Implications of Unequal-Mass Kinematics for the Regge-Pole Model of Vector-Meson Production*

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Application of the Regge-pole model to vector-meson production by incident pseudoscalars is reexamined in the light of recent theoretical advances in the treatment of unequal-mass processes. It is found that in the absence of conspiracy between trajectories at $t=0$, the kinematic factors separated from the helicity amplitudes before Reggeization tend to suppress the effects of parity $=(-1)^J$ exchanges relative to parity $=(-1)^{J+1}$ contributions, in the region $|t|\leq 0.2$ GeV^2. This helps account for the experimental observation that, although α_π is less than $\alpha_{A2}, \alpha_\omega \cdots$, π exchange tends to dominate the forward peak wherever it is possible (production of ρ and K^* mesons). This kinematic suppression also helps explain the failure of ρ exchange to dominate in ω production.

I. INTRODUCTION

It has traditionally been difficult to interpret high-energy experiments on vector-meson production in terms of the Regge-pole model. In particular, the data for forward production of ρ mesons appear to be dominated by π exchange at all energies despite the fact that the ω and A_2 trajectories have larger $\alpha(t)$'s. Another difficulty is that ω production differential cross sections and density matrices at small t bear scant resemblance to those predicted on the basis of ρ trajectory exchange.

From the theoretical standpoint, exact application of the model has been uncertain because (as in all unequal-mass processes) there is a region about $t_{\min}$ in which $|\cos\theta_t|$ remains small.[1,2] However, recent developments in the Regge-pole theory of unequal-mass scattering[3] and the construction of helicity amplitudes free of kinematical singularities[4] now make possible a straightforward parametrization of inelastic reactions in which the particles have arbitrary spins and masses.[5] Within this framework, the Regge asymptotic form of a helicity amplitude is[5]

$$f_{cd;ab}{}^t=\frac{[1\pm e^{-i\pi\alpha(t)}]}{\sin\pi\alpha(t)}K(t)(\sin\tfrac{1}{2}\theta_t)^{|\lambda-\mu|}(\cos\tfrac{1}{2}\theta_t)^{|\lambda+\mu|}\times\gamma(t)\left(\frac{s}{s_0}\right)^{\alpha-M},\quad(1)$$

where

$$\lambda=a-b,\quad \mu=c-d,\quad M=\max(|\lambda|,|\mu|).$$

Here $K(t)$ is a known function of t containing kinematic singularities of the amplitude, and $\gamma(t)$ is a smooth function of t.

The purpose of this paper is to point out implications in this formalism of the fact that parity conservation at the π-vector-Regge vertex requires a helicity change ($M\geq 1$) for $P=(-1)^J$ exchanges, but allows the possibility of no helicity change ($M=0$) for exchange of a trajectory with $P=(-1)^{J+1}$. This restriction on M means that every amplitude for exchange of parity $P=(-1)^J$ will contain factors of the form

$$(\sin\tfrac{1}{2}\theta_t)^{|\lambda-\mu|}(\cos\tfrac{1}{2}\theta_t)^{|\lambda+\mu|},$$

which decrease rapidly in the forward direction. A detailed study of the factors

$$K(t)(\sin\tfrac{1}{2}\theta_t)^{|\lambda-\mu|}(\cos\tfrac{1}{2}\theta_t)^{|\lambda+\mu|}$$

leads to the conclusion that, in the absence of conspiracy between trajectories at $t=0$, this effect will suppress $P=(-1)^J$ exchanges relative to the other parity in the region $|t|\leq 0.2$ GeV^2. This accounts qualitatively for π dominance in ρ production, and the masking of ρ exchange in ω production. Although these kinematic factors are implicit in previous work,[1] where the entire $d_{\lambda\mu}{}^J$ function was calculated, the application of daughter trajectories alters the treatment of dynamics and makes the role of purely kinematical effects more explicit.

It should be emphasized at this point that these kinematic effects are only one element of a complete Regge-pole description of the reactions, and that there is no reason for them to completely determine the relative importance of various exchanges, particularly at lower energies. Other well-known explanations of π dominance in such reactions as $\pi N\to\rho N$ include: (a) nearness of the π pole to the s-channel physical region in t, and (b) the relatively large strength of couplings at the vertices. These facts certainly play an important role. The kinematic effect described here is quite distinct, however, and is by no means confined to π exchange. In this paper only the kinematic effects will be discussed, with no quantitative attempt to evaluate the relative importance of such factors as pole position and coupling strength.

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[1] R. Thews, Phys. Rev. 155, 1624 (1967).
[2] E. Fiset, Nuovo Cimento 35, 472 (1965).
[3] D. Freedman and J-M. Wang, Phys. Rev. Letters 17, 569 (1966).
[4] Ling-Lie Wang, Phys. Rev. 142, 1187 (1966).
[5] Ling-Lie Wang, Phys. Rev. 153, 1664 (1967).

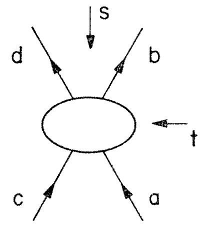

FIG. 1. Labeling of particle kinematics.

Section II contains a review of the methods used to obtain a formula for helicity amplitudes of unequal-mass processes in the Regge-pole model. The main kinematic effect which discriminates between exchanges of different parities is explained in Sec. III, and Sec. IV contains a detailed discussion of all kinematic contributions to vector-meson production reactions. Section V is devoted to a qualitative comparison with experiment, and further problems are indicated in Sec. VI.

