can be exchanged in the reaction $K^-p \to \bar{K}^0 n$, which is related by $SU(3)$ to the reaction $K^-p \to \Lambda\eta$.]

Isotopic-spin-$\frac{1}{2}$ N^*'s (including the nucleon) can be exchanged in the u channel; see Fig. 8(c). There is no evidence that such exchanges are important here.

C. The Branching Ratio $\Gamma(\eta \to \text{neutrals})/\Gamma(\eta \to \pi^+\pi^-\pi^0)$

The cross section for $K^-p \to \Lambda\eta \to (p\pi^-)(\pi^+\pi^-\pi^0)$, averaged over the momentum distribution of the data, is 37 ± 3 μb. The corresponding cross section with $\eta \to$ neutrals is 148 ± 11 μb, but a systematic overestimation of the $\eta \to$ neutrals of as much as 16% may have been made (see Sec. III). Thus, using all the data, we find that the branching ratio is between 3.2 and 3.8, with a statistical error of 0.4. However, we can substantially reduce the systematic uncertainty in this result by restricting our analysis to events in the forward peak ($\cos\theta_{K\eta} > 0.6$), where the effective resolution-function width in the neutral mass-squared spectra is much narrower than for the entire sample. We find, by the same technique as above, that the branching ratio is between 3.5 and 3.7 with a statistical error of 0.5. Our best estimate of the branching ratio, where we have included both systematic and statistical error, is

$$\Gamma(\eta \to \text{neutrals})/\Gamma(\eta \to \pi^+\pi^-\pi^0) = 3.6\pm0.6\,.$$

This value is consistent with the latest compiled value[4] of 3.25 ± 0.4.

It should be remembered that $\Gamma(\eta \to \text{neutrals})$ is slightly smaller than the sum of the $\pi^0\pi^0\pi^0$, $\gamma\gamma$, and $\pi^0\gamma\gamma$ modes because Dalitz decays (electron-positron pairs at the decay vertex) can occur in all these modes.

These same events have been used to set an upper limit for the branching ratio $\Gamma(\eta \to \pi^+\pi^+\pi^0\gamma)/\Gamma(\eta \to \pi^+\pi^-\pi^0)$ of 7%.[8]

ACKNOWLEDGMENTS

We thank Professor M. Lynn Stevenson and Professor Luis Alvarez for their encouragement and support, and we thank the members of the Scanning and Measuring Group for their contribution to this work.

[8] S. M. Flatté, Phys. Rev. Letters **18**, 976 (1967).

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Determination of the Spin and Parity of the $Y_1^*(1660)$†

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We report experimental evidence for a spin of $\frac{3}{2}$ and negative parity for the $Y_1^*(1660)$, based on a study of a production experiment, $K^-p \to Y_1^*(1660)^+ + \pi^- \to \Sigma^\pm\pi^\mp\pi^+\pi^-$, in the region 2.1 to 2.7 GeV/c. The spin was determined by an Adair analysis of the $Y_1^*(1660)$ decay angular distributions, and the parity determination was based on a Dalitz–Miller type of analysis of the $Y_1^*(1660)$ decay into $\Sigma\pi\pi$, involving three interfering processes and some background.

INTRODUCTION

Of the many established hyperon resonances, the $Y_1^*(1660)$ has a peculiar history in that although its existence has long since been established,[1,2] attempts to measure its spin-parity quantum numbers have as yet been inconclusive, and in some instances have provided contradictory results.[3] In this paper, we report experimental evidence for a spin of $\frac{3}{2}$ and negative parity for the $Y_1^*(1660)$ or $\Sigma(1660)$.[4] The data were obtained from an analysis of the reactions

$$K^-p \to \Sigma^+\pi^+\pi^-\pi^-\,, \quad (1)$$

$$K^-p \to \Sigma^-\pi^+\pi^+\pi^-\,, \quad (2)$$

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[1] G. Alexander, L. Jacobs, G. R. Kalbfleisch, D. H. Miller, G. A. Smith, and J. Schwartz, in *Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962* (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 373.

