Theory of Angular Correlations for Two Nucleons Emitted in the Nuclear Absorption of Slow Pions*

DANIEL S. KOLTUN
Department of Physics and Astronomy, University of Rochester, Rochester, New York
(Received 24 March 1967)

A theory is developed for the angular correlation for two fast nucleons emitted when a nucleus absorbs a slow pion, based on the assumption that the reaction is direct. It is shown that for a quite general class of models for the two-nucleon absorption mechanism, there are selection rules which relate the final angular momenta of the two nucleons to the angular momenta carried initially by the two nucleons in the target nucleus. The selection rules involve the relative and center-of-mass motions, separately, of the nucleon pair. The angular correlation of interest is therefore defined with respect to the angle between the relative and c.m. momenta of the emitted nucleons. The selection rules provide a direct connection between the observed angular correlation and the two-nucleon angular momentum structure of the target nucleus. Some examples are given for the application of this theory to the study of nuclear structure.

I. INTRODUCTION

SEVERAL authors[1] have expressed the idea that study of the reaction in which a pion is absorbed by a nucleus with the emission of two energetic nucleons should yield information about the structure of the nucleus. The idea is based on the assumption that the reaction is direct, in that the pion interacts only with the ejected nucleon pair. Evidence that this assumption may be reasonable, at least for light nuclear targets, is given by experiments which show (1) the distribution of the summed momentum of the two ejected nucleons is consistent with estimates of the two-nucleon momentum distribution in the target,[2,3] and (2) the probability of the $(\pi^-,2p)$ reaction, which would involve at least three target nucleons, is small compared to that of the $(\pi^-,2n)$ and (π^-,np), which can involve only two.[3]

In this paper we shall demonstrate the connection between the angular distribution of the ejected nucleon pairs and the angular-momentum structure of the target nucleus, based on the assumption that the reaction is direct. We shall not consider any specific model for the reaction, but shall assume that some kind of two-nucleon impulse approximation is valid. The features of this approximation will be discussed in Sec. III.

The central argument is based on two considerations: First, that for pions moving slowly with respect to the target nucleus, there will be little transfer of angular momentum to the center of mass (c.m.) of the absorbing nucleon pair, which is also moving slowly with respect to the target. Combined with conservation of angular momentum and parity for the entire system, this limitation is equivalent to a set of approximate selection rules for transfer of angular momentum to the c.m. system and relative coordinates, separately, of the nucleon pair. Second, the angular correlation of two emitted particles is related to their individual and total angular momenta. Since the selection rules for the absorption control the relative and c.m. angular momenta, the angular correlation of interest is that between the *relative* and *total* momenta of the two emitted nucleons. (See Fig. 1.) Thus a study of this correlation, together with the selection rules, provides a means of measuring the relative and c.m. angular momenta of the nucleon pair in the initial target.

If the initial pion were at rest with respect to the absorbing nucleon pair, no linear or orbital angular momentum would be transferred to the c.m. of the pair, measured, for example, with respect to the target. However, both linear and angular momentum would be transferred in relative coordinates of the pair, consistent with conservation of energy, angular momentum, and parity, giving us the selection rules mentioned above.

In this paper we shall consider only pions, initially moving slowly with respect to the target nucleus, as, for example, in a π atomic bound state. For a not-too-

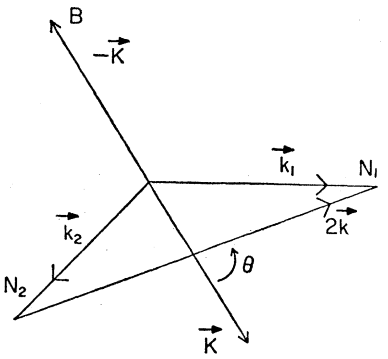

FIG. 1. Kinematics for the reaction $A(\pi,N_1N_2)B$ in total c.m. system; the angular correlation discussed in this paper is defined for the angle θ, between the relative momentum $\mathbf{k}$ and c.m. momentum $\mathbf{K}$ of the two emitted nucleons, N_1 and N_2.

* Work supported in part by the U. S. Atomic Energy Commission.

[1] T. Ericson, Phys. Letters **2**, 278 (1962); in *Proceedings of the International Conference on High Energy Physics and Nuclear Structure, CERN, 1963* (CERN, Geneva, 1963), p. 47; M. Jean, Nuovo Cimento Suppl. **2**, 400 (1964).

[2] H. Davies, H. Muirhead, and J. N. Woulds, Nucl. Phys. **78**, 663 (1966); M. E. Nordberg, Jr., K. F. Kinsey, and R. L. Burman, in *Proceedings of the Williamsburg Conference on Intermediate Energy Physics, Williamsburg, 1966* (William and Mary College, Williamsburg, Virginia, 1966), p. 207.

[3] M. E. Nordberg, Jr., and R. L. Burman (private communication).

high nuclear charge, the motion of a bound pion with respect to the target is negligible, so that the relative momentum of the pion and nucleon pair is

$$\mathbf{k}_{\rm rel}\simeq(\mu/2m)\mathbf{K}, \tag{1.1}$$

where the c.m. momentum $\mathbf{K}$ is the sum of the momenta of the two nucleons (either before or after absorption, since there is no momentum transfer), and $\mu/m(=0.146)$ is the ratio of the masses of the pion and nucleon.

For $K\neq 0$, we expect some violation of the selection rules. We estimate the change $\Delta\mathfrak{L}$ of the pair c.m. angular momentum, to be limited by[4]

$$\Delta\mathfrak{L}\lesssim k_{\rm rel}R, \tag{1.2}$$

where R is an "interaction length," which gives the mean relative separation of the two nucleons in interaction with the pion. This length should be of the order of the inverse of the relative momentum transfer. For a light nucleus, we expect $K\lesssim 1$ F$^{-1}\simeq$200 MeV/$\hbar c$ and $R\lesssim 1$ F. According to (1) and (2), this leads us to an estimate: $\Delta\mathfrak{L}\lesssim 0.07$ which strongly favors the selection rule $\Delta\mathfrak{L}=0$.