II. METHOD OF PARAMETRIZATION

For the elastic scattering of spinless particles, the contribution of the leading Regge pole, obtained from a Sommerfeld-Watson transformation on the amplitude, takes the form

$$\bar{\beta}(t)\frac{[1\pm e^{-i\pi\alpha(t)}]}{\sin\pi\alpha(t)}(q_tk_t)^{\alpha(t)}P_\alpha(\cos\theta_t),$$

where

$$\cos\theta_t=\frac{s}{2q_tk_t}+\frac{t-2(m_a^2+m_c^2)}{4q_tk_t}. \tag{2}$$

As $s\to\infty$, $(q_tk_t)^{\alpha(t)}P_\alpha(\cos\theta_t)\to s^{\alpha(t)}$. Thus, in this case, those trajectories with the highest $\alpha(t)$ should dominate, regardless of the value of (negative) t.

For processes in which the external masses at a single vertex are unequal (Fig. 1),

$$\cos\theta_t=\frac{2t(s-m_a^2-m_c^2)+(t+m_a^2-m_b^2)(t+m_c^2-m_d^2)}{\{[t-(m_a-m_b)^2][t-(m_a+m_b)^2][t-(m_c-m_d)^2][t-(m_c+m_d)^2]\}^{1/2}}. \tag{3}$$

At $t=t_{\min}$, $\cos\theta_t=-1$. Within a region about $t_{\min}$, often referred to as the forward cone, $|\cos\theta_t|$ is small and the replacement of $P_\alpha(\cos\theta_t)$ by $(\cos\theta_t)^\alpha$ is no longer justified. In other words, when the external masses at a single vertex are unequal, it is no longer clear which term in the expansion in powers of s of $(q_tk_t)^{\alpha(t)}P_\alpha(\cos\theta_t)$ dominates at low t. The ambiguity has been resolved in the spinless case by the work on daughter trajectories,[3] which demonstrates that those terms in the expansion which are singular at $t=0$ will be canceled by the contributions of the daughters. Hence the $s^{\alpha(t)}$ behavior of the Regge-pole contribution is preserved, and the resulting amplitude contains only singularities of a dynamical nature.

When the external particles have spin, some care must be used in applying the above method. In order to carry through the procedure, it was necessary to write the invariant amplitude for the process in the form of a dispersion integral with the location of the cuts and the discontinuities across them determined solely by the dynamics of the process. Thus a major problem in the case with spin is to determine which pieces of the physical amplitude can be expressed in this manner. It is most convenient to find expressions closely related to the t-channel helicity amplitudes, because the differential cross section and the density matrices can all be expressed simply in terms of these. It has been found[4] that if $f_{cd;ab}$ is the t-channel helicity amplitude for the problem, defined by

$$\langle p_c,c;p_d,d|S-1|p_a,a;p_b,b\rangle =i(2\pi)^4\delta(p_c+p_d-p_a-p_b)f_{cd;ab}\times[p_a{}^0p_b{}^0p_c{}^0p_d{}^0]^{-1/2}, \tag{4}$$

and

$$\bar{f}_{cd;ab}=f_{cd;ab}/(\sin\tfrac{1}{2}\theta_t)^{|\lambda-\mu|}(\cos\tfrac{1}{2}\theta_t)^{|\lambda+\mu|}, \tag{5}$$

where $\lambda=a-b$, $\mu=c-d$, then

$$\bar{f}_{cd;ab}{}^t\pm\bar{f}_{-c-d;ab}{}^t=K_\pm(t)C_\pm(s,t), \tag{6}$$

where $K_\pm(t)$ may be obtained from the formulas in Ref. 4, and $C(s,t)$ is analytic except for dynamical singularities. C may then be written in the form of a dispersion integral and the application of methods similar to those used in the case of unequal-mass spinless scattering yields an asymptotic form.[6] The result is that C takes the form

$$\gamma_\pm(t)s^{\alpha-M}[1\pm e^{-i\pi\alpha(t)}]/\sin\pi\alpha(t),$$

where $M=\max(|\lambda|,|\mu|)$ and $\gamma_\pm(t)$ is expected to be a smooth function of t. Hence the s dependence of each $f_{cd;ab}$ arises from two sources: (i) the factor C, in which the $s^{\alpha-M}$ behavior is preserved by contributions of daughter trajectories; and (ii) the kinematic factors $(\sin\frac{1}{2}\theta_t)^{|\lambda-\mu|}(\cos\frac{1}{2}\theta_t)^{|\lambda+\mu|}$. At large t, these kinematic factors contribute the remaining s^M, but in the "cone" at low t this growth is suppressed when the masses are unequal.

To place these conclusions for helicity amplitudes on the same intuitive basis as those for spinless scattering, it is helpful to examine some of the details involved in the replacement of $\bar{f}_{cd;ab}{}^t\pm\bar{f}_{-c-d;ab}{}^t$ by

$$K_\pm(t)\gamma_\pm(t)s^{\alpha-M}[1\pm e^{-i\pi\alpha(t)}]/\sin\pi\alpha(t)$$

for a particular exchange. Because of the action of the

[6] M. Gell-Mann, M. Goldberger, F. Low, E. Marx, and F. Zachariasen, Phys. Rev. 133, B145 (1964).

