IV. DISCUSSION

The preceding calculations have shown that to fourth order in the field-detector coupling there is no contribution to the mean correlation between the outputs of two photodetectors from the photon field commutator. Consequently, for quantized fields whose average behavior corresponds to that of a classical field, there can be no difference between the results of semiclassical and fully quantum theories of intensity correlations.

In conclusion, we note that the above result does not mean that the zero-point fluctuations of the field cannot play any role in correlation experiments. However such contributions will be at least eighth order in the field-detector coupling and will involve the reflection of light from one detector into the other. At normal intensities, the experimental arrangement can be constructed so as to make such processes extremely negligible.

ACKNOWLEDGMENT

The author is grateful to Professor H. Heffner for his encouragement, advice, and critical reading of the manuscript.

PHYSICAL REVIEW VOLUME 163, NUMBER 5 25 NOVEMBER 1967

Symmetrization Postulate of Quantum Mechanics

MICHAEL FLICKER AND HARVEY S. LEFF*
Case Western Reserve University, Department of Physics, Cleveland, Ohio
(Received 19 May 1967; revised manuscript received 21 August 1967)

On the basis of reasonable physical assumptions, it is proved that for systems of identical spinless particles, the permissible nonrelativistic Schrödinger wave functions are either all symmetric or all antisymmetric. The proof is a generalization of that given by Girardeau. Although connectivity properties of the configuration space play an important role in the proof, it is not true, as was previously believed, that a connected space is always necessary. In particular, for one-dimensional systems, connectivity of the configuration space is not necessary for the proof to hold.

I. INTRODUCTION

ONE of the important superselection principles of quantum mechanics is the so-called symmetrization postulate. This states that physically realizable states for collections of identical particles are represented by either symmetric or antisymmetric wave functions. A number of highly mathematical "proofs" of this statement have appeared in the literature in recent years. Most recently, Girardeau[1] pointed out that these proofs "involve mathematical assumptions which are either in conflict with known physical principles or at least do not follow directly from such principles." Girardeau then presented a proof of the symmetrization postulate explicitly using the condition that the configuration space be connected. Major emphasis was placed on a class of one-dimensional counterexamples to the symmetrization postulate, for which the configuration space is *not* connected.

In this paper, we present a more general proof of the symmetrization postulate for spinless particles. We show that connectivity of the configuration space is, in fact, not necessary for the proof to hold. In particular, it is not necessary in one-dimensional problems with hard-core particles. For dimensionality greater than 1, connectivity appears to be necessary, but we have neither proved nor disproved this. We believe that the present discussion puts the question of connectivity in its proper perspective.

Our approach is similar, in some respects, to that advanced over thirty years ago by Witmer and Vinti.[2] We do not require (as Girardeau does) the existence of a nondegenerate energy level. Rather, we define permissible wave functions solely in terms of permutation properties. Our hypotheses contain Girardeau's assumption of a nondegenerate level as a special case.

In Sec. II, we discuss the meaning of "permissible" wave functions. Section III contains the proof of the symmetrization postulate for spinless particles. Section IV consists of an explicit discussion of one-dimensional problems.

II. DEFINITION OF PERMISSIBLE STATES

The first underlying assumption of this discussion is that the position probability density, for identical particles, can be written as $p(\mathbf{x}_1,\cdots,\mathbf{x}_N)=\psi^*(\mathbf{x}_1,\cdots,\mathbf{x}_N)\times\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N)$,[3] and that $p(\mathbf{x}_1,\cdots,\mathbf{x}_N)d\mathbf{x}_1\cdots d\mathbf{x}_N$ is the

* Supported in part by the U. S. Atomic Energy Commission.
[1] M. D. Girardeau, Phys. Rev. 139, B500 (1965).

[2] E. E. Witmer and J. P. Vinti, Phys. Rev. 47, 538 (1935). Unfortunately, this early treatment does not make full use of the superposition principle. In particular, the right-hand side of Witmer and Vinti's Eq. (11) is identically zero because of this principle.