To make these arguments more quantitative, one must adopt a more specific model of the pion-nuclear interactions, which will not be done in this work. But most models in use depend on a short-range nucleon-nucleon correlation to provide the relative momentum transfer, which would agree with our estimate of R, above.

In any case, the theory to be presented here can itself be used to test the validity of the selection rules by comparing with experiments the predicted two-nucleon angular correlation for a target whose angular-momentum structure is thought to be known.

In the following section, and in Appendix A, we derive the general form for the angular correlation, in terms of the algebra of angular-momentum coupling. In Sec. III, we consider the particular structure of the two-nucleon impulse model for pion absorption. In Sec. IV we obtain the selection rules for this model, and in Sec. V work out several simple examples of angular correlations based on these selection rules. In Appendix B, we show as an example that a specific absorption model, which has received much attention,[5] does lead to the selection rules. The discussion in Sec. VI concludes the paper.

II. ANGULAR CORRELATION

We consider the amplitude for the reaction $A(\pi,N_1,N_2)B$, where A, B are the target and residual nuclei, respectively, and N_1N_2 are the two fast nucleons. We assume that the amplitude is given by the first-order matrix element of an absorption operator T

$$M=\langle B,N_1,N_2|\,T\,|A,\pi\rangle, \tag{2.1}$$

where the initial and final states do not depend on T. We require that T be a scalar under rotations and parity inversions. If the pion is initially in a state with quantum numbers α, we may take the matrix element of T between the pion state α and the pion vacuum state

$$T_\alpha\equiv\langle 0|\,T\,|\alpha\rangle, \tag{2.2}$$

thus defining a transition operator depending explicitly on nucleon variables only. If the initial pion state α has orbital angular momentum q with projection μ, then T will transform under rotations of the *nuclear* coordinates, as a tensor of rank $T_\mu{}^q$, since T must be a scalar in nuclear-pion coordinates. The negative intrinsic parity of the pion means that T_α carries parity $=(-1)^{q+1}$. The transition amplitude (2.1) can then be expressed as a *nuclear* matrix element, which we shall write out more fully, using $\mathbf{K}$ and $\mathbf{k}$ for the c.m. and relative momenta of the emitted nucleon pair (see Fig. 1)

$$\begin{aligned}&M=\langle\mathbf{K},\mathbf{k},m_1,m_2,J_BM_BX_B|\,T_\alpha\,|J_AM_AX_A\rangle,\\ &\mathbf{K}=\mathbf{k}_1+\mathbf{k}_2,\quad \mathbf{k}=(\tfrac{1}{2})(\mathbf{k}_1-\mathbf{k}_2),\end{aligned} \tag{2.3}$$

where m_1, m_2 are the spin projections along some axis of the emitted nucleons, and J_BM_B are the spins and spin projections of the final and initial target states, respectively. We use X_B, X_A to denote all internal quantum numbers of the final and initial target states.

For an experiment in which we measure the momenta of the two ejected nucleons, but not the initial or final spin projections of any of the particles (or initial orientation of the pion), we are concerned with the quantity

$$P(k,K,\theta)=([q][J_A])^{-1}\sum_{m_1,m_2,M_B,M_A,\mu}|M|^2, \tag{2.4}$$

where M is given in (2.3). The pion has orbital angular-momentum q with projection μ. We use the notation $[l]\equiv 2l+1$. Notice that P depends only on the magnitudes k, K, and on the relative angle θ.

The momenta k and K are related by energy conservation:

$$\begin{aligned}Q&=\frac{k_1^2}{2m}+\frac{k_2^2}{2m}+\frac{|\mathbf{k}_1+\mathbf{k}_2|^2}{2W_B}\\ &=(2m)^{-1}[2k^2+(\tfrac{1}{2}+m/W_B)K^2],\end{aligned} \tag{2.5}$$

where m, W_B are the masses of the nucleon and the final nucleus B, respectively. Q is the energy transfer

$$Q=(W_\pi+W_A-W_B-2m)c^2, \tag{2.6}$$

where W_π, W_A are the masses of the pion and target nucleus A, respectively.

[4] In units of $\hbar$.

[5] P. Huguenin, Z. Physik **167**, 416 (1962); G. M. Shklyarevskii, Zh. Eksperim. i Teor. Fiz. **45**, 698 (1963) [English transl.: Soviet Phys.—JETP **18**, 480 (1964)]; R. I. Jibuti and T. I. Kopaleishvii, Nucl. Phys. **55**, 337 (1964); R. M. Spector, Phys. Rev. **134**, B101 (1964); T. Kohmura, Progr. Theoret. Phys. (Kyoto) **34**, 234 (1965); Y. Sakamoto, Nucl. Phys. **87**, 414 (1966); Il-Tong Cheon, Phys. Rev. **145**, 794 (1966); I. Cheon, Y. Sakamoto, and C. Nguyen-Trung, Progr. Theoret. Phys. (Kyoto) **34**, 574 (1965).

The distribution of particles for the unpolarized experiment can be expressed as a function of K and $Z=\cos\theta$:

$$\frac{\partial^2 W(K,Z)}{\partial K \partial Z} = \text{const} \times kK^2 P(k,K,\theta), \tag{2.7}$$

with the restrictions (2.5) and (2.6).

