Rydberg energy. In natural units this means $V \gg 1/137$. If this condition is not fulfilled, the interference terms in all the above cross sections must be modified.[2]

Nonrelativistic cross sections, like those in Eq. (13), have been applied by several authors[1–3] in the theory of ionization and excitation of atoms with electrons. These authors consider the atomic electron as moving isotropically in the atom with a velocity corresponding to the binding energy. The ionization process is then considered as a free collision between the incident and the bound electron with an energy transfer exceeding the binding energy (impulse approximation).

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g_I/g_J Ratios of Rb^{85} and Rb^{87}

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An optical-pumping experiment has been performed to measure $g_I/g_J(Rb^{85})$ and $g_I/g_J(Rb^{87})$. The measurements were made in a magnetic field of 40 G. The field was produced by a precision-wound solenoid surrounded by three coaxial magnetic shields. The magnetic field varied by three parts in 10^6 over the volume of the optical pumping cells. The optical-pumping cells were filled with various pressures of helium and neon buffer gases. The measured ratios did not depend upon the nature or pressure of the buffer gas. The results of the measurements were $g_I/g_J(Rb^{85}) = -1.46648(8)\times10^{-4}$; $g_I/g_J(Rb^{87}) = -4.96997(9)\times10^{-4}$.

I. INTRODUCTION

RECENTLY, an optical pumping experiment was performed by White, Hughes, Hayne, and Robinsin[1] to measure g_I/g_J for Rb^{85} and Rb.[87] The results of this experiment are in disagreement with an earlier value for $g_I/g_J(Rb^{85})$ obtained by Penselin, Cohen, Moran, and Winkler[2] using an atomic beam. The experiment reported here is an optical-pumping experiment similar to that of White *et al.* It was used to measure $g_I/g_J(Rb^{85})$ and $g_I/g_J(Rb^{87})$ and was undertaken in order to test the relative validity of the $g_I/g_J(Rb)$ values obtained by the two groups mentioned above. There are two basic reasons why $g_I/g_J(Rb)$ is an interesting number. In the last few years several precision optical-pumping experiments have been performed to measure the magnetic moments of the free electron,[3] the proton,[4] and the hydrogen atom[3,5] in terms of the electronic magnetic moment of the rubidium atom. In such experiments one must have a reliable value for $g_I/g_J(Rb)$ in order to extract the desired magnetic-moment ratio from the measured transition frequencies. In addition, measurements of $g_I/g_J(Rb^{85})$ and $g_I/g_J(Rb^{87})$ lead to values for the nuclear magnetic moments $\mu_I(Rb^{85})$ and $\mu_I(Rb^{87})$ of the two isotopes.

In Sec. II the theory of the experiment is discussed. Then the apparatus is described. Finally, the measurements are discussed, and the results are compared with results of previous experiments. A corrected value for $g_J(Rb^{85})/g_J(H)$ is given.

II. THEORY

In this experiment, rubidium atoms in a magnetic field are polarized by illuminating them with circularly polarized optical resonance radiation incident along the axis of the magnetic field. This creates differences in the populations of the various Zeeman levels in the ground state of the rubidium atoms. Applying an rf field to the atoms at one of the Zeeman transition frequencies tends to equalize the populations of the two Zeeman levels involved. This changes the amount of light absorbed by the atoms. The Zeeman transitions are detected by monitoring the intensity of the light transmitted by the optical pumping cell containing the rubidium atoms.

The energy levels in the ground states of Rb^{85} and Rb^{87} as a function of an applied static magnetic field are given by the Breit-Rabi formula. They are shown in Fig. 1 and Fig. 2, respectively. The nuclear spin of Rb^{85} is $\frac{5}{2}$ and the nuclear spin of Rb^{87} is $\frac{3}{2}$. The ratio g_I/g_J can be obtained by measuring appropriate pairs of Zeeman transition frequencies. In this experiment, two such pairs were measured in each run so that the sign of the polarization of the pumping light could be reversed with no decrease in the signal-to-noise ratio.