[2] L. W. Alvarez, M. H. Alston, M. Ferro-Luzzi, D. O. Huwe, G. R. Kalbfleisch, D. H. Miller, J. J. Murray, A. H. Rosenfeld, J. B. Shafer, F. T. Solmitz, and S. G. Wojcicki, Phys. Rev. Letters **10**, 184 (1963).

[3] A review of the experimental situation regarding the $Y_1^*(1660)$ spin and parity is given in the rapporteur's talk by M. Ferro-Luzzi, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, September, 1966* (University of California Press, Berkeley, California, 1967), p. 183.

[4] We will henceforth use this notation of A. H. Rosenfeld, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, P. Soding, S. G. Wohl, M. Roos, and W. J. Willis, Rev. Mod. Phys. **39**, 1 (1967).

for incident K^- beam momenta in the region 2.1–2.7 GeV/c. The pictures were taken in the Berkeley 72-in. hydrogen bubble chamber and analyzed by use of the Alvarez group program system.[5] The exposure had a K^- pathlength equivalent of about 20 events/μb. We found 2814 and 2253 events which fitted reactions (1) and (2), respectively. The events have been weighted to correct for biases in detecting short-lived and small-angle decay Σ's. From this sample, we were able to select a rather large and clean subsample (435 events) of the quasi-two-body reaction

$$K^-p \rightarrow \Sigma(1660)^+ + \pi^- \rightarrow \Sigma^\pm\pi^\mp\pi^+\pi^-, \quad (3)$$

by use of several criteria for the $(\Sigma\pi\pi)^+$ particle combinations: (a) a $(\Sigma\pi\pi)^+$ mass selection; criterion: that the $(\Sigma\pi\pi)^+$ invariant mass be between 1.58 and 1.74 GeV; (b) a $\Lambda(1405)$ selection; criterion: that the $(\Sigma\pi\pi)^+$ system include a $(\Sigma\pi)^0$ combination with an

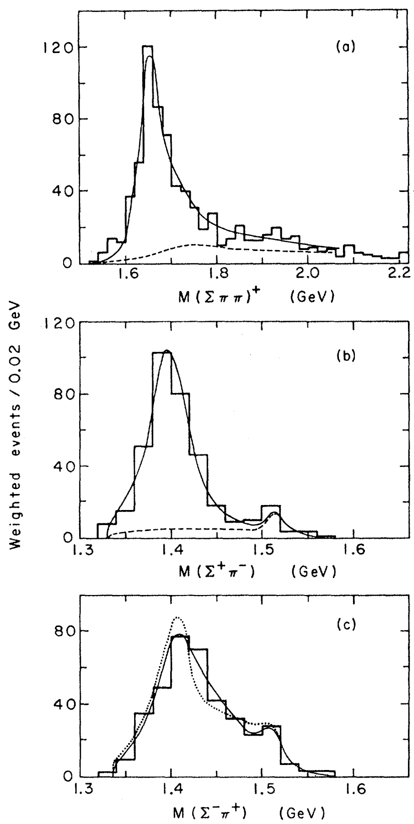

FIG. 1. (a) Mass plot of the $(\Sigma\pi\pi)^+$ system satisfying selection criteria (b) and (c); (b) mass plot of the $(\Sigma^+\pi^-)$ system satisfying selection criteria (a) and (c); (c) mass plot of the $(\Sigma^-\pi^+)$ system satisfying selection criteria (a) and (c), two combinations plotted per event. The solid curves are the results of the best fit for the negative-parity hypothesis for the $\Sigma(1660)$. The dashed curves in (a) and (b) are the corresponding estimates of the non-$\Sigma(1660)$ background, and the dotted curve in (c) is the result of the best fit for positive parity for the $\Sigma(1660)$.

[5] A. H. Rosenfeld and W. E. Humphrey, Ann. Rev. Nucl. Sci. **13**, 103 (1963).