We are interested in the angular correlation $F(\theta)$ given by (2.7) for fixed value of K, or, up to a constant by $P(k,K,\theta)$ for fixed K and appropriate k

$$F(\theta)=P(k,K,\theta); \quad \text{fixed } k, K. \tag{2.8}$$

In order to express $F(\theta)$ in compact form, it is convient to introduce into the matrix element (2.3) a complete set of intermediate states (of the total nuclear system) with spin J_c, spin projection M_c, and internal quantum numbers X_c:

$$M=\sum_{J_c M_c X_c} \langle \mathbf{K},\mathbf{k},m_1,m_2,J_B,M_B,X_B | J_c,M_c,X_c\rangle \times \langle J_c M_c X_c | T_\alpha | J_A M_A X_A\rangle. \tag{2.9}$$

Since T_α transforms as a spherical tensor T_μ^α, we may reduce

$$\langle J_c M_c X_c | T_\alpha | J_A M_A X_A\rangle = (-1)^{2q}\langle J_c X_c \| T_\alpha \| J_A M_A\rangle \times (J_A q M_A \mu | J_c M_c), \tag{2.10}$$

using the standard vector coupling coefficient. We choose the internal quantum numbers for the state labeled X_c as follows: Let $\mathfrak{L}$, $\mathfrak{l}$, be the c.m. (measured with respect to the target c.m.) and relative angular momentum, respectively, of the two emitted nucleons, and S the two-nucleon spin, all referring to the final state. Then these are vector-coupled:

$$\begin{aligned} \mathfrak{L}+\mathfrak{l}&=\mathbf{L},\\ \mathbf{L}+\mathbf{S}&=\mathbf{I},\\ \mathbf{I}+\mathbf{J}_B&=\mathbf{J}_c, \end{aligned} \tag{2.11}$$

where L, I are the two-nucleon orbital and total angular momentum, respectively. Then we have

$$|J_c M_c X_c\rangle = |\mathfrak{L},\mathfrak{l},L,S,I,J_B X_B, J_c M_c\rangle, \tag{2.12}$$

where X_c stands for all the quantum numbers and vector coupling displayed in (2.11) as well as the magnitude of the momenta $\mathbf{K}$, $\mathbf{k}$. In this construction, it is convenient not to antisymmetrize the emitted nucleons with the nucleons in the residual target.

$F(\theta)$ may be calculated by standard techniques of the theory of angular correlations[6] as given in more detail in Appendix A. The result is

$$F(\theta)=\sum_{J_c X_c X_{c'}} [J_c]([J_A][q])^{-1/2}\delta_{S,S'}\delta_{I,I'}\delta_{L,L'} \times \langle J_c X_c \| T_\alpha \| J_A X_A\rangle \langle J_c X_c \| T_\alpha \| J_A X_A\rangle^* \times \sum_r (-1)^{L+r-\mathfrak{l}-\mathfrak{L}'} W(\mathfrak{L}'\mathfrak{l}'\mathfrak{L}\mathfrak{l}; Lr) \times c^r(\mathfrak{l}',\mathfrak{l}) c^r(\mathfrak{L}',\mathfrak{L}) P_r(\cos\theta), \tag{2.13}$$

where

$$c^r(\mathfrak{l}',\mathfrak{l}) \equiv 4\pi([\mathfrak{l}'][\mathfrak{l}])^{1/2}(\mathfrak{l}'\mathfrak{l}000|r0),$$

W is the usual Racah coefficient, and P_r is the Legendre function.

Equation (2.13) is completely general; it is of interest only if we can find restrictions on the final states X_c connected to the target state through T_α. Then the angular correlation $F(\theta)$ will have a maximum complexity given by the vector restrictions on the arguments of the Racah coefficient (Fig. 2). Thus r obeys both inequalities,

$$r \le 2\mathfrak{l}_{\max}, \quad r \le 2\mathfrak{L}_{\max}, \tag{2.14}$$

for the maximum relative and c.m. orbital angular momenta of the two nucleons in the final state. Also the c^r coefficients guarantee that both $(\mathfrak{l}+\mathfrak{l}'+r)$ and $(\mathfrak{L}+\mathfrak{L}'+r)$ are even numbers (parity).

It is clear that $F(\theta) \to$ constant as K or $k \to 0$.

III. IMPULSE APPROXIMATION

Let us now consider the specific properties of the two-nucleon impulse approximation for the absorption process, which was discussed in the Introduction.

By a two-nucleon absorption process, we mean that the interaction operator T, Eq. (2.1), or its nuclear part T_α, Eq. (2.2), is a sum of operators on no more than two nucleons

$$T_\alpha=\sum_i T_\alpha^{(1)}(i)+\tfrac{1}{2}\sum_{i\neq j} T_\alpha^{(2)}(i,j), \tag{3.1}$$

where i, j stand for the coordinates or momenta and spins of the ith and jth nucleons. This encompasses a variety of pion-nuclear interactions considered in various models: one-nucleon interactions have been considered by many authors[5] and two-nucleon terms due to meson rescattering have also been treated[7] for meson-absorption processes. The two-nucleon model of Eck-

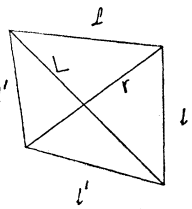

FIG. 2. The quadrilateral represents the recoupling through the Racah coefficient of Eq. (2.13). Each of the triangles represents a "triangle inequality" restriction on the three angular momenta which compose it.

[6] L. J. B. Goldfarb, in *Nuclear Reactions, I*, edited by P. M. Endt and M. Demeur (North-Holland Publishing Company, Amsterdam, 1959), p. 159; S. Devons and L. J. B. Goldfarb, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 42, p. 362. We use, however, the phase conventions of Brink and Satchler, Ref. 10.

[7] D. S. Koltun and A. Reitan, Phys. Rev. **141**, 1413 (1966); (to be published).

stein[8] and Divakaran[9] are of this type, although the interaction $T(ij)$ is not given explicitly, since the effect of two-nucleon correlation is included in an effective two-nucleon absorption interaction.

Since T_α is a sum of one- and two-nucleon operators, the absorption amplitude M (2.3) can be reduced to a sum of two-nucleon matrix elements of T_α, multiplied by overlap integrals (or form factors) for removing two nucleons from the target. However, in the *two-nucleon impulse approximation*, we further assume that the only important two-nucleon matrix elements connect the two emitted nucleons with two nucleons in the target; we neglect processes in which T_α connects a nucleon in target A with a nucleon in the residual target B. This is based on the notion that while two-nucleon correlations are essential to the absorption process, three nucleon and higher correlations are less probable, so that if two nucleons interact through $T_\alpha^{(2)}(ij)$, a third nucleon is not likely to receive sufficient momentum transfer to be ejected at high energy.

