being counted. In order to compare with theory it is customary to apply a correction factor $(1+\delta)^{-1}$ to the measured cross section to obtain an idealized cross section without radiation. The correction must take into account extra virtual photons as well as radiated real photons in order to avoid divergences. The real-photon part of the correction depends on the detection apertures, since these limit the range of photon momenta that will be counted.

Because of the large difference in masses, we may ignore radiation by the hadrons compared to radiation by the electrons. The similarity of the electron currents in elastic and inelastic scattering then makes the form of the radiative correction the same for each. In elastic scattering the electron momentum at a given angle is constrained by two-body kinematics. In electroproduction the detection of the pion at fixed momentum and angle imposes an analogous constraint on the electron momentum. Therefore, the aperture-dependent parts of the radiative corrections for elastic scattering and electroproduction are quite similar provided the same electron spectrometer is used in both measurements.

Making the peaking approximation in the evaluation of the radiation probability, we obtain

$$\delta_e=\frac{\alpha}{\pi}\left\{\frac{13}{6}\ln\left(\frac{-k^2}{m^2}\right)-\frac{28}{9}+\left[\ln\left(\frac{-k^2}{m^2}\right)-1\right]\times\ln\left[\frac{E}{E'}\left(\frac{\Delta E'}{E'}\right)^2\right]\right\}$$

for the correction to the elastic cross section, and

$$\delta_{e\pi}=\frac{\alpha}{\pi}\left\{\frac{13}{6}\ln\left(\frac{-k^2}{m^2}\right)-\frac{28}{9}+\left[\ln\left(\frac{-k^2}{m^2}\right)-1\right]\times\ln\left[\frac{dE'}{dE}\left(\frac{W}{M}\right)^2\left(\frac{E^2}{E}\right)^2\left(\frac{\Delta E'}{E'}\right)^2\right]\right\}$$

for the electroproduction correction (E' and k^2 are not the same in corresponding measurements though). These corrections agree within 0.02 with more accurate calculations.[50] For our data $\delta=-0.10$ typically. The net effect of the radiative correction on the measured ratio of electroproduction and elastic scattering was never more than 2%. We have ignored both corrections and have estimated the contribution to the error in the ratio to be $\pm 2\%$.

[50] For the correction to elastic scattering see N. Meister and D. R. Yennie, Phys. Rev. **130**, 1210 (1963); for the electroproduction correction see A. Bartl and P. Urban, Acta Phys. Austriaca **24**, 139 (1966).

PHYSICAL REVIEW VOLUME 163, NUMBER 5 25 NOVEMBER 1967

Determination of the S-Wave $\pi\pi$ Phase Shifts in the ρ Region from 4.16-GeV/c π^-p Interactions*

P. B. JOHNSON, L. J. GUTAY, R. L. EISNER, P. R. KLEIN, R. E. PETERS, R. J. SAHNI, W. L. YEN, AND G. W. TAUTFEST
Department of Physics, Purdue University, Lafayette, Indiana
(Received 19 July 1967)

The results of a calculation of the $T=0$ and $T=2$ S-wave $\pi\pi$ elastic scattering phase shifts in the ρ region are presented. Two solutions are found for both. One set for δ_0^0 does not pass through 90°. The physically acceptable solution for the δ_0^2 phase shift is found to be small ($|\delta_0^2|<20°$). The absorption corrections are essential for the determination of δ_0^0.