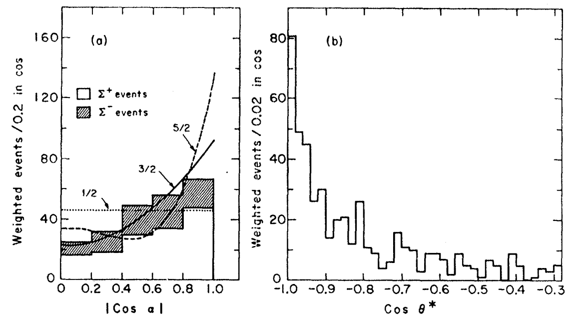

FIG. 2. $\Sigma(1660)$ angular distributions for incident K^- beam momenta in the region 2.45 to 2.7 GeV/c and satisfying selection criteria (a) and (b). (a) Decay angular distribution plotted as a function of the Adair angle as defined in the text, for events whose production $\cos\theta^* \leqslant -0.9$ (Ref. 11). The curves represent the predictions of the different spin hypotheses as labeled; (b) production $\cos\theta^*$ distribution.

invariant mass between 1.36 and 1.45 GeV[6]; (c) an angular selection; criterion: that the $(\Sigma\pi\pi)^+$ production angle with respect to the incident K^- in the center-of-mass (c.m.) system, θ^*, be such that $\cos\theta^* < -0.7$ for the events at 2.1 GeV/c and $\cos\theta^* < -0.8$ for the higher incident momenta (2.45–2.7 GeV/c).[7]

No event of reaction (1) had more than one $\Sigma^+\pi^+\pi^-$ combination satisfying criteria (a) and (c) or (b) and (c) at the same time.

Figure 1(a) shows the $(\Sigma\pi\pi)^+$ mass plot of combinations satisfying criteria (b) and (c) only. A pronounced enhancement around 1.66 GeV is clearly visible above a rather small background. Figure 1(b) shows the $\Sigma^+\pi^-$ mass distribution for events of reaction (1) satisfying the criteria (a) and (c) only. It shows an enhancement at 1.405 GeV that demonstrates the dominance of the $[\Lambda(1405)+\pi]$ decay mode of the $\Sigma(1660)$, as reported previously but with smaller statistics.[8]

SPIN DETERMINATION

In a formation experiment, Bastien and Berge[9] studied the $\Sigma(1660)$ and concluded that its spin was not $\frac{1}{2}$ but was most likely $\frac{3}{2}$. Using the Adair analysis[10] in our production experiment, we find spin $\frac{1}{2}$ and spin $\frac{5}{2}$ incompatible with our data, whereas the spin-$\frac{3}{2}$ hypothesis fits the data extremely well.

In Fig. 2 we used only those events with incident beam momenta in the region 2.45–2.7 GeV/c[7] and satis-

[6] In the $\Sigma^-\pi^+\pi^+$ system, where there are two possible $(\Sigma^-\pi^+)$ pairs, the selection criterion is that either pair have a mass in the $\Lambda(1405)$ region.

[7] These $\cos\theta^*$ cuts were chosen because they give an optimum $\Sigma(1660)$ signal over the background. The $\Sigma(1660)$ production distribution is more sharply peaked at 2.45 to 2.7 GeV/c than at 2.1 GeV/c; therefore, the higher-momentum data are better suited to Adair analysis.

[8] P. Eberhard, F. T. Shively, R. R. Ross, D. M. Siegel, J. R. Ficenec, R. I. Hulsizer, D. W. Mortara, M. Pripstein, and W. P. Swanson, Phys. Rev. Letters **14**, 466 (1965).

[9] P. L. Bastien and J. P. Berge, Phys. Rev. Letters **10**, 188 (1963).