[3] We emphasize that this is an assumption. For an opposing view see W. Pauli, *Handbuch der Physik*, edited by H. Geiger and K. Scheel (Julius Springer Verlag, Berlin, 1934), Vol. 24.

probability that there is a particle in $(\mathbf{x}_1, \mathbf{x}_1+d\mathbf{x}_1)$, a particle in $(\mathbf{x}_2, \mathbf{x}_2+d\mathbf{x}_2)$, and so forth. This assumption differs from the one for distinguishable particles in that, for distinguishable particles, the order in which one writes the position coordinates defines a labelling of *particular* particles. That is, for distinguishable particles $p(\mathbf{x}_1,\cdots,\mathbf{x}_N)d\mathbf{x}_1\cdots d\mathbf{x}_N$ is not simply the joint probability that there is one particle in each of the intervals $(\mathbf{x}_i, \mathbf{x}_i+d\mathbf{x}_i)$, where $i=1, \cdots, N$. Rather, it is the probability that a *particular* particle lies in the first interval, a second *particular* particle lies in the second interval, and so forth. With the above interpretation for identical particles, it is clear that $p(\mathbf{x}_1,\cdots,\mathbf{x}_N)$ must be permutation-invariant. Consequently,

$$P|\psi|^2=|\psi|^2, \tag{1}$$

where P is any permutation of the dynamical variables, i.e., the arguments of ψ.

In addition to this, we also require that if $\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N)$ is a permissible wave function (the criteria for permissibility will be stated shortly), then $P\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N)$ is also permissible. The motivation for requiring this was given by Witmer and Vinti in one of the first papers on the symmetrization postulate.[2] Briefly, if, for a symmetric Hamiltonian, ψ is an eigenfunction of Schrödinger's equation, then $P\psi$ must also be an eigenfunction corresponding to the same energy. Also, the dynamical states represented by ψ and $P\psi$ cannot be distinguished by any observation. Thus, it is *reasonable* to require that $P\psi$ be permissible if ψ is.

Our precise definition of permissibility can be stated formally in the following way, using the Dirac bra-ket formalism.

For a given Hamiltonian H, which is symmetric in the single particle variables, those states having the following properties will be permissible states: (i) *The permissible states $|\psi\rangle$ form a subspace S, of the Hilbert space of H.* (ii) *If $|\psi\rangle$ is contained in S, then $P|\psi|^2=|\psi|^2$ for all P, where $\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N)=\langle\mathbf{x}_1,\cdots,\mathbf{x}_N|\psi\rangle$.* (iii) *If $|\psi\rangle$ is contained in S, then $\mathbf{U}_P|\psi\rangle$ is contained in S for all P. $\mathbf{U}_P$ is the unitary operator which corresponds to the permutation operator P in the space of wave functions ψ. Specifically,*

$$\mathbf{U}_P|\psi\rangle=\int\cdots\int d\mathbf{x}_1\cdots d\mathbf{x}_N|\mathbf{x}_1,\cdots,\mathbf{x}_N\rangle P\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N).$$

The above hypotheses are couched in the language of abstract Hilbert space for purposes of generality. In statements (ii) and (iii), and in the remainder of this paper, the configuration space representation is used explicity. One expects that the analogs of (ii) and (iii), in the momentum-space representation, are also valid. However, we have not been able to determine the relationship of these conditions with our analysis in the configuration-space representation. Hypothesis (i) is a statement of the superposition principle in the abstract Hilbert space S. As a consequence, the superposition principle holds for wave functions in the configuration-space representation, irrespective of any considerations of connectivity properties. This is important in our analysis.

Finally, we point out that implicit in our interpreta tion of $\psi^*\psi$ is the assumption that ψ must be continuous. It is not necessary to assume the continuity of any derivatives of ψ. Thus, our discussion even applies to the one-dimensional δ-function model[4] for which the first derivative of ψ is discontinuous.