The two pairs of transitions chosen for Rb^{85} were ($F=3$, $M=3\leftrightarrow F=3$, $M=2$), ($F=2$, $M=1\leftrightarrow F=2$, $M=2$), and ($F=3$, $M=-2\leftrightarrow F=3$, $M=-3$), ($F=2$, $M-2\leftrightarrow F=2$, $M=-1$). From the Breit-Rabi formula

* This research was conducted while pursuing a postdoctoral resident research associateship.
[1] C. W. White, W. M. Hughes, G. S. Hayne, and H. G. Robinson, Bull. Am. Phys. Soc. **12**, 507 (1967).
[2] S. Penselin, T. Moran, V. W. Cohen, and G. Winkler, Phys. Rev. **127**, 524 (1962).
[3] L. C. Balling and F. M. Pipkin, Phys. Rev. **139**, A19 (1965).
[4] R. L. Driscoll, Phys. Rev. **136**, A54 (1964).
[5] G. S. Hayne, E. S. Ensberg, and H. G. Robinson, Bull. Am. Phys. Soc. **10**, 28 (1965).

one readily obtains the following expression for $g_I/g_J(Rb^{85})$:

$$\frac{g_I}{g_J}(Rb^{85})=\frac{\nu(F=3,\ M=-2\leftrightarrow F=3,\ M=-3)-\frac{1}{2}\Delta\nu^{85}[(1-\frac{4}{3}x+x^2)^{1/2}+x-1]}{\Delta\nu^{85}x+\nu(F=3,\ M=-2\leftrightarrow F=3,\ M=-3)-\frac{1}{2}\Delta\nu^{85}[(1-\frac{4}{3}x+x^2)^{1/2}+x-1]}, \tag{1}$$

where

$$x=\frac{(-g_J+g_I)\mu_0 H_0}{h\Delta\nu^{85}}=\frac{\bar{\nu}_1(\bar{\nu}_1+\Delta\nu^{85})}{\Delta\nu^{85}(\bar{\nu}_1+\frac{1}{3}\Delta\nu^{85})}, \tag{2}$$

$$\mu_{I/I}=g_I\mu_0, \tag{3}$$

$$\mu_{J/J}=g_J\mu_0, \tag{4}$$

and

$$\bar{\nu}_1=\nu(F=3, M=-2\leftrightarrow F=3, M=-3)+\nu(F=2, M=-2\leftrightarrow F=2, M=-1). \tag{5}$$

Here μ_0 is the Bohr magneton, H_0 is the static magnetic field, and $\Delta\nu^{85}$ is the hyperfine splitting of Rb^{85}. Similarly,

$$\frac{g_I}{g_J}(Rb^{85})=\frac{\nu(F=3,\ M=3\leftrightarrow F=3,\ M=2)-\frac{1}{2}\Delta\nu^{85}[1+x-(1+\frac{4}{3}x+x^2)^{1/2}]}{\Delta\nu^{85}x+\nu(F=3,\ M=3\leftrightarrow F=3,\ M=2)-\frac{1}{2}\Delta\nu^{85}[1+x-(1+\frac{4}{3}x+x^2)^{1/2}]}, \tag{6}$$

where

$$x=\frac{\bar{\nu}_2(\Delta\nu^{85}-\bar{\nu}_2)}{\Delta\nu^{85}(\frac{1}{3}\Delta\nu^{85}-\bar{\nu}_2)}, \tag{7}$$

and

$$\bar{\nu}_2=\nu(F=3, M=3\leftrightarrow F=3, M=2)+\nu(F=2, M=1\leftrightarrow F=2, M=2). \tag{8}$$

For Rb^{87} the two pairs of observed transitions $(F=2, M=2\leftrightarrow F=2, M=1)$, $(F=1, M=0\leftrightarrow F=1, M=1)$ and $(F=2, M=-1\leftrightarrow F=2, M=-2)$, $(F=1, M=-1\leftrightarrow F=1, M=0)$ yield $g_I/g_J(Rb^{87})$ in a similar way.

$$\frac{g_I}{g_J}(Rb^{87})=\frac{\nu(F=2,\ M=2\leftrightarrow F=2,\ M=1)-\frac{1}{2}\Delta\nu^{87}[1+x-(1+x+x^2)^{1/2}]}{\Delta\nu^{87}x+\nu(F=2,\ M=2\leftrightarrow F=2,\ M=1)-\frac{1}{2}\Delta\nu^{87}[1+x-(1+x+x^2)^{1/2}]}, \tag{9}$$

where

$$x=\frac{\bar{\nu}_3(\Delta\nu^{87}-\bar{\nu}_3)}{\Delta\nu^{87}(\frac{1}{2}\Delta\nu^{87}-\bar{\nu}_3)}, \tag{10}$$

and

$$\bar{\nu}_3=\nu(F=2, M=2\leftrightarrow F=2, M=1)+\nu(F=1, M=0\leftrightarrow F=1, M=1). \tag{11}$$