[10] R. Adair, Phys. Rev. **100**, 1540 (1955).

fying the selection criteria (a) and (b). Figure 2(a) shows the decay angular distribution of the $\Sigma(1660)$ events whose production $\cos\theta^* \leqslant -0.9$, plotted as a function of the cosine of the angle α between the direction of the $(\Sigma\pi)^0$ system in the $(\Sigma\pi\pi)^+$ rest frame and that of the incident proton in the over-all c.m. system.[11] The predicted distributions for spin $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ hypotheses, assuming the Adair condition[10] is valid and the dominant decay mode is $[\Lambda(1405)+\pi]$ for these events, are shown in Fig. 2(a) normalized to the total number of events. The fit for either spin $\frac{1}{2}$ or $\frac{5}{2}$ has a χ^2 confidence level $\leqslant 0.1\%$, whereas the spin-$\frac{3}{2}$ hypothesis fits the data with a χ^2 confidence level of 50%.

The Adair analysis seems justified, since Fig. 2(b) shows that the production of $\Sigma(1660)$ does not tend to vanish or even decrease in the very backward direction, but, on the contrary, most of the events are produced at the extreme backward angles. Moreover, the same type of decay distribution (not shown) as in Fig. 2(a), but for the events of Fig. 2(b) lying between $\cos\theta^* = -0.9$ and $\cos\theta^* = -0.55$, shows much less anisotropy than the one of Fig. 2(a). Therefore, the distribution in Fig. 2(a) can be considered as having the features characteristic of the production at 180°, where the Adair analysis is truly valid.

The predicted $\Sigma(1660)$ decay distributions for spin $\frac{3}{2}$ and $\frac{5}{2}$ are based on assumption of a spin $\frac{1}{2}$ for the $\Lambda(1405)$, as determined by Kim.[12] It should be noted that in Fig. 2(a) each bin of the histogram represents a sum over all possible decay angles of the $(\Sigma\pi)^0$ system, so that interference effects between the $[\Lambda(1405)+\pi]$ decay mode and other decay modes with a $(\Sigma\pi)^0$ system of different spin parity from that of the $\Lambda(1405)$ are integrated out, while those with the $(\Sigma\pi)^0$ system having the same spin parity as the $\Lambda(1405)$ give the same predictions as shown in Fig. 2(a) for the $[\Lambda(1405)+\pi]$ decay mode alone.

Henceforth, spin $\frac{3}{2}$ is assumed in this paper.

PARITY DETERMINATION

Previous efforts[3] to measure the parity of the $\Sigma(1660)$, in both formation[13,14] and production[15–17] experiments, have yielded some contradictory results.[18,19]

Our method consists essentially of a Dalitz-Miller type of analysis[20] of the $\Sigma(1660)$ decay, which predicts a depopulation of events for negative parity and a relative enhancement of events for positive parity about the point on the Dalitz plot where the Σ^- is at rest in the $\Sigma^-\pi^+\pi^+$ rest frame.[21] We call that point the strategic point from now on. More specifically, we consider the distributions of the $\Sigma^\pm$, $\pi^\mp$, and π^+ particles with respect to each other in the $(\Sigma\pi\pi)^+$ combination satisfying criteria (a) and (c) and compare them with the predictions[22] when negative or positive parity is assumed for the $\Sigma(1660)$. Those predictions are expected to be very different around the strategic point. Only information pertaining directly to the $\Sigma(1660)$ decay properties is included in the analysis,[23] and any information that would depend also on the production mechanism is ignored.

The $\Sigma(1660)$ decay into $\Sigma^+\pi^+\pi^-$ and $\Sigma^-\pi^+\pi^+$ was considered to occur via a π^+ and a $(\Sigma\pi)^0$ system, which is the superposition of three states, namely,

(I) $\Lambda(1405)$ with spin $\frac{1}{2}$, negative parity, isospin 0[12];

(II) a nonresonant $(\Sigma\pi)^0$ system [i.e., a matrix element independent of the $(\Sigma\pi)^0$ mass] with spin $\frac{1}{2}$, negative parity, isospin 1—analogous to that deduced by Humphrey and Ross[24] in the analysis of $K^-+p \to \Sigma^\pm\pi^\mp$; and

(III) $\Sigma(1385)^0$ with spin $\frac{3}{2}$, positive parity, isospin 1.

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[11] For those $\Sigma^-\pi^+\pi^+$ events which have both $\Sigma^-\pi^+$ combinations in the $\Lambda(1405)$ mass region, both combinations were plotted with a weight of $\frac{1}{2}$ assigned to each combination.

