Semidirect Isospin Mixing in Deuteron-Induced Reactions

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A new mechanism which leads to substantial violation of isotopic invariance in "direct" processes involving deuterons is suggested. The T-forbidden (d,α) cross section resulting from this mechanism is nearly comparable in magnitude with that from the formation of isotopically impure compound intermediate states. This new mechanism involves transitions from the $S=1$ state of the deuteron to the $S=0$, $T=1$ "state." These transitions occur because the nucleon-target interaction contains a spin-orbit part; however, the Coulomb distortion of the proton wave function in the intermediate states causes an asymmetry in the spin-orbit potentials seen by the neutron and proton. Thus the spin of one nucleon is flipped more often than that of the other. Because the electromagnetic interaction merely provides the asymmetry factor (which is due to the long-range nature of the Coulomb potential), while it is the nuclear forces which actually flip the spins, this source of $T=1$ admixture is far more important than two other "direct" mechanisms which have been suggested, namely, Coulomb polarization of the deuteron internal wave function, or preliminary (allowed) deuteron pickup leading to a $T=0$ state of the residual nucleus, followed by Coulomb excitation to a $T=1$ final state. Estimates of the forbidden reaction cross sections resulting from either the preferential spin-flip or the Coulomb-excitation mechanism are presented, and an experimental test of the new hypothesis is proposed. Finally, it is concluded from this investigation that the preferential spin-flip mechanism can explain the recent observations of Meyer-Schützmeister *et al.*, both qualitatively and quantitatively.

I. INTRODUCTION

REACTIONS such as $C^{12}(d,\alpha)B^{10}$, leading to (forbidden) $T=1$ isotopic spin states of the residual nucleus, are commonly assumed to take place through the formation of isotopically impure (compound) intermediate states,[1] which can subsequently decay through the forbidden channel. In this model, the cross section for production of the T-forbidden B^{10} state is an unequivocal measure of the $T=1$ admixture of the compound intermediate level excited by the reaction. Evidence of a direct mechanism for such reactions has been reported by Meyer-Schützmeister, von Ehrenstein, and Allas.[2] They have found that at deuteron energies above $\sim$11.3 MeV, the differential cross section for production of the 1.74 MeV, $J^\pi=0^+$, $T=1$ state in B^{10} resembles typical direct-reaction angular distributions. Two possible mechanisms to account for isospin mixing in a direct process were proposed by MEA. Neither seems capable of explaining the observed violation.

The first of these mechanisms is polarization of the wave function of the incident deuteron by the Coulomb field of the target.[3] Such polarization would introduce $S=1$, $T=1$ odd-l components in the deuteron internal wave function. This explanation is insufficient for several reasons. In the first place, examination of the Born amplitude for deuteron pickup shows that the vertex function for $\alpha \rightarrow d+d$ is required. This function is basically the projection of the α-particle internal (four-nucleon) wave function onto two-deuteron internal wave functions. The essential point is that the nucleons in an unpolarized α particle all have zero relative angular momentum, so that odd-l components of the internal wave function of the incident deuteron will make no contribution of order $Z/137$ to the amplitude. In order for the deuteron polarization mechanism to contribute, the emergent α particle must also be polarized, which means that the amplitude for the process would be reduced from the allowed amplitude by a factor of order $(Z/137)^2$ [for the $C^{12}(d,\alpha)B^{10}$ reaction, the factor is $60/(137)^2=0.003$] so that the T-forbidden cross section from this mechanism would be unobservably small. Secondly, even disregarding the preceding argument, Drachman has shown[4] that the ratio of $l=1$ to $l=0$ components in the wave function of the polarized incident deuteron is already too small to account for the observed effect. (In fact, since the adiabatic approximation Drachman used is a low-energy treatment, it substantially overestimates the polarization at incident energies above the Coulomb barrier.) The combination of these two arguments seems to eliminate the polarization mechanism as a candidate for the source of the observed isospin violation.

The other mechanism suggested in MEA is that the incident deuteron first picks up a deuteron from the target, leaving the B^{10} nucleus in a $T=0$, 1^+ state; the departing α particle then Coulomb-excites the residual nucleus to the $T=1$, 0^+ state. Since the transition is $1^+ \rightarrow 0^+$, the electromagnetic multipole is[5] $M1$; it is straightforward to estimate that the effect of such final $M1$ transitions is too small to explain the data.

