Effect of Stripping Channels upon the Elastic Scattering of Deuterons for the Nucleus of Calcium

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Stripping channels are phenomenologically coupled to the incident deuteron channel and the effect upon the elastic cross section is calculated for incident energies at 7, 11, 12.8, 14.3, and 22 MeV. Only the stripping to the $2p$ states in ^{41}Ca is explicitly considered, and the corresponding stripping cross sections are also obtained. The spins of all particles are set equal to zero, and the nonorthgonality between stripping and deuteron channels is ignored to a large extent. It is found that the deuteron amplitude is significantly reduced in the nuclear interior compared to that calculated with an optical model. This reduction has an effect upon the stripping cross section which is reminiscent of a cutoff in distorted-wave Born approximation calculations, but does not appreciably affect the main stripping peaks. The agreement with experiment is not unreasonable.

I. INTRODUCTION

THE description of pickup and stripping reactions by the distorted-wave Born-approximation (DWBA) method[1] is in very good agreement with experiment. In particular, the improvement over the plane-wave Butler description is very satisfying. This success of the DWBA method in describing the deuteron nucleus interaction is remarkable in view of the fact that there are various serious approximations involved in the calculation. One of those consists in ignoring entirely the internal degrees of freedom of the deuteron, despite the fact that large distortions are expected to occur in the internal deuteron wave function when the deuteron's center of mass approaches the surface of the target nucleus. Another somewhat paradoxical feature is the relatively large probability, at least according to the optical-model description, with which this undistorted deuteron is expected to be found in the interior of the nuclear region[2–4] during the elastic-scattering process. This probability is larger than that for alpha particles and reveals itself in that the elastic scattering of deuterons is sensitive to changes in the optical potential at small distances, while α-particle scattering is not. This "nonabsorption" of deuterons in the nuclear interior is at first sight surprising since, in view of the small binding energy of the deuteron, one would expect that it should not survive as readily as α particles in the interior region of the nucleus.

It is the purpose of this paper to suggest that the nonabsorption of the deuterons in the nuclear interior is a consequence of the optical-model procedure of neglecting the deuteron's internal degrees of freedom. Stated in another way, a procedure is here described in which the deuteron channel is assumed to be strongly absorbed in the nuclear interior and at the same time a mechanism is introduced which takes into account, at least approximately, the independent motion of the two nucleons in the nuclear interior. After the two nucleons have traversed the nuclear interior independently from each other, they, or some other nucleons, recombine into a deuteron. This process is found to contribute a significant portion to the elastic-deuteron-scattering amplitude.

Previous attempts of incorporating the internal degrees of freedom of the deuteron into stripping calculations consist in introducing a complete set of states of relative motion of the neutron and proton in the presence of the neutron-proton potential but in the absence of the target-nucleus potential.[5,6] This procedure is well suited to describe the deuteron internal distortion at large distances, and interesting results have been obtained by this approach. The starting point of the present considerations consists in choosing as a complete set of states the eigenstates of a nucleon in the potential of the target or daughter nucleus.[7] The main difficulty with this method is the great complication which would be required to describe the wave function in the deuteron channel, but it is well suited to describe the

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[1] A recent discussion of DWBA theories can be found in notes of lectures given by G. R. Satchler, in *Lectures in Theoretical Physics*, edited by P. D. Kunz, D. A. Lind, and W. E. Brittin (University of Colorado Press, Boulder, Colorado, 1965), Vol. VIIIC. Compare also N. Austern, in *Selected Topics in Nuclear Theory*, edited by F. Janouch (International Atomic Energy Agency, Vienna, 1963).
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[7] G. Rakavy, Nucl. Phys. **7**, 553 (1958); M. Tanifuji, *ibid.* **58**, 81 (1964); A. J. Kromminga, K. L. Lim, and I. E. McCarthy, Phys. Rev. **157**, 770 (1967); S. T. Butler, R. G. L. Hewitt, B. H. J. McKellar, and R. M. May, Ann. Phys. (N. Y.) **43**, 282 (1967). A set of coupled equations for a (t,p) reaction somewhat similar to the one arrived at in the present paper was given by A. P. Stamp, Nucl. Phys. **83**, 232 (1966).

stripping channels, in which, for example, the neutron is in a bound state and the proton in a continuum state. The stripping channels thus play the role of describing internal degrees of freedom of the deuteron, particularly while the deuteron is in the nuclear "interior."

The aim of this paper is to see what effects are introduced into the elastic-deuteron wave function if explicit account is taken of the stripping channels. This is done in the simplest possible manner, ignoring to a large extent the mathematical complications due to the nonorthogonality of the stripping and deuteron channels. In the calculation of the matrix elements which couple the stripping and deuteron channels the deuteron-channel wave function is replaced by the product of an undistorted internal deuteron wave $\phi_d(r)$ times a distorted center-of-mass motion wave $z_d(R)$, and the breakup channels are neglected altogether. One thereby arrives at a simple set of coupled equations similar to those encountered in inelastic-scattering calculations. If one wanted to treat the deuteron channel rigorously, and thus avoid the nonorthogonality problem, it appears that one would have to include the breakup channels so as to make the set of neutron-proton wave functions complete. This would lead to a three-body problem, and might be approached by Faddeev-type coupled-equation methods.[8] The calculation is very involved but results have already been achieved along these lines.[9]

Preliminary reports of the calculation here described have been presented[10] previously. The present calculation differs from the older results[10] mainly in that the ingoing wave boundary condition employed in the deuteron channel is now replaced by the more conventional "regular-at-the-origin" treatment. The strong absorption is achieved by employing a large volume imaginary potential. The energy range of the incident deuteron has been extended to 7, 11, 12.8, 14.3, and 22 MeV, and stripping differential cross sections are obtained in each case.

II. THE COUPLED EQUATIONS

Despite the fact that there is no good derivation of the equations used, some effort was made to see to what extent it would be possible to take into account the nonorthogonality difficulty. An account of this effort is given below, and the reader interested in the final results is invited to bypass the rest of this section.

The target nucleus is considered initially as an inert core without internal degrees of freedom. They are later reintroduced phenomenologically by means of an imaginary part in the deuteron-nucleus potential. The daughter nucleus resulting from the stripping process is described by the various excited states of a nucleon in the potential of the target nucleus. The complete $(A+2)$-nucleon wave function is written as the product of the supposedly known target ground-state wave function $\phi_A(\xi_1,\cdots,\xi_A)$ and an unknown two-particle function $u(\mathbf{r}_n,\mathbf{r}_p)$:

$$\psi(\xi_1\xi_2,\cdots,\xi_A,\mathbf{r}_n\mathbf{r}_p)\approx\phi_A(\xi_1,\cdots,\xi_A)u(\mathbf{r}_n\mathbf{r}_p). \quad (1)$$

As can be easily seen, the nucleon coordinates can then be eliminated from the Schrödinger equation and one obtains

$$(K_n+K_p+V_A{}^n(\mathbf{r}_n)+V_A{}^p(\mathbf{r}_p)+V(\mathbf{r}_n,\mathbf{r}_p)-E)u(\mathbf{r}_n,\mathbf{r}_p)=0, \quad (2)$$

where K_n and K_p are the kinetic-energy operators $K_i=-(\hbar^2/2m_i)\nabla_i{}^2$ acting on the neutron and proton coordinates, respectively, and $V_A{}^n$ and $V_A{}^p$ are the real neutron-nucleus and proton-nucleus potentials obtained by averaging the nucleon-nucleon potential, which acts between the incoming nucleon and the target nucleons, over the target wave function,

$$V_A{}^i(\mathbf{r})=\langle\phi_A(\xi)\,|\,[\sum_j V(\mathbf{r},\xi_j)]\phi_A(\xi)\rangle. \quad (3)$$

After stripping, the daughter nucleus is described by the wave functions

$$\phi_{A+n}{}^i(\xi_1,\cdots,\xi_A,\mathbf{r}_n)\approx\phi_A(\xi_1\cdots\xi_A)\varphi_n{}^{(i)}(\mathbf{r}_n), \quad (4)$$

in which $\varphi_n{}^{(i)}$ are the solutions of the equation

$$[K_n+V_A{}^n-\epsilon_n{}^{(i)}]\varphi_n{}^{(i)}(\mathbf{r}_n)=0, \quad (5)$$

which describe the states of a neutron bound to the target core. The amplitude of the stripped proton, given in general by $\langle\phi_{A+n}{}^i|\psi\rangle$, is in this case given by the overlap integral of $\varphi_n{}^{(i)}$ and u,

$$f_p{}^{(i)}(\mathbf{r}_p)=\langle\varphi_n{}^{(i)}(\mathbf{r}_n)\,|\,u(\mathbf{r}_n,\mathbf{r}_p)\rangle. \quad (6)$$

The deuteron component of u is given by

$$f_D(\mathbf{R})=\langle\phi_d(\mathbf{r})\,|\,u(\mathbf{r}_n,\mathbf{r}_p)\rangle, \quad (7)$$

where $\mathbf{r}$ and $\mathbf{R}$ denote, respectively, $\mathbf{r}_n-\mathbf{r}_p$ and $\frac{1}{2}(\mathbf{r}_n+\mathbf{r}_p)$, and where ϕ_d is the internal deuteron ground-state wave function, which satisfies the equation

$$(K_r+V(r)-\epsilon_d)\phi_d(r)=0. \quad (8)$$

The function u contains, in addition to the components mentioned above, a certain amount of compound nucleus as well as breakup amplitudes. The former have exponentially decreasing asymptotic amplitudes when r_n and r_p go to infinity independently, and the ampli-

[8] L. D. Faddeev, Zh Eksperim. i Teor Fiz. **39**, 1459 (1961) [English transl.: Soviet Phys.—JETP **12**, 1014 (1961)]; Dokl. Akad. Nauk SSSR **138**, 565 (1961); **145**, 301 (1962) [English transls.: Soviet Phys.—Doklady **6**, 384 (1961); **7**, 600 (1963)].

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[10] G. H. Rawitscher, Phys. Letters **21**, 444 (1966); and in *Proceedings of the International Conference on Nuclear Physics, Gallinburg, 1966* (Academic Press, Inc., New York, 1967).

tudes for the latter decrease asymptotically as $r_n^{-1}r_p^{-1}$. The operator $\langle\varphi|$ applied on a function $|\psi\rangle$ which was used in Eqs. (3), (6), and (7) is defined as

$$\langle\varphi(\mathbf{r})|\psi(\mathbf{r},\mathbf{R})\rangle=\int\varphi^*(\mathbf{r})\psi(\mathbf{r},\mathbf{R})d^3r, \tag{9}$$

where the integration is carried out over all the variables contained in the function φ.

The operators $\langle\varphi_n^{(i)}|$ and $\langle\phi_d|$ could now be applied to Eq. (2). The result would be

$$[K_R-E+\epsilon_d]f_D(R)=-\langle\phi_d|(V_A^n+V_A^p)u\rangle, \tag{10}$$

$$[K_p+V_A^p-E+\epsilon_n^{(i)}]f_p^i(r_p)=-\langle\varphi_n^{(i)}|V(n,p)u\rangle. \tag{11}$$

In writing the above equations, use is made of the relation

$$\langle\phi|H_\phi\psi\rangle=\langle H_\phi^+\phi|\psi\rangle, \tag{12}$$

where H_ϕ is the Hamiltonian of which $|\phi\rangle$ is an eigenstate. There are additional surface terms in Eq. (12) whenever $|\phi\rangle$ does not vanish asymptotically, as is the case when the eigenvalue lies in the continuum.