III. PROOF OF THE SYMMETRIZATION POSTULATE

We now prove that for all permissible wave functions ψ, either

$$P\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N)=(-1)^P\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N)$$

or

$$P\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N)=\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N).$$

$(-1)^P$ is $(+1, -1)$ accordingly, as the permutation P is (even, odd). Our analysis does not predict which of the above situations prevails, but only that *one* of them does.

It follows from (ii) that

$$P\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N)=C_{P\psi}(\mathbf{x}_1,\cdots,\mathbf{x}_N)\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N), \tag{2}$$

where

$$|C_{P\psi}(\mathbf{x}_1,\cdots,\mathbf{x}_N)|=1. \tag{3}$$

The requirement that S must be a subspace of a Hilbert space implies that if $|\phi\rangle$ and $|\psi\rangle$ are contained in S, then $a|\phi\rangle+b|\psi\rangle$ is contained in S for all complex numbers a and b. Therefore,

$$P|a\phi+b\psi|^2=|a\phi+b\psi|^2. \tag{4}$$

Since this must be true for all complex a and b, one can conclude after a bit of algebra that

$$(P\phi)^*P\psi=\phi^*\psi. \tag{5}$$

However, from Eq. (2),

$$P\phi=C_{P\phi}\phi \quad\text{and}\quad P\psi=C_{P\psi}\psi.$$

Thus, Eq. (5) requires that

$$(C_{P\phi}^*C_{P\psi}-1)\phi^*\psi=0. \tag{6}$$

Since $|C_{P\phi}|=1$, Eq. (6) can be written as

$$(C_{P\psi}-C_{P\phi})\phi^*\psi=0. \tag{7}$$

In order to fully appreciate the significance of Eq. (7), we classify all points in the $3N$-dimensional configuration space as follows: (a) a point $(\mathbf{x}_1,\cdots,\mathbf{x}_N)$ is a type-α point if all allowable wave functions vanish at that point; (b) a point $(\mathbf{x}_1,\cdots,\mathbf{x}_N)$ is a type-β point if at least one allowable wave function does not vanish at

[4] See, for example, E. H. Lieb and W. Liniger, Phys. Rev. **130**, 1605 (1963); M. Girardeau, J. Math. Phys. **1**, 516 (1960).

that point (i.e., a point is a type-β point if it is not a type-α point). Thus, from Eq. (7), we see that, at each type-β point, C_P is independent of ψ and ϕ for all pairs ψ and ϕ which are not zero at that point. For wave functions which vanish at any specific type-β point, one can consistently define C_P to have the value obtained for nonvanishing wave functions. Since the above argument is independent of P, we have shown that, at all type-β points

$$C_{P\psi}(\mathbf{x}_1,\cdots,\mathbf{x}_N)=C_{P\phi}(\mathbf{x}_1,\cdots,\mathbf{x}_N) \tag{8}$$

for all P. A discussion of the functions C_P at type-α points is not necessary since Eq. (7) is trivially satisfied at these points. Our procedure, so far, has been equivalent to that of Girardeau.

We now deviate from Girardeau's method. In what follows, all configuration points under discussion will be of type β unless explicitly stated otherwise. Equation (2) can be written as

$$P\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N)=C_P(\mathbf{x}_1,\cdots,\mathbf{x}_N)\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N). \tag{9}$$

Operating on $P\psi$ with another permutation operator Q, and recalling from (iii) that $P\psi$ is permissible if ψ is, we obtain

$$Q(P\psi)=C_Q(\mathbf{x}_1,\cdots,\mathbf{x}_N)(P\psi) \tag{10}$$

or

$$C_{QP}(\mathbf{x}_1,\cdots,\mathbf{x}_N)\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N)=C_Q(\mathbf{x}_1,\cdots,\mathbf{x}_N)\times C_P(\mathbf{x}_1,\cdots,\mathbf{x}_N)\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N). \tag{11}$$