parity operator on helicity states[7]

$$P|J,\lambda_1,\lambda_2\rangle=\eta_1\eta_2(-1)^{J-s_1-s_2}|J,-\lambda_1,-\lambda_2\rangle\,, \quad (7)$$

a Regge pole of parity P will couple to the particular linear combination of vector-pseudoscalar-meson states $|J,\lambda_1\lambda_2\rangle+(-1)^{J+1}P|J,-\lambda_1-\lambda_2\rangle$ [where P is either $(-1)^J$ or $(-1)^{J+1}$]. Thus its contribution to the partial-wave amplitude obeys

$$T_{-c-d;ab}{}^J(t)=P(-1)^{J+1}T_{cd;ab}{}^J(t)\,,$$

and hence for exchange of a spin-J object,

$$\bar{f}_{cd;ab}{}^t\pm\bar{f}_{-c-d;ab}{}^t =\sum_J T_{cd;ab}{}^J\left[\frac{d_{\lambda\mu}{}^J(\theta)}{(\sin\frac{1}{2}\theta_t)^{|\lambda-\mu|}(\cos\frac{1}{2}\theta_t)^{|\lambda+\mu|}} \pm\frac{P(-1)^{J+1}d_{\lambda-\mu}{}^J(\theta)}{(\sin\frac{1}{2}\theta_t)^{|\lambda+\mu|}(\cos\frac{1}{2}\theta_t)^{|\lambda-\mu|}}\right]. \quad (8)$$

Expansion of the $d_{\lambda\mu}{}^J(\theta)$ in powers of $\cos\theta_t$ shows that a given Regge pole contributes with maximum strength to only one of the amplitudes (8); its contribution to the other amplitude is proportional to a smaller power of s. For large s, therefore, if the contribution is mainly to the $(\pm)$ amplitude, $\bar{f}_{cd;ab}{}^t\approx(\pm)\bar{f}_{-c-d;ab}{}^t$. To determine whether the important amplitude is the sum or the difference, it is convenient to use the properties

$$d_{\lambda\mu}{}^J(\theta)=(-1)^{\lambda-\mu}d_{\mu\lambda}{}^J(\theta)=(-1)^{\lambda-\mu}d_{-\lambda-\mu}{}^J(\theta) \quad (9)$$

to obtain

$$d_{\lambda\mu}{}^J=e^{i\phi_1}d_{\lambda_1\lambda_2}{}^J(\theta),\quad d_{\lambda-\mu}{}^J=e^{i\phi_2}d_{\lambda_3\lambda_4}{}^J(\theta)\,,$$

where ϕ_1, ϕ_2, and the λ_i's are defined by the requirement that $\lambda_1+\lambda_2$, $\lambda_1-\lambda_2$, $\lambda_3+\lambda_4$, and $\lambda_3-\lambda_4$ all be greater than or equal to zero. For $d_{\lambda_1\lambda_2}{}^J$ functions in which the indices have this property,

$$d_{\lambda_i\lambda_j}{}^J=(\sin\tfrac{1}{2}\theta_t)^{\lambda_i-\lambda_j}(\cos\tfrac{1}{2}\theta_t)^{\lambda_i+\lambda_j} \times P_{J-M}{}^{(\lambda_i-\lambda_j,\lambda_i+\lambda_j)}(\cos\theta_t)\,; \quad (10)$$

hence,

$$\bar{f}_{cd;ab}{}^t\pm\bar{f}_{-c-d;ab}{}^t =\sum_J T_{cd;ab}{}^J[e^{i\phi_1}P_{J-M}{}^{(|\lambda-\mu|,|\lambda+\mu|)}(\cos\theta_t) \pm P(-1)^{J+1}e^{i\phi_2}P_{J-M}{}^{(|\lambda+\mu|,|\lambda-\mu|)}(\cos\theta_t)], \quad (11)$$

and the amplitude with the most important contribution has sign $(\pm)$ if $e^{i(\phi_2-\phi_1)}(-1)^{J+1}P$ is $(\pm)$.

Systematic application of the method of Ref. 6 shows that $\gamma_\pm(t)$ contains functions of $\alpha(t)$. These are not important except at those places where they require the amplitude to vanish; in general, an amplitude derived from $d_{\lambda\mu}{}^J$ must vanish for all integer J smaller than $|\lambda|$ or $|\mu|$. In vector-meson production at low negative t, this means that all $P=(-1)^J$ exchanges (which have $M=1$, as shown in Sec. III) must vanish at $\alpha=0$. Because $\alpha=0$ occurs for $|t|\gtrsim0.5$ GeV^2 for the ρ, ω, and A_2 exchanges considered here, this effect may be considered independently of forward kinematic effects and is not of central interest to this paper.

For our purposes, the important thing to notice is the form

$$f_{cd;ab}=g(t)(\sin\tfrac{1}{2}\theta_t)^{|\lambda-\mu|}(\cos\tfrac{1}{2}\theta_t)^{|\lambda+\mu|}s^{\alpha(t)-M}. \quad (12)$$

At the lowest physical $|t|$ for the s-channel process, $\cos\theta_t\to-1$, $\cos\frac{1}{2}\theta_t\to0$.[8] Also, $(d\cos\theta_t)/ds\to0$ as $t\to0$ for any unequal-mass process. For such low $|t|$, therefore, the s dependence of the amplitude is reduced by the maximum helicity in the t channel M. If for some reason the trajectory with largest $\alpha(t)$ cannot have $M=0$, a lower trajectory may account for the observations at small $|t|$. This is exactly what happens in vector-meson production.

In order to distinguish between the simple cone effect ($|\cos\theta_t|\sim1$) due only to unequal masses, and this effect (which depends also on a particular treatment of the spin kinematics), the reduction in power of s will be referred to as spin suppression.