The purpose of this article is to point out the existence of a direct mechanism which can account for the

* Supported in part by the National Science Foundation.
[1] D. H. Wilkinson, Phil. Mag. 1, 379 (1956).
[2] L. Meyer-Schützmeister, D. von Ehrenstein, and R. G. Allas, Phys. Rev. 147, 743 (1966). Henceforth this reference or its authors will both be referred to as MEA.
[3] Y. Hashimoto and W. P. Alford, Phys. Rev. 116, 981 (1959).
[4] R. J. Drachman, Phys. Rev. Letters 17, 1017 (1966).
[5] L. R. B. Elton, *Introductory Nuclear Theory* (Sir Isaac Pitman and Sons, Ltd., London, 1959), p. 198.

magnitude and qualitative behavior of the T-violating differential cross section. The reaction amplitude may be written as the sum of a "direct" and of a "compound" part. The details of this splitting depend on the experimental energy resolution. The "direct" part of the amplitude will be a matrix element of the form

$$\langle \alpha, B^{10} | V_f | d, C^{12}; \Phi_i^+ \rangle$$

where Φ_i^+ is the wave function of the n-p-C^{12} system, treated as elementary particles interacting via two-body optical potentials. The state Φ_i^+ generally contains a component describing the quasielastic transition to the $S=0, T=1$ state of the n-p system. (Hereafter, this "state" will be denoted by φ.) On the other hand, the ground state of C^{12} may be written as a superposition of $T=0$ states of B^{10} coupled to a deuteron, and of $T=1$ states of B^{10} coupled to a φ, both giving over-all $T=0$. The reaction $C^{12}(d,\alpha)B^{10}$ ($T=1$) may be thought of as φ pickup by the φ component of the incident three-body optical-model wave function Φ_i^+. (Since this mechanism requires a preliminary transition from d to φ before the direct φ pickup takes place, it may be called "semi-direct.") The cross section for production of the $T=1$ final state will therefore measure the φ component of the C^{12} ground state, as well as the isospin impurity of highly excited N^{14} states.

The remainder of this paper is organized as follows: in Sec. II, the reaction amplitude is nonrigorously split into "compound" and "direct" parts, assuming "intermediate"[6] energy resolution. Section II is not intended as a justification of the three-body optical model; rather, its purpose is to indicate heuristically the origin of compound and direct processes, and to show how the reaction mechanism can change in a relatively narrow energy interval.

Section III has been included for completeness: It is a brief account of the construction of the wave function Φ_i^+ using a previously derived[7] three-particle theory of deuteron reactions which has subsequently been modified to account for Coulomb effects.[8] The important formulas in this section are Eqs. (3.9) through (3.11), and (3.17). The first part of Sec. IV is a derivation of the (direct) allowed and forbidden (d,α) reaction amplitudes, using the wave function (3.17). The important equations in this derivation are (4.2) and (4.9) through (4.15). Next, the forbidden amplitude is approximated in a way which allows the determination of the ratio of forbidden to allowed cross sections. Using the fact that the form of the $M1$ transition matrix element (suggested in MEA) is virtually identical to that of expression (4.14) for the preferential spin-flip amplitude, a rather good estimate of the ratio of these two amplitudes is obtained in (4.18). The necessary inclusion of the φ intermediate states, which is the subject matter of this paper, has experimentally observable consequences for the breakup reaction $C^{12}(d,np)C^{12}$; the rest of Sec. IV describes a possible experiment for directly observing the $d \rightarrow \varphi$ transition. Finally, the ideas and results of this paper are summarized in Sec. V.

Appendix A contains an estimate of the leading term for the magnetic-dipole-interaction mechanism suggested in MEA. Appendix B is a brief derivation of the (d,np) breakup amplitude given by the theory described in Sec. III and Ref. 7.