[12] J. K. Kim, Phys. Rev. Letters **14**, 29 (1965).

[13] M. Taher-Zadeh, D. J. Prowse, P. E. Schlein, W. E. Slater, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **11**, 470 (1963).

[14] D. Berley, P. L. Connolly, E. L. Hart, D. C. Rahm, D. L. Stonehill, W. B. Thevenet, W. J. Willis, and S. S. Yamamoto, in *Proceedings of the Twelfth International Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965), Vol. 1, p. 565.

[15] A. Leveque, M. Ville, P. J. Negus, W. M. Blair, A. L. Grant, I. S. Hughes, R. M. Turnbull, A. A. Z. Ahmad, S. Baker, L. Celnikier, S. Misbahuddin, I. O. Skillicorn, J. G. Loken, R. L. Sekulin, J. H. Mulvey, A. R. Atherton, G. B. Chadwick, W. T. Davies, J. H. Field, P. M. D. Gray, D. E. Lawrence, L. Lyons, A. Oxley, C. A. Wilkinson, C. M. Fisher, E. Pickup, L. K. Rangan, J. M. Scarr, and A. M. Segar, Phys. Letters **18**, 69 (1965).

[16] G. W. London, R. R. Rau, N. P. Samios, S. S. Yamamoto, M. Goldberg, S. Lichtman, M. Prime, and J. Leitner, Phys. Rev. **143**, 1034 (1966).

[17] Y. Y. Lee, D. D. Reeder, and R. W. Hartung, Phys. Rev. Letters **17**, 45 (1966).

[18] Attempts to determine the $\Sigma(1660)$ parity by using its $\Lambda\pi$ decay mode in formation experiments (Refs. 13 and 14) have led to contradictory results, due principally to its small decay branching ratio into $\Lambda\pi$ and the presence of relatively large nonresonant amplitudes. Other attempts have been made in production experiments (Refs. 16 and 17) using the $[\Lambda(1405)+\pi]$ decay mode and comparing the spin alignment of the $\Sigma(1660)$ to the prediction for either parity hypothesis according to a K^* exchange model. Those determinations (favoring negative parity) are based on the assumption that a magnetic dipole transition would dominate the K^*-p-$\Sigma(1660)$ vertex if the $\Sigma(1660)$ parity were positive. However, an electric quadrupole transition (which was not mentioned) at that vertex would be allowed for positive parity and would produce an alignment compatible with their data (Ref. 19). Finally, attempts to determine the $\Sigma(1660)$ parity using a Dalitz and Miller analysis of the $\Sigma^-\pi^+\pi^+$ Dalitz plot of the $\Sigma(1660)$ have suffered from a lack of statistics (Refs. 15 and 16).

[19] Professor J. D. Jackson, University of Illinois (private communication).

[20] R. H. Dalitz and D. H. Miller, Phys. Rev. Letters **6**, 562 (1961).

[21] In the events of our whole sample, the Σ particles have lab momenta between 400 and 1200 MeV/c. When an event is situated at the strategic point, the Σ^- lab momentum is about 700 MeV/c. The weights of our events around 700 MeV/c do not indicate any singularity in the Σ^- detection efficiency. No singularity is expected there and our results should be insensitive to small errors in computing the weights.

[22] Philippe Eberhard and Morris Pripstein, Lawrence Radiation Laboratory Report No. UCRL-17682, 1967 (unpublished).

[23] These decay properties are independent of the $\Sigma(1660)$ density matrix and hence are independent of the production process, production angle, and beam momentum. Combining data obtained in different conditions of production, one still deals with the same distributions for the $\Sigma(1660)$ decay and with an average contribution of the background terms.

[24] W. E. Humphrey and R. R. Ross, Phys. Rev. **127**, 1305 (1962).