If the stripping and deuteron channels were orthogonal to each other, i.e., if the overlap integral

$$\langle\varphi_n^{(i)}(\mathbf{r}_n)f_p^{(i)}(\mathbf{r}_p)|\phi_d(\mathbf{r})f_D(\mathbf{R})\rangle$$

were equal to zero, then the expansion for u,

$$u\approx\sum_i\varphi_n^{(i)}f_p^{(i)}+\phi_d(r)f_D(R), \tag{13}$$

would be valid and Eqs. (10) and (11) would give rise to the following set of coupled equations:

$$[K_R+\langle\phi_d(V_A^n+V_A^p)\phi_d\rangle-E+\epsilon_d]f_D=-\sum_i\langle\phi_d(V_A^n+V_A^p)f_p^{(i)}\varphi_n^{(i)}\rangle, \tag{14a}$$

$$[K_p+V_A^p-E+\epsilon_n^i]f_p^{(i)}+\sum_j\langle\varphi_n^{(i)}V(n,p)\varphi_n^{(j)}\rangle f_p^{(j)}=-\langle\varphi_n^{(i)}|V(n,p)\phi_d f_D\rangle. \tag{14b}$$

The unsatisfactory aspect of Eqs. (14) is that the coupling terms between the stripping and the deuteron channels are not the same in the two equations, and, in addition, the nonorthogonality between the deuteron and stripping channels has been totally ignored in the underlying Eq. (13).

Numerically the term $\langle\varphi|V(n,p)|\phi_d f_D\rangle$ is quite easy to handle because of the short-range nature of the neutron-proton potential. On the other hand, the function $f_p^{(i)}$ can undergo various oscillations within the range of $V_A^n+V_A^p$ and hence significant cancellations are expected to occur in the integral

$$\langle\phi_d|(V_A^n+V_A^p)f_p^{(i)}\varphi_n^{(i)}\rangle,$$

making it thus hard to calculate. For this reason some of the cancellations will be taken out explicitly by subjecting Eq. (2) to transformations which are different from those described above. These transformations are reminiscent of those used by Buck and Hill[11] in another context. The result is similar to Eqs. (14) with the exception that $V_A^n+V_A^p$ is replaced approximately by $V(n,p)$ on the right-hand side of Eq. (14b), and that f_D is replaced by a new function, called z_d, which asymptotically equals f_D, and for which the nonorthogonality problems are somewhat less severe. Additional interaction terms appear which are believed to represent the cancellations mentioned above and which are neglected. The steps are described below.

The function u is expanded in a complete set of orthonormal eigenfunctions of the operator $K_n+V_A^n$, given by the solutions of Eq. (5). The index i is replaced by $\mathbf{k}$ to label the states in the continuum, while i is reserved for the discrete states:

$$u(r_n,r_p)=\sum_{i=1}^m f_p^i(r_p)\varphi_n^i(r_n)+\int f_p^{\mathbf{k}}(r_p)\varphi_n^{\mathbf{k}}(r_n)d^3k. \tag{15}$$

The transformations become more transparent if they are described in terms of projection operators. One set of projection operators is obtained from the solution of Eq. (5), i.e., $P_n^{(i)}=\varphi_n^{(i)}\rangle\langle\varphi_n^{(i)}$, and another set is obtained from the proton eigenstates ψ of the equation

$$[K_p+V_{A+1}^p-\epsilon_p^{(i)}]\psi_p^{(i)}(r_p)=0, \tag{16}$$

and is given by $P_p^{(i)}=\psi_p^{(i)}\rangle\langle\psi_p^{(i)}$. It is further convenient to introduce the bound and unbound neutron projection operators $P_n=\sum_i P_n^{(i)}$ and $Q_n=1-P_n$, respectively, as well as similar operators for the proton states: $P_p=\sum_i P_p^{(i)}=1-Q_p$. The sums in the above expressions run over all the bound states. The operators $P_n^{(i)}$ and Q_n are orthogonal, i.e., $P_n^{(i)}Q_n=Q_nP_n^{(i)}=0$ but $P_n^{(i)}P_p^{(j)}\neq P_p^{(j)}P_n^{(i)}$. In the operator language one has, for example, $P_n^{(i)}u=f_p^{(i)}\varphi_n^{(i)}$ and $Q_nu=\int f_p^{\mathbf{k}}\times\varphi_n^{\mathbf{k}}d^3k$. The Hamiltonian

$$H_0=T_p+V_A^p+T_n+V_A^n \tag{17}$$

has the property that it commutes with both $P_n^{(i)}$ and $P_p^{(i)}$,as can be seen by considering operations of the type given by Eq. (12). As a result, H_0 also commutes with P_n and P_p, and hence also with Q_n and Q_p. By multiplying operators $P_n^{(i)}$, Q_n, and Q_pQ_n to the left of the Schrödinger equation $(H_0+V-E)u=0$, and after commuting these operators with H_0, together with some straightforward regrouping of terms which involve the identity $Q_n=1-P_n$, one obtains

$$(H^0-E)P_n^{(i)}u=-P_n^{(i)}Vu, \tag{18a}$$

$$(H^0-E)Q_nu=-Vu+P_nVu, \tag{18b}$$

$$(H^0-E+V)Q_pQ_nu=-VP_nu-VP_pQ_nu+P_pVu+P_nVu-P_pP_nVu. \tag{18c}$$

The quantity Q_nu contains those states for which the

[11] B. Buck and A. D. Hill, Nucl. Phys. **A95**, 663 (1967).

neutrons are unbound. Among these, $P_p^{(i)}Q_nu$ selects the bound proton states and Q_pQ_nu describes the "continuum-continuum" states in which both protons and neutrons have asymptotic values which vanish like r_p^{-1} and r_n^{-1}, respectively. The latter contains both the breakup states and the deuteron states. The deuteron component in Q_pQ_nu is given by

$$z_d(R)=\langle\phi_d|Q_pQ_nu\rangle. \tag{19}$$

The approximation is now made of neglecting the breakup states in Q_pQ_nu, i.e.,

$$Q_pQ_nu\approx\phi_d(r)z_d(R). \tag{20}$$

The resulting approximation for u,

$$u\approx P_nu+P_pQ_nu+\phi_d(r)z_d(R), \tag{21}$$

is inserted on the right-hand sides of Eqs. (18), and Eqs. (18a), (18b), and (18c) are operated upon on the left by $\langle\varphi_n^{(i)}(r_n)|$, $\langle\psi_p^{(i)}(r_p)|$, and $\langle\phi_d(r)|$, respectively. Furthermore, the quantities $\langle\psi_p^{(i)}|P_nVu\rangle$ and $\langle\phi_d|\times(P_p+P_n-P_pP_n)VU\rangle$ are neglected, since, as discussed in the Appendix, they are expected to be small compared to the other terms, because of cancellations expected to occur in the integrals. This is probably the weakest link in the argument. The result is

$$[K_p+V_A^p-E+\epsilon_n^{(i)}]f_p^{(i)}+\sum_j\langle\varphi_n^{(i)}|V\varphi_n^{(j)}\rangle f_p^{(j)}$$
$$=-\langle\varphi_n^{(i)}|V\sum_j\psi_p^{(j)}y_n^{(j)}\rangle-\langle\varphi_n^{(i)}V\phi_dz_d\rangle, \tag{22}$$

$$[K_n+V_A^n-E+\epsilon_p^i]y_n^{(i)}+\sum_j\langle\psi_p^{(i)}|V\psi_p^{(j)}\rangle y_n^{(j)}$$
$$=-\langle\psi_p^{(i)}|V\sum_j f_p^{(j)}\varphi_n^{(j)}\rangle-\langle\psi_p^i|V\phi_dz_d\rangle, \tag{23}$$

$$[K_R+\langle\phi_d|(V_A^n+V_A^p)\phi_d\rangle+\epsilon_d-E]z_d$$
$$=-\langle\phi_d|V(\sum f_p^{(i)}\varphi_n^{(i)}+y_n^{(i)}\psi_p^{(i)})\rangle. \tag{24}$$

Here the quantity y_n is defined as

$$y_n^{(i)}=\langle\psi_p^{(i)}|Q_nu\rangle. \tag{25}$$

The asymptotic behavor of $y_n^{(i)}$ and z_d should be equal to that of $f_n^{(i)}=\langle\psi_p^{(i)}|u\rangle$ and $f_d=\langle\phi_d|u\rangle$, respectively. The neglect of the breakup terms in Eqs. (20) and (21) affects only those terms which arise from the right-hand sides of Eqs. (18). The nonorthogonality introduced by Eqs. (19) and (21) does not affect the left-hand sides of Eqs. (22)–(25), but presumably enters only in the matrix elements on the right-hand sides of these equations. The errors, however, are hard to assess, particularly those due to neglecting the quantities $\langle\psi_p^{(i)}|P_nVu\rangle$ and $\langle\phi_d|(P_p+P_n-P_pP_n)Vu\rangle$, as is further discussed in the Appendix.

The Eqs. (22)–(25) are quite similar to the coupled equations used to describe inelastic-scattering processes, with the addition that transitions to and from the stripping channel are also included. The approximations introduced from this point on are, therefore, similar to the ones made for the DWBA, or coupled-channel, treatment of inelastic scattering. The matrix elements $\langle\varphi_n^i|V\varphi_n^j\rangle$, $\langle\varphi_n^i|V\psi_p^j\rangle$, and $\langle\varphi_n^i|V\phi_dz_d\rangle$ represent, respectively, transitions between inelastic states, charge-exchange transitions, and transitions between stripping and inelastic channels. Most of the channels i and j can be expressed at least formally by means of Green's functions in terms of a few chosen channels according to a method described by Feshbach.[12] The resulting potentials in the equations for the few channels which remain are complex and nonlocal. The results, written for the case $i=j=1$, are

$$(K_p+U_{A+1}{}^p-E_p)f_p(\mathbf{r}_p)=-\langle\varphi_n|U(\mathbf{r}_n,\mathbf{r}_p)\phi_dz_d\rangle,$$
$$(K_n+U_{A+1}{}^n-E_n)y_n(\mathbf{r}_n)=-\langle\psi_p|U(\mathbf{r}_n,\mathbf{r}_p)\phi_dz_d\rangle,$$
$$(K_R+U_D-E_d)z_d(\mathbf{R})=-\langle\phi_d|U[\varphi_nf+\psi_py]\rangle. \tag{26}$$

The potentials $U_{A+1}{}^p$ and $U_{A+1}{}^n$ are complex and replace the terms containing $j\neq i$ in the first lines of Eqs. (22) and (23), respectively, as well as the charge-exchange processes which are not included here explicitly. However, if excitation of analog states is involved in the reaction, then the charge-exchange processes are non-negligible[13,14] and need to be treated explicitly,[15] for instance by means of the isotopic spin-dependent Lane[16] potentials. This case is discussed in the Appendix, where it is shown that the resulting equations give rise to stripping amplitudes which are in agreement with the results of Zaidi and Brentano[14] to first order in the stripping transitions.