In this paper, we present the results of a calculation of the $I=0$ and $I=2$ S-wave $\pi\pi$ elastic scattering phase shifts based on a method reported previously.[1] The motivation for this investigation has been to include the effects of an $I=2$, S-wave amplitude and to test the consistency of the model at a different incident momentum. The data is taken from two-prong π^-p interactions at an incident pion momentum of 4.16 GeV/c from an exposure of the Lawrence Radiation Laboratory 72-in. hydrogen bubble chamber. The two reactions considered are

$$\pi^-p\rightarrow\pi^+\pi^-n\,, \quad (1)$$

$$\pi^-p\rightarrow\pi^0\pi^-p\,. \quad (2)$$

The sample consists of approximately 4400 events from reaction (1) and 2900 events from reaction (2). The selection of these events and other details about this experiment are discussed elsewhere.[2]

The absorption-modified one-pion-exchange model has been quite successful in predicting the differential

* Work supported in part by U. S. Atomic Energy Commission.
[1] L. J. Gutay, P. B. Johnson, F. J. Loeffler, R. L. McIlwain, D. H. Miller, R. B. Willmann, and P. L. Csonka, Phys. Rev. Letters **18**, 142 (1967).

[2] R. L. Eisner, P. B. Johnson, P. R. Klein, R. E. Peters, R. J. Sahni, W. L. Yen, and G. W. Tautfest, Phys. Rev. (to be published).

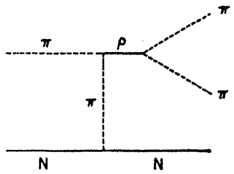

FIG. 1. The Feynman diagram for the one-pion-exchange process $\pi^- N \rightarrow \rho N$.

cross sections and decay angular distributions of the reaction

$$\pi N \rightarrow \rho N \tag{3}$$

over a range of incident momenta from 2.0 to 8.0 GeV/c.[3–8] Where this model is applicable, single-pion production is described by the one-pion-exchange diagram of Fig. 1. The upper vertex in Fig. 1 can be considered as a $\pi\pi$ elastic scattering in $J=1$ state. To account for the observed asymmetry in the ρ decay an S-wave amplitude is included. The S-P wave interference and the absorption effects in ρ decay have been introduced previously.[1,9] When both pions are on the mass shell, the angular distributions for $\pi^+\pi^-$ and $\pi^0\pi^-$ elastic scattering can be written as[10]

$$\frac{d\sigma}{d\Omega} = \lambda\!\!\!^{-2}\{(4/9)\sin^2\delta_0{}^0 + \tfrac{1}{9}\sin^2\delta_0{}^2 + (4/9)\cos(\delta_0{}^0 - \delta_0{}^2) \times \sin\delta_0{}^0 \sin\delta_0{}^2 + [4\cos(\delta_0{}^0 - \delta_1{}^1)\sin\delta_0{}^0 \sin\delta_1{}^1 + 2\cos(\delta_0{}^2 - \delta_1{}^1)\sin\delta_0{}^2 \sin\delta_1{}^1]\cos\theta + 9\sin^2\delta_1{}^1\cos^2\theta\}, \tag{4}$$

and

$$\frac{d\sigma}{d\Omega} = \lambda\!\!\!^{-2}\{\sin^2\delta_0{}^2 + 6\cos(\delta_0{}^2 - \delta_1{}^1)\sin\delta_0{}^2 \sin\delta_1{}^1 \cos\theta + 9\sin^2\delta_1{}^1\cos^2\theta\}, \tag{5}$$

respectively, where $\delta_J{}^I$ denotes the phase shift with isotopic spin I and angular momentum J, and θ is the scattering angle.

We note that Eqs. (4) and (5) cannot be applied directly to reactions (1) and (2), because the Chew-Low result,[11] which relates the $\pi\pi$ cross section to the πN cross section is rigorous only at the pole $\Delta^2 = -\mu^2$ (Δ^2 is the square of the four-momentum transfer to the outgoing nucleon and μ is the mass of a charged pion). Therefore, to determine $\delta_0{}^0$, $\delta_0{}^2$, and $\delta_1{}^1$, one should, ideally, carry out an extrapolation in Δ^2 for the relevant amplitudes, from the boundary of the physical region to the pole. This method, however, requires prohibitive statistics. The statistical errors in $\sigma_{\pi\pi}$, determined by Carmony and Van de Walle,[12] by extrapolating to the pole, are of the order of 50% although their sample consists of approximately 2100 events with $\Delta^2 \leq 7\mu^2$. The S-wave $I=0$ cross section is expected to be $\lesssim 15\%$ of that of the P-wave in the ρ region (700 to 800 MeV). Thus to obtain a sufficient statistical accuracy for the determination of the S-wave cross section, we would need more than an order of magnitude increase in our data. The $\pi\pi$ angular distribution provides more information than the $\sigma_{\pi\pi}$, but the extrapolation of the angular distribution requires further increase in the statistics.