Since for any type-β point there is at least one nonvanishing function ψ, we may conclude that

$$C_{QP}(\mathbf{x}_1,\cdots,\mathbf{x}_N)=C_Q(\mathbf{x}_1,\cdots,\mathbf{x}_N)C_P(\mathbf{x}_1,\cdots,\mathbf{x}_N). \tag{12}$$

One can also operate directly on Eq. (9) with the operator Q yielding

$$C_{QP}(\mathbf{x}_1,\cdots,\mathbf{x}_N)=C_P(Q\mathbf{x}_1,\cdots,Q\mathbf{x}_N)C_Q(\mathbf{x}_1,\cdots,\mathbf{x}_N). \tag{13}$$

It follows that

$$C_P(Q\mathbf{x}_1,\cdots,Q\mathbf{x}_N)=C_P(\mathbf{x}_1,\cdots,\mathbf{x}_N). \tag{14}$$

Since any permutation can be written as a product of transpositions, it is convenient, for the moment, to treat only the latter. For a transposition which is represented by the operator P, Eq. (12) implies

$$C_{PP}(\mathbf{x}_1,\cdots,\mathbf{x}_N)=[C_P(\mathbf{x}_1,\cdots,\mathbf{x}_N)]^2. \tag{15}$$

Since $C_{PP}=1$ when P is a transposition, it follows that

$$C_P(\mathbf{x}_1,\cdots,\mathbf{x}_N)=\pm 1 \tag{16}$$

for all transpositions P. It is well known that any transposition P_{ij} can be written as a product of other transpositions (P_{ij} transposes $\mathbf{x}_i$ and $\mathbf{x}_j$). For example,

$$P_{ij}=P_{1i}P_{2j}P_{12}P_{2j}P_{1i}, \tag{17}$$

from which it follows, using Eqs. (14) and (15), that

$$C_{ij}(\mathbf{x}_1,\cdots,\mathbf{x}_N)=C_{12}(\mathbf{x}_1,\cdots,\mathbf{x}_N) \tag{18}$$

for all i and j. Here, we are using a new notation in which the double subscript on C refers to a single transposition. Thus, at each point, the C_{ij} functions, for all i and j, are the same. Consequently, from Eq. (16), either $C_{ij}=+1$ for all i and j or $C_{ij}=-1$ for all i and j. Finally, for an arbitrary permutation represented by the operator P, which can be written as a product of transposition operators, we have

$$C_P(\mathbf{x}_1,\cdots,\mathbf{x}_N)=(-1)^P \quad \text{or} \quad (+1)^P. \tag{19}$$

$(-1)^P$ is negative when P represents an odd permutation of $(\mathbf{x}_1,\cdots,\mathbf{x}_N)$ and is positive otherwise.

If ψ is a solution of Schrödinger's equation, then $P\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N)=C_P(\mathbf{x}_1,\cdots,\mathbf{x}_N)\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N)$ is also a solution. As stated in Sec. II, all permissible solutions must be continuous. By definition, for a type-β point, $(\mathbf{x}_1',\cdots,\mathbf{x}_N')$, there exists *at least one* permissible wave function ψ such that $\psi(\mathbf{x}_1',\cdots,\mathbf{x}_N')\neq 0$. Since ψ must be continuous, there exists a neighborhood $\mathfrak{N}(\mathbf{x}_1',\cdots,\mathbf{x}_N',\delta)$ [δ is the diameter of the neighborhood about $(\mathbf{x}_1',\cdots,\mathbf{x}_N')$] for which $\psi\neq 0$. Clearly, $C_P\psi\neq 0$ in $\mathfrak{N}(\mathbf{x}_1,\cdots,\mathbf{x}_N,\delta)$. Thus,

$$C_P(\mathbf{x}_1,\cdots,\mathbf{x}_N)=\frac{C_P(\mathbf{x}_1,\cdots,\mathbf{x}_N)\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N)}{\psi(\mathbf{x}_1,\cdots,\mathbf{x}_N)}$$

is continuous at $(\mathbf{x}_1',\cdots,\mathbf{x}_N')$. It follows that for all type-β points which can be connected by a continuous curve, passing only through type-β points, C_P must be a constant. As we have seen, this constant must be either $+1$ or -1.