Also,

$$\frac{g_I}{g_J}(Rb^{87})=\frac{\nu(F=2,\ M=-1\leftrightarrow F=2,\ M=-2)-\frac{1}{2}\Delta\nu^{87}[(1-x+x^2)^{1/2}+x-1]}{\Delta\nu^{87}x+\nu(F=2,\ M=-1\leftrightarrow F=2,\ M=-2)-\frac{1}{2}\Delta\nu^{87}[(1-x+x^2)^{1/2}+x-1]}, \tag{12}$$

where

$$x=\bar{\nu}_4(\bar{\nu}_4+\Delta\nu^{87})/\Delta\nu^{87}(\bar{\nu}_4+\tfrac{1}{2}\Delta\nu^{87}), \tag{13}$$

and

$$\bar{\nu}_4=\nu(F=2, M=-1\leftrightarrow F=2, M=-2)+\nu(F=1, M=-1\leftrightarrow F=1, M=0). \tag{14}$$

The above equations enable one to calculate (g_I/g_J)-(Rb^{85}) and $(g_I/g_J)(Rb^{87})$ from the measured transition frequencies.

III. APPARATUS

A block diagram of the apparatus is shown in Fig. 3. The rubidium D-1 line was used to optically pump the rubidium atoms. The circular polarizer could be adjusted to transmit either left- or right-circularly polarized light. The rf power used to drive the Zeeman transitions was amplitude-modulated by a coaxial relay which switched the rf on and off at a rate of 10 Hz. The fluctuations in the intensity of the light transmitted by the optical-pumping cell were detected by a photocell, amplified, and displayed on an oscilloscope and lock-in detector.

The rf power was produced by a stable frequency synthesizer, and the frequency was measured by a frequency counter. The frequency counter utilized a 1 MHz standard frequency provided by the Boulder Laboratory. The time scale used was the atomic time scale in which the hyperfine splitting of Cs^{133} is 9 192 631 770 Hz.

The static magnetic field of approximately 40 G was produced by a solenoid situated inside three concentric

TABLE I. A summary of the measurements of the ratio $g_I/g_J(\mathrm{Rb}^{85})$. Two pairs of Zeeman transitions were measured in each run so that the sign of the circular polarization of the pumping light could be reversed. The results for both polarizations were averaged for each run. The results from each run were weighed equally and averaged. This yields $g_I/g_J(\mathrm{Rb}^{85})=-1.46648\times10^{-4}$. The standard deviation for the distribution of the ratios obtained from each run is 7×10^{-9}.

			$(F=3, M=-2 \leftrightarrow F=3, M=-3)$ $(F=2, M=-2 \leftrightarrow F=2, M=-1)$			$(F=3, M=3 \leftrightarrow F=3, M=2)$ $(F=2, M=1 \leftrightarrow F=2, M=2)$			
Buffer gas	Buffer gas pressure at 25°C (mm Hg)	$\Delta\nu^{85}$ (Hz)	No. of meas.	$(g_I/g_J)(\mathrm{Rb}^{85})$	Std. dev.	No. of meas.	$g_I/g_J(\mathrm{Rb}^{85})$	Std. dev.	Average $g_I/g_J(\mathrm{Rb}^{85})$
Neon	172	3 035 762 604	24	-1.46649×10^{-4}	11×10^{-9}	24	-1.46657×10^{-4}	17×10^{-9}	-1.46653×10^{-4}
Neon	172	3 035 762 585	24	-1.46641×10^{-4}	17×10^{-9}	19	-1.46639×10^{-4}	17×10^{-9}	-1.46640×10^{-4}
Neon	172	3 035 762 613	18	-1.46654×10^{-4}	14×10^{-9}	21	-1.46656×10^{-4}	17×10^{-9}	-1.46655×10^{-4}
Neon	172	3 035 762 594	15	-1.46636×10^{-4}	18×10^{-9}	18	-1.46652×10^{-4}	27×10^{-9}	-1.46644×10^{-4}
Neon	105	3 035 750 836	15	-1.46644×10^{-4}	11×10^{-9}	19	-1.46644×10^{-4}	22×10^{-9}	-1.46644×10^{-4}
Neon	105	3 035 750 836	21	-1.46642×10^{-4}	17×10^{-9}	15	-1.46637×10^{-4}	8×10^{-9}	-1.46639×10^{-4}
Helium	208	3 035 800 992	21	-1.46643×10^{-4}	27×10^{-9}	17	-1.46658×10^{-4}	32×10^{-9}	-1.46650×10^{-4}
Helium	120	3 035 771 878	13	-1.46656×10^{-4}	7×10^{-9}	16	-1.46654×10^{-4}	15×10^{-9}	-1.46655×10^{-4}