II. COMPOUND VERSUS DIRECT REACTION MECHANISMS

We consider the scattering of a neutron, a proton, and C^{12}. The reactions we are most interested in are $C^{12}(d,\alpha)B^{10}$ and $C^{12}(d,d)C^{12}$. The transition amplitude for $C^{12}(d,\alpha)B^{10}$ may be written

$$\langle \alpha, B^{10} | a^+(E) | d, C^{12} \rangle = \langle \mathbf{k}_\alpha', I_f M_f; \alpha, B^{10} | V_f | \mathbf{K}_d, \nu; d, C^{12+} \rangle, \quad (2.1)$$

where

$$(E-H)|d,C^{12+}\rangle=0,$$
$$H=H_i+V_i=H_f+V_f,$$
$$V_i=V_{n,C^{12}}+V_{p,C^{12}},$$
$$V_f=V_{\alpha,B^{10}},$$
$$(E-H_i)|d,C^{12}\rangle=(E-H_f)|\alpha,B^{10}\rangle=0,$$

and

$$|d,C^{12+}\rangle=|d,C^{12}\rangle+\lim_{\eta\to 0}(E+i\eta-H_i)^{-1}V_i|d,C^{12+}\rangle. \quad (2.2)$$

Following Feshbach and others,[6,9] we define the projection operator onto the ground state of C^{12}: $\mathbf{P}=|C^{12}\rangle\langle C^{12}|$ and let $\mathbf{R}=\mathbf{1}-\mathbf{P}$. Then, since $\mathbf{R}|d,C^{12}\rangle\equiv 0$, we have

$$\mathbf{P}|d,C^{12+}\rangle=\mathbf{P}|d,C^{12}\rangle+\mathbf{P}(E+i\eta-H_i)^{-1} \times[\mathbf{P}V_i\mathbf{P}+\mathbf{P}V_i\mathbf{R}(E+i\eta-\mathbf{R}H\mathbf{R})^{-1}\mathbf{R}V_i\mathbf{P}]|d,C^{12+}\rangle, \quad (2.3)$$

and

$$\mathbf{R}|d,C^{12+}\rangle=\mathbf{R}(E+i\eta-\mathbf{R}H\mathbf{R})^{-1}\mathbf{R}V_i\mathbf{P}|d,C^{12+}\rangle. \quad (2.4)$$

Now we assume that at what Feshbach calls "intermediate" energy resolution,[6] the effective potential becomes

$$\mathbf{P}V_i\mathbf{P}+\mathbf{P}V_i\mathbf{R}(E+i\eta-\mathbf{R}H\mathbf{R})^{-1}\mathbf{R}V_i\mathbf{P} \simeq |C^{12}\rangle\left[V_n^{\text{opt.}}+V_p^{\text{opt.}}+\sum_\lambda \frac{|v_\lambda\rangle\langle v_\lambda|}{E+i\eta-\mathcal{E}_\lambda}\right]\langle C^{12}|, \quad (2.5)$$

where the $|v_\lambda\rangle$ are vertex functions for forming the intermediate structure resonances from the entrance

[6] H. Feshbach, A. K. Kerman, and R. H. Lemmer, Ann. Phys. (N. Y.) **41**, 230 (1967). A list of references in which Feshbach has employed the channel elimination technique is given in this paper.

[7] J. V. Noble, Phys. Rev. **157**, 939 (1967); and Ph.D. thesis, Princeton University, 1966 (unpublished).

[8] J. V. Noble, Phys. Rev. **161**, 945 (1967).

[9] T. Teichmann and E. P. Wigner, Phys. Rev. **87**, 123 (1952).

channel, and where the $\mathcal{E}_\lambda$ are the energies of these states. (We assume some sort of energy-averaging so that only a few of these intermediate states will be important at any given energy.) Approximating the contribution of $\mathbf{R}|d,C^{12+}\rangle$ by its most rapidly varying part,

$$\mathbf{R}|d,C^{12+}\rangle \simeq \sum_\lambda \frac{\mathbf{R}|\phi_\lambda\rangle\langle v_\lambda|\Phi_i^+\rangle}{E+i\eta-\mathcal{E}_\lambda},$$

we find that

$$\langle\alpha,B^{10}|a^+(E)|d,C^{12}\rangle \simeq \langle\alpha,B^{10}|V_f|C^{12},\Phi_i^+\rangle + \sum_\lambda \frac{\langle\alpha,B^{10}|\omega_\lambda^+(E)\rangle\langle v_\lambda|\Phi_i^+\rangle}{E-\mathcal{E}_\lambda-\Delta_\lambda^+(E)}. \quad (2.6)$$