The potentials U_D and U_{A+1} should, of course, be determined simultaneously by fitting to experiment the elastic-scattering cross sections of nucleon-nucleus and deuteron-nucleus scattering, as well as charge-exchange reactions, and stripping reactions, calculated from the coupled equations [Eqs. (26)]. However, according to preliminary rough estimates, the presence of the deuteron channels is expected to have a much smaller effect on the elastic nucleon-nucleus scattering than the effect which the stripping channels are found to have upon the elastic scattering of deuterons. Therefore the potential U_{A+1} is taken equal to the local "measured" nucleon-nucleus optical potentials, although this procedure is open to criticism.[17] The potential U_D is to be chosen phenomenologically. Although to zero order in the nucleon-nucleus interaction U_D is

[12] H. Feshbach, Ann. Phys. (N. Y.) **5**, 357 (1958), Sec. 2.
[13] C. F. Moore, C. E. Watson, S. A. A. Zaidi, J. J. Kent, and J. G. Kulleck, Phys. Letters **17**, 926 (1966).
[14] S. A. A. Zaidi and P. von Brentano, Phys. Letters **13**, 151 (1967).
[15] T. Tamura and C. E. Watson (to be published); T. Tamura, Phys. Rev. Letters **19**, 321 (1967).
[16] A. M. Lane, Nucl. Phys. **35**, 676 (1962).
[17] B. Buck and J. R. Rook, Nucl. Phys. **A92**, 513 (1967).

given by[18] the real function $\langle\phi_d|V_A^n+V_A^p|\phi_d\rangle$, the point of view taken here is that the potential should be strongly imaginary in the nuclear interior in analogy to the optical potential encountered for the description of α-particle nucleus scattering. This strong absorption should describe both the effect of compound-nucleus formation and the belief that the two-body correlation of the neutron and proton which exists in the undistorted deuteron cannot be maintained in the presence of the other nucleons in the nuclear interior. This view is contrary to one frequently adopted in the literature.

A comment concerning the meaning of "strong" and "weak" absorption of the incident particle in the nuclear interior is useful at this point. In the optical-model language, the degree of absorption of a particle is characterized by the smallness of the amplitude of the nuclear wave function in the nuclear interior. The radial wave functions are usually damped exponentially and hence the absorption can be expressed in terms of a mean free path. Another criterion which describes the absorption is given in terms of the elastic-scattering characteristics. The angular distribution for strongly absorbed particles present regularly spaced diffraction minima which vary in a regular and monotonic manner with energy, while for weakly absorbed particles a new minimum can appear in between two previously existing ones as the energy increases. Although this phenomenon has not yet been unambiguously tied to the presence of waves which are transmitted through the nuclear interior, there are good reasons to suspect that this is the case.[4] The effect also shows up in the angular-momentum dependence of the reflection coefficients. They have a smooth monotonic L dependence in the strong absorption case and a zigzaglike behavior for small L values (which appears to be necessary[2]) in the weak absorption case. The amount of absorption, as discussed above, depends on the size of both the real and imaginary parts of the optical-model potential, as well as on the mass and energy of the incident particle. The imaginary potential alone may, therefore, be a misleading indicator of the degree of absorption taking place. As a matter of fact, the magnitude of the imaginary potential (volume) for deuterons is not far different from that for α particles, yet the former are usually far less strongly absorbed than the latter.

The coupling potential $U(n,p)$ which arises, just as potentials U_{A+1} and U_D, as a result of the formal elimination of states φ_i and ψ_i from Eqs. (22), (23), and (24), also are complex and nonlocal. This can be ascribed to the presence of transitions which occur first to intermediary states φ_i or ψ_i and from there to the final states via an inelastic or charge-transition interaction. Evidence for stripping via intermediary inelastic states has been seen experimentally[19] and studied theoretically.[20] Although the corrections introduced are found to be non-negligible they will, nevertheless, be ignored for the present. As a result the potential $U(n,p)$ is replaced by the elementary nucleon-nucleon interaction potential $V(n,p)$. Even for the case of highly deformed nuclei, for which inelastic transitions occur with high probability in both the incident and the exit channels, the above-mentioned calculations[20] indicate that the contribution to stripping due to inelastic transitions is not excessively large. It would nevertheless be interesting to re-examine this point for the deformed nuclei.

III. NUMERICAL RESULTS FOR ^{40}Ca

The nucleus of calcium is suitable for an application of the equations presented in Sec. II because the stripping reactions have a large cross section (about 10% of the geometric cross section) and the bulk of the stripping proceeds to only a few $2p$ states in ^{41}Ca. The resulting coupled equations are relatively easy to handle because with an angular-momentum transfer of one unit, only three radial waves are coupled at one time. Furthermore, the stripping and elastic-deuteron scattering on Ca has been extensively examined both experimentally[21–24] and as a test of the applicability of the DWBA.[21–25]

The basic equations are Eqs. (26). By means of additional simplifying assumptions now to be discussed, these equations are brought into the form of Eqs. (28) below. The spin of all particles involved is set equal to zero, and the various p states found experimentally in ^{41}Ca at excitation energies of 1.95, 2.47, 3.95, and 3.98 MeV are coalesced into a single p state. The Q value of the reaction is taken equal to 4 MeV ,which is close to the Q values for the two $p_{3/2}$ states ($Q=4.19$ and 3.67 MeV). Comparison with experimental stripping cross sections is made by summing the two experimental $p_{3/2}$ stripping cross sections and multiplying the result with a factor $\frac{3}{2}$ in order to account for the statistical weight of the missing $p_{1/2}$ cross sections, since experimentally these are not known as well. The sum of the spectroscopic factors for the two $p_{3/2}$ transitions is found in DWBA calculations[21] to be close to unity, and hence a

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unit spectroscopic factor (suitably adjusted for the lack of spin) is also assumed here. The stripping to the ground state of ^{41}Ca($f_{7/2}$) has a cross section which is about 10% of the stripping to the p states, and therefore it is neglected. A further approximation consists in including in the coupled Eqs. (26) only *one* stripping channel explicitly (the $2p$ state discussed above) and assuming that the other stripping channels are represented by functions which are sufficiently similar to the one treated explicitly. To account for these implicit channels, the sum $\varphi f+\psi y$ in the last of Eqs. (26) is replaced by $N\varphi_n f_p$, φ_n representing the bound $2p$ neutron and f_p the corresponding proton amplitude. The factor N is chosen rather arbitrarlily to have the value 3 so as to represent the (d,n) stripping to the mirror states in which a proton is captured and a neutron emitted, plus the presence of an additional amount of stripping amplitude ($\frac{1}{2}$ for each neutron and proton case). A change in the choice of the value for N affects the parameters of the potential U_D. If N is increased above unity the range parameters in U_D also increase, and the peaks and valleys of the stripping cross sections move to smaller angles. The calculated cross sections are not very sensitive to the choice for N, a value of $N=1$ also giving acceptable results, but the agreement with experiment appears to improve if its value is larger than unity.

Finally, the zero-range approximation is also used, i.e.,

$$V(\mathbf{r})\phi_d(\mathbf{r})\approx -D\delta(\mathbf{r}),$$
$$D=(1.5)^{1/2}D_0\approx(1.5)^{1/2}\times 10^2 \text{ MeV F}^{3/2}, \tag{27}$$

where the factor 1.5 makes up for part of the finite range effects, as is discussed in Ref. 21.

With the approximations discussed above, Eqs. (26) become

$$(K_1+U_{A+1}-E_p)f_p(\mathbf{r}_p)=D\varphi_n^*(\mathbf{r}_p)z_d(\mathbf{r}_p),$$
$$(K_R+U_D-E_D)z_d(\mathbf{R})=ND\varphi_n(\mathbf{R})f_p(\mathbf{R}). \tag{28}$$

The main difference between Eqs. (28) and those used to describe inelastic transition amplitudes is that in the present case the complex potentials and the masses of the scattered particles are not the same in the two channels, and the coupling potential is not given by the derivative of an optical potential but is proportional to the bound-neutron wave function φ_n. This latter feature is also shared by the coupled equations employed by Breit and co-workers[26] in describing transfer of a neutron between two nitrogen nuclei. The potential U_D is to be found by fitting of the elastic and stripping cross sections to experiment; the other quantities U_{A+1}, $D\varphi_n$, and N are assumed to be fixed *a priori* by the theory as discussed in Sec. II. In particular, U_{A+1} is taken to be the "measured" proton-^{41}Ca elastic-optical potential, the parameters being the same as those employed by Lee *et al.*[21]

The reduction of Eqs. (28) to coupled equations between radial waves is accomplished after introducing the partial-wave expansions[1,27]

$$\varphi_n{}^{(m)}(\mathbf{r}_n)=U_l(r_n)Y_l{}^m(\hat{r}_n), \tag{29}$$

$${}^{(m)}f_p(\mathbf{r}_p)=r_p{}^{-1}\sum_{L,M}{}^{(m)}f_{pL}{}^M(r_p)Y_L{}^M(\hat{r}_p), \tag{30}$$

$$z_d(\mathbf{R})=[(4\pi)^{1/2}/k_DR]\sum_J(2J+1)^{1/2}i^J \times e^{i\sigma_J{}^D}[f_J{}^D(R)Y_J{}^0(\hat{R})]. \tag{31}$$

The direction of the Z axis is taken parallel to the incident deuteron momentum. The orbital angular momentum of the captured neutron is l, its projection along the Z axis is m, and the deuteron radial waves f_D are normalized such that asymptotically

$$f_J{}^D\sim e^{iK_J{}^D}\sin(\phi_J{}^D+K_J{}^D), \tag{32}$$

where the (complex) nuclear phase shift is denoted by K^D and ϕ is given by

$$\phi_J{}^D=k_DR-\tfrac{1}{2}J\pi+\sigma_J{}^D-\eta^D\ln(2k_DR). \tag{33}$$

Here σ_J and η are quantities[28] which arise because of the Coulomb interaction. The asymptotic deuteron wave number is $k_D=[2M_DE_D/\hbar^2]^{1/2}$. It is convenient to define a combination $F_{(Ll)J}(r)$ of proton waves which is independent of m and M,

$$F_{(Ll)J}(r)=[k_D/(i^Je^{i\sigma_J}(\sqrt{4\pi}))]\times\sum_{m,M}{}^{(m)}f_{pL}{}^M\begin{pmatrix} l & L & J \\ m & M & 0\end{pmatrix}, \tag{34}$$

where the objects in parentheses represent Wigner 3-j symbols. The resulting radial wave equations are

$$\left[-\frac{\hbar^2}{2m_p}\left(\frac{d^2}{dr^2}-\frac{L(L+1)}{r^2}\right)+U_{A+1}-E_p\right]\times F_{(Ll)J}(r)=V_lC_{(Ll)J}f_J{}^D(r),$$
$$\left[-\frac{\hbar^2}{2m_D}\left(\frac{d^2}{dr^2}-\frac{J(J+1)}{r^2}\right)+U_D-E_D\right]f_J{}^D(r)=N\times V_l\sum_L C_{(Ll)J}F_{(Ll)J}(r), \tag{35}$$

where[27]

$$V_l(r)=DU_l(r)[(2l+1)/4\pi]^{1/2}, \tag{36}$$

$$C_{(Ll)J}=(2L+1)^{1/2}\begin{pmatrix} l & L & J \\ 0 & 0 & 0\end{pmatrix}. \tag{37}$$

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In the present application $l=1$; therefore for each value of J, L runs over the two values $J-1$ and $J+1$.