cylindrical magnetic shields. The solenoid-shield system used in this experiment is similar to a system described in full detail elsewhere.[6] The solenoid is 12.8 in. in diam and 36 in. long. It is equipped with field-correction coils which improve the homogeneity of the field. The innermost magnetic shield is constructed of $\frac{1}{8}$-in.-thick soft iron and is 36.5 in. long and 14.5 in. in diam. End plates of $\frac{1}{8}$-in.-thick soft iron cover the ends of this shield. A 3-in.-diam hole was cut in the center of each end plate to allow the light beam to pass through. The next shield is 40 in. long and 16.5 in. in diam and is constructed of 0.05-in. molypermalloy. The outermost shield of the same material is 42 in. long and 18.5 in. in diam. The innermost shield is equipped with demagnetization windings through which a 60 Hz current (~30A) is passed to demagnetize the shield. The magnetic field of approximately 40 G was found to be homogeneous to three parts in 10^6 over the 30 cm^3 volume of the optical-pumping cells.

The current for the solenoid was approximately 2A and was provided by a current-regulated supply. Current stabilization was achieved by comparing the voltage drop across a 10Ω reference resistor, in series with the solenoid, with the voltage of a series string of mercury cells and feeding back the difference voltage to the transistors which controlled the output current of the power supply. The short-term stability (5 min) was of the order of three parts in 10^7.

The optical-pumping cells were pyrex cylinders 30 cm^3 in volume. Rubidium was distilled into them, and they were filled with various pressures of helium and neon buffer gas.

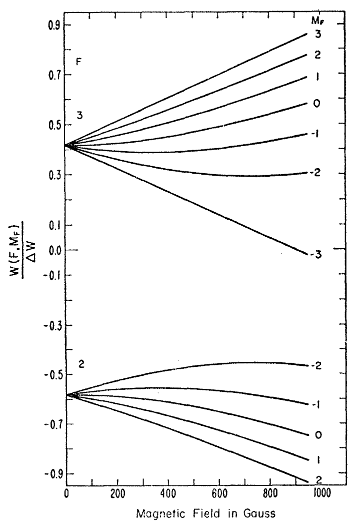

FIG. 1. The energy levels of a Rb^{85} atom in its ground state as a function of the magnetic field.

[6] R. J. Hanson and F. M. Pipkin, Rev. Sci. Instr. 36, 179 (1965).

IV. MEASUREMENTS

The transition frequencies for Rb^{87} centered around 26 MHz and those for Rb^{85} around 17.5 MHz. The full

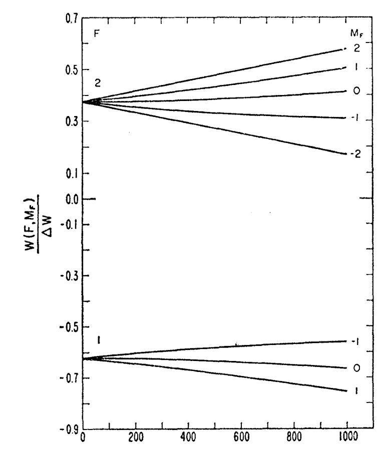

FIG. 2. The energy levels of a Rb^{87} atom in its ground state as a function of the magnetic field.

TABLE II. A summary of the measurements of the ratio $g_I/g_J(Rb^{87})$. Two pairs of Zeeman transitions were measured in each run so that the sign of the circular polarization of the pumping light could be reversed. The results for both polarizations were averaged for each run. The results from each run were then weighed equally and averaged. This yields $g_I/g_J(Rb^{87}) = -4.96997\times10^{-4}$. The standard deviation for the distribution of the ratios obtained from each run is 8×10^{-9}.