To circumvent the statistical problem which arises from the extrapolation, Selleri suggested how the $\pi\pi$ angular distribution should be modified to correct for off-mass-shell effects.[13] Selleri was able to calculate the

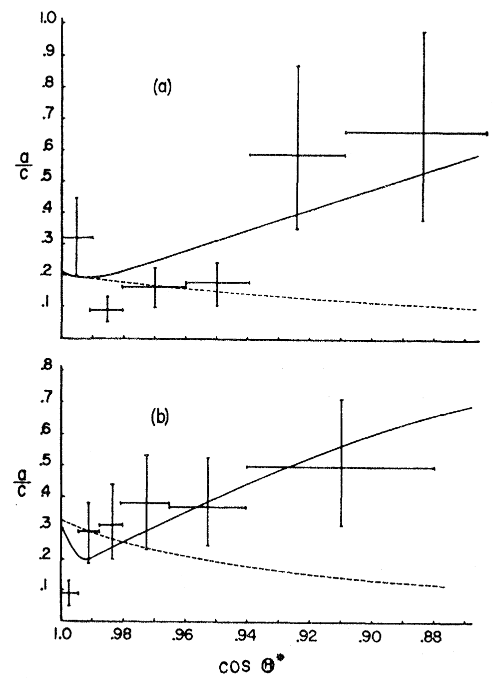

FIG. 2. The ratio of the isotropic term to the coefficient of $\cos^2\theta$ term as function of $\cos\Theta^*$. The dashed and the solid curve are calculated from Selleri's model and the S-P wave absorption model, respectively. (a) Data at 2.7 GeV/c; (b) data at 4.16 GeV/c.

[3] K. Gottfried and J. D. Jackson, Nuovo Cimento **34**, 735 (1964); L. Durand III and Y. T. Chiu, Phys. Rev. **139**, B646 (1965).
[4] J. D. Jackson, J. T. Donohue, K. Gottfried, R. Keyser, and B. E. Y. Svensson, Phys. Rev. **139**, B428 (1965).
[5] J. D. Jackson, Rev. Mod. Phys. **37**, 484 (1965).
[6] N. Schmitz, CERN Report No. 65-24, Vol. 1, 1965 (unpublished).
[7] D. H. Miller, L. J. Gutay, P. B. Johnson, F. J. Loeffler, R. L. McIlwain, R. J. Sprafka, and R. B. Willmann, Phys. Rev. **153**, 1423 (1967).
[8] Two recent works have shown that ω exchange as well as π exchange contributes to $\pi^- p \rightarrow \rho^- p$: W. L. Yen, R. L. Eisner, L. J. Gutay, P. B. Johnson, P. R. Klein, R. E. Peters, R. J. Sahni, and G. W. Tautfest, Phys. Rev. Letters **18**, 1091 (1967), from $\pi^- p$ at 4.16 GeV/c; I. Derado, J. A. Poirier, N. N. Biswas, N. M. Cason, V. P. Kenney, and W. D. Shephard, Phys. Letters **24B**, 112 (1967), from $\pi^- p$ at 8 GeV/c.
[9] L. Durand III and Y. T. Chiu, Phys. Rev. Letters **14**, 329 (1965); P. L. Csonka and L. J. Gutay (to be published). D. Griffiths and R. J. Jabbur, Phys. Rev. **157**, 1371 (1967); M. Bander and G. L. Shaw, *ibid.* **155**, 1675 (1967).
[10] Equations (4) and (5) reflect the observation that in our data we find no evidence for partial waves with $J > 1$ for dipion effective masses less than 1 GeV/c^2.