The outgoing proton waves are obtained for each neutron orientation m in terms of the functions $F_{(Ll)J}$ by means of the relation

$$^{(m)}f_{pL}{}^{M}=[(\sqrt{4\pi})/k_D]\sum_J i^L e^{i\sigma_J}(2J+1)\times\begin{pmatrix} l & L & J \\ m & M & 0\end{pmatrix}F_{(Ll)J}. \quad (38)$$

The coupled equations (35) are solved numerically with the boundary conditions that the radial waves are regular at the origin and at large distances f^D has both ingoing and outgoing waves, as determined by Eq. (32), while $F_{(Ll)}$ has only outgoing waves. The numerical procedure adopted consists of the iterative employment of the Green's-function method,[29]

$$F_{(Ll)J}{}^{(n+1)}(r)=\int_0^\infty G_L{}^p(r,r')V_l(r')C_{(Ll)J}f_J{}^{(n)}(r')dr',$$
$$f_J{}^{(n+1)}(r)=N\int_0^\infty G_J{}^D(r,r')V_l[\sum_L C_{(Ll)J}\times F_{(Ll)J}{}^{(n)}(r')]dr'. \quad (39)$$

The Green's functions are given by

$$G_L{}^s(r,r')=-\frac{2m_s}{\hbar^2}\frac{1}{k_s}\hat{H}_L{}^s(r_>)\hat{f}_L{}^s(r_<), \quad (40)$$

where s stands for either p or D and the functions $\hat{f}$ and $\hat{H}$ are, respectively, the regular and irregular solutions of the homogeneous equations obtained from Eqs. (35) by setting the right-hand sides equal to zero.

The asymptotic r dependence is given by

$$\hat{f}_L{}^s\sim\sin(\phi_L{}^s+\hat{K}_L{}^s)\exp(i\hat{K}_L{}^s),$$
$$\hat{H}_J{}^s\sim\exp(i\phi_L{}^s). \quad (41)$$

In order not to lose accuracy, the functions $\hat{H}_L$ are obtained from the functions $\hat{G}$ and $\hat{f}$ by the relation

$$\hat{H}_L=(\hat{G}_L+i\hat{f}_L)/\{\tfrac{1}{2}[1+\exp(2iK_L)]\}, \quad (42)$$

where $\hat{G}(r)$ is a solution of the homogeneous equation with the boundary condition $\hat{G}_L\sim\cos(\phi_L)$ obtained by numerical integration from large distances towards the origin.

The zeroth order (n=0) functions used in starting the iterations, described by Eqs. (39), are carried out by means of the Gauss procedure.[30] With 96 points in the integration interval the accuracy of the final phase shifts was found to be good to about the third figure after the decimal point, which is sufficient for the present purpose, although between 6 and 12 iterations might be required for the lower partial waves. The advantage of this method is the economy in memory space required to store the various wave functions. The calculations were carried out on an IBM7040, with 18 500 words required for the object program, inclusive of subroutines. (This leaves 8300 words of unused IBM7040 memory.) A typical run takes 4–8 min.

TABLE I. Test of the iteration method.

Incident channel		Coupled channel		Angular momentum	Number of iterations
S	C	S	C	L	
0.05764[a]	1.1736	0.08833	0.0666	0	6
0.05772[b]	1.1736	0.08833	0.0666		
−0.10082[a]	1.1757	0.01301	0.1321	1	8
−0.10065[b]	1.1758	0.01296	0.1322		
−0.25427[a]	1.0130	−0.12135	0.09875	2	4
−0.25420[b]	1.0130	−0.12140	0.09879		
−0.23483[a]	0.7425	−0.21647	−0.02781	3	3
−0.23476[b]	0.7426	−0.21654	−0.02783		

[a] Calculated via the uncoupled equations as described in the text.
[b] Calculated by iterations of the coupled equations $S=\sin[R_e(2K_L)]\times\exp[-\mathrm{Im}(2K_L)]$; $C=1-\cos[R_e(2K_L)]\exp[-\mathrm{Im}(2K_L)]$.

A test of the solution of the coupled equations by means of the iterative Green's function method described above was carried out for the particular case in which the masses, energies, and potentials in both channels are taken equal to each other (N is set equal to unity), and the angular-momentum transfer l of the bound neutron is taken equal to zero. In this case only the term with $L=J$ appears in Eq. (35) and the two coupled radial equations can be uncoupled[29] by introducing two new functions which are the sum and difference of the two original radial functions. The potentials in the corresponding uncoupled Schrödinger equations are $U_{A+1}\pm V_l$ respectively. The uncoupled equations are solved numerically by standard methods and from the complex phase shifts the phase shifts of the original coupled wave functions are obtained, and compared with those calculated with the Green's-function method. The integration for the latter were carried out with Simpson's rule. A more detailed description of the method can be found in Ref. 31. Table I shows a comparison of the results.

The differential stripping cross sections are calculated simultaneously with the elastic-scattering cross section in each numerical run, and the total stripping cross section is also obtained. The results of the differential stripping cross section were checked against a code developed by Smith [32] which used as input the asymptotic values of the proton wave functions obtained from the coupled

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TABLE II. Potential parameters.

	Deuteron		
Coupled channel	$V_0=120.7$ MeV	$r_0=0.906$ F	$a=0.846$ F
Optical model Z	$V_0=120.7$ MeV	$r_0=0.966$ F	$a=0.846$ F
Optical model $Z2$	$V_0=112.0$ MeV	$r_0=1.00$ F	$a=0.90$ F
Coupled channel	$W_0=60.0$ MeV	$r_0'=1.40$ F	$a'=0.50$ F
Optical model Z	$W_D=16.4$ MeV	$r_D=1.46$ F	$a_D=0.492$ F
Optical model $Z2$	$W_D=18.0$ MeV	$r_D=1.55$ F	$a_D=0.47$ F
	Proton		
	$V=(60-0.5E_p)$ MeV	$r_0=1.20$ F	$a=0.65$ F
	$W_D=11$ MeV	$r_D=1.25$ F	$a_D=0.47$ F
	Neutron		
Woods-Saxon well	$V_{0N}=59.64$	$r_{0N}=1.21$ F	$a_N=0.65$ F
Square well	$V_{0N}=40.8$ MeV	$r_{0N}=1.49$ F	$a_N=\infty$
Binding energy $=6.41$ MeV			

Eqs. (28). In addition, the DWBA results obtained with the code by setting the coupling factor N in Eq. (28) equal to zero, and taking for U_D the usual optical-model value, are in good agreement with the theoretical curves of Lee *et al.*[21] Of course, neutron wave functions bound to a Woods-Saxon potential rather than square-well potential are used in this comparison.

The optical potentials U_{A+1} and U_D are parametrized as follows:

$$U_D=V_0(1+e^x)^{-1}+iW_0(1+e^{x'})^{-1}+U_c, \quad (43)$$

$$U_{A+1}=V_0(1+e^x)^{-1}+4iW_Dd/dx(1+e^{x_D})^{-1}+U_c, \quad (44)$$

where

$$x=(r-r_0A^{1/3})/a, \quad x'=(r-r_0'A^{1/3})/a', \quad (45)$$

$$x_D=(r-r_DA^{1/3})/a_D. \quad (46)$$

The Coulomb potential U_C is the same for the proton and the deuteron. It is produced by a uniform charge distribution of radius $R_C=4.4$ F$\approx(1.3$ F$)A^{1/3}$. Comparison runs employing an optical-model calculation for the elastic case and the DWBA method for the stripping cross section are also carried out. The radial part of the neutron wave function bound to ^{41}Ca, $U_l(r)$ defined in Eq. (29), was calculated as the $2p$ eigenfunction for both a square well of radius R_N and depth V_N, and for a Woods-Saxon well of depth V_{0N}, radius $1.21\times(40)^{1/3}$, and diffuseness $a_N=0.65$ F. The neutron-binding energy E_n was taken equal to the experimental separation energy -6.41 MeV. The value of R_N, and consequently V_N, was adjusted so that the negative-energy tails of the square-well and Woods-Saxon wave functions were nearly equal to each other. The value of R_N thus obtained is 5.1 F, which is close to the radius (5.5 F) beyond which the Woods-Saxon potential is smaller than the binding energy. The value of the parameters is given in Table II.

IV. RESULTS

In addition to the parameters for the complex potentials U_{A+1} and U_D, and the neutron potential V_N, given in Table II, the following parameters also enter into the numerical solution of Eqs. (28). The number of stripping channels $N=3$; the radius of the uniform charge distribution $R_C=4.4$ F; a radius $R_{CC}=2$ F such that the coupling potential vanishes for distances less than R_{CC}; and the matching radius $R_{\text{match}}=13$ F beyond which the coupling potential can be neglected. Of all these, only the parameters which describe U_D are assumed to be freely adjustable, the others being chosen as described in Sec. III.

The imaginary part of U_D is a volume Woods-Saxon potential. Its depth of 60 MeV is chosen large enough so as to absorb the deuteron channel wave functions which penetrate into the nuclear interior. The values of the diffuseness a and depth V_0 of the real well are taken equal to those of the optical potential Z obtained by Bassel *et al.*[22] Only the radii r_0 and r_0' of the real and imaginary deuteron wells were varied independently so as to obtain a reasonable fit to both the elastic and stripping cross sections. The parameters for the optical potential Z are also listed in Table II, and it is seen that the values of r_0 and r_0' found for U_D are significantly smaller than the corresponding values for potential Z. A plot of the volume and the surface potentials which represent the imaginary part of the coupled-channel and the optical-model potentials of the deuteron, respectively, shows that the two potentials cross each other near the maximum of the surface potential, the volume potential being the smaller one beyond this point. The part of the volume potential in this tail region is found to be significant. This is seen by means of coupled-channel calculations in which the imaginary part of the deuteron potential is obtained as the sum of a short-range volume potential plus a surface potential equal to that of the optical potential Z but having a variable radius. It is found that the elastic cross section is a sensitive function of the position of the maximum of the surface part, and its value turns out to be smaller by about 0.5 F than that of the optical potential Z, which is consistent with the difference between the values of r_0', given in Table II.

An average optical deuteron potential was found by Lee *et al.*[21] Its parameters have no energy dependence and the results fit the elastic-deuteron-scattering cross sections fairly well in the energy range from 7 to 12 MeV, and the corresponding stripping cross section has an energy dependence which is in reasonable agreement with experiment. This potential is called $Z2$ and is used as a basis for comparison with the coupled-channel results. The parameters are listed in Table II. The reaction cross sections are listed in Table III.