			$(F=2, M=-1 \leftrightarrow F=2, M=-2)$ $(F=1, M=-1 \leftrightarrow F=1, M=0)$			$(F=2, M=2 \leftrightarrow F=2, M=1)$ $(F=1, M=0 \leftrightarrow F=1, M=1)$			
Buffer gas	Buffer gas pressure at 25°C (mm Hg)	$\Delta\nu^{87}$ (Hz)	No. of meas.	$g_I/g_J(Rb^{87})$	Std. dev.	No. of meas.	$g_I/g_J(Rb^{87})$	Std. dev.	Average $g_I/g_J(Rb^{87})$
Neon	172	6 834 751 077	14	-4.96989×10^{-4}	27×10^{-9}	14	-4.97006×10^{-4}	25×10^{-9}	-4.96998×10^{-4}
Neon	172	6 834 751 099	17	-4.97001×10^{-4}	32×10^{-9}	22	-4.97006×10^{-4}	25×10^{-9}	-4.97004×10^{-4}
Neon	172	6 834 751 121	21	-4.96992×10^{-4}	16×10^{-9}	18	-4.97021×10^{-4}	27×10^{-9}	-4.97006×10^{-4}
Neon	105	6 834 724 372	16	-4.96983×10^{-4}	11×10^{-9}	16	-4.97001×10^{-4}	31×10^{-9}	-4.96992×10^{-4}
Neon	105	6 834 724 372	17	-4.96967×10^{-4}	16×10^{-9}	16	-4.96996×10^{-4}	15×10^{-9}	-4.96982×10^{-4}
Neon	60	6 834 706 435	15	-4.96984×10^{-4}	17×10^{-9}	20	-4.96997×10^{-4}	32×10^{-9}	-4.96991×10^{-4}
Helium	208	6 834 839 241	15	-4.96996×10^{-4}	19×10^{-9}	15	-4.97006×10^{-4}	29×10^{-9}	-4.97001×10^{-4}
Helium	120	6 834 772 784	15	-4.96994×10^{-4}	20×10^{-9}	15	-4.97013×10^{-4}	19×10^{-9}	-4.97003×10^{-4}

widths at half-maximum of the Rb^{85} and Rb^{87} lines were 100 and 120 Hz, respectively. The measurements were carried out at a temperature of approximately 50°C for Rb^{85} and at 60°C for Rb^{87}. These temperatures optimized the signal-to-noise ratios of the transition signals in the lower hyperfine multiplet which were smaller than the signal-to-noise ratios in the upper multiplet. At those temperatures there was some line broadening due to Rb-Rb spin-exchange collisions. In addition, the lines were somewhat rf broadened. The same rf power was used for both the strong and the weak Zeeman transition signals.

The magnetic-field inhomogeneity for each run was determined by measuring the width of a Rb^{87} transition at low rf power and at 30°C. Under these conditions the width of the observed line was determined by the field inhomogeneity. At 26 MHz the magnetic-field–dependent linewidth was approximately 70 Hz for all runs.

Each run consisted of about 30 measurements, half of which were made with left-circularly polarized light and the other half with right-circularly polarized light. The sign of the circular polarization determined which pair of Zeeman transitions was measured. The results of both polarizations were then averaged. A single measurement was made in the following way. The peak of one transition was located five times, then the second transition peak five times, and finally the first transition peak five times again in order to compensate for magnetic field drift.

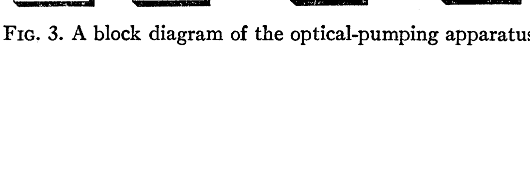

FIG. 3. A block diagram of the optical-pumping apparatus.

The results of the measurements for Rb^{85} and Rb^{87} are summarized in Tables I and II, respectively. g_I/g_J was calculated on the basis of the measurements with the aid of a computer. The hyperfine splitting for each optical pumping cell was calculated using existing pressure-shift data.[7] There is no discernable dependence of the results upon buffer gas or buffer gas pressure. Neon filled cells were used more extensively because of better signal-to-noise ratios for the transitions in the lower hyperfine multiplet.