The derivation of expression (2.6) [in terms of the various quantities defined in (2.7) and (2.8)] from the basic definition (2.1) and the assumption (2.5) is a matter of straightforward algebra and has been given many times previously,[9,10] so we omit it here. In (2.6), $|\Phi_i^+\rangle$ is a pure three-body wave function describing the interaction of n, p and C^{12} via two-body optical potentials; it is asymptotic to the wave function of the noninteracting deuteron-C^{12} system, and satisfies the Lippmann-Schwinger equation

$$|\Phi_i^+\rangle = |d\rangle + (E+i\eta-H_0-V_{np})^{-1}[V_n^{\text{opt.}}+V_p^{\text{opt.}}]|\Phi_i^+\rangle. \quad (2.7)$$

In (2.7), H_0 is the kinetic energy operator of $n+p+C^{12}$. The level shift is given by (we have assumed $\langle v_{\lambda'}|v_\lambda\rangle \propto \delta_{\lambda'\lambda}$)

$$\Delta_\lambda^+(E) = \langle v_\lambda|(E+i\eta-H_0-V_{np}-V_n^{\text{opt.}}-V_p^{\text{opt.}})^{-1}|v_\lambda\rangle, \quad (2.8)$$

and has a negative-definite imaginary part.[10] The "final vertex function" appearing in (2.6) is given by

$$|\omega_\lambda^+(E)\rangle = V_f(E+i\eta-H_0-V_{np}-V_n^{\text{opt.}}-V_p^{\text{opt.}})^{-1}|C^{12},v_\lambda\rangle + V_f\mathbf{R}|\phi_\lambda\rangle. \quad (2.9)$$

Clearly, if $|\phi_\lambda\rangle$ is a state of definite spin and parity, and if only one such state is important at a given energy, then the expression

$$\tfrac{1}{3}\sum_{M_f,\nu}\left|\frac{\langle\mathbf{k}_\alpha' I_f M_f;\alpha B^{10}|\omega_\lambda^+(E)\rangle\langle v_\lambda|\Phi_i^+;\mathbf{K}\nu\rangle}{E-\mathcal{E}_\lambda-\Delta_\lambda^+(E)}\right|^2$$

(which is proportional to the cross section arising from the formation and decay of one intermediate state) will be symmetric about $\theta=\cos^{-1}(\hat{k}_\alpha'\cdot\hat{K})=90°$. The differential cross sections given in MEA for excitation of the $T=1$, 0^+ state in B^{10} are not symmetric about 90° at any of the incident energies. Above ~10.5 MeV there is substantial forward-backward asymmetry. The forward peaking observed at energies above 11.4 MeV is best described in terms of a direct-reaction model, for which the amplitude is just the first term in (2.6): $\langle\alpha,B^{10}|V_f|C^{12},\Phi_i^+\rangle$. It is this amplitude with which the next section will be concerned.

To briefly summarize: In this section we have seen how the (d,α) reaction amplitude may be split into a "direct" and a "compound" contribution. It should be emphasized that this separation is entirely artificial, depends on the energy resolution and is, in a sense, superfluous because the many-level R-matrix formulation[11] is complete at any resolution. Nevertheless, it is useful to split the amplitude for heuristic reasons, and to indicate that under some circumstances the two mechanisms can interfere in an observable way.

The isospin violating (d,α) cross sections observed by MEA seem to have the compound mechanism (symmetric about $\theta=90°$) behavior at lower deuteron energies, and the direct (forward-peaked) behavior at higher energies. The magnitude of the total cross section decreases rapidly, reaching a minimum around $E=11.4$ MeV, and then increases again. Such qualitative behavior may be explained by just the compound-direct interference effect mentioned above.

III. THE THREE-BODY OPTICAL MODEL

The mathematical techniques for constructing the scattering wave function of three particles interacting via two-body optical potentials have been reviewed in a previous paper,[7] and there now exists a considerable body of literature on the subject. In a more recent publication, the modification of the theory of Ref. 7 necessary in order to include the proton-target Coulomb potential were described in considerable detail.[8] For this reason, the present discussion of deuteron scattering will be limited to a brief review.