It is quite possible that one will not be able to connect all type-β points in this manner. This will happen if type-α points are distributed so that the configuration space is separated into disjoint β regions. If this situation prevails, the above argument fails. Physically, this might happen if singular interactions exist both between particles of a given species and between particles of different species (e.g., between the molecules of a gas and those of the container walls). Figure 1 illustrates such a situation for two discs in a two-dimensional square box. Classically, it is impossible to get from configuration (a) to configuration (b). Quantum mechanically, we may argue that no paths, containing only

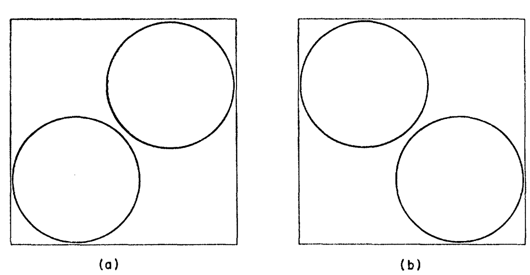

FIG. 1. Two discs near close packing. The representative points in configuration space for states (a) and (b) cannot be connected by any path containing only type-β points.

type-β points, exist which connect these two configurations. Thus, the two configurations *could have different symmetries.* It is emphasized that this situation is caused by the existence of container walls. Were there no walls, the discs could be moved freely from configuration (a) to configuration (b). Our conclusion at this point is that connectivity of the configuration space is sufficient to ensure that permissible wave functions are either symmetric or antisymmetric. This conclusion is in complete agreement with Girardeau. In two or more dimensions, the connectivity restriction may or may not be satisfied, depending upon the system's density. In the next section, we show that for one-dimensional problems there is no such density dependence, and that *connectivity is not necessary.*

To close this section, we point out some of the similarities and differences between our proof and that of Girardeau. Requirements (i) and (ii) for permissible wave functions are common to both proofs. Requirement (iii) is a generalization of Girardeau's assumption of a nondegenerate energy level. To see this, suppose that we had required that there be *only one* $\psi=\psi_0$ such that $P\psi_0$ is contained in S for all P. We could conclude then that Eq. (12) holds only at points where $\psi_0\neq 0$. Now, it is convenient to classify points where $\psi_0=0$ in the following way. We denote such points by the shorthand notation γ and denote a *finite* neighborhood of diameter δ about γ by $\mathfrak{N}(\gamma,\delta)$.

Case (i) Points γ for which *all* $\mathfrak{N}(\gamma,\delta)$ contain a subset of points of finite measure on which $\psi_0=0$.

Case (ii) Points γ for which *at least one* $\mathfrak{N}(\gamma,\delta)$ exists in which the points for which $\psi_0=0$ form a set of measure zero.

Each γ included in case (ii) is the limit of a continuous set of points on which $\psi_0\neq 0$. Invoking Eq. (12) and using the continuity of $C_P(\mathbf{x})$, we find that for case (ii) points,

$$C_{QP}(\gamma)=C_Q(\gamma)C_P(\gamma). \qquad (20)$$

If the zeros of ψ_0 are all case (ii) points, then it is sufficient, for our proof, to have but one $\psi=\psi_0$ with the property that $P\psi_0$ is permissible if ψ_0 is. Since Girardeau's assumption of one nondegenerate level is a special case of the latter statement, our proof contains his when case (i) points do not occur for ψ_0.

If ψ_0 has case (i) points then, to our knowledge, there is no way to demonstrate Eq. (20) and the proof cannot be completed.[5] We have required that $P\psi$ be permissible for all, rather than for just one, wave function ψ on the basis of this fact as well as on the basis of the motivation discussion of Sec. II. It is well to point out that if Girardeau's single nondegenerate level has an eigenstate with case (i) points, then his proof also cannot be completed. Thus, the present proof is not more restrictive than Girardeau's. On the contrary, it appears to be more general.