This approximation is in part amenable to experimental test, in the relative probability for the reaction $(\pi^-,2p)$ to either (π^-,pn), or (π^-,nn), since the first must involve a three-nucleon process, while the second and third may proceed through a two-nucleon process.

We note that the two-nucleon interaction (3.1) does not directly imply the two-nucleon impulse approximation, in the sense just discussed; the latter is a statement about the nucleus and is based on the notion of an "independent-pair approximation" for nuclear structure.

This approximation is most easily formulated in terms of the reduced matrix element (2.10):

$$\langle J_c X_c\|T_\alpha\|J_A X_A\rangle=\sum_{I_i,X_i}(-1)^{J_c+I_i-q-J_B}([J_A][I_f])^{1/2}\times W(I_f I_i J_c J_A,qJ_B)\langle I_f X_f\|\tilde{T}_\alpha\|I_i X_i\rangle\times\langle I_i X_i;J_B X_B|\}J_A X_A\rangle, \quad (3.2)$$

where

$$\tilde{T}_\alpha=T_\alpha^{(1)}(1)+T_\alpha^{(1)}(2)+T_\alpha^{(2)}(1,2) \quad (3.3)$$

and

$$\langle J_A X_A\{|I_i X_i;J_B X_B\rangle$$

is the coefficient of fractional parentage (c.f.p.)[10] for the two-nucleon state I_i, X_i which leaves the residual nucleus in the state J_B, X_B. The two-nucleon state I_i, X_i has total angular momentum I_i, and internal structure given by X_i, which we shall take to stand for $\mathfrak{L}_i$, $\mathfrak{l}_i$, the c.m. and relative orbital angular momenta, and S_i the spin, of the two nucleons, with the vector coupling

$$\mathfrak{L}_i+\mathfrak{l}_i=\mathbf{L}_i,\qquad \mathbf{L}_i+\mathbf{S}_i=\mathbf{I}_i. \quad (3.4)$$

We have added the index f to the quantum numbers (2.11) referring to the final two-nucleon state. Finally, the Racah recoupling in (3.2) takes care of the fact that J_A and J_c are coupled states:

$$\mathbf{I}_f+\mathbf{J}_B=\mathbf{J}_c,\qquad \mathbf{I}_i+\mathbf{J}_B=\mathbf{J}_A. \quad (3.5)$$

What have been left out of (3.2) in making the impulse approximation, are terms with c.f.p. to states of the residual nucleus other than J_B, X_B.

IV. SELECTION RULES

The qualitative argument of the Introduction was that for slow pions there is little probability of angular momentum transfer to the c.m. of the absorbing nuclear pair. This gives us the approximate selection rules:

$$\Delta\mathfrak{L}=0, \quad \text{(favored)} \quad (4.1a)$$

$$\Delta\mathfrak{L}=1, \quad \text{(possible, but not favored)}. \quad (4.1b)$$

This means that the two-nucleon–transition operator $\tilde{T}_\alpha$ [Eq. (3.3)] transforms under rotations of the c.m. coordinates $\mathbf{R}$, like a spherical tensor of rank zero for case (4.1a) and rank one for case (4.1b), with even or odd parity in $\mathbf{R}$, respectively.

We may find selection rules for $\mathfrak{l}$, S, and isospin T of the two-nucleon system, by combining Eq. (4.1) with the fact that $\tilde{T}_\alpha$ transforms under rotations of the complete two-nucleon system as a tensor of rank q and parity $(-1)^{q+1}$, and is symmetrical in nuclear coordinates.

We give the results for initial pions in s-state ($q=0$) and p-state ($q=1$):

(1) For s wave, we have $\Delta I=0$ and a parity change: This requires $\tilde{T}_\alpha$ to transform as a vector in spin and in isospin. Thus

$$\Delta\mathfrak{L}=0,\ \Delta\mathfrak{l}=1,\ \text{and}\ TS=1,0\leftrightarrow 11=0,1\leftrightarrow 11 \quad (4.2a)$$

$$\Delta\mathfrak{L}=1,\ \Delta\mathfrak{l}=0,2,\ \text{and}\ TS=11\leftrightarrow 11=11\leftrightarrow 00\ \ 10\leftrightarrow 01 \quad (4.2b)$$

with $\Delta L=1$ for both cases.

(2) For p wave, $\Delta I=1$ and no parity change:

$$\Delta\mathfrak{L}=0,\ \Delta\mathfrak{l}=0,2,\ TS=11\leftrightarrow 11,\ 11\leftrightarrow 01,\ 10\leftrightarrow 01 \quad (4.3a)$$

$$\Delta\mathfrak{L}=1,\ \Delta\mathfrak{l}=1,3,\ TS=11\leftrightarrow 10,\ 11\leftrightarrow 01 \quad (4.3b)$$

with $\Delta L=0$, 2 for both cases.

One notices some simple features of these selection rules. Set (4.2a) corresponds to absorption of the pion in an s wave with respect to the two nucleons, and (4.2b) to a relative p wave, although both correspond

[8] S. G. Eckstein, Phys. Rev. **129**, B413 (1963).
[9] P. P. Divakaran, Phys. Rev. **139**, B387 (1965).
[10] See, e.g., D. M. Brink and G. R. Satchler, *Angular Momentum* (Clarendon Press, Oxford, England, 1962), p. 84; or A. de Shalit and I. Talmi, *Nuclear Shell Theory* (Academic Press Inc., New York, 1963), p. 359.

to an s-wave pion with respect to the *target*. Similarly, (4.3a) corresponds to a p-wave pion relative to the two nucleons and has the same selection rules for the internal pair variables $\mathfrak{l}$, S, T as (4.2b). Finally, (4.3b) corresponds to pion s and d wave relative to the two nucleons.