Those three states involve three decay processes in interference with one another. In the $\Sigma^-\pi^+\pi^+$ combinations, there are two possible $(\Sigma\pi)^0$ systems, and the resulting matrix element has been symmetrized according to Bose statistics. In addition, we considered that some of the $(\Sigma\pi\pi)^+$ combinations in our sample were due to background processes not involving the $\Sigma(1660)$ resonance. Because most of them are $(\Sigma\pi\pi)^+$ systems of different spin parity from that of the $\Sigma(1660)$, they were considered as not interfering with processes I, II, and III, and they are of the following types:

(IV) The bulk of background events in the sample of $\Sigma^+\pi^+\pi^-$, approximated by a phase-space distribution;

(V) A phase-space background for the $\Sigma^-\pi^+\pi^+$ system that could be different in magnitude from that in process IV because the reflections of other final-state resonances in reactions (1) and (2) are different;

(VI) $\Lambda(1520)+\pi^+$ phase-space distribution followed by a $\Lambda(1520)$ decay into $\Sigma^\pm\pi^\mp$ with a branching ratio expected from the ratio of available phase space and a $\Lambda(1520)$ width of 20 MeV.

A complete mathematical description of the model is given in Ref. 22.

Process I has been shown as the dominant one[8]; however, if it were the only process in the $\Sigma(1660)$ decay, the ratio of Σ^+/Σ^- events in our sample would be expected to be between 1.1 and 1.2, and not 1.8, as found experimentally by us and by others.[16,25] A combination of processes I, III, IV, V, and VI alone cannot adjust that ratio and still explain the rest of the data. Process II, though, can adjust that ratio without perturbing any other distribution substantially. Process III, on the other hand, distorts the distribution on the Dalitz plot and changes the prediction around the strategic point, as seen in Fig. 3, where two curves for the positive-parity hypothesis are shown, resulting from two fits, one with and one without introducing process III.[26] The presence of processes IV–VI is evident when one looks at distributions (not shown) of events when the $(\Sigma\pi\pi)$ mass is >1.74 GeV and selection (c) but not (b) is made.

Alternative modes were considered for processes II and III, where the $(\Sigma\pi\pi)^+$ system had the same spin parity as the $\Sigma(1660)$, hence interfering with process I, but was not resonating at a $(\Sigma\pi\pi)$ mass of 1660 MeV.[27]

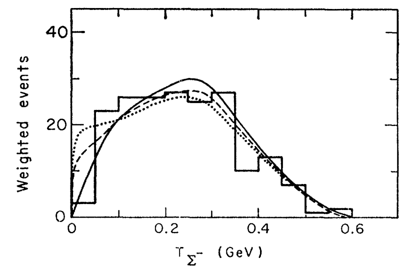

FIG. 3. Distribution of Σ^- kinetic energy in the $\Sigma^-\pi^+\pi^+$ rest frame for $\Sigma(1660)$ events satisfying selection criteria (a) and (c) defined in the text. The solid curve is the result of the best fit for the negative-parity hypothesis. The dashed curve is the result of the best fit for positive parity and the dotted curve corresponds to the fit for positive parity but with process III turned off.

Process II always gave a much worse fit to the data when the alternative mode [i.e., as non-$\Sigma(1660)$ background] was considered, whether process III was in an alternative mode or not. Process III gave a slightly worse fit for the alternative mode in both cases of parity. The curves in Figs. 1 and 3 all correspond to processes II and III considered as decay modes of the $\Sigma(1660)$.

The intensities[22] of processes I–VI and relative phases of processes I–III were adjusted to fit at the same time the 660 $\Sigma^+\pi^+\pi^-$ and the $\Sigma^-\pi^+\pi^+$ combinations satisfying criterion (c) and $\Sigma\pi\pi$ mass range 1.58–1.86 GeV. The reason for using a broader selection than selection (a) is to improve the determination of the parameters controlling processes IV–VI.