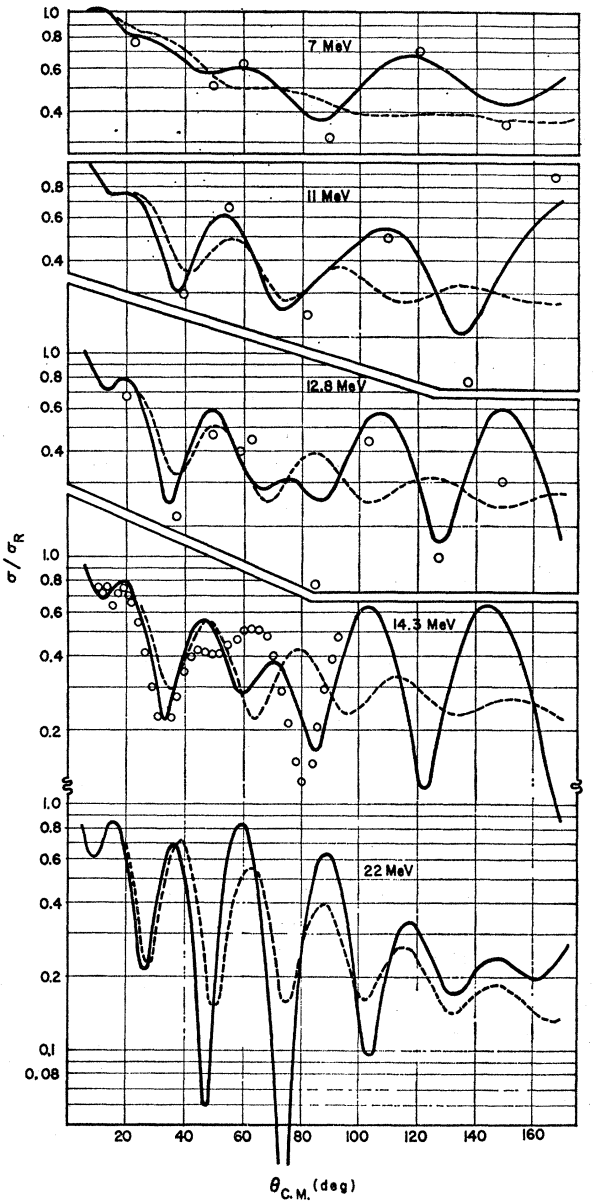

FIG. 1. Elastic deuteron ^{40}Ca scattering cross sections. The ordinate shows the ratio to Rutherford cross section, the abscissa shows the c.m. scattering angle. The lab energies are indicated; the curves are calculated using the coupled-channel equations with parameters given in Table II, the bound neutron being the solution of the square-well potential. The circles represent maxima or minima in the experimental values of σ/σ_R. At 14.3 MeV all experimental points are shown. The dotted lines represent results obtained if the coupling potential is set equal to zero.

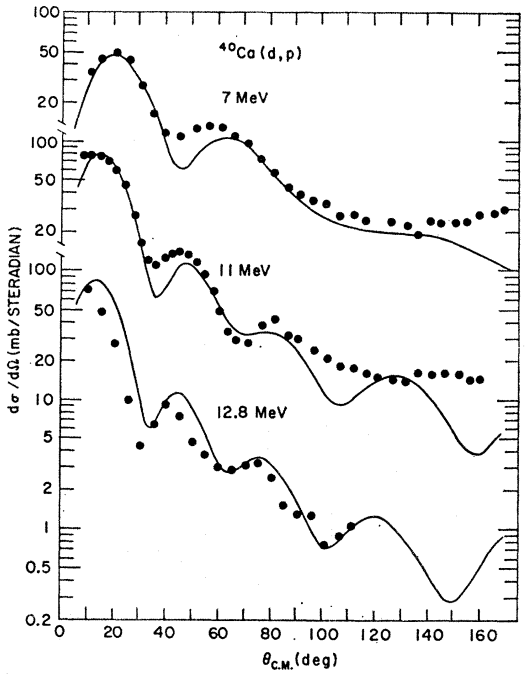

FIG. 2. Stripping cross sections calculated with the coupled-channel equations. The parameters are given in Table II. The neutron in ^{41}Ca is bound to the Woods-Saxon potential. The experimental (Refs. 21, 23) points represent the sum of the two lowest $p_{3/2}$ cross sections (Q of 4.19 and 3.67 MeV) multiplied by the factor $\frac{3}{2}$ to take into account the lack of spin in the theoretical calculations.

The elastic and stripping cross sections are shown by the solid lines in Figs. 1 and 2. The only energy dependence of the parameters is the one prescribed[21] for the proton potential. The experimental points are from Refs. 21–24.

The elastic cross sections obtained from Eqs. (28) in the absence of the coupling potential (but using the same value of U_D) are shown by the dotted lines in Fig. 1. Comparison with the solid line shows that the coupling potential V_1 has a considerable effect upon the cross section. The position of the minima and maxima is altered relative to the equal spacings characteristic for strongly absorbed particles reflected only from the sur-

TABLE III. Reaction cross sections.

	Coupled channel		Optical model $Z2$	
E_D (MeV)	Stripping[a] (mb)	Reaction[b] (mb)	Stripping[a] (mb)	Reaction[b] (mb)
7	91.8	1020	104.8	953
11	106.6	1252	99.7	1216
12.8	104.5	1292	96.6	1271
14.3	101.1	1310	93.7	1302
22.0	75.8	1321	68.5	1399

[a] The stripping reaction populates the p states in ^{41}Ca described by a neutron bound to a Woods-Saxon well, as discussed in the text.
[b] In mb, including all reaction channels.

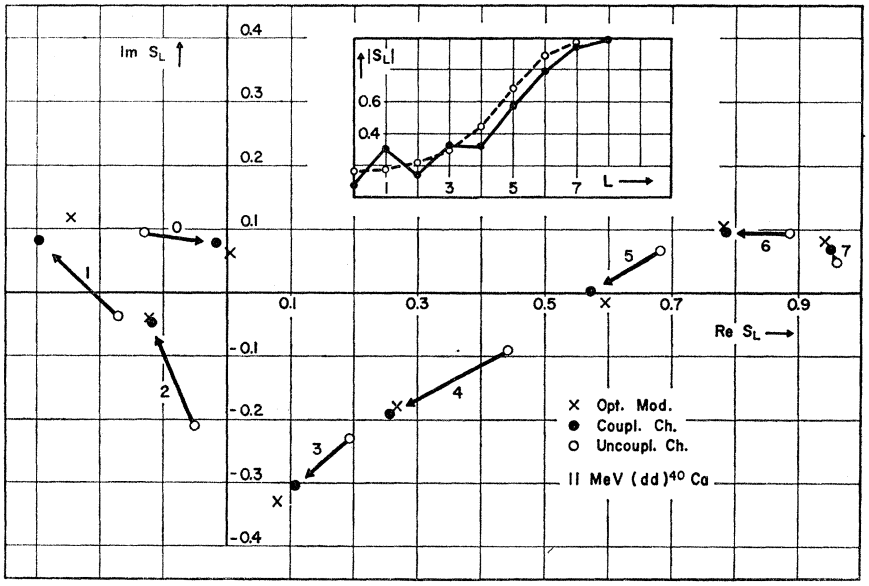

FIG. 3. Scattering elements for elastic d-^{40}Ca scattering at 11 MeV. The quantities S_L are defined in Eq. (47), and the numbers 1, 2, ···, 7 represent angular momenta. The insert shows the absolute values S_L, i.e., the reflection coefficients. The neutron is bound to a square well.

face of the interaction region, which is what the dotted lines represent, and the peak-to-valley ratio is increased. In Fig. 3 the scattering elements, also called reflection coefficients,

$$S_L = \exp(2iK), \tag{47}$$

are compared. Again it is seen that the effect of the coupling potential is to add a non-monotonic L dependence to the regularly varying values obtained for the uncoupled (and strongly absorbing) case. The values of S_L obtained for the optical potential $Z2$ have an L dependence surprisingly similar to the results for the coupled equation. As discussed previously,[2-4] the nonmonotonic L dependence of the scattering elements in the optical-model description is due to the interference of the two components of the partial wave which are reflected, respectively, from the surface (Coulomb barrier) and the interior $[L(L+1)/r^2]$ of the optical potential. Stated in three-dimensional language, the phenomenon corresponds to the interference of the waves which are transmitted through the interior of the nucleus with those which are diffracted from the surface. This interference probably gives rise to the interesting feature observed[33,34] in the elastic scattering of deuterons which is the appearance of a new valley as either the incident energy or the mass number increase. For ^{40}Ca the new valley forms at about 60° at an energy of 12.8 MeV. The present calculation does reproduce this phenomenon but the valley occurs at too large an angle. This blemish is consistent with the approximation of taking too large a neutron binding energy for the additional channels represented by the factor N in Eq. (28), since the larger the distance where the bulk of the coupling to the stripping channels occurs, the more forward the interference angle should go. Neglecting the breakup channels should give rise to an error in the same direction. An indication for the validity of this interpretation is given by the fact that in the older calculations,[10] for which the neutron square well had a radius of 4.6 F, rather than the presently used value of 5.1 F, the new valley had not yet appeared at $E_d=12.8$ MeV. The nonmonotonic L dependence added to the deuteron phase shifts on account of the coupling to the proton channels disappears if the protons are also strongly absorbed[10] and the dependence becomes monotonic.

It is easy to understand the above-mentioned properties by semianalytical methods, but a rigorous formulation which obtains an equivalent local optical potential in terms of the potentials which occur in the coupled equations, is still lacking. Of course the equivalent nonlocal potential is easily written down in terms of Green's functions and the coupling potential. It appears that the nonlocality of the latter potential is not of the form[35] $U(\mathbf{r}+\mathbf{r}')H(\mathbf{r}-\mathbf{r}')$ commonly adopted as a basis for the nonlocal energy-approximation calculations[36]; however, this point should be investigated in some detail.

A comparison between the coupled channel and the optical-model distorted deuteron waves is presented in

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[35] F. G. Perey and B. Buck, Nucl. Phys. 32, 353 (1962).

[36] F. G. Perey, in *Proceedings of the Rutherford Jubilee International Conference, Manchester, England, 1961*, edited by J. B. Birks (Heywood and Company, Ltd., London, 1962); P. J. A. Buttle and L. J. B. Goldfarb, Proc. Phys. Soc. (London) 83, 701 (1964); G. Bencze and J. Zimanyi, Phys. Letters 9, 246 (1964); F. G. Perey and D. Saxon, *ibid.* 10, 107 (1964).

Fig. 4 for the case $L=0$. It is clear that the optical-model wave has a larger amplitude at small distances and is of a "standing-wave" character. The standing-wave character is revealed by the occurrence of the near zeros in the absolute value of the partial wave, and is due to the smallness of the imaginary part of the optical potential in the nuclear interior, while, by contrast, an ingoing character is associated with a large imaginary potential.[4] The difference between these two deuteron wave functions also embodies the difference between the stripping cross sections obtained by the two methods of calculation, because the $T_{(dp)}$ matrix for stripping in the coupled-channel case can be written as[37]

$$T_{(dp)}=\langle \chi_p^{(-)}(\mathbf{r}_p)\varphi_n(\mathbf{r}_n)\,|\,V(\mathbf{r}_n,\mathbf{r}_p)\phi_d(\mathbf{r})z_d(\mathbf{R})\rangle,$$

where $\chi_p^{(-)*}$ is the solution of

$$(K_1+U_{A+1}-E_p)\chi_p^{(-)*}=0,$$

and has the boundary condition of an incident plane wave plus an outgoing spherical wave. The DWBA expression for $T_{(dp)}$ is formally identical to the one above,

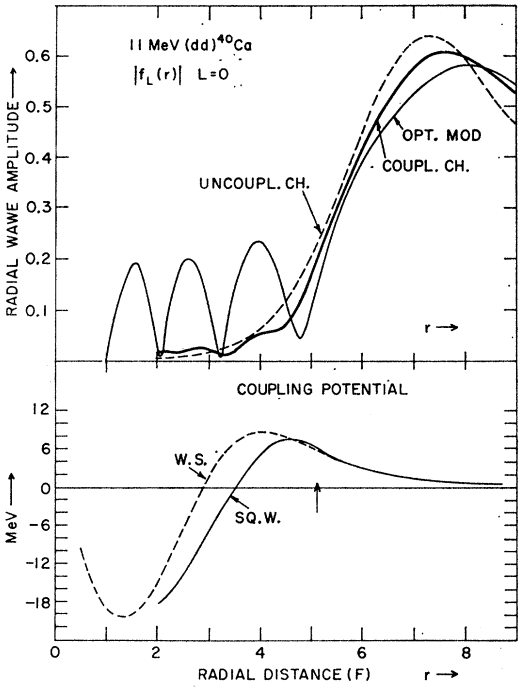

FIG. 4. Absolute values of the radial deuteron wave functions for $L=0$ are shown in the top half of the figure. The asymptotic value of the radial waves are $\exp(-iK_L)\sin(\phi_L+K_L)$, Eq. (32) text, where K_L is the complex-scattering phase shift. (The neutron is bound to the square well.) In the lower half of the figure the radial wave coupling potential $V_1(r)$, Eq. (36), is shown for both the cases in which the neutron is bound to a square well (SQ.W.) and a Woods-Saxon well (W.S). These potentials are proportional to the bound-neutron wave functions. For $r<2F$, V_1 was set equal to zero in the calculations.