[11] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).
[12] D. D. Carmony and R. T. Van de Walle, Phys. Rev. **127**, 959 (1962).
[13] F. Selleri, Phys. Letters **3**, 76 (1962).

$\pi\pi$ P-wave cross section from the data in the physical region and was able to reproduce its resonant behavior in the ρ region. A recent analysis[1] at 2.7 GeV/c and the present work at 4.16 GeV/c indicate that the Selleri suggestion does not correctly predict the Δ^2 dependence of the S-wave amplitude (see Fig. 2). Nevertheless, Selleri's approach points in the right direction. In lieu of extrapolation we have to make an assumption for the form of the function which relates the pion-nucleon cross section to the pion-pion cross section when $\Delta^2 \neq -\mu^2$.

The method of calculating S-wave phase shifts from reactions (1) and (2) requires the knowledge of two functions: (a) the dipion decay angular distribution as a function of Δ^2 and (b) the P-wave amplitude as a function of the dipion effective mass. Following Selleri's suggestion,[13] we incorporate the Δ^2 dependence in Eqs. (4) and (5) as

$$\frac{d^3\sigma_{\pi n}{}^i}{d\Delta^2 d(\cos\theta)dw} = R(\Delta^2,w)2\pi\lambda^2\{F_0{}^i(\Delta^2,w)A_i(w) + F_1{}^i(\Delta^2,w)B_i(w)\cos\theta + F_2{}^i(\Delta^2,w)C_i(w)\cos^2\theta\}, \quad i=1,2; \quad (6)$$

where w denotes the $\pi\pi$ effective mass and

$$\begin{aligned} A_1 &= (4/9)\sin^2\delta_0{}^0 + \tfrac{1}{9}\sin^2\delta_0{}^2 + (4/9)\cos(\delta_0{}^0-\delta_0{}^2)\sin\delta_0{}^0\sin\delta_0{}^2, \\ B_1 &= 4\cos(\delta_0{}^0-\delta_1{}^1)\sin\delta_0{}^0\sin\delta_1{}^1 + 2\cos(\delta_0{}^2-\delta_1{}^1)\sin\delta_1{}^1\sin\delta_0{}^2, \quad (7) \\ C_1 &= 9\sin^2\delta_1{}^1 = C_2, \\ A_2 &= \sin^2\delta_0{}^2, \\ B_2 &= 6\cos(\delta_0{}^2-\delta_1{}^1)\sin\delta_0{}^2\sin\delta_1{}^1. \end{aligned}$$

Note that A_i, B_i, and C_i are the on-mass-shell terms from Eqs. (4) and (5). The difference between the procedure presented below and that of Selleri is that in our case the Δ^2-dependent factors F_i are determined from the absorption model.

This model makes definite predictions about the off-mass-shell behavior of the dipion decay angular distribution. The predictions are most conveniently expressed in terms of the density matrix elements. The decay angular distribution of the dipion system is written as

$$W(\theta,\Phi) = \frac{1}{4\pi} + \frac{3}{4\pi}[(\rho_{00}-\rho_{11})(\cos^2\theta - \tfrac{1}{3}) - \sqrt{2}\,\mathrm{Re}\rho_{10}\sin2\theta\cos\Phi - \rho_{1,-1}\sin^2\theta\cos2\Phi] + \frac{\sqrt{3}}{4\pi}[-2\sqrt{2}\,\mathrm{Re}\rho_{10}{}^{\mathrm{int}}\sin\theta\cos\Phi + 2\,\mathrm{Re}\rho_{00}{}^{\mathrm{int}}\cos\theta], \quad (8a)$$

$$U(\theta) = \int d\Phi\, W(\theta,\Phi) = \tfrac{1}{2}[1 + (\rho_{00}-\rho_{11})(3\cos^2\theta - 1) + 2\sqrt{3}\,\mathrm{Re}\rho_{00}{}^{\mathrm{int}}\cos\theta], \quad (8b)$$