III. A KINEMATIC PECULIARITY OF VECTOR-MESON PRODUCTION

Application of the parity operator to a pion-vector-meson helicity state gives

$$P|JM;\lambda0\rangle=(-1)^{J+1}|JM;-\lambda0\rangle. \quad (13)$$

Thus the state in which the vector meson has helicity zero can couple only to the systems with "unnatural parity," $P=(-1)^{J+1}$. [This will be true for the production by pseudoscalar mesons of any system with "natural" parity, $P=(-1)^J$.] Hence, $M=1$ is the smallest value of M possible for exchange of a Regge trajectory with $P=(-1)^J$. This means that in any reaction where the π trajectory is the $P=(-1)^{J+1}$ exchange with highest α, its contribution will dominate the cross section at large s and sufficiently small t despite the possibility of natural parity exchanges with higher-lying trajectories. For larger t, the $\sin\frac{1}{2}\theta_t$ and $\cos\frac{1}{2}\theta_t$ terms are proportional to $s^{1/2}$; thus for large enough t and s, the normal hierarchy of trajectories should be seen.[9]

[7] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) 7, 404 (1959).

[8] The factor $(\cos\frac{1}{2}\theta_t)^{|\lambda+\mu|}$ ensures that only those amplitudes with $\lambda=-\mu$ will be nonzero in the backward direction of the t channel. This is just a statement of angular-momentum conservation for the t-channel amplitude, and it is true regardless of the external masses. The external masses do, however, play an important role when this factor is evaluated in the s-channel physical region. For the case of equal masses, $\cos\theta_t$ assumes its largest value at $t=0$, $\cos\theta_s=1$, and $\cos\frac{1}{2}\theta_t\sim\sqrt{s}$ at this point. However, when the masses are unequal in either initial or final state of the t channel (or both), $\cos\theta_s=1$ implies $\cos\theta_t=-1$ and t-channel amplitudes with $\lambda\neq-\mu$ are suppressed also in the forward direction of the s channel.

[9] A more general discussion of spin-parity combinations for which this sort of restriction on M occurs can be found in Ref. 5, Appendix 3.

IV. EXPLICIT PARAMETRIZATION FOR THE PROCESSES

Without examining the other pieces of the amplitude in great detail, we have pinpointed a possible explanation for the experimental observation that pseudoscalar exchange tends to dominate the most forward directions in any vector-meson-production reaction in which it is possible, while natural parity exchanges become more important at larger t.[10] The $(\sin\frac{1}{2}\theta_t)^{|\lambda-\mu|}(\cos\frac{1}{2}\theta_t)^{|\lambda+\mu|}$ pieces contain, of course, only a portion of the t dependence of the amplitude. To be able to say that natural parity exchanges are kinematically suppressed at low t, one must make certain that $K(t)$ does not tend to cancel the half-angle effects. To this end, $K(t)$ has been listed for the cases of interest (Table I).

Clearly, all threshold factors and the vector-pseudoscalar pseudothreshold $[(t-(m_V-m_\pi)^2)^{1/2}]$ factors will be smooth enough in the region of small negative t to be absorbed into $\gamma(t)$. The only portions of $K(t)$ which might be important at low t are factors of $t^{1/2}$ and of $[t-(M_N-M_\Delta)^2]^{1/2}$. From Table I, one sees that the dominant amplitudes for both types of parity exchange contain the same $N\bar{\Delta}$ pseudothreshold factor $[t-(M-M_\Delta)^2]^{-1/2}$. Hence this factor can be neglected in the study of relative enhancement; the only parts of $K(t)$ which might compete with the spin-suppression effects are powers of $t^{1/2}$. Most of those listed in Table I fall into three categories:

(i) Factor of $t^{1/2}$ in the π exchange $\lambda=0$, $\mu=0$ amplitude found by factorization of residues. At first glance, one might expect this to be comparable to the $\cos\frac{1}{2}\theta_t$ terms which appear in natural parity exchanges. However, the π exchange amplitude is strongly affected by the π pole, so that the over-all behavior at small t may be approximated by $t^{1/2}/(t-\mu^2)$ which suppresses the amplitude only for $|t|<0.02$ GeV^2.

TABLE I. Kinematic factors $K(t)$ for helicity amplitudes. In this table $\lambda=\lambda_{N'}-\lambda_N$; $\mu=\lambda_V$; $N'=N, \Delta$; $\tau_{\pi V}=[(t-(m_V-m_\pi)^2)\times(t-(m_V+m_\pi)^2)]^{1/2}$.

			$\pi B\to VB$		
P	μ	λ	Solely from crossing matrix	From factorization of residues	Net
$(-1)^{J+1}$	0	0	$t^{-1/2}\tau_{\pi V}^{-1}$	t (π trajectory)	$t^{1/2}\tau_{\pi V}^{-1}$
	1	0	$(t-4M^2)^{1/2}t^{-1/2}$	t (π trajectory)	$t^{1/2}(t-4M^2)^{1/2}$
	0	1	$(t-4M^2)^{1/2}t^{-1/2}$		$(t-4M^2)^{1/2}t^{-1/2}$
	1	1	$(t-4M^2)^{1/2}$		$(t-4M^2)^{1/2}$
$(-1)^J$	1	0	$\tau_{\pi V}$		$\tau_{\pi V}$
	1	1	$\tau_{\pi V}t^{-1/2}$	t	$t^{1/2}\tau_{\pi V}$

$\pi B\to V\Delta$

All factors come from crossing matrix.