[37] The relation between the T matrix and the solution of the coupled equations is discussed in the Appendix for the case that charge exchange also is present in the equations.

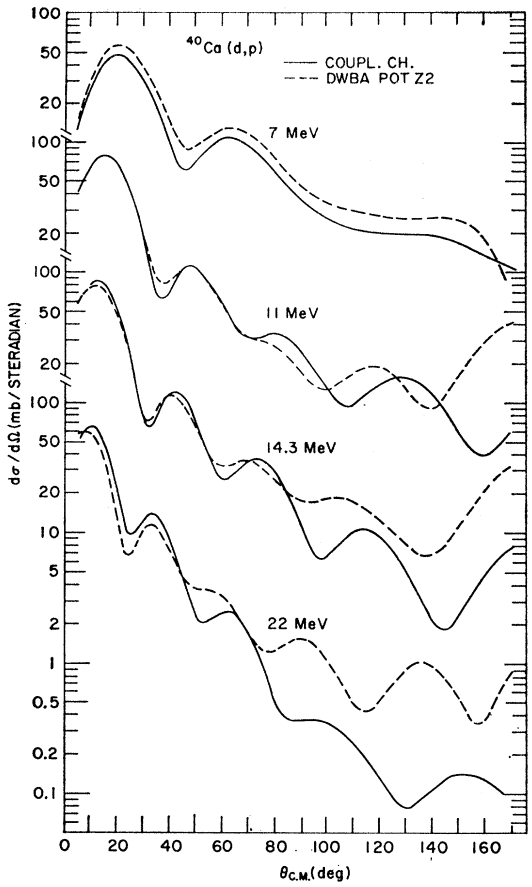

FIG. 5. Comparison of coupled channel and DWBA stripping cross section. The DWBA calculations are based on (Ref. 22) potential $Z2$ described in Table II. Both calculations use the same neutron wave function bound to a Woods-Saxon well, the parameters of which are given in Table II.

the only difference being that instead of $z_d(R)$, the optical-model wave function $\chi_d(R)$ is used.

Figures 5 and 6 show that the two calculations give nearly the same energy dependence for the peak of the

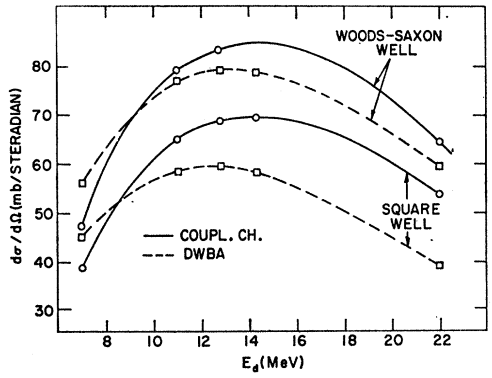

FIG. 6. Peak values of the differential $2p$ stripping cross section versus energy of the incident deuteron for both the coupled channel and the DWBA calculations. The latter is based on potential $Z2$. The neutron is bound either to a square or a Woods-Saxon well, as indicated in the figure. The curves are drawn through the theoretical points to guide the eye.

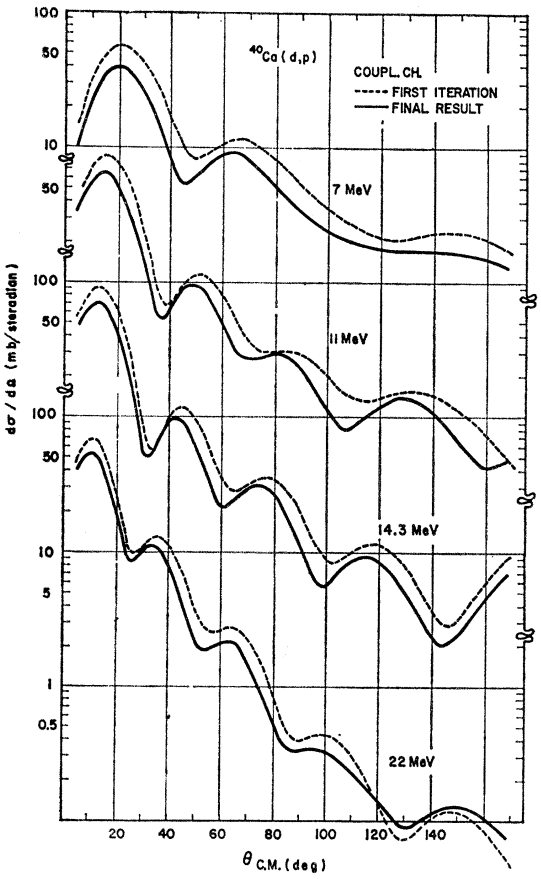

FIG. 7. The dotted curve illustrates the stripping cross section obtained from a DWBA calculation in which the deuteron optical potential is taken equal to the complex-deuteron potential employed in the coupled-channel calculation. This cross section is also equal to the result obtained in the first iteration of the solution of the coupled equations. The solid line shows the final coupled-channel stripping cross section. The neutron is bound to a square well in these calculations. Comparison of the two curves illustrates the effect which the presence of the coupling term V_1 has on the stripping cross section.

stripping cross section. This is to be expected since the bulk of the cross-section peak comes from the large distance contribution of the stripping overlap integrals, where optical-model and coupled-equation waves are very similar. The DWBA cross-section peaks increase somewhat more slowly with energy than the coupled-channel results very likely because of the cancellations in the stripping overlap radial integrals which arise from the contributions at small distances, and which are absent in the coupled-channel case. The angular distribution of the stripping cross section, Fig. 5, shows a difference between the two methods of calculation. This difference increases with angle and with energy, and reflects the difference between the wave functions at small distances. The difference is similar to the effect of introducing a radial cutoff into the DWBA calculations. The coupled-channel results have a tendency to present deeper minima and falloff faster with angle. Since the main difference between the two methods of calculation arises from the difference between the deuteron wave functions at small distances, it would be interesting to examine experiments which in the DWBA method of calculation are sensitive to contributions from small distances. The DWBA calculations of the j dependence[38] of stripping cross sections may be an example of such a situation. Ideally, the coupled-channel calculation of the j dependence should include the breakup of the deuteron as well as the antisymmetrization of the wave functions, since all of these effects may influence the stripping cross section at large angles. The stripping cross section is affected to some extent by the presence of the coupling potential, as is shown in Fig. 7. The stripping cross section is less sensitive to the nuclear interior when calculated by the coupled-equation method as comparted to the DWBA method. This is illustrated in Fig. 8.

V. SUMMARY AND CONCLUSIONS

An attempt is made to include explicitly the presence of the stripping channels in the calculation of elastic-deuteron-nucleus scattering. The resulting equations are of the coupled-channel type, and in the case of d-Ca interaction, the stripping channels are found to give rise to a large feedback to the deuteron channel. The coupled equations are approximate since they do not explicitly take into account the nonorthogonality of the stripping and deuteron channels, the antisymmetrization of the nucleons in the incident deuteron and those in the target nucleus, and the breakup of the deuteron. These effects may have been taken into account implicitly to a certain but unknown extent by the use of a phenomenological complex potential in the deuteron channel. The imaginary part of this potential is assumed to produce a large volume absorption, in analogy to the large absorption encountered in the case of alpha-nucleus scattering. In the proton channel the complex potential is taken equal to the "measured" proton-optical-model potential. The main result is that this system of coupled equations does produce roughly the same features which exist in the experimental elastic-deuteron-scattering angular distributions. Notable among these features is the appearance of a new valley beyond 12.8 MeV and the associated fact that the position of the valleys do not correspond to what is expected from simple diffraction scattering.

The presence of the coupling to the stripping channels seems to demand a deuteron volume absorption which is larger than that used in the conventional optical-model description. This is found by running a coupled-

[38] L. L. Lee, Jr. and J. P. Schiffer, Phys. Rev. **136**, B405 (1964); J. P. Schiffer, L. L. Lee, Jr., A. Marinov, and C. Mayer-Boricke, *ibid.* **147**, 829 (1966); C. M. Glashausser, Ph.D. dissertation, Princeton University (unpublished). Inclusion of the d-state component in the internal deuteron wave function markedly improves the DWBA fit of the j dependence at forward angles, as is shown by R. C. Johnson and F. D. Santos, Phys. Rev. Letters **19**, 364 (1967).

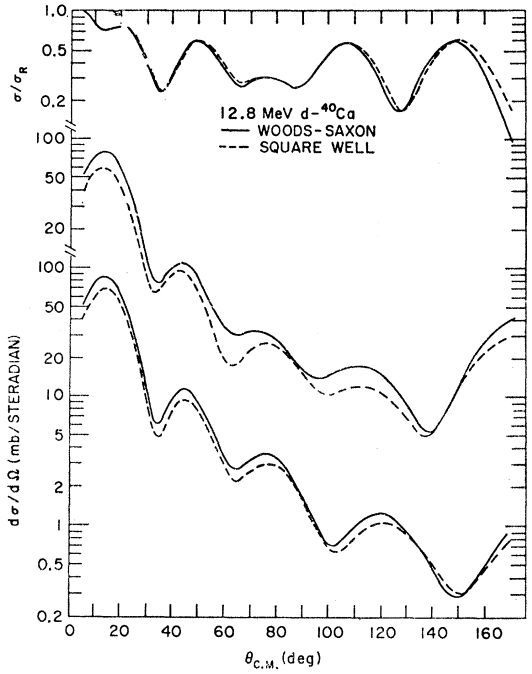

FIG. 8. Sensitivity of the cross sections to the choice of the radial dependence of the coupling potential V_1. The solid curves are obtained using a neutron bound to the Woods-Saxon well, the dotted curves are based on a square well. Both coupling potentials V_1 derived from these neutron wave functions are shown in the bottom half of Fig. 4. The top two curves illustrate the elastic cross section (ratio to Rutherford cross section), the two middle curves represent stripping cross sections obtained with the DWBA method based on the deuteron optical potential $Z2$, and the two lower curves illustrate the coupled-channel result for the stripping cross section. It is seen that the DWBA results are more sensitive to the choice of V_1 than the coupled-channel results.

channel case in which the deuteron potential U_D is replaced by the optical potential $Z2$ which only has a surface absorption and which, without coupling, gives a reasonable fit to the elastic cross section. The result is that in the elastic-scattering cross section the peak-to-valley ratios are increased significantly (almost a factor of two beyond the second cross-section valley). If a volume imaginary potential is then added to the deuteron potential, the peak-to-valley ratio is again decreased.