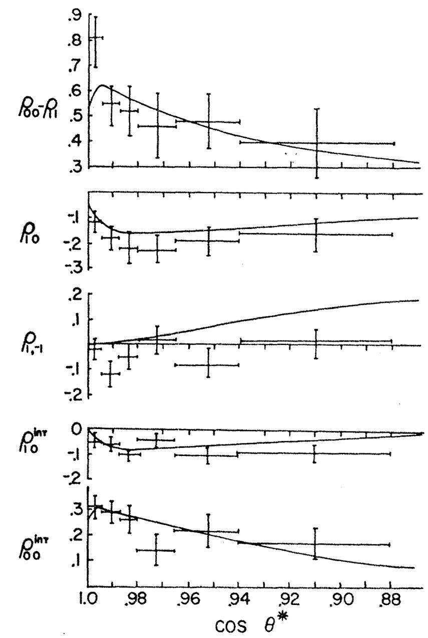

FIG. 3. ρ^0 density matrix elements plotted as functions of the c.m. production angle Θ^*. The solid curves are the predictions of the S- and P-wave absorption model.

where the matrix element notation is the same as that of Ref. 7, θ is the angle between the incoming and outgoing π^- in the dipion rest frame, and Φ is the Treiman-Yang angle.

In addition to the usual absorption parameters,[2,4] a variable a_s is introduced. If we assume a resonant S-wave $\pi\pi$ interaction, a_s would be related to the width of the resonance.[9] It should be emphasized that the absorption model modified here to include an S-wave amplitude does not assume an $I=0$, $J=0$ resonance. The parameter a_s is adjusted to give best fit to the data.

A comparison between the absorption model for reaction (1) and the data at 4.16 GeV/c is shown in Fig. 3. The experimental points are obtained from a maximum-likelihood fit of the data to Eq. (8a) as a function of Θ^*, the c.m. production angle. The errors shown are statistical and are based upon the observation that for sufficient data, approximately 100 events per data point in our case, the likelihood function is nearly Gaussian. In general, the agreement between the experiment and the theory in Fig. 3 is seen to be good. However, the theory does not predict the peaking of $\rho_{00}-\rho_{11}$ at small Θ^*, a discrepancy which is seen also in the 2.7-GeV/c data.[1] The disagreement at 4.16 GeV/c is more serious because it occurs at the limit of the physical region. When we use the absorption model value of $\rho_{00}-\rho_{11}$ at $\Theta^*=0$, we do not obtain a solution for $\delta_0{}^0$ throughout much of the region $0.70 \leq w \leq 0.84$ GeV/c^2. Thus at this limit, we have used the experimental value $\rho_{00}-\rho_{11}=0.81$, in our calculations.

In the application of the absorption model to reaction (2), we have found it necessary to consider the following points. The $N^{*++}(1238)$ production cross section amounts to approximately 9% of reaction (2).[2] Such contamination contributes a well-known bias toward forward decay angles in the dipion system. We have made a subtraction of this effect by removing all events whose $p\pi^0$ effective mass lies in the N^* band, 1238 ± 65 MeV. This deletes 52 events from a sample of 730 events with $\Delta^2\leq 20\mu^2$ and $0.60\leq w\leq 0.84$ GeV. We find that removing these possible N^* events does not change our results. Secondly, evidence for ω exchange as well as π exchange has been found in reaction (2).[8] The experimental $\cos\Theta^*$ dependence of $\rho_{00}-\rho_{11}$ agrees best with a mixture of π and ω exchange.[8] It should be noted that ω exchange cannot contribute to an S-wave amplitude because parity and angular momentum conservation forbid the reaction $\omega+\pi^-\rightarrow\pi^0+\pi^-$ in a dipion S-state. To estimate δ_0^2 in our data we have assumed that $\rho_{00}^{\rm int}$ comes only from S- and P-wave interference.