P	μ	λ	
$(-1)^{J+1}$	0	0	$\tau_{\pi V}^{-1}[t-(M+M_\Delta)^2]^{-1}[t-(M-M_\Delta)^2]^{-1/2}$
	0	1	$t^{-1/2}[t-(M+M_\Delta)^2]^{-1/2}$
	0	2	$\tau_{\pi V}[t-(M-M_\Delta)^2]^{1/2}t^{-1}$
	1	0	$t^{-1/2}[t-(M+M_\Delta)^2]^{-1/2}$
	1	1	$[t-(M+M_\Delta)^2]^{-1/2}$
	1	2	$\tau_{\pi V}[t-(M-M_\Delta)^2]^{1/2}t^{-1/2}$
$(-1)^J$	1	0	$\tau_{\pi V}t^{-1/2}[t-(M-M_\Delta)^2]^{-1/2}$
	1	1	$\tau_{\pi V}[t-(M-M_\Delta)^2]^{-1/2}$
	1	2	$(\tau_{\pi V})^2t^{-1/2}[t-(M+M_\Delta)^2]^{1/2}$

In contrast, the poles for natural parity exchanges do not have much effect on the amplitudes, and the effects of spin suppression are appreciable out to $|t|\approx 0.2$ GeV^2.[11] Hence, for the range $0.02\leq|t|\leq 0.2$ GeV^2, the π exchange can be expected to dominate natural parity exchanges.[12]

(ii) Factors of $t^{-1/2}$ in $\pi B\to V\Delta$ reactions. In this reaction where all the masses are unequal, $\sin\frac{1}{2}\theta_t$ behaves near $t=0$ like $t^{1/2}$. (In contrast, for $\pi B\to VB$, $\sin\frac{1}{2}\theta_t\sim\frac{1}{2}\sqrt{2}$ near $t=0$.) Because of this behavior, each parity-conserving combination $\bar{f}_{cd;ab}\pm\bar{f}_{-c-d;ab}$ has a kinematic factor $t^{-\xi/2}$, $\xi=\max(|\lambda-\mu|, |\lambda+\mu|)$.[4,13]

It happens, however, that when cross sections are formed in a single Regge-pole model, these amplitudes contribute only the factor $(\sin\frac{1}{2}\theta_t)^{2\xi'}$, where $\xi'=\min(|\lambda-\mu|, |\lambda+\mu|)$. Thus in order to remove kinematic zeros in the cross section, it is necessary to have only a kinematic factor of $t^{-\xi'/2}$ from each of the amplitudes. This is the factor which appears in Table I. It follows from the above discussion that these factors do not make the amplitude increase at small t (because they are canceled by the $t=0$ zero in $\sin\frac{1}{2}\theta_t$), neither do they cancel the zero in $\cos\frac{1}{2}\theta_t$ due to the cone effect.

(iii) Factors of $t^{1/2}$ in the $P=(-1)^J$, $|\lambda|=1$, $|\mu|=1$ and $P=(-1)^{J+1}$, $|\mu|=1$, $\lambda=0$ amplitudes for $\pi B\to VB$. These are implied by the factorization condition when the kinematic singularity $(t^{-1/2})^{\min(|\lambda-\mu|,|\lambda+\mu|)}$ justified in (ii) is used for the $\pi\pi\to VV$ amplitude.

Hence, for single Regge-pole models, none of the kinematic factors in $K(t)$ will cancel the spin-suppression effect in the region $0.02\leq|t|\leq 0.2$ GeV^2.[14] The $t^{1/2}$ factors are the kinematic factors which have the greatest effect on the behavior in the region of interest; thus, for purposes of fitting, the amplitudes

[10] S.-U. Chung *et al.*, Phys. Rev. Letters **16**, 481 (1966).

[11] An estimate of the region in which suppression is most important may be obtained by at least two methods: (i) Calculate $\cos^2\theta_t$ for the s in question as a function of t. It is found that it rises sharply from 1 at $t_{\min}$ to some $(\cos^2\theta_t)_0$, and then remains close to this value for the rest of the range considered ($|t|\leq 0.5$ GeV^2). The rise for $2.5\leq p_\pi\leq 8$ GeV/c occurs principally in the region $|t|\leq 0.2$ GeV^2. (ii) Calculate $(d\cos\theta_t)/ds$ as a function of t. This increases rapidly from 0 at $t=0$ as $|t|$ increases in the physical region. Above $|t|\approx 0.2$ GeV^2, the rate of increase

$$\left[\frac{d}{d(-t)}\frac{d}{ds}(\cos\theta_t)\right]$$

is appreciably slower than it is below this value. This criterion depends only on the masses involved, not on the energy of the experiment.

[12] This is the one place in the present paper where the nearness of the π pole is brought in explicitly.

[13] Y. Hara, Phys. Rev. **136**, B507 (1964).

[14] This has also been noticed by B. Diu and M. Le Bellac, Phys. Letters **24B**, 416 (1967).

may be approximated by the form

$$h(t)f(t^{1/2})\left[\frac{1\pm e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}\right](\sin\tfrac{1}{2}\theta_t)^{|\lambda-\mu|}\times(\cos\tfrac{1}{2}\theta_t)^{|\lambda+\mu|}\left(\frac{s}{s_0}\right)^{\alpha(t)-M}, \quad (14)$$

where $f(t^{1/2})$ is found in Table I, and $h(t)$ is more slowly varying.

A number of comments about this parametrization are in order:

(a) In both types of reactions considered, the product of kinematic factors $K(t)(\sin\frac{1}{2}\theta_t)^{|\lambda-\mu|}(\cos\frac{1}{2}\theta_t)^{|\lambda+\mu|}$, when evaluated in the physical region of the s channel, has the same phase (up to a sign) for all helicity amplitudes corresponding to a given trajectory. This means that relative phases between helicity amplitudes are determined solely by the $\gamma(t)$'s.