In Appendix B we work out the specific example of a simple model absorption operator, from which we obtain the selection rules above.

V. EXAMPLES

In this section we shall work out some specific cases to show the connection between the structure of the target and the angular distribution of nucleon pairs, through the selection rules of the previous section.

We shall first suppose that the target nucleus is well described by the shell model, with the two absorbing nucleons both assigned to the first p shell. If we further suppose that the radial wave functions are those for a harmonic oscillator, we may easily find[11] the possible orbital angular momenta in c.m. and relative coordinates, and their sum, to the

	$\mathfrak{L}_i$	$\mathfrak{l}_i$	L_i	
S	0	0	0	(5.1a)
D	0	2	2	(5.1b)
D	2	0	2	(5.1c)
P	1	1	1	(5.1d)

Consider the possible transition for absorption of an s-state pion, under the strong selection rules (4.2a). We may obtain the final-state angular momenta:

$\mathfrak{L}_f$	$\mathfrak{l}_f$	L_f	
0	1	1	(5.2a)
0	1,3	1,3	(5.2b)
2	1	1,2,3	(5.2c)
1	0,2	1,2,3	(5.2d)

Now suppose the initial two-nucleon total spin is $I_i=0$, with isospin $T_i=1$. Then only components (5.1a) $^1S_0$, and (5.1d) $^3P_0$ are present in the initial state. The selection rules (4.2a) give

$$T,S=1,0\rightarrow 1,1\,, \tag{5.3a}$$

$$T,S=1,1\rightarrow 1,0 \text{ or } 0,1\,. \tag{5.3b}$$

If the final two-nucleon isospin is $T_f=1$, there is no final state in (5.2d) with $I_f=0$, so that only (5.2a) will contribute to the absorption. It follows from (2.13), (2.14) that the angular distribution will be isotropic, that is, $F(\theta)$ will be constant, since $\mathfrak{L}_f=0$.

[11] E.g., T. A. Brody and M. Moshinsky, *Tables of Transformation Brackets* (Monografias del Instituto de fisica, Mexico City, 1960).

A transition of this type would be the case for $A(\pi^-,2n)B$ with $J_A=J_B=0$, and $T_A=0$, $T_B=1$, as, for example, in

$$\begin{aligned} &O^{16}(J,T=00)\rightarrow N^{14*}(0,1)\,,\\ &C^{12}(0,0)\rightarrow B^{10*}(0,1)\,. \end{aligned} \tag{5.4}$$

A nonisotropic angular correlation for these transitions would imply either (1) violation of the strong selection rules (4.2a), or (2) the presence of components in the initial target state, with angular-momentum structure not given by the harmonic-oscillator shell model.

As a second example, consider absorption of an s-state pion by a p-shell nucleon pair with $I_i=1$, $T_i=0\rightarrow I_f=1$, $T_f=1$, as in

$$\begin{aligned} &Li^6(JT=1,0)\rightarrow He^4(0,0)\,,\\ &O^{16}(0,0)\rightarrow N^{14}(1,0)\,. \end{aligned} \tag{5.5}$$

Again using the strong selection rules (4.2a), we find that the contribution of (5.1d) is eliminated by the spin-isospin rules.

Thus the components of the initial state which contribute to the transition are $^3S_1$: (5.1a), and $^3D_1$, which is given by a linear combination of (5.1b) and (5.1c). The contributions of these two components are in proportion to the c.f.p.'s [see Eq. (3.2)]:

$$\begin{aligned} a_S&=\langle {}^3S_1;\,J_B=0,X_B|\}J_A=0,X_A\rangle\,,\\ a_D&=\langle {}^3D_1;\,J_B=0,X_B|\}J_A=0,X_A\rangle\,. \end{aligned} \tag{5.6}$$

The two nucleon transition amplitudes in Eq. (2.3) may be written as

$$\begin{aligned} M_k&=\langle I_f,X_k\|\tilde{T}_\alpha\|I_iX_i\rangle\\ &=\langle I_f;\mathfrak{L}_f\mathfrak{l}_fL_fS_f\|\tilde{T}_\alpha\|I_i;\mathfrak{L}_i\mathfrak{l}_iL_iS_i\rangle\,, \end{aligned} \tag{5.7}$$

where X_i may be $^3S_1$ or $^3D_1$, and X_k stands for one of the four possible sets of final-state quantum numbers:

	$\mathfrak{L}_f$	$\mathfrak{l}_f$	L_f	S_f		
M_1:	0	1	1	1	from $^3S_1$	
M_2:	0	1	1	1	from $^3D_1$	(5.8)
M_3:	2	1	1	1		
M_4:	2	1	2	1		

The amplitudes M_3 and M_4 are not, in fact, independent, since they differ only in the value of L_f; all other quantum numbers are constant. If we use the fact that $\tilde{T}_\alpha$ transforms as a scalar in I, and can be decomposed into a scalar in c.m., vector in relative coordinates, and vector in spin, we may decompose the reduced matrix elements (5.7) for M_3, M_4, and obtain the ratio

$$\frac{M_3}{M_4}=-(\tfrac{1}{3})^{1/2}\frac{W(1L_iS_fS_i;\,1I)W(\mathfrak{l}_f\mathfrak{l}_i1L_i;\,1\mathfrak{L}_f)}{W(2L_iS_fS_i;\,1I)W(\mathfrak{l}_f\mathfrak{l}_i2L_i;\,1\mathfrak{L}_f)}\,. \tag{5.9}$$