After confirming the approximate validity of the absorption model for reactions (1) and (2), we calculate the momentum-transfer–dependent terms in Eq. (6). Let a_i denote the isotropic term, b_i the coefficient of $\cos\theta$ and c_i the coefficient of the $\cos^2\theta$ term in Eq. (6). We find that the Δ^2 dependence of the ratios

$$\frac{a_i}{c_i}=\frac{F_0{}^i(\Delta^2,w)}{F_2{}^i(\Delta^2,w)}\frac{A_i(w)}{C_i(w)},\qquad \frac{b_i}{c_i}=\frac{F_1{}^i(\Delta^2,w)}{F_2{}^i(\Delta^2,w)}\frac{B_i(w)}{C_i(w)},\tag{9}$$

are contained in the ratios of $F_k{}^i(\Delta^2,w)$. From Eq. (8b) we can also obtain the ratios $F_0{}^i/F_2{}^i$ and $F_1{}^i/F_2{}^i$. For example,

$$\frac{F_1{}^i(\Delta^2)}{F_2{}^i(\Delta^2)}=\frac{\mathrm{Re}\rho_{00}{}^{\rm int}(\Delta^2)}{\rho_{00}(\Delta^2)-\rho_{11}(\Delta^2)}\times\left[\frac{\mathrm{Re}\rho_{00}{}^{\rm int}(\Delta^2)}{\rho_{00}(\Delta^2)-\rho_{11}(\Delta^2)}\right]^{-1}_{\Delta^2=-\mu^2},\tag{10}$$

which we can calculate from the absorption model.

At this point, we make the assumption that the normalized dipion angular distribution is identical to the normalized on-mass-shell $\pi\pi$ angular distribution at the limit of the physical region. At our energy this limit is only $1.25\mu^2$ from the pole. We incorporate this into our calculation by setting $F_1{}^i/F_2{}^i=F_0{}^i/F_2{}^i=1$ at $\Delta=\Delta_0$, where Δ_0 is the momentum transfer at the limit of the physical region. This assumption is not as stringent a requirement as assuming the validity of the Chew-Low formula at this limit. The remaining unknowns in Eq. (6), the on-mass-shell terms $A_i(w)$, $B_i(w)$, and $C_i(w)$, are determined from a maximum-likelihood fit to the experimental $\pi\pi$ angular distribution as a function of increasing $\pi\pi$ effective mass.

To satisfy requirement (b) we have restricted our calculations to values of w for which the P-wave phase shift $\delta_1{}^1$ can be described by a Breit-Wigner form with energy-dependent width,

$$\tan\delta_1{}^1=\frac{w_r}{w_r{}^2-w^2}\frac{2(q/q_r)^3\Gamma_r}{1+(q/q_r)^2},\tag{11}$$

where $w_r\approx 770$ MeV, q and q_r are the momenta of the decay pions at effective mass w and the resonance peak, respectively, and $\Gamma_r=125$ MeV. For our data the effective-mass limits are taken to be $0.60\leq w\leq 0.84$ GeV.[14]

The phase shifts are obtained from the equations

$$\tan\delta_1{}^1=\frac{\sin2\delta_0{}^2}{3(B/C)_2-2\sin^2\delta_0{}^2},$$

$$\tan\delta_1{}^1=\frac{2\sin2\delta_0{}^0}{9[(B/C)_1-\frac{1}{3}(B/C)_2]-4\sin^2\delta_0{}^0}.\tag{12}$$

Equations (12) yield two sets of solutions $\delta_0{}^I$ and $(\delta_0{}^I)'=\pi/2-(\delta_0{}^I-\delta_1{}^1)$, but the isotropic terms $(A/C)_i$, if accurately determined, could distinguish between the two sets. There is also the indistinguishable ambituity $\delta_0{}^I\pm\pi$.

The values obtained for $\delta_0{}^2$ are shown in Fig. 4. At 760 MeV the value $(B/C)_2$ is such that Eq. (12) does not have a solution; instead the upper and lower limits on $\delta_0{}^2$ and $\delta_0{}^{2\prime}$ are indicated.