RESULTS OF THE PARITY FIT

The width of the $\Lambda(1405)$ was first considered as 35 MeV and the $\Sigma(1660)$ width as 60 MeV. Then the $\Lambda(1405)$ width was added as a new parameter. When the $\Sigma(1660)$ parity was considered as negative, the best estimate became approximately 50 MeV whether or not processes II and/or III were considered with their alternative mode. For positive parity of the $\Sigma(1660)$, the best estimate stayed around 35 MeV for all cases. The $\Lambda(1405)$ width was then fixed at its best value for each case of parity and the $\Sigma(1660)$ width was adjusted. The best estimate for it became about 80 MeV in all cases for negative parity and 110 MeV for positive parity.

In either parity case, the natural logarithm of the likelihood[28] $\mathcal{L}$ decreases by more than 7.3 if process III is turned off, by more than 15 if both processes II and III are turned off. These results illustrate the necessity of including interference effects between processes I and II and between I and III, for the events around the $\Sigma(1660)$ mass.

[25] W. E. Slater, P. M. Dauber, P. E. Schlein, D. H. Stork, and H. K. Ticho, Bull. Am. Phys. Soc. **10**, 1196 (1965).

[26] Therefore, the presence of process III can quite possibly simulate the wrong $\Sigma(1660)$ parity assignment if it is ignored in the $\Sigma^-\pi^+\pi^+$ Dalitz-plot analysis. A proper analysis of this process requires that the Σ^+ data as well as the Σ^- data be fitted, since the interference term between processes I and III has opposite sign for the Σ^+ and Σ^- events.

[27] Our model here assumes that the effect of the processes in their alternative modes can be averaged over all production angles and beam momenta. This is valid if these processes act mainly via their interference with the $\Sigma(1660)$ decay. None of the fits attributed more than 6% of the events to the noninterfering term of any process in its alternative mode, in agreement with the above condition.

[28] Since we are dealing with weighted events, we used a likelihood such that its natural lograithm is of the form

$$\sum_n (W_n \ln p_n)/\bar{W},$$

where W_n and p_n are the weight and probability, respectively, of the nth event, and $\bar{W}$ is the average weight ($\bar{W}=1.28$ for our events).

More important is the difference between the logarithm of $\mathcal{L}$ obtained for the positive- and negative-parity hypotheses. The difference *always* favors negative parity. For the best fit for both parity assignments, that difference is 13.5.[28] For the constrained fits, that is, when the fits were constrained by either turning off process III, or processes II and III and/or constraining the width of the $\Lambda(1405)$ to be 35 MeV and the $\Sigma(1660)$ width to be 60 MeV, the difference was always greater than 11, favoring negative parity.

We also constructed a χ^2 to compare the probability distributions for both parity assignments.[29,30] We computed the χ^2 for the sample used in the fit [$\Sigma\pi\pi$ mass included between 1.58 and 1.86 GeV and criterion (c)] and for the more restricted sample satisfying selection criteria (a) (i.e., $\Sigma\pi\pi$ mass between 1.58 and 1.74 GeV) and (c). Both χ^2 were similar in each case and we quote here the χ^2 referring to the smaller sample, i.e., satisfying criteria (a) and (c). Comparing the best fits of each parity assignment, we obtain a χ^2 of 17.5 for an expected χ^2 of 1 if the positive-parity hypothesis was correct.[29] For the constrained fits, described in the previous paragraph, the χ^2 for positive parity are always greater than 15.4.[29] On the other hand, the best fit for the negative-parity hypothesis has a χ^2 of 0.1 for an expected χ^2 of 1, and all the constrained fits have χ^2 less than 2.3.[30] We conclude, therefore, that the parity of the $\Sigma(1660)$ is negative.

From the best fit for negative parity we obtain the following results for the amounts of the various processes, expressed as a percentage of the total numbers of events in our $\Sigma(1660)$ sample defined by selection criteria (a) and (c):

69% for $\Sigma(1660) \to [\Lambda(1405)+\pi]$ (i.e., process I),
4% for $\Sigma(1660) \to [(\Sigma^\pm\pi^\mp)_{I=1,\, S\text{-wave}}+\pi^+]$ (i.e., process II),
5% for $[\Sigma(1385)+\pi]$ background (i.e., process III),
17% for the total noninterfering background (processes IV–VI),
+11% for the amount of interference between various processes in the Σ^+ events,
−6% for the amount of interference between various processes in the Σ^- events.