The present results show that an alternative to the description of elastic-deuteron scattering by means of a local optical model which has weak volume absorption is a coupled-channel treatment in which the deuterons are strongly absorbed in the nuclear volume. The $\Delta l=1$ stripping cross sections which are obtained with the coupled-channel method are surprisingly similar to the conventional DWBA results in the angular region of the first maxima. The deviations which occur at large angles are progressively larger as the incident deuteron energy increases. Although the coupled-channel equations discussed above do not give as good a fit to the elastic deuteron cross section than does the optical model, it is nevertheless hoped that the present results provide encouragement for additional work along the same lines. Once the breakup channels are included into the calculation in a realistic way, then the nonorthogonality problems mentioned above will be eliminated and the resulting coupled equations will be more rigorous. It is hoped that a further test of coupled-channel calculations will be provided by the analysis of the j dependence in stripping,[38] the energy dependence of the stripping reaction, polarization phenomena involving deuteron rearrangement reactions, and the charge-exchange influence of analog states upon stripping cross sections.[13–15]

ACKNOWLEDGMENTS

It is a pleasure to thank Professor A. Schmidt-Rohr at the Max Planck Institut in Heidelberg, Germany, for stimulating discussions concerning deuteron-nucleus interactions. Useful conversations with Dr. B. Buck, Dr. C. R. Johnson, and Dr. Y. Hahn are also gratefully acknowledged. The hospitality of the Brookhaven National Laboratory during the summer of 1967 is most appreciated.

APPENDIX A

The coupling terms neglected in Eqs. (18) will now be discussed. The terms which are neglected are $\langle\psi_p^{(i)}|P_nVu\rangle$, $\langle\phi_d|P_pVu\rangle$, $\langle\phi_d|P_nVu\rangle$, and $\langle\phi_dP_pP_nVu\rangle$. The terms which are kept are $\langle\psi_p^{(i)}|Vu\rangle$, $\langle\phi_d|VP_nu\rangle$, and $\langle\phi_d|VP_pQ_nu\rangle$. What distinguishes the terms which are kept from those which are neglected is the occurrence in the latter of one or two additional projection operators to the left of V. For example, the $\langle\phi_dP_nVu\rangle$ term, expressed in terms of the bound-neutron eigenstates defined in Eq. (5), is given by

$$\langle\phi_d|p_nVu\rangle=\int\phi_d^*(r_n-r_p)\sum_j\varphi_n^j(r_n)\int\varphi_n^{j*}(r_n')\times V(r_n',r_p)u(r_n',r_p)d^3r_n'd^3(r_n-r_p). \quad \text{(A1)}$$

The interaction $V(\mathbf{r}_n\mathbf{r}_p)$ is of short range and for the present discussion it is replaced by $V_0\delta(\mathbf{r}_n-\mathbf{r}_p)$. and Eq. (A1) becomes

$$\langle\phi_dP_nVu\rangle\approx V_0\int\phi_d^*(r_n-r_p)[\sum_j\varphi_n^j(r_n)\varphi_n^{j*}(r_p)]\times u(r_p,r_p)d^3(r_n-r_p). \quad \text{(A2)}$$

The non-neglected term $\langle\phi_dVP_nu\rangle$ to be compared with $\langle\phi_dP_nVu\rangle$ is then approximated by

$$\langle\phi_dVP_nu\rangle\approx V_0\phi_d^*(0)\sum_j f_p^{(j)}(r)\varphi_n^j(r). \quad \text{(A3)}$$

If the sum $\sum_j\varphi_n^j\varphi_n^{j*}$ in Eq. (A2) were equal to a delta function $\delta(\mathbf{r}_n-\mathbf{r}_p)$, then expressions (A3) and (A2) would be comparable and the approximation of neglecting one of the two terms would be totally unjustified.

The completeness relation of the functions φ_n is

$$\sum_j \varphi_n{}^j(r_1)\varphi_n{}^{j*}(r_2)+\int \varphi_n{}^k(r_1)\varphi_n{}^{k*}(r_2)d^3k=\delta(r_1-r_2),$$

and hence, because of the absence of the continuum states from the sum $\sum_j \varphi_n{}^j\varphi_n{}^{j*}$, its value is not equal to a delta function. If only a few terms contribute, the value of the sum is an oscillatory function of the distance r between the two coordinates r_1 and r_2 which is expected to be nonvanishing over a region of nuclear dimension. Since, in addition, the function $\phi_d(\mathbf{r}_n-\mathbf{r}_p)$ in a slowly varying function of $\mathbf{r}_n-\mathbf{r}_p$ ($\approx r^{-1}\exp(-r/4.3\text{ F})$ while $u(\mathbf{r}_p,\mathbf{r}_p)=u(\mathbf{r}_n,-\mathbf{r},\mathbf{r}_n-\mathbf{r})$ is an oscillatory function of $\mathbf{r}$, cancellations are expected in the integral contained in Eq. (A2). Such cancellations are not present in the integral which leads to Eq. (A3) and hence $\langle\phi_d V P_n u\rangle$ is expected to be larger than $\langle\phi_d P_n V u\rangle$. As it stands, this is not, as remarked in the text, a very compelling argument.

APPENDIX B

The presence of charge-exchange effects will now be incorporated into the coupled stripping-deuteron equations. The application in mind is that of the reaction $^{90}\text{Zr}(d,p)^{91}\text{Zr}$ which can also proceed first via the stripping to the analog channel $^{90}\text{Zr}(d,n)(^{91}\text{Nb})^A$, and then via charge exchange $(^{91}\text{Nb})^A(n,p)^{91}\text{Zr}$. Here $(^{91}\text{Nb})^A$ denotes the analog state of ^{91}Zr. A state with a good isotopic spin T and Z component T_Z is denoted $|T,T_Z\rangle$. The target-nucleus isotopic-spin wave function is $|T_0,T_0\rangle$, and those of the proton and neutron are $|\frac{1}{2},-\frac{1}{2}\rangle$ and $|\frac{1}{2},\frac{1}{2}\rangle$, respectively. The deuteron is given by $|00\rangle=[|\frac{1}{2},-\frac{1}{2}\rangle_1\times|\frac{1}{2},\frac{1}{2}\rangle_2-|\frac{1}{2},\frac{1}{2}\rangle_1|\frac{1}{2},-\frac{1}{2}\rangle_2]/\sqrt{2}$.

The derivation of the coupled equations [Eqs. (28)] is now repeated after inserting the isotopic-spin functions for the various channels. As a guide, the first step is to obtain the isotopic-spin extension of Eq. (11). The complete wave function $\psi(\xi,1,2)$ is taken as

$$\psi(\xi,1,2)=f_n(2)\phi_p(\xi,1)|\alpha\rangle+f_p(2)\phi_n(\xi,1)|\beta\rangle+f_D(R)\phi_d(r)\phi_0(\xi)|\gamma\rangle, \quad \text{(B1)}$$

where

$$|\alpha\rangle=|T_0+\tfrac{1}{2},\ T_0-\tfrac{1}{2}\rangle_{\xi,1}|\tfrac{1}{2},\tfrac{1}{2}\rangle_2 \quad \text{(B2)}$$

describes the isotopic-spin function of the situation where particle 1 is a proton bound to the core into an analog state of $|T_0-\frac{1}{2},\ T_0-\frac{1}{2}\rangle$, and where particle 2 is a neutron. The spatial wave functions are, respectively, $\phi_p(\xi,1)$ and $f_n(2)$.

Similarly, state $|\beta\rangle$ describes the isotopic spin wave of the normal stripping channel

$$|\beta\rangle=|T_0+\tfrac{1}{2},\ T_0+\tfrac{1}{2}\rangle_{\xi,1}|\tfrac{1}{2},-\tfrac{1}{2}\rangle_2, \quad \text{(B3)}$$

and $|\gamma\rangle$ describes the deuteron channel

$$|\gamma\rangle=|T_0,T_0\rangle_\xi|0,0\rangle_{1,2}. \quad \text{(B4)}$$

The states $\phi_p(\xi,1)|T_0+\frac{1}{2},\ T_0-\frac{1}{2}\rangle$ and $\phi_n(\xi,1)|T_0+\frac{1}{2},\ T_0+\frac{1}{2}\rangle$ are eigenstates of the Hamiltonian $K_1+\hat{V}(1,\xi)+H_C(\xi)$ with eigenvalues ϵ_p and ϵ_n, respectively, where H_C is the Hamiltonian of the target nucleus and $\hat{V}(1,\xi)$ is the interaction of particle 1 with all the target nucleons. Since these two states are analogs of each other, ϵ_n is more negative than ϵ_p by approximately the Coulomb energy Δ of one proton in the core nucleus. State $\phi_0(\xi)|T_0,T_0\rangle$ is an eigenstate of $H_C(\xi)$ corresponding to the eigenvalue ϵ_C. The other excited states which should also appear in Eq. (B1) have already been left out with the understanding that the various potentials which appear in the Schrödinger equation are complex. In the isotopic-spin notation the nucleon-nucleus potentials become

$$\hat{V}(2,\xi)=\sum_{i=1}^{A}[V_0(2,\xi_i)+V_1(2,\xi_i)\mathbf{t}_2\cdot\mathbf{t}_{\xi_i}], \quad \text{(B5)}$$

and the Hamiltonian H_C in the Schrödinger equation, $(\hat{H}-\hat{E})\psi=0$, is given by

$$\hat{H}=K_1+K_2+H_C(\xi)+\hat{V}_A(1,\xi)+\hat{V}_A(2,\xi)+V(1,2).$$

Inserting Eq. (B1) into the Schrödinger equation given above and carrying out the operations $\langle\phi_p|\langle\alpha|$ and $\langle\phi_n|\langle\beta|$, one obtains

$$\begin{aligned}[K_2+\epsilon_p-E]\langle\alpha\phi_p|\psi\rangle&=-\langle\alpha\phi_p|(\hat{V}(2,\xi)+\hat{V}(1,\xi))\psi\rangle,\\ [K_2+\epsilon_n-E]\langle\alpha\phi_n|\psi\rangle&=-\langle\beta\phi_n|(\hat{V}(2,\xi)+\hat{V}(1,\xi))\psi\rangle.\end{aligned} \quad \text{(B6)}$$

The expressions $\langle\alpha\phi_p|\psi\rangle$ and $\langle\beta\phi_n|\psi\rangle$ are replaced by $f_n(2)$ and $f_p(2)$, respectively.