(b) The entire residue takes the form

$$K(t)(P_{\pi V}P_{NN'})^{\alpha-M}\gamma(t).$$

Near each fermion-antifermion threshold, it is observed to have the behavior (Ref. 5, footnote 8)

$$(t-4M^2)^{L_i} \quad \text{or} \quad [t-(M+M_\Delta)^2]^{L_i}, \quad (15)$$

where L_i is the smallest permissible orbital angular momentum for the parities and spins involved.

Near the pseudothreshold $t=(M-M_\Delta)^2$ it has the behavior $[t-(M-M_\Delta)^2]^{L'}$, where L' is the lowest possible orbital angular momentum for a pair of fermions with intrinsic parity+. In other words, the pseudothreshold behavior is the same as threshold behavior for a particle with negative mass and opposite parity to one of the fermions.

The behavior at the boson threshold is normal and is identical to that at the pseudothreshold. Presumably this is because the antibosons have the same parities as the bosons.

(c) Because we are approximating all amplitudes by Regge poles, the residues $K(t)\gamma(t)(P_{\pi V}P_{NN'})^{\alpha-M}$ must obey the factorization condition. This places additional constraints on the analytic pieces $\gamma(t)$.[5] The additional powers of t discussed above are the minimum required by the comparison

$$[\beta_{\lambda,\mu}{}^{\pi N\to VN}]^2=\beta_{\mu,\mu}{}^{\pi\pi\to VV}\beta_{\lambda,\lambda}{}^{NN\to NN}$$

[provided that the factor $(t^{-1/2})^{\min(|\lambda-\mu|,|\lambda+\mu|)}$ is usedi n computing $\beta^{\pi\pi\to VV}$]. This provides information only at $t=0$, because all the residues automatically have the proper behavior at thresholds.

(d) The kinematic factors obtained from crossing-matrix and factorization considerations agree with those found by considering all perturbation-theory graphs for the exchange.[15]

(e) The kinematical factors in Table I are (with the exception of the $|\lambda|=1$, $\mu=0$ amplitude of $\pi B\to VB$, as explained in Ref. 15) those which would apply in the case of a single Regge-pole exchange, i.e., when only one parity-conserving helicity amplitude is important to highest power of s. This situation may be characterized by the requirement that all subsidiary conditions[16,17] imposed on the helicity amplitudes by kinematics at $t=0$ are satisfied by the vanishing of individual regularized helicity amplitudes at this point, rather than by conspiracy between amplitudes.

Removal of this requirement would allow the amplitudes for $\pi B\to V\Delta$ and $\pi\pi\to VV$ to have a singularity of order $(t^{-1/2})^{|\lambda|+|\mu|}$; also the additional t's listed for some $\pi B\to VB$ amplitudes would not be required. The coefficients of the singular terms would in this case have to satisfy conspiracy relationships. In the s-channel physical region, the resulting singularities of the $|\lambda|=|\mu|$ helicity amplitudes would cancel the spin-suppression dip due to half-angle factors, and there would be no kinematic reason for $P=(-1)^{J+1}$ dominance of the forward peak. However, all the experimental evidence to date on vector-meson production is consistent with the no-conspiracy hypothesis; hence we conclude that any contributions actually containing the additional singularities are small enough to be neglected in these reactions. This is the point of view taken in the analysis in Sec. V.

Relations between the amplitudes at points other than $t=0$ (thresholds and pseudothresholds) have not been considered.

V. APPLICATION TO DATA

For the purpose of illustration, we consider only the reactions $\pi N\to\omega N$, $\pi N\to\rho N$, $\pi N\to\omega\Delta$, and $\pi N\to\rho\Delta$ because the number of possible exchanges in these is sharply limited by G parity and isospin. Identical kinematic statements can be made about the processes $KN\to K^*N$, $KN\to K^*\Delta$; the density matrices and differential cross sections for these reactions have the same qualitative behavior as those for ρ production.[18]

The differential cross section for production of ρ

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[15] The unexpected $t^{-1/2}$ for $P=(-1)^{J+1}$, $\mu=0$, $\lambda=1$ can be obtained from the amplitude $\epsilon^\mu\bar{N}\gamma_5\gamma_\mu N$ by remembering that the longitudinal ϵ^μ vector is singular at $t=0$ for this problem (Ref. 13)

$$\epsilon_0{}^\mu=(1/m_V)(P_V;P_V{}^0\hat{P}_V).$$

Because the contribution of this amplitude to the cross section must vanish in the forward direction ($t=t_{\min}\neq0$) due to angular-momentum conservation, the presence or absence of this $t^{-1/2}$ is not of major importance to the effect discussed here. It should be noted, however, that if only the amplitude $\bar{f}_{00;\frac{1}{2}-\frac{1}{2}}{}^t$ is excited (corresponding to standard treatment of a single Regge-pole exchange with the appropriate quantum numbers), a thorough study of subsidiary conditions at $t=0$ (Ref. 16) requires that $\bar{f}_{00;\frac{1}{2}-\frac{1}{2}}{}^t$ behave like $t^{1/2}$ rather than $t^{-1/2}$; this further suppresses it in the forward direction.

[16] H. Hogaasen and Ph. Salin, CERN Report No. TH. 788 (unpublished).

[17] G. Cohen-Tannoudji, A. Morel, and H. Navelet, Saclay Report, 1967 (unpublished).