Several determinations of the $I=2$, $\pi\pi$ cross sections have been made from πp interactions assuming that the Chew-Low relation is valid for small Δ^2. In $\pi^-p\rightarrow N^{*++}\pi^-\pi^-$ interactions at 2.75 GeV/c, Alitti *et al.*[15] find $|\delta_0{}^2|\approx 0$ to 5° from 700 to 900 MeV and Armenise *et al.*[16] deduce $|\delta_0{}^2|\approx 13°$ and $|\delta_2{}^2|\approx 5°$ from 640 to 860 MeV in the reaction $\pi^+p\rightarrow n\pi^+\pi^+$ at 2.75 GeV/c. These results are generally compatible with each other as well as with the present analysis. The solution of Baton and Regnier[17] is plotted as an inset on Fig. 4. Recently Walker *et al.*,[18] combining π^-p data from 2 to 8 GeV, obtained $\delta_0{}^2$ decreasing slowly from threshold to $\approx -20°$ in the 300- to 900-MeV interval, as plotted on Fig. 4. Considering the fact that no rapid rise in $\pi^-\pi^-$ cross section was found in Ref. 15, we agree with Walker's choice on the physically acceptable set of $\delta_0{}^2$ (the negative set in Fig. 4), though its magnitude is somewhat smaller in our case.

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[14] The cutoff at 0.84 GeV/c^2 is necessary because the P-wave amplitude begins increasing again at that point. The lower limit is approximately 1.4 full widths from the resonance peak.

[15] Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento **35**, 1 (1965).

[16] Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento **37**, 361 (1965).

[17] J. P. Baton and J. Regnier, Nuovo Cimento **36**, 1149 (1965).

[18] W. D. Walker, J. Carroll, A. Garfinkel, and B. Y. Oh, Phys. Rev. Letters **18**, 630 (1967).

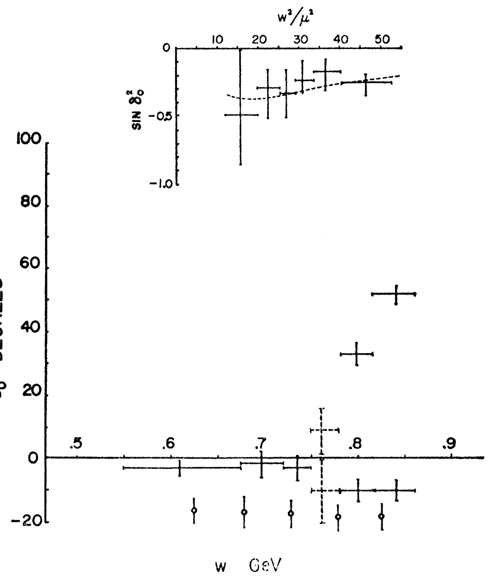

FIG. 4. $I=2$, S-wave phase shift δ_0^2 as a function of the dipion effective mass w. Crossed solid error bars: this experiment; circles with error bars: Ref. 18; crossed dashed error bars: no solution (see text). The insert is from Ref. 17.

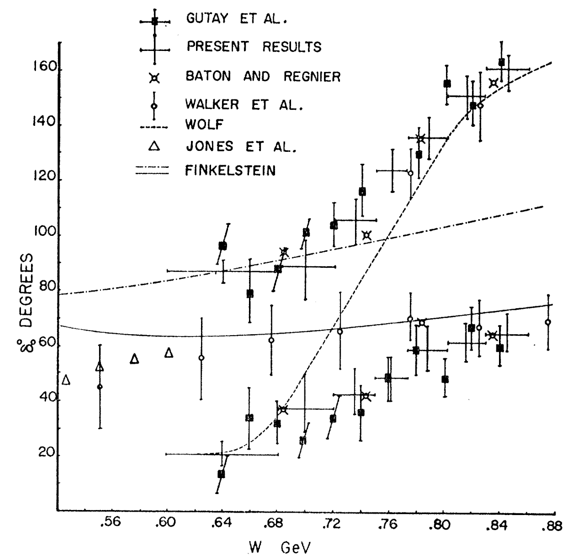

FIG. 5. $I=0$, S-wave phase shift δ_0^0, as a function of dipion effective mass w. The results of six experimental analyses and the theoretical predictions of Finkelstein are presented.