Inserting the resulting values of (3.2) into (2.13) for the four cases in (5.8), we obtain an angular distribu-

tion of the form

$$F(\theta)=A_0P_0(\cos\theta)+A_2P_2(\cos\theta),$$
$$A_0=g(0101;10)\{a_S{}^2|M_1|^2+a_D{}^2|M_2|^2 +a_Da_S2\,\mathrm{Re}M_2{}^*M_1\}+g(2121;10)a_D{}^2|M_3|^2 +g(2121;20)a_D{}^2|M_4|^2,$$
$$A_2=g(2121;12)a_D{}^2|M_3|^2 +g(2121;22)a_D{}^2|M_4|^2+g(2101;12) \times 2\,\mathrm{Re}(a_SM_1+a_DM_2)a_DM_3{}^*, \quad (5.10)$$

where we use

$$g(\mathfrak{L}'\mathfrak{l}'\mathfrak{L}\mathfrak{l};Lr)=(-1)^{L+r-\mathfrak{l}-\mathfrak{L}'}W(\mathfrak{L}'\mathfrak{l}'\mathfrak{L}\mathfrak{l};Lr) \times c^r(\mathfrak{l}',\mathfrak{l})c^r(\mathfrak{L}',\mathfrak{L}). \quad (5.11)$$

In the case of $O^{16}\rightarrow N^{14}$ [Eq. (5.5)] there are, in principle, three final states based on the p shell, with $J=1$, $T=0$. Suppose we start with an assumption of the values of the c.f.p.'s a_S and a_D for each final state, based, say, on a spectroscopic calculation of the levels of N^{14}. Then a measurement of A_0, A_2 for all three states gives six relations for the three independent complex amplitudes M_i, through [Eq. (5.10)]. On the other hand, if the transition amplitudes are given, the A_0, A_2 give a_s and a_D for each state. If only relative intensities were measured, then one fewer quantity would be obtained. Such measurements seem possible with the ground and 3.95-MeV states of N^{14} as final states, since they are well-separated levels. Note that the measurements would have to be made for the same k, K for each level, which can only approximately be satisfied [Eqs. (2.5), (2.6)].

One can go further, with more specific assumptions about the details of the two-nucleon amplitudes M_k(5.7). For example, if the only interaction in the final state is assumed to be that between the two outgoing nucleons "on energy shell," then the complex phase of each M_k is given by $\exp(i\delta_{\mathfrak{l},j})$, where $\delta_{\mathfrak{l},j}$ is the phase shift for the appropriate $\mathfrak{l}_f$, $j_f(\mathbf{j}=\mathfrak{l}+\mathbf{S})$ which are unique for each M_k in (5.8). There will be additional phases ±1, $\pm i$ from the phase conventions for the initial and final states. Under these assumptions, there are only three real unknowns in the M_k's to be determined using Eq. (5.10).

VI. DISCUSSION

The theory presented above rests on two assumptions: (1) that the absorption process can be treated in a two-nucleon impulse approximation, as described in Sec. III, and (2) that the absorption mechanism is such that the selection rules of Sec. IV are well obeyed. The possibility of confirming these assumptions depends on the ability to measure the angular correlations with sufficient accuracy to detect the presence of angular complexity higher than that predicted by the theory. A minimum requirement is energy resolution sufficient to separate excited states of the residual target.

In making the above assumptions, we have neglected three-nucleon processes, whose contribution is not easily estimated. However, as mentioned in the Introduction, there is evidence that these effects are small. Possibly a more serious interfering process is the distortion of the correlation by scattering of the emitted nucleons by the residual nucleus; this effect might be included in a numerical calculation of a specific absorption model, but is difficult to estimate qualitatively. One does not expect these distortions to be large in light nuclei. Large deflections of the outgoing nucleons would also smear out the angular distribution in the *laboratory* angle between $\mathbf{k}_1$ and $\mathbf{k}_2$. However, experiments[2] on Li and O targets show narrow distributions in this angle, peaked at 180°, with widths consistent with no deflection of outgoing nucleons.

It should be noted that the angular correlations of interest [Eq. (2.8)] are also functions of K. The K dependence reflects the total momentum distribution of the absorbing nucleon pair in the target nucleus, which, in turn depends on the angular-momentum quantum numbers $\mathfrak{L}$ of the initial pair. In general, the maximum of the momentum distribution will occur at higher K for increasing $\mathfrak{L}$; only $\mathfrak{L}=0$ contributes at $K=0$. Thus a study of the angular correlation as a function of K may help separate the contribution of the different values of $\mathfrak{L}$ in the initial target. However, we also expect the selection rules to be less exact with increasing K [see Eq. (1.1)], which introduces ambiguities into this analysis.

In this paper we have considered only the absorption of pions with negligible momentum with respect to the target nucleus. The theory may be applied directly to the case of pions of moderate momentum by averaging over the momentum direction. We again measure the distribution $P(k,K,\theta)$ [Eq. (2.4)] for fixed k, K, defined in the c.m. system of the pion+target, with the energy-conservation restriction (2.5); Q now includes the pion kinetic energy. However, there are several complications: (1) The direction average smears out the K dependence of the angular correlation, since a sphere of momenta in the target, of radius equal to the pion momentum, now contributes to a given final momentum K. (2) The selection rules (4.1) will become more complicated with increasing momentum. (3) Each partial wave of the pion (with respect to the target) has its own set of selection rules; the number of waves and resultant complexity of the distributions increases with pion momentum. As a result, it is much harder to get at the nuclear structure information, for fast pions than it is for slow.

Instead of averaging over the direction of the pion momentum, one could consider the angular correlation for a fixed relation between $\mathbf{K}$ and the pion momentum $\mathbf{q}$. This is not the correlation calculated in Sec. II, since the projection μ of the pion orbital angular momentum is now fixed. The problem is similar to that treated,

usually at higher energies, in the "peripheral" or "pole" approximations.[12] Then the angular correlation is analogous to the "decay correlation" of a resonant state. However, in these high-energy cases, one knows the spin of the resonant state, while in the present case, one wants to determine this quantity. Further, because of the high momentum brought in by the pion, one does not have the simple angular-momentum selection rules which allow determination of the angular-momentum structure of the target.

In conclusion, it has been shown how the measurement of the angular correlation of the emitted nucleons in the reaction $A(\pi,N_1N_2)B$ under the kinematic limits discussed, leads to information about the two-nucleon angular momentum structure of the target, A. Used "in reverse," the theory presented here also provides a new means of studying the mechanism by which two nucleons absorb a pion by separating the contributions of different two-nucleon angular momentum states.