We write the proton-neutron-target Hamiltonian as

$$H=H_0+V_n+V_p+V_{np}+U_p, \quad (3.1)$$

where H_0 is the kinetic energy, V_n and V_p are the neutron target and proton-target optical potentials, respectively, V_{np} is the neutron-proton potential and U_p is the proton-target Coulomb potential, which is taken to be that for point charges. It is important to realize that despite its weakness, the long-range nature of the Coulomb potential can produce effects which might seem disproportionately large.[10] To understand such effects requires treating the Coulomb interaction to all orders of perturbation theory.

The scattering wave function Φ_i^+ described in Sec. II satisfies the Lippmann-Schwinger equation

$$|\Phi_i^+\rangle = |d;\mathbf{K}\nu\rangle + (E+i\eta-H_0-V_{np})^{-1}(V_n+V_p+U_p)|\Phi_i^+\rangle, \quad (3.2)$$

where $|d;\mathbf{K}\nu\rangle$ is the wave function of a deuteron with

[10] J. V. Noble, Phys. Rev. **148**, 1528 (1966).

[11] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958); G. E. Brown, *ibid.* **31**, 893 (1959).

z projection of spin ν, and with momentum $\mathbf{K}$ relative to the target. Equation (3.2) is only a formal relation because the Coulomb distortion of the incident plane wave must somehow be included. One way to do this is to define the Coulomb potential $\tilde{U}_d$ acting on the n-p center of mass by

$$\tilde{U}_d = Ze^2[\tfrac{1}{2}|\mathbf{r}_n+\mathbf{r}_p|]^{-1}[1+2m/M_c]^{-1}, \quad (3.3)$$

where m is the nucleon mass and M_c the target mass. We now define $|\phi_d^+\rangle$ to be the outgoing wave solution of the Schrödinger equation

$$(E-H_0-V_{np}-\tilde{U}_d)|\phi_d^+\rangle=0, \quad (3.4)$$

asymptotic to $|d;\mathbf{K}\nu\rangle$. [Since (3.4) separates, $|\phi_d^+\rangle$ is just a product wave function.] Equation (3.4) implies the relation

$$|\Phi_i^+\rangle=|\phi_d^+\rangle+(E+i\eta-H_0-V_{np}-\tilde{U}_d)^{-1} \times(V_n+V_p+U_p-\tilde{U}_d)|\Phi_i^+\rangle. \quad (3.5)$$

Defining the Coulomb-modified deuteron elastic scattering operator by

$$\tilde{U}_{dd}^+(E)|\phi_d^+\rangle=(V_n+V_p+U_p-\tilde{U}_d)|\Phi_i^+\rangle, \quad (3.6)$$

we find the integral equation

$$\begin{aligned}\tilde{U}_{dd}^+(E)|\phi_d^+\rangle&=(E-H_0-\tilde{U}_d)\\ &\times(E+i\eta-H_0-V_n-V_p-U_p)^{-1}\\ &\times\{(V_n+V_p+U_p-\tilde{U}_d)|\phi_d^+\rangle\\ &+(V_n+V_p+U_p-\tilde{U}_d)(E+i\eta-H_0-\tilde{U}_d)^{-1}\tilde{t}_{np}^+(E)\\ &\times(E+i\eta-H_0-\tilde{U}_d)^{-1}\tilde{U}_{dd}^+(E)|\phi_d^+\rangle\}. \quad (3.7)\end{aligned}$$

In (3.7), $\tilde{t}_{np}(W)$ is the n-p scattering matrix in the presence of $\tilde{U}_d$, defined by the equation

$$\tilde{t}_{np}(W)=V_{np}+V_{np}(W-H_0-\tilde{U}_d)^{-1}\tilde{t}_{np}(W). \quad (3.8)$$

Equation (3.7) has no mathematical pathologies[7,8] and has a completely continuous[12] kernel so that, given the ability to calculate the Green's function $(E+i\eta-H_0-V_n-V_p-U_p)^{-1}$, (3.7) is a suitable general starting point for the calculation of Φ_i^+. In Ref. 8 it is shown how the above Green's function may be constructed in terms of solutions of Fredholm integral equations formally similar to the Faddeev[13] equations; for the present purpose it is enough to know that this construction can in principle be accomplished.

