+\sin^2\theta)+\tfrac{1}{2}Y\sin^2\theta]^2\int\frac{d^4k}{k^2}, \quad (7)$$

where μ, Q, and Y are, respectively, the mass, charge, and hypercharge of the hadron.

Note added in proof. In this discussion we have tacitly assumed, with Bjorken, that all (W-hadron scattering) amplitudes satisfy unsubtracted dispersion relations, whereas Regge theory indicates that some of them need subtractions. It would be interesting, though difficult, to explore the consequences of subtractions and/or Schwinger terms in our arguments. Intuitively, we expect such to make the divergences worse.

As mentioned in the Introduction, the quadratic divergence is not surprising really, because, in general, current algebras are true in free theories: Certainly the box graphs for bare nucleons are quadratically divergent. For pions, the same is true, remembering that the time-ordered product now also contains contact graphs where the W-mesons meet on the pion line.

For multiple W exchange between hadrons, we content ourselves with a rough power-containing argument, just as we would for the bare ladder graphs. Graphs analogous to Fig. 1, but with N mesons exchanged, involve time-ordered products of N currents between hadron states. We can exhibit the large-momentum dependence of such structures in analogy with the two-current form, e.g.,[7] by replacing each $e^{ik\cdot x}$ by $(ik_0)^{-1}\partial_0 e^{ik\cdot x}$ and integrating by parts to generate a power series in $(k_0)^{-1}$. In the case of three currents,

$$\int\int d^4x_1d^4x_2e^{ik_1\cdot x_1}e^{ik_2\cdot x_2}\langle B|T\{j_{\mu_1}(x_1)j_{\mu_2}(x_2)j_{\mu_3}(0)\}|A\rangle \rightarrow -\frac{1}{k_{01}k_{02}}\int d\mathbf{x}_1\int d\mathbf{x}_2e^{-i\mathbf{k}_1\cdot\mathbf{x}_1}e^{-i\mathbf{k}_2\cdot\mathbf{x}_2}\times\langle B|\{[[j_{\mu_1}(\mathbf{x}_1),j_{\mu_2}(\mathbf{x}_2)],j_{\mu_3}(0)]+[j_{\mu_2}(\mathbf{x}_2),[j_{\mu_1}(\mathbf{x}_1),j_{\mu_3}(0)]]\}|A\rangle, \quad (8)$$

and, in general for N currents, we find the time-ordered product goes down like $(k_0)^{-N+1}$. Thus, counting momenta, the exchange of N W-mesons between hadrons is divergent like Λ^{2N-2}, where Λ is an invariant cutoff mass. As above, we note that this degree of divergence is exactly that of, say, a ladder of bare nucleons. Inclusion of the strong interactions doesn't seem to help at all.

[5] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

[6] The only model in which the leading divergence would be zero is one for which the commutator in Eq. (3) vanished, which would appear to disagree with experiment.

[7] Alternatively, one can write spectral representations following Bjorken. Essentially, for each θ-function in the time ordering, there is another energy denominator in the representation.

III. NUCLEON-NUCLEON SCATTERING TO FOURTH ORDER

In Sec. II, our discussion was limited to ladder-type graphs to all orders in both the weak and the strong interactions. At each order in the weak interactions there are, however, other graphs, namely vertex and self-energy corrections, which could conceivably cancel the ladder-graph divergences. In this Section, we want to study these other graphs, for definiteness, in the case of nucleon-nucleon scattering to fourth order. We note that, in general, the cancellation is not possible.

The processes that need to be considered in addition to the ladder graphs are shown in Fig. 2. All blobs are time-ordered products of purely hadronic currents. Because the usual theory[4] has only charged W mesons, we have no 3-W vertices. Moreover, the 4-W vertex that might contribute to the W renormalization is absent by normal ordering. Note that we cannot write all the graphs as currents with closed W-loops, so, at least in the case of Fig. 2(d), we cannot make a definite statement about divergence.

It is best to consider separate cases. For proton-proton scattering, none of the graphs of Fig. 2 are present (by quantum numbers), so the quadratic divergence of the box graphs persists. In the case of charge-exchange neutron-proton scattering, on the other hand, all the graphs of Fig. 2 contribute (along with the graphs of Fig. 1), so we proceed to discuss them. Fig. 2(a) is quadratically divergent by our previous methods. After the vertex renormalization however, the divergence is only logarithmic and cannot cancel the box-graph divergence. A similar statement applies to the W-renormalization graphs of Fig. 2(c). The graphs of Fig. 2(c) are quadratically divergent with a coefficient depending on the hadronic states H, but again, after nucleon renormalization, the divergence is only logarithmic. The graphs of Fig. 2(d) are not obviously divergent at all (because our methods only work for closed W-loops). In perturbation theory, with say, a nucleon-antinucleon loop, the process would

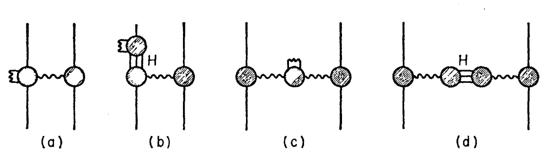

FIG. 2. Other strong corrections to fourth-order nucleon-nucleon scattering: H stands for hadronic intermediate states.

again be logarithmically divergent after renormalization. We consider it highly unlikely, although not inconceivable, that the graphs of Fig. 2(d) can cancel the divergence of Fig. 1 for n-p scattering; most probably, the quadratic divergence persists, just as for p-p scattering.

In conclusion, we have argued that the nonrenormalizability of the weak interactions persists for nonleptonic processes, even with all strong form factors. The arguments clearly go through as well for semileptonic processes; e.g., in the case of β decay, the N-W exchange process is also Λ^{2N-2} divergent. In the case of fourth-order purely leptonic processes our reasoning also applies, with the observation that there are no strong form factors in any graphs except Figs. 2(c) and (d).

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APPENDIX

In Sec. III, we have omitted discussion of some processes which correspond to polynomials in ν, (energy) —and which depend in a more explicit way on how the W meson is coupled into the theory. As in the text, it is unlikely that these additional divergences will conspire to cancel the ones already discussed. In fact, they are in general more divergent than Λ^2. As an example, consider the process in which the nucleons exchange an A_0 (neutral axial vector) line with a W^+W^- intermediate state. In a quark model, where all hadrons are composite, such processes are already included in the double time-ordered product (Fig. 1), but in a generalized Yang-Mills model such one-meson-exchange graphs can appear. This particular set is divergent like Λ^4 after renormalization, thus overpowering the divergences discussed in the text. (Note that current conservation will not reduce this divergence, because the axial current is not conserved—not do the corresponding pion-exchange graphs reduce the divergence.) Another way of saying this is that, because of current nonconservation, the divergence of a time-ordered product of two currents is not just equal to the current vertex. Even in the case of ρ_0 exchange (for which the current is conserved), it is not likely that the divergence (now Λ^2) cancels the box-graph divergence.