ACKNOWLEDGMENT

The author wishes to thank Professor J. B. French for helpful discussions, and for a careful reading of the manuscript.

APPENDIX A

We derive Eq. (2.13) for the angular distribution using the *density* and *efficiency* matrix formalism.[6]

$$\langle J_cM_cX_c|\rho|J_c'M_c'X_c'\rangle = ([q][J_A])^{-1}\sum_{M_A,\mu}\langle J_cM_cX_c|T_\alpha|J_AM_AX_A\rangle \times\langle J_AM_AX_A|T_\alpha^+|J_c'M_c'X_c'\rangle, \quad \text{(A1)}$$

and the efficiency matrix ϵ

$$\langle J_cM_cX_c|\epsilon|J_c'M_c'X_c'\rangle = \sum_{m_1,m_2,M_B}\langle J_cM_cX_c|\mathbf{K},\mathbf{k},m_1,m_2,J_BM_BX_B\rangle \times\langle\mathbf{K},\mathbf{k},m_1,m_2,J_BM_BX_B|J_c'M_c'X_c'\rangle. \quad \text{(A2)}$$

So that, Eqs. (2.4) and (2.8) may be written

$$F(\theta)=\mathrm{Tr}\rho\epsilon. \quad \text{(A3)}$$

It is useful to introduce the irreducible tensor forms of ρ

$$\rho_\mu^\lambda(JX,J'X')=\sum_M(-1)^{\lambda-J'-M}\langle JMX|\rho|J'M'X'\rangle \times(JJ'-MM'|\lambda\mu), \quad \text{(A4)}$$

and similarly for ϵ. Since ρ in (A1) is a scalar, having no preferred direction, only ρ_0^0 survives in (A4):

$$\rho_0^0(J_cX_c,J_cX_c')=([J_c]/[J_A][q])^{1/2}\langle J_cX_c\|T_\alpha\|J_AX_A\rangle \times\langle J_cX_c'\|T_\alpha\|J_AX_A\rangle^*, \quad \text{(A5)}$$

and the trace (A3) becomes

$$F(\theta)=\sum_{J_c,X_c,X_c'}\rho_0^0(J_cX_c,J_cX_c')\epsilon_0^0(J_cX_c',J_cX_c). \quad \text{(A6)}$$

The total efficiency tensor ϵ_0^0 may be decomposed successively into pairs of coupled tensors

$$\epsilon_0^0(J_cX_c,J_cX_c') = \sum_\lambda[\lambda][J_c]\begin{Bmatrix} I & J_B & J_c \\ I' & J_B & J_c \\ \lambda & \lambda & 0\end{Bmatrix}\sum_\mu(\lambda\lambda\mu,-\mu|00) \times\epsilon_\mu^\lambda(IX_I,I'X_I')\epsilon_{-\mu}^\lambda(J_B,J_B) = ([J_c]/[I])^{1/2}\epsilon_0^0(IX_I,IX_I')\delta_{I,I'}, \quad \text{(A7)}$$

where we have used the Wigner $9-j$ symbol, and

$$\epsilon_\mu^\lambda(J_B,J_B)=[J_B]^{1/2}\delta_{\lambda,0}\delta_{\mu,0}, \quad \text{(A8)}$$

since M_B is not measured in (A2).

Further decomposition yields

$$\epsilon_0^0(J_cX_c,J_cX_c')=[J_c]^{1/2}\delta_{S,S'}\delta_{I,I'}\delta_{L,L'}\sum_r[r][L]^{1/2} \times\begin{Bmatrix}\mathfrak{L} & l & L \\ \mathfrak{L}' & \mathfrak{L}' & L \\ r & r & 0\end{Bmatrix}\sum_\mu(rr\mu,-\mu|00)\epsilon_\mu^r(\mathfrak{L},\mathfrak{L}')\epsilon_{-\mu}^r(\mathfrak{l},\mathfrak{l}'). \quad \text{(A9)}$$

Decomposition of the plane-wave in the $\hat{k}$ direction gives

$$\epsilon_{-\mu}^r(\mathfrak{l},\mathfrak{l}')=c^r(\mathfrak{l},\mathfrak{l}')(4\pi/[r])^{1/2}Y_{r,-\mu}(\hat{k})i^{\mathfrak{l}-\mathfrak{l}'}(-1)^{r-\mathfrak{l}'} \quad \text{(A10)}$$

and a similar expression for $\epsilon_\mu^r(\mathfrak{L},\mathfrak{L}')$, in $\hat{K}$, with

$$c^r(\mathfrak{l},\mathfrak{l}')=4\pi([\mathfrak{l}][\mathfrak{l}'])^{1/2}(\mathfrak{l}\mathfrak{l}'00|r0). \quad \text{(A11)}$$

Using the addition theorem on $Y_{r,\mu}(\hat{K})Y_{r,-\mu}(\hat{k})$ we obtain

$$\epsilon_0^0(J_cX_c',J_cX_c) = [J_c]^{1/2}\sum_r(-1)^{L+r-\mathfrak{l}-\mathfrak{L}'}\delta_{S,S'}\delta_{I,I'}\delta_{L,L'} \times W(\mathfrak{L}'\mathfrak{l}'\mathfrak{L}\mathfrak{l};Lr)c^r(\mathfrak{l}',\mathfrak{l})c^r(\mathfrak{L}',\mathfrak{L})P_r(\hat{k}\cdot\hat{K}). \quad \text{(A12)}$$

Finally, inserting (A5) and (A12) into (A6), we obtain the result Eq. (2.13).