The matrix elements on the right-hand side of Eq. (B6) are evaluated employing Eq. (B5) and by defining new potentials $U_0(2)$ and $U_1(2)$ as follows:

$$\int\phi_p{}^*(\xi,1)[\sum_1^{A+1}V_0(2,i)+V_1(2,i)\mathbf{t}_1\cdot\mathbf{t}_2]\phi_p(\xi,1)d(\xi)d(1)\equiv U(2)=U_0(2)+2U_1(2)\mathbf{T}_1\cdot\mathbf{t}_2. \quad \text{(B7)}$$

Here $\mathbf{T}_1$ is the isotopic spin operator for the system $A+1$. Assuming that the wave functions ϕ_p and ϕ_n are sufficiently similar to each other, then the nuclear part of potential U_0 and U_1 have the same value if ϕ_p is replaced by ϕ_n in Eq. (B7). Also, making use of the isotopic spin identities

$$\begin{aligned}|\alpha\rangle&=\frac{-|-\rangle+(2T_2)^{1/2}|+\rangle}{\hat{T}_2},\\ |\beta\rangle&=\frac{(2T_1)^{1/2}|-\rangle+|+\rangle}{\hat{T}_1},\end{aligned} \quad \text{(B8)}$$

where $|\pm\rangle=|T_1\pm\frac{1}{2},\ T_1-\frac{1}{2}\rangle$, $T_1=T_0+\frac{1}{2}$, and $\hat{T}_1=(2T_1+1)^{1/2}$, one obtains

$$\begin{aligned}\langle\alpha U\alpha\rangle&=U_0+(T_1-1)U_1,\\ \langle\beta U\beta\rangle&=U_0-T_1U_1,\\ \langle\alpha U\beta\rangle&=(2T_1)^{1/2}U_1.\end{aligned} \quad \text{(B9)}$$

Insertion of Eqs. (B9) into Eq. (B6) leads to the customary Lane equations for the parts of $|\psi\rangle$ involving the functions $f_n|\alpha\rangle$ and $f_p|\beta\rangle$.

The evaluation of the matrix element in Eq. (B6) involving $|\gamma\rangle$ will now be discussed. The spatial part of this matrix element contains terms of the form

$$M_{dn}(2)=\int \phi_p^*(\xi,1)[\sum_i V(\xi_i,2)+V(1,2)] \times\phi_0(\xi)\phi_d(r)f_D(R)d(\xi)d(1). \quad \text{(B10)}$$

This expression is similar to the matrix element usually encountered in stripping calculations. The function $\phi_p(\xi,1)$ is replaced by $(S)^{1/2}\phi_0(\xi)\varphi_p(1)$, where S is a suitable spectroscopic factor and φ_p is a single-particle bound-state wave function. The integration over ξ can now be performed. The potential $\sum_i V(\xi_i,2)$ which would give rise to core excitations is frequently neglected and the potential $V(1,2)$ which remains is made responsible for the stripping process. It may be interesting to note that if $\sum_i V(\xi_i,2)$ is kept, even in the absence of core excitations, then after integration over ξ it leads to a potential $U'(2)$ which is a function of (2) alone and which does not influence the subsequent integration over (1). The latter gives rise to an overlap integral of the stripping and the continuum-continuum channels described in the text, which are orthogonal to each other if the projection-operator technique is used.

Keeping only $V(1,2)$ in Eq. (B10), and further neglecting the isotopic spin dependence of $V(1,2)$, one obtains

$$[K_2+U_0+(T_1-1)U_1-E_n]f_n(2)+(2T_1)^{1/2}U_1f_p(2) =-M_{dn}(2)/(2T_0+1)^{1/2}, \quad \text{(B11a)}$$

$$[K_2+U_0-T_1U_1-E_p]f_p(2)+(2T_1)^{1/2} \times U_1f_n(2)=+M_{dp}(2), \quad \text{(B11b)}$$

where $E_n=E-\epsilon_p$; $E_p=E-\epsilon_n$.

The factors multiplying the quantities M_{dn} and M_{dp} on the right-hand sides are obtained by carrying out the isotopic-spin matrix elements $\langle\alpha|\gamma\rangle$ and $\langle\beta|\gamma\rangle$, which are equal to $(2)^{-1/2}(2T_0+1)^{-1/2}$ and $-(2)^{-1/2}$, respectively. The factor $(2)^{-1/2}$ is dropped because it represents the antisymmetrization carried out between particles 1 and 2 in the deuteron, which however has not been carried out in the stripping channels. The function $M_{dn}(2)$ has been defined in Eq. (B10). The counterpart M_{dp} is obtained from Eq. (B10) by replacing $\phi_p^*(\xi,1)$ by $\phi_n(\xi,1)$.

In order to obtain the third of the coupled equations, the projection-operator technique discussed in the text must again be used. The equation obtained from Eq. (18c) by including the isotopic spin dependence of the potential and wave functions is now multiplied to the left by $\langle\gamma\phi_0\phi_d(r)|$. Replacing $\langle\gamma\phi_0\phi_d|Q_pQ_nu\rangle$ by $z_d(R)$ and neglecting matrix elements similar to those discussed in Appendix A, one obtains the desired equation. The three coupled equations are

$$(K_R+U_D-E_D)f_D=\langle\phi_d|V\varphi_nf_p\rangle -\langle\phi_d|V\varphi_py_n\rangle/\hat{T}_0, \quad \text{(B12)}$$

$$(K_2+U_0+(T_1-1)U_1-E_n)y_n+\hat{T}_0U_1f_p =-\langle\varphi_p|V\phi_df_D\rangle/\hat{T}_0, \quad \text{(B13)}$$

$$(K_2+U_0-T_1U_1-E_p)f_p+\hat{T}_0U_1y_n=\langle\varphi_n|V\phi_df_D\rangle, \quad \text{(B14)}$$

where $\hat{T}_0=(2T_0+1)^{1/2}$.

The single-particle bound-neutron and bound-proton functions are φ_n and φ_p, respectively. These should be multiplied by spectroscopic factors, which however have been left out for simplicity.

The functions y_n and f_p have asymptotically only outgoing waves, while f_D has an incident plane wave plus an outgoing wave. The asymptotic amplitude for f_p which results from the three equations above leads to a transition matrix element T_{dp} written down by Zaidi and Brentano,[14,15]

$$T_{dp}=\langle\chi_p^{(-)}\varphi_n|V(1,2)\phi_df_D\rangle -\langle\chi_n^{(-)}\varphi_p|V(1,2)\phi_df_D\rangle/\hat{T}_p. \quad \text{(B15)}$$

Here $\chi_p^{(-)*}$ and $\chi_n^{(-)*}$ are the solution of the charge-exchange equations

$$[K_2+U_0+(T_1-1)U_1-E_n]\chi_n^{(-)*} +\hat{T}_0U_1\chi_p^{(-)*}=0, \quad \text{(B16)}$$
$$[K_2+U_0-T_1U_1-E_p]\chi_p^{(-)*}+\hat{T}_0U_1\chi_n^{(-)*}=0,$$

with the boundary conditions that $\chi_p^{(-)*}$ asymptotically has an incident plane wave plus an outgoing wave, while $\chi_n^{(-)*}$ has only an outgoing wave. In order to see how Eq. (B15) follows from the coupled Eqs. (B12)–(B14) it is sufficient to apply the operation $\langle\chi_n^{(-)}|$ to Eq. (B13) and $\langle\chi_p^{(-)}|$ to Eq. (B14). The sum of both operations gives

$$\int y_n[(K_2+U_0+(T_1-1)U_1-E_n)\chi_n^{(-)*}+\hat{T}_0U_1\chi_p^{(-)*}]d^3r+\int f_p[(K_2+U_0-T_1U_1-E_p)\chi_p^{(-)*}+\hat{T}_0U_1\chi_n^{(-)*}]d^3r$$
$$+\int[\chi_n^{(-)*}K_2y_n-y_nK_2\chi_n^{(-)*}]d^3r+\int[\chi_p^{(-)*}K_2f_p-f_pK_2\chi_p^{(-)*}]d^3r$$
$$=\langle\chi_p^{(-)}\varphi_n|V(1,2)\phi_df_D\rangle-\langle\chi_n^{(-)}\varphi_p|V(1,2)\phi_df_D\rangle/\hat{T}_0.$$

The first two lines vanish on account of Eqs. (B16), the third line vanishes because the asymptotic behavior of both $\chi_n^{(-)*}$ and y_n is outgoing, and the fourth line gives rise to a surface term which is equal to the transition matrix elements T_{dp}.[39]

If the feedback of the stripping mechanism upon the deuteron channel is negligible, then the right-hand side of Eq. (B12) can be set equal to zero and the functions f_d can be obtained from a conventional optical-model deuteron potential. However, the case where the analog state has a spectroscopic factor close to unity is interesting because then the coupling term in Eq. (B12) cannot be neglected. In this case, as the incident deuteron energy is varied near the threshold of the analog channel, the opening of the latter should introduce a particular energy dependence in f_D. The energy dependence might in turn reflect itself in the stripping cross section to other channels which are not related to the neutron channel via charge exchange, and the results may differ from those obtained by Tamura and Watson.[15]

[39] A. Messiah, *Quantum Mechanics* (John Wiley & Sons, Inc., New York, 1962), Eq. (XIX.9).

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Study of the Low Levels of Si^{27}

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The low-lying excited levels of Si^{27} have been studied by the $Si^{28}(He^3,\alpha\gamma)Si^{27}$ reaction. The particle-γ angular-correlation method of Litherland and Ferguson was used. Most probable spin assignments for the 0.96-, 2.17-, and 2.65-MeV levels were determined as $\frac{3}{2}$, $\frac{7}{2}$, and $\frac{5}{2}$, respectively. Transitions from the 3.54- and 4.13-MeV levels appeared to have isotropic γ-ray angular correlations, and decayed primarily through the well established $J=\frac{1}{2}$ level at 0.78 MeV. A spin assignment of $\frac{1}{2}$ is likely for these two levels, although a $J=\frac{3}{2}$ assignment cannot be rigorously ruled out. The branching scheme for the decay of the 3.80-MeV level suggests a spin of $\frac{3}{2}$ for this level although an angular correlation was not carried out. The levels at 2.87 and 2.91 MeV were not resolved, and an angular correlation for these levels was not carried out. The γ-ray spectra for the doublet indicate a strong ground-state transition for both levels. Mixing ratios and branching ratios for many of the γ-ray decays were measured for the first nine levels. Strong- and weak-coupling model calculations were carried out, and the results indicate significant evidence for Si^{28} core-excitation configuration in the Si^{27} levels below 3.00 MeV. Many transition properties predicted by the strong-coupling (rotational) model are in conflict with measured values.

I. INTRODUCTION

THE $1d2s$ shell is now a well established mass region in which nuclei manifest collective excitation properties. In particular, the sequence of spins and some of the spectroscopic properties of low-lying levels in many of the odd-A nuclei show a rotational behavior which is at least approximately accounted for by the coupling of a single particle to a statically deformed core as in the Nilsson model.[1–4] In some cases, particularly those in which there is not a large energy gap between the lowest two odd particle levels, the nucleus may be better described in terms of coupling of an odd particle to two or more different states of the core rather than to a statically deformed one. The different states of the core are due to one or more of the core nucleons occupying one of the excited, but low-lying, single-particle states. As Bhatt[4] has pointed out, such a situation might occur in the $1d2s$ shell near the crossing of the 5th and 9th Nilsson orbits. There is some experimental evidence that Ne^{23}, Al^{25}, Mg^{25}, and Al^{27} have their first excited states (orbit 9) separated about 1 MeV or less from their ground state (orbit 5). For example, Thankappan[5] has described some of the properties of the low-lying levels of Al^{27} in terms of a proton hole coupled

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