[18] R. George *et al.*, Nuovo Cimento **46**, 539 (1966); CERN Report No. 66-18 (unpublished).

mesons, both with and without an isobar, possesses a well-defined diffraction peak at all energies in the range 2.5–8-GeV/c incident-pion momentum.[19–21] The spin-density matrices of the ρ's produced closely resemble those for elementary one-pion exchange ($\rho_{00}=1$; $\mathrm{Re}\rho_{10}=0$, $\rho_{1-1}=0$) at the lowest physical t, but as t increases the value of ρ_{00} decreases and that of ρ_{1-1} increases.[21] This makes the density matrices look somewhat more like those expected from elementary vector-meson exchange ($\rho_{00}=\mathrm{Re}\rho_{10}=0$).

Trajectories which may be exchanged in this reaction are π, $A_2(R)$, and ω (in non-charge-exchange $\pi N \to \rho N$). Because the A_2 and ω have natural parity, it is expected that they will be suppressed at small t and that the forward peak is produced by π exchange. In fact, the region $0.05 \leq |t| \leq 0.4$ GeV^2 agrees fairly well with an amplitude of the form

$$\begin{aligned} \pi N \to VN \quad & t^{1/2} g(t)[1+e^{-i\pi\alpha_\pi(t)}] \\ & \times (s/s_0)^{\alpha_\pi(t)}/\sin\pi\alpha_\pi(t)\,, \\ \pi N \to V\Delta \quad & \bar{g}(t)[1+e^{-i\pi\alpha_\pi(t)}] \\ & \times (s/s_0)^{\alpha_\pi(t)}/\sin\pi\alpha_\pi(t)\,, \end{aligned} \tag{16}$$

over the experimental range of s. Deviations from this at larger t become more pronounced at the higher energies, as expected (the region over which suppression is appreciable decreases as the energy increases).[11,22]

In contrast, the differential cross section for production of ω mesons is quite flat as a function of t at all energies measured.[19,20,23] Among well-known trajectories, only the ρ can contribute, but none of the differential cross sections show the expected dip at $t=0.5$ GeV^2.[24] Furthermore, the density matrix elements consistently disagree with ρ-exchange dominance: Pure ρ exchange implies $\rho_{00}=0$, but the measured values average to $\frac{1}{2}$; and a reasonable model of the ρ-nucleon-isobar vertex[25] predicts Δ density matrix elements which also disagree with the data.[26]

This behavior is at least plausible in the light of the above kinematic separation. The ρ exchange is suppressed at the lowest t by kinematics and at $t=-0.5$ GeV^2 by a zero in the amplitude. It is not surprising, therefore, that it should be entirely masked by background effects, such as the possible exchange of a B meson ($\alpha_B \gtrsim \alpha_\rho - 1$).[27] If these are the only causes of suppression, the ρ-trajectory contribution should rise above the background at large t and s. This would probably produce a small bump in the differential cross section above $t=-0.5$ GeV^2.

A further indication that spin-suppression factors play a major role is provided by comparing the cross section for $\pi^+N^+ \to \omega N^{*++}$ with that for $\pi^+N^+ \to \pi^0 N^{*++}$ at the same energy. Both reactions are expected to be dominated by ρ exchange, but whereas $\pi^+N^+ \to \pi^0 N^{*++}$ differential cross sections consistently show a forward peak and a dip at $\alpha_\rho=0$,[19,20] the $\pi^+N^+ \to \omega N^{*++}$ distribution in t is flat, as mentioned above. Plots of $\cos\theta_t$ versus t for the two reactions show that the region over which spin suppression might affect $\pi p \to \pi N^*$ is considerably smaller ($|t| \leq 0.05$ GeV^2) than the comparable region for $\pi N \to \omega N^*$, and that much of the ρ peak for $\pi p \to \pi N^*$ falls within the "suppressed" region for ω production.

The shapes of the density matrix elements near $t=0$ are also strongly influenced by the suppression factors. In the coordinate system in which these are normally measured,[28] the density matrix elements can be expressed entirely in terms of t-channel helicity amplitudes, $\rho_{ij} = \sum_\lambda T_\lambda{}^i T_\lambda{}^j / \sum_{k,\lambda} |T_\lambda{}^k|^2$ (i, j refer to the helicity of the particle under consideration; λ runs over all sets of other helicities in the problem). Unless a particular model is used, the behavior of the ρ_{ij} as a function of t is quite complex. However, the behavior for the lowest possible t is easily found (Table II).

TABLE II. Behavior as $t \to t_{\min}$ of density matrix elements. Each expression may be multiplied by an arbitrary constant.

$\pi B \to VB$			
Parity of exchanged trajectory	ρ_{00}	ρ_{10}	ρ_{1-1}
$(-1)^{J+1}$	1	$\left\lvert\frac{\sin\theta_t}{s}\right\rvert$	$\left\lvert\frac{\sin^2\theta_t}{s^2}\right\rvert$
$(-1)^J$	0	0	$\lvert\sin^2\theta_t\rvert$

$\pi B \to V\Delta$	ρ_{00}	ρ_{10}	ρ_{1-1}	ρ_{33}	ρ_{31}	ρ_{3-1}
$(-1)^{J+1}$	1	$\left\lvert\frac{\sin\theta_t}{st^{1/2}}\right\rvert$	$\left\lvert\frac{\sin^2\theta_t}{ts^2}\right\rvert$	$\frac{1}{s^2}$	$\left\lvert\frac{\sin\theta_t}{t^{1/2}s}\right\rvert$	$\left\lvert\frac{\sin^2\theta_t}{ts^2}\right\rvert$
$(-1)^J$	0	0	$\left\lvert\frac{\sin^2\theta_t}{t}\right\rvert$	1	$\left\lvert\frac{\sin\theta_t}{t^{1/2}}\right\rvert$	$\left\lvert\frac{\sin^2\theta_t}{t}\right\rvert$