APPENDIX B

Consider as a specific model of the absorption operator (3.1) the Gallilean-invariant form of the static model pion-nucleon interaction[5]

[12] E.g., J. D. Jackson, Rev. Mod. Phys. **37**, 484 (1965); I. S. Shapiro, V. M. Kolybasov, and G. R. Augst, Nucl. Phys. **61**, 353 (1965).

$$T_\alpha^{(1)}(i) \propto \boldsymbol{\sigma}(i)\cdot\{\nabla\phi_\alpha(\mathbf{r}_i)-(\mu/m)\mathbf{p}_i\phi(\mathbf{r}_i)\}\tau_-(i)\,, \quad \text{(B1)}$$

where $\boldsymbol{\sigma}(i)$, $\tau_-(i)$, $\mathbf{p}_i$ are the spin, isospin, and momentum operators, respectively, of the ith nucleon, and $\phi_\alpha(\mathbf{r}_i)$ is the wave function of the pion, in the initial state α, at the position of the same nucleon.

To a first approximation, for a pion bound in an atomic state to a nucleus of small change Z, the wave function within the nucleus is given by

$$\phi_{nlm}(\mathbf{r}) \propto r^l Y_{lm}(\theta,\phi)\,. \quad \text{(B2)}$$

This is equivalent to neglecting the pion momentum.

Now consider the operator $\tilde{T}_\alpha$ for the pion in an s state ($\alpha=n,0,0$) in the approximation (B2).

$$\tilde{T}_S \propto \boldsymbol{\sigma}(1)\cdot\mathbf{p}_1\tau_-(1)+\boldsymbol{\sigma}(2)\cdot\mathbf{p}_2\tau_-(2)\,,$$

which can also be written

$$\tilde{T}_S \propto \tfrac{1}{4}\{(\boldsymbol{\sigma}(1)+\boldsymbol{\sigma}(2))(\tau_-(1)-\tau_-(2))+(\boldsymbol{\sigma}(1)-\boldsymbol{\sigma}(2))\times(\tau_-(1)+\tau_-(2))\}\cdot(\mathbf{p}_1-\mathbf{p}_2)+\tfrac{1}{4}\{(\boldsymbol{\sigma}(1)+\boldsymbol{\sigma}(2))\times(\tau_-(1)+\tau_-(2))+(\boldsymbol{\sigma}(1)-\boldsymbol{\sigma}(2))\times(\tau_-(1)-\tau_-(2))\}\cdot(\mathbf{p}_1+\mathbf{p}_2)\,. \quad \text{(B3)}$$

It is easily verified that the first term of (B3) gives the selection rules (4.2a), while the second term gives (4.2b). For nucleons moving slowly in the target, we expect the nuclear matrix elements to satisfy

$$\langle f|\mathbf{p}_1-\mathbf{p}_2|i\rangle \gg \langle f|\mathbf{p}_1+\mathbf{p}_2|i\rangle\,, \quad \text{(B4)}$$

which gives the general selection rule (4.1). This is in part due to the final-state kinematics ($K\ll k$) and in part to the role of the two-nucleon correlation, which enhances the transfer of momentum in the relative coordinates, but not, presumably, in the c.m. coordinates.

A similar argument can be constructed for a p-state pion, which leads to the rules (4.3).

PHYSICAL REVIEW VOLUME 162, NUMBER 4 20 OCTOBER 1967

Second-Order Contribution to the Binding Energy of Closed-Shell Nuclei with the Tabakin Potential*

A. K. KERMAN AND M. K. PAL†
Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 24 May 1967)

Hartree-Fock calculations have been performed previously on the binding energy of closed-shell nuclei using Tabakin's separable nonlocal two-nucleon potential. In this paper, we report on an evaluation of the second-order correction to the binding energy of O^{16} and Ca^{40}. Including the second-order terms, we obtain binding energies of 6.7 and 10.9 MeV, respectively, for these two nuclei.

I. INTRODUCTION

TWO distinct kinds of two-nucleon potentials have been used in recent years in nuclear structure studies. The Yale potential of Lassila *et al.*[1] and the Hamada-Johnston potential[2] belong to the first category. These potentials were constructed to fit the properties of the deuteron and two-nucleon scattering data over a large energy range; they both contain a repulsive hard core and hence give rise to two-nucleon wave functions that are very strongly correlated at a small separation distance. In order to use these potentials in nuclear-structure work, one has to go through the painstaking process of evaluating the effective matrix elements (i.e., the K-matrix elements of Brueckner's many-body theory[3]). The problems that this kind of calculation encounters for finite nuclei have been overcome in recent years and the effective matrix elements for the harmonic-oscillator shell-model states are now available from the work of Kuo and Brown,[4] Becker and McKellar,[5] and Shakin *et al.*[6] These matrix elements have been used in conventional spectroscopic calculations in Ref. 4 and by Lawson *et al.*[7] and Waghmare and Shakin.[8] Successful results have also been obtained[9,10] in Hartree-Fock (HF) calculations

* This work is supported in part through funds provided by the U. S. Atomic Energy Commission under Contract AT(30-1)2098.

† Present address: Department of Physics and Institute of Theoretical Science, University of Oregon, Eugene, Oregon.

[1] K. E. Lassila, M. H. Hull, Jr., H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. **126**, 881 (1962).

[2] T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962).

[3] K. A. Brueckner and J. L. Gammel, Phys. Rev. **109**, 1023 (1958). This work lists all the earlier references by Brueckner and collaborators.

[4] T. T. S. Kuo and G. E. Brown, Nucl. Phys. **85**, 40 (1966).

[5] R. L. Becker and A. D. McKeller, Phys. Letters **21**, 201 (1966).

[6] C. M. Shakin, Y. R. Waghmare, and M. H. Hull, Jr. (to be published).

[7] R. D. Lawson, M. H. MacFarlane, and T. T. S. Kuo, Phys. Letters **22**, 168 (1966).

[8] Y. R. Waghmare, C. M. Shakin, and J. P. Svenne, Bull. Am. Phys. Soc. **11**, 321 (1966).

[9] C. M. Shakin, J. Svenne, and Y. R. Waghmare, Phys. Letters **21**, 209 (1966); also (to be published).

[10] M. K. Pal and A. P. Stamp, Phys. Rev. **158**, 924 (1967).