Table II indicates a systematic difference between the results for one circular polarization and for the other. This is probably due principally to light-intensity shifts of the Zeeman transitions which are a function of the polarization of the pumping light.[1,8] Spin-exchange frequency shifts may also contribute, since the lines are broadened by Rb-Rb collisions.[9]

The principal source of error in the measurements appears to be the inhomogeneity of the static magnetic field which creates asymmetry in the line shape. Changes in the cell position, in the orientation of the shield end plates, or in the current in the correction coil, produced changes in the spatial dependence of the field across the optical pumping cell without changing the magnetic-field–dependent linewidth. Such alterations produced systematic changes in the measured values of g_I/g_J. Such changes were made in a random way before each run in the expectation that the resulting spread in the g_I/g_J values would reflect the error due to this effect.

The use of both circular polarizations eliminates any error due to spin-exchange frequency shifts. Errors due

[7] P. L. Bender, E. C. Beaty, and A. R. Chi, Phys. Rev. Letters 1, 311 (1958).
[8] M. Arditi and T. R. Carver, Phys. Rev. 124, 800 (1961).
[9] L. C. Balling, R. J. Hanson, and F. M. Pipkin, Phys. Rev. 133, A607 (1964).

to imperfect knowledge of the hyperfine splitting in each bulb are completely negligible. Measurements of the Zeeman transitions as a function of light intensity have placed an upper limit to the error in g_I/g_J due to light intensity shifts not averaged out by the change in circular polarization. This limit is $\pm 4\times 10^{-9}$. To obtain the estimated error this limit was added in quadrature to the standard deviation in the g_I/g_J values from the various runs, giving an error of $\pm 8\times 10^{-9}$ for $g_I/g_J(\mathrm{Rb}^{85})$ and $\pm 9\times 10^{-9}$ for $g_I/g_J(\mathrm{Rb}^{87})$.

Thus one obtains

$$g_I/g_J(\mathrm{Rb}^{85}) = -1.46648(8)\times 10^{-4} \qquad (15)$$

and

$$g_I/g_J(\mathrm{Rb}^{87}) = -4.96997(9)\times 10^{-4}. \qquad (16)$$

These numbers can be compared with the results of White *et al.*, which are

$$g_I/g_J(\mathrm{Rb}^{85}) = -1.466496(10)\times 10^{-4} \qquad (17)$$

and

$$g_I/g_J(\mathrm{Rb}^{87}) = -4.969917(7)\times 10^{-4}. \qquad (18)$$

The ratio of the estimated error in the experiment reported here to the error quoted by White *et al.*, is approximately equal to the ratio of the magnetic-field inhomogeneities in the two experiments.[10] The result of Penselin *et al.*, for Rb^{85} is

$$(g_I/g_J)(\mathrm{Rb}^{85}) = -1.466764(3)\times 10^{-4}. \qquad (19)$$

Measurements[3] of $g_J(\mathrm{Rb}^{85})/g_J(\mathrm{H})$ using $g_I/g_J(\mathrm{Rb}^{85}) = -1.466764\times 10^{-4}$ give

$$g_J(\mathrm{Rb}^{85})/g_J(\mathrm{H}) = 1\times(23.7\pm 0.1\times 10^{-6}). \qquad (20)$$

Using the optical-pumping value

$$g_I/g_J(\mathrm{Rb}^{85}) = -1.466496(10)\times 10^{-4}$$

yields

$$g_J(\mathrm{Rb}^{85})/g_J(\mathrm{H}) = 1+(23.6\pm 0.1\times 10^{-6}). \qquad (21)$$

V. CONCLUSIONS

It appears that the atomic beam value for $g_I/g_J(\mathrm{Rb}^{85})$ is incorrect although the reason for the discrepancy is not apparent. The error has a relatively small effect on the ratio $g_J(\mathrm{Rb}^{85})/g_J(\mathrm{H})$.

Note added in proof. The rf discharge oscillator shown in Fig. 3 has no function in this experiment. Also, the photocell used was a 925 photocell.

ACKNOWLEDGMENT

I wish to express my appreciation to Dr. P. L. Bender for many helpful discussions during the course of this work.

[10] H. G. Robinson (private communication).