[19] Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Phys. Rev. **138**, B897 (1965).
[20] Aachen-Berlin-CERN Collaboration, Phys. Letters **19**, 608 (1965).
[21] D. H. Miller *et al.*, Phys. Rev. **153**, 1423 (1967).
[22] An additional indication of the nature of the exchanges is the zero in ω exchange at $\alpha=0$, $t\approx-0.5$ GeV^2. If ω exchange is important, this should produce a sharp dip in the cross section at this point. Because the relative importance of ω exchange at this t is expected to increase with energy, the dip should become more prominent as s increases. So far, a dip at this place has been observed only in $\pi^+p \to \rho^+p$ at 4 GeV/c. (See Ref. 19.)
[23] H. Cohn, W. Bugg, and G. Condo, Phys. Letters **15**, 344 (1965).
[24] Ling-Lie Wang, Phys. Rev. Letters **16**, 756 (1966).
[25] R. Dashen and S. C. Frautschi, Phys. Rev. **152**, 1450 (1966); L. Stodolsky and J. Sakurai, Phys. Rev. Letters **11**, 90 (1963).
[26] Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Nuovo Cimento **35**, 659 (1965).
[27] M. Barmawi, Phys. Rev. **142**, 1088 (1966); Phys. Rev. Letters **16**, 595 (1966). To obtain a rough estimate of the position of the B trajectory, assume that it is a straight line with a slope of 1. In order that it assume the value $+1$ at $t=(1.22)^2$ GeV^2, α_B must be $t-0.49$. Recent fits to the ρ trajectory give $\alpha_\rho \approx t+0.5$. Hence, $\alpha_\rho - 1 \lesssim \alpha_B$ and the ρ and B contributions will have similar s dependence in the low-t region.
[28] K. Gottfried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964).

These general results replace, in a Regge-pole model, the expectations for pure π or vector-meson exchange obtained from field theory. The elements ρ_{1-1}, ρ_{31}, and ρ_{3-1} are predicted to have a definite shape near the lowest t regardless of the parity exchanged; this shape can be seen most clearly in K^* production.[18,29] Additional assumptions must be made to obtain values for the constants which multiply these low-t shapes, or to find the shapes at larger t.[30]

It is interesting to note that the suggested shapes strongly resemble the behavior near $t=0$ of density matrices obtained on the basis of the absorption model.[31] This behavior agrees with the bulk of data to date. The agreement of shape at low t between experiment and the two theories should thus not be viewed as a triumph of a particular dynamical model, but simply as an indication that the kinematical constraints have been handled properly in both theories. Until recently it was not clear how to incorporate these into a Regge-pole model. Hence the agreement in the density matrices provides a great deal of support for the methods of Refs. 3 and 5.

The spin-suppression effect is also expected to occur in tensor-meson production. Because 2^+ mesons have natural parity, the helicity-0 state (and hence any $M=0$ amplitudes) can be populated only by unnatural parity exchange. As in vector-meson production, unnatural parity exchanges should thus dominate others in the forward peak.

Although relatively few data are available on the production of tensor mesons, recent work[19,32] at 4 GeV/c provides some indications that spin-suppression effects may explain features of these processes also. Consider, for example, the reactions $\pi^- p \to f_0 n$, and $\pi^+ p \to A_2 p$. In $\pi^- p \to f_0 n$, the π and A_2 exchanges are allowed. These are the same contributions expected in $\pi^- p \to \rho_0 n$; it is found experimentally that the cross sections for the two reactions have sharp forward peaks, indicating π dominance.[23] In contrast, the A_2 production differential cross section is relatively flat, resembling that for ω production.[19] A_2 production is like ω production in that the only well-accepted trajectories which can contribute (ρ and f_0 to $\pi N \to A_2 N$; ρ to $\pi N \to \omega N$) have natural parity.

VI. DISCUSSION

Careful treatment of kinematic effects and the use of daughter trajectories are only two elements of a complete Regge-pole model of these reactions. Several theoretical problems remain. For example:

(i) The $\pi^+ p \to \rho^0 N^{*++}$ cross section at 8 GeV/c seems almost flat from $|t|_{\min}$ to $|t|=0.1$ GeV^2.[20] This can not be predicted from the kinematic factors for π exchange in Table I.

(ii) It is extremely difficult to obtain purely theoretical estimates of the magnitudes of Regge-pole couplings. Some of the couplings can be obtained at the particle pole by comparison with Feynman graphs, but their variation with t can not at present be predicted. For those couplings which vanish at the particle pole, there is no known way to theoretically estimate magnitudes at any t.

Because these questions are not resolved, it is believed that a least-squares fitting program would be premature. The ambiguity mentioned in (ii) makes it essentially impossible to compare, for example, the magnitudes of ρ and B exchange in ω-production processes. Until such a comparison can be made, one cannot be certain how much of the suppression of natural parity exchanges is due to the spin-suppression effect discussed in this paper. Within the present Regge formalism, however, the $(\sin\frac{1}{2}\theta_t)^{|\lambda-\mu|}(\cos\frac{1}{2}\theta_t)^{|\lambda+\mu|}s^{\alpha-M}$ dependence will continue to play an important role.

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[29] J. Friedman and R. Ross, Phys. Rev. Letters **16**, 485 (1966).

[30] If a particular model is so restrictive that the cross sections drop in the forward direction like $\sin^2\theta_t$, the predicted shapes at $t=0$ of some of the density matrix elements will be different from those in Table II. Experimental cross sections do not seem to have this sort of behavior.

[31] J. D. Jackson *et al.*, Phys. Rev. **139**, B428 (1965).

[32] Aachen-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Nuovo Cimento **31**, 729 (1964).