IV. ONE-DIMENSIONAL PROBLEMS

Our formulation is particularly suitable for analyzing one-dimensional problems. For such cases, we denote configurations by the N numbers $(x_1,\cdots,x_N)$, in which case Eq. (14) becomes

$$C_P(Qx_1,\cdots,Qx_N)=C_P(x_1,\cdots,x_N). \qquad (21)$$

The fact that connectivity is not necessary for one-dimensional problems is well illustrated by the example of a system of two identical hard rods. For this system, the allowable wave functions $\psi(x_1,x_2)$ vanish when $|x_2-x_1|<d$, where d is the hard rod length. Thus, one cannot connect the point (a,b) with the point (b,a) without passing through type-α points. It thus appears on the surface that since the configuration space is not connected, the preceding proof does not go through. However, this is not so. The shaded region in Fig. 2 consists solely of type-α points. We may get from any point (a,b) above the shaded region to its reflection (b,a) below the shaded region by permuting the two arguments. Since $C_P(x_1,x_2)=C_P(Qx_1,Qx_2)$, it follows that $C_P(x_1,x_2)=C_P(x_2,x_1)$. Thus, C_P has the same value below the shaded region as it does above this region. This argument can be extended directly to a system of N hard rods. We conclude that C_P is either $(-1)^P$ at all type-β points, or is $(+1)$ at all such points. No explicit appeal to connectivity properties is necessary in one dimension. Apparently, this is so because the disjoint β regions are related to one another via interparticle permutations.

In light of the above discussion, we now consider the one-dimensional counterexamples of Girardeau. The systems of interest consist of N particles with hard-core repulsion and arbitrary attractive forces. The Bose (i.e., completely symmetric) solutions to Schrödinger's equation corresponding to energy E are denoted by $\psi_E^{(s)}(x_1,\cdots,x_N)$. Girardeau has pointed out that the

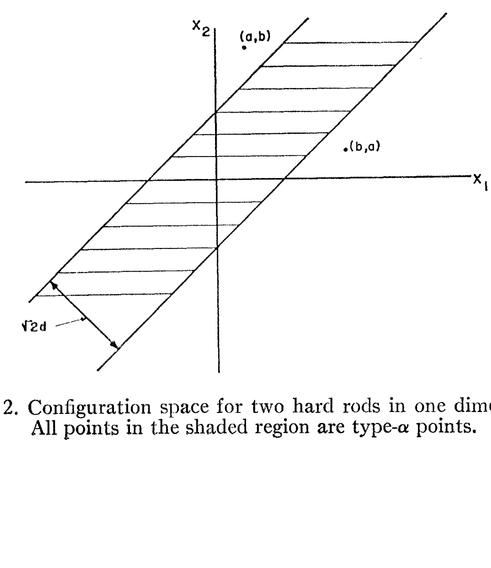

FIG. 2. Configuration space for two hard rods in one dimension. All points in the shaded region are type-α points.

[5] This statement is directed primarily at type-β points only. If ψ_0 vanishes on a set of finite measure, and all other permissible wave functions vanish on the same set, then our prior discussion of type-α points holds.

functions

$$\phi_E(x_1,\cdots,x_N)=[\prod_{j<l}{}' \operatorname{sgn}(x_j-x_l)]\psi_E^{(s)}(x_1,\cdots,x_N) \quad (22)$$

are also solutions to Schrödinger's equation with the same energy. Here, $\operatorname{sgn}x\equiv|x|/x$ and the primed product may extend over any number of the $N(N-1)/2$ particle pairs. The various possible ϕ_E functions are neither symmetric nor antisymmetric in general. These functions each satisfy hypothesis (i) and thus appear to be candidates as counterexamples to the symmetrization postulate. *We wish to emphasize that these wave functions are not permissible and thus are not counterexamples.* To see this, we now show that these wave functions violate either hypothesis (ii) or hypothesis (iii), or both. For example, if one considers the class of functions defined by