As an independent check of our model, we have calculated from the results of a study[31] of the reaction $K^-p \to \Lambda\pi^+\pi^-\pi^0$, in the same bubble-chamber exposure, that the amount of $[\Sigma(1385)+\pi]$ in our data sample should be $\leqslant 5.3\%$, which is consistent with the result for process III from our best fit.

Finally, we plotted for each fit, using the program FAKE,[32] the distributions expected for various invariants in the $(\Sigma\pi\pi)^+$ system and compared them with the data for real events satisfying criteria (a) and (c). Some of the curves for the best fits are shown along with the histograms in Figs. 1 and 3. All positive-parity predictions—for the best fit and for the constrained fits—fitted the data poorly, especially in the bins of the histograms corresponding to the strategic point on the Dalitz plot.[21] With our determination of the widths and with all processes I–VI turned on, the best fit for negative parity is acceptable on every histogram (including the many histograms not shown) while the expectations for the other cases fit poorly at one region or another of our data.

CONCLUSIONS

1. The spin of the $\Sigma(1660)$ is $\frac{3}{2}$ if the Adair analysis is valid for our data.
2. The $\Sigma(1660)$ parity is negative if our model with processes I–VI can approximate the mechanism of decay $\Sigma(1660) \to \Sigma\pi\pi$ and the background in our sample.
3. The decay mode $\Sigma(1660) \to \Sigma\pi\pi$ is dominated by the process $\Sigma(1660) \to \Lambda(1405)+\pi$.
4. The decay $\Sigma(1660) \to \Sigma\pi\pi$ cannot be characterized solely by the $\Lambda(1405)+\pi$ decay mode. Our model gives an excellent fit to the data for negative parity.
5. Our best estimate for the width of the $\Lambda(1405)$ is 50 ± 8 MeV.
6. Our best estimate for the $\Sigma(1660)$ width is 75 ± 10 MeV.

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We acknowledge with pleasure the continuing stimulus and support of Professor L. W. Alvarez and the essential work of the Bubble Chamber Operating Group under the direction of R. D. Watt. Three of us (M.P., W.P.S., and U.E.K.) would also like to thank Professor E. Goldwasser for his encouragement. We are indebted to Professor J. D. Jackson for many stimulating discussions.

[29] The χ^2 is defined as follows:

$$\chi^2 = (\sum_n q_n)^2 / \sum_n q_n^2,$$

with $q_n = W_n[p^-(\tau_n) - p^+(\tau_n)]/p^+(\tau_n)$. The summation over n extends to all events of the sample. W_n is the weight of the nth event and τ_n is its configuration. $p^-(\tau)$[or $p^+(\tau)$] is the probability function of the configuration τ, for the parameters determined by a fit, when parity minus [or plus] is assumed.

If $p^+(\tau)$ were the true distribution of the weighted events, then $\sum_n q_n$ would have an expectation value equal to

$$\begin{pmatrix}\text{normalization}\\ \text{factor}\end{pmatrix} \times \int \frac{p^-(\tau)-p^+(\tau)}{p^+(\tau)} p^+(\tau) d\tau = 0.$$

The standard deviation could then be approximated by $(\sum_n q_n^2)^{1/2}$ and our χ^2 would have a one-degree-of-freedom probability distribution. The exact expressions for $p^-(\tau)$ and $p^+(\tau)$ are given in Ref. 22.

[30] If $p^-(\tau_n)$ and $p^+(\tau_n)$ are interchanged in the definition of q_n (hence in the definition of χ^2) in Ref. 29, then $\sum_n q_n$ is expected to be zero and χ^2 is expected to have a one-degree-of-freedom χ^2 distribution if the negative-parity hypothesis is true.

[31] Daniel M. Siegel, Lawrence Radiation Laboratory (private communication).

[32] G. R. Lynch, Lawrence Radiation Laboratory Report No. UCRL-10335, 1962 (unpublished).