The values of δ_0^0 obtained in various experiments and the theoretical calculation of Finkelstein[19] are plotted on Fig. 5. The results of this experiment are in excellent agreement with Baton and Regnier[17] and with our 2.7-GeV/c results. Our solutions are in disagreement with the result of Wolf[20] who relied on Selleri's model. The result of Jones *et al.*[21] and Walker *et al.*[18] coincide, which is surprising since Jones *et al.*[21] did not introduce absorption effects. Our nonresonant solution for δ_0^0 is in qualitative agreement with that presented by Walker *et al.*,[18] except that Walker's values tend to be larger.

To study the origin of the difference, we note that the following equations of Ref. 18:

$$\begin{aligned}&\sin\delta_0^2 \cos(\delta_1^1-\delta_0^2)+3x_{\min}^{0-} \sin\delta_1^1=0\,,\\&\sin\delta_0^2 \cos(\delta_1^1-\delta_0^2)+2 \sin\delta_0^0 \cos(\delta_1^1-\delta_0^0)\\&\qquad\qquad+9x_{\min}^{+-} \sin\delta_1^1=0\,,\end{aligned}\tag{13}$$

reduce to our Eqs. (10) with $x_{\min}=-B/2C$.

If the difference between our solution and that of Ref. 18 were due to the neglect of $(B/C)_2$ in Eq. (12), as suggested by the authors of Ref. 18, their solution and our solution of δ_0^0 would obviously intersect when $x_{\min}^{0-}$ changes sign. This is not the case however. Since δ_0^0 and δ_0^2 are determined from identical equations by both groups, one would expect that the parameters $x_{\min}$ or equivalently $B/2C$ are different.

The absorption model predicts that the dipion production and decay angular distributions are a function of the incident π^- momentum and the $\pi\pi$ effective mass. Thus the Δ^2 dependence of B/C varies as a function of the incident beam momentum. Therefore, it is surprising that using one constant correction factor, as was done in Ref. 18, to experimental data with incident beam momenta between 2–8 GeV/c and dipion effective masses from 300 to 900 MeV for both $\pi^+\pi^-$ and $\pi^-\pi^0$ angular distributions, their values of δ_0^0 are similar to ours.

Studying the $\pi\pi$ interaction, we have reached the following conclusions:

(1) The absorption-modified one-pion-exchange model with the inclusion of S-wave $\pi\pi$ scattering is able to predict rather well the Δ^2 dependence of the dipion decay angular distribution in the ρ^0 region. As such it forms a basis for extrapolating the angular distribution to the limit of the physical region.

(2) Consistent results for δ_0^0 are obtained at the different momenta 2.7 and 4.16 GeV/c.

(3) The lower set of δ_0^0 which is slightly favored by the data does not exhibit resonant behavior in the region $0.6\leq w\leq 0.84$ GeV.

(4) $|\delta_0^2|$ is less than 20° in the region $0.6\leq w\leq 0.8$ GeV.

(5) Calculations of $\pi\pi$ amplitudes from πN interactions which do not take into account absorption effects tend to overestimate the S-wave and under-estimate the P-wave phase shifts.

We wish to thank D. D. Carmony for several helpful conversations.

[19] J. Finkelstein, Phys. Rev. **145**, 1185 (1966).
[20] G. Wolf, Phys. Letters **19**, 328 (1965).
[21] L. W. Jones, D. O. Caldwell, B. Zacharov, D. Harting, E. Bleuler, W. C. Middlekoop, and B. Elsner, Phys. Letters **21**, 590 (1966).