$$\operatorname{sgn}(x_j-x_l)\psi_E^{(s)}(x_1,\cdots,x_N), \quad (23)$$

where *all* (j,l) pairs and all possible $\psi_E^{(s)}$ are included, then hypothesis (ii) is violated, but (iii) is not. If one considers the class of functions for which the prefactor in (23) contains *only one* specific (j,l) pair, then hypothesis (iii) is violated, while (ii) is not. For the class of functions for which the prefactor in (23) contains two or more, but not all, (j,l) pairs, then both hypotheses (ii) and (iii) are violated. One may note that the various ϕ_E functions are linearly independent so that each energy eigenvalue E is multiply degenerate. Therefore, these functions violate Girardeau's assumption of one nondegenerate level.[6]

ACKNOWLEDGMENTS

We would like to thank Dr. L. L. Foldy and Dr. P. M. Pearle for a number of interesting discussions. One of us (H.S.L.) would like to acknowledge correspondences with Dr. W. F. Parks and Dr. J. C. T. Pool on related aspects of this problem.

[6] In a private communication from Professor M. D. Girardeau, it was pointed out that his proof can be carried out without the assumption of a nondegenerate level. Instead, one can assume the existence of at least one permissible wave function which is real and which vanishes only at type-α points. With this assumption, the ϕ_E functions do not violate any of Girardeau's requirements other than that of connectivity, and thus represent *bona fide* counterexamples. Also, if this assumption is made, we can no longer say that his requirements are a special case of ours.

PHYSICAL REVIEW VOLUME 163, NUMBER 5 25 NOVEMBER 1967

Scalar Gravitational Radiation from Binary Stars and Planetary Orbits

R. E. MORGANSTERN
Department of Physics, State University of New York, Stony Brook, New York
(Received 14 June 1967)

The gravitational radiation arising from the relative orbital motion of a two-body gravitationally bound system is analyzed on the basis of the Brans-Dicke theory. This theory predicts radiation by a scalar field as well as the usual tensor field. It is found that for small eccentricities the quadrupole scalar radiation due to the circular component of the orbit is the dominant contribution of the scalar radiation field, and that this is almost a hundred times smaller than tensor quadrupole radiation.

I. INTRODUCTION

RECENT observations of solar oblateness by Dicke *et al.*[1] have cast some doubt on the correctness of Einstein's general relativity theory. This observed solar oblateness provides an additional perihelion precession for the planet Mercury which makes the general-relativity prediction about 8% too large. It is, however, in agreement with the Brans-Dicke theory,[2] which for a particular choice of the scalar interaction coupling parameter ($\omega\sim 6$) reduces the general relativity precession by 8%. This observation fixes the coupling parameter in the Brans-Dicke theory. However, because the interpretation of this observation is rather involved and is not without controversy, it may be useful to look for other locally observable consequences of scalar gravitational fields.

We consider here the scalar wave radiation arising from the relative orbital motion of a two-body system; more specifically, the scalar radiation from planetary orbits and binary star systems will be investigated. Brill[3] has calculated the radiation of scalar waves from a planetary orbit, and has found, in the monopole approximation, that the radiation rate is proportional to e^2, where e is the eccentricity of the orbit.

However, as will become apparent in the following sections, there are quadrupole contributions to the radiation for both circular and elliptical components of the orbit, and in fact the circular component dominates

[1] R. H. Dicke and M. H. Goldenberg, Phys. Rev. Letters **18**, 313 (1967).

[2] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961); C. Brans, *ibid.* **125**, 2194 (1961); R. H. Dicke, *ibid.* **125**, 2163 (1961).

[3] D. R. Brill, in *Evidence for Gravitational Theories, Enrico Fermi Course XX* (Academic Press Inc., New York, 1962), p. 63.