The fact that the triplet state of the deuteron is weakly bound while the singlet state is nearly bound allows the n-p scattering matrix to be well represented by the simple form[7]

$$t_{np}(W)\simeq-|v_1\rangle\hat{\tau}_1(W)\Lambda^T\langle v_1|-|v_0\rangle\hat{\tau}_0(W)\Lambda^S\langle v_0|. \quad (3.9)$$

That is, we approximate V_{np} by one separable term in the ${}^3S_1$ state, and by one separable term in the ${}^1S_0$ state, and we neglect n-p scattering in states with $l_{np}>0$. [In (3.9), Λ^T and Λ^S are, respectively, the triplet and singlet spin-projection operators.] Putting (3.9) into (3.7) leads to the following set of coupled linear integral equations[7]:

$$\begin{aligned}\langle\mathbf{K}'S'\nu'|\tilde{T}(W)|\mathbf{K}1\nu\rangle&=\langle\mathbf{K}'S'\nu'|\tilde{B}(W)|\mathbf{K}1\nu\rangle\\ &-\sum_{S''=0}^{1}\sum_{\nu''=-S''}^{S''}\int d\mathbf{K}''\langle\mathbf{K}'S'\nu'|\tilde{B}(W)|\mathbf{K}''S''\nu''\\ &\times\tau_{S''}(W-(2M)^{-1}K''^2)\langle\mathbf{K}''S''\nu''|\tilde{T}(W)|\mathbf{K}1\nu\rangle. \quad (3.10)\end{aligned}$$

In (3.10) the object $\tilde{B}(W)$ has been defined by

$$\begin{aligned}\langle\mathbf{K}'S'\nu'|\tilde{B}(W)|\mathbf{K}S\nu\rangle&=\int d\mathbf{q}'\int d\mathbf{q}\langle v_{S'}|\mathbf{q}'\rangle\\ &\times\langle\phi_{\mathbf{K}'}{}^c\mathbf{q}'S'\nu'|(W-H_0-V_n-V_p-U_p)^{-1}\\ &\times(V_n+V_p+U_p-\tilde{U}_d)|\phi_{\mathbf{K}}{}^c\mathbf{q}S\nu\rangle\\ &\times\frac{\langle\mathbf{q}|v_S\rangle}{W-(2M)^{-1}K^2-m^{-1}q^2}, \quad (3.11)\end{aligned}$$

and $\tilde{T}(W)$ is defined similarly, with the exception of replacing $(W-H_0-V_n-V_p-U_p)^{-1}(V_n+V_p+U_p-\tilde{U}_d)$ in (3.11) by $U_{dd}(W)$. The system of notation used to write the matrix elements in (3.10) and (3.11) is

$$\mathbf{K}=\mathbf{k}_n+\mathbf{k}_p, \quad (3.12a)$$

$$\mathbf{q}=(\mathbf{k}_n-\mathbf{k}_p)/2, \quad (3.12b)$$

and clearly ($\hbar=1$),

$$H_0|\mathbf{K},\mathbf{q}\rangle=[(2M)^{-1}K^2+m^{-1}q^2]|\mathbf{K},\mathbf{q}\rangle, \quad (3.13)$$

where the deuteron reduced mass M is given by

$$M=2mM_c/(M_c+2m). \quad (3.14)$$

The Coulomb-modified plane wave $|\phi_{\mathbf{K}}{}^c\rangle$ is the solution of the equation

$$[\nabla^2+K^2-2M\tilde{U}_d(R)]\langle\mathbf{R}|\phi_{\mathbf{K}}{}^c\rangle=0. \quad (3.15)$$

On the energy shell [that is, with $W=(2M)^{-1}K^2-\epsilon_d+i\eta$] the deuteron internal wave function is given by

$$\psi_d(\mathbf{q})=\langle\mathbf{q}|v_1\rangle/[W-(2M)^{-1}K^2-m^{-1}q^2], \quad (3.16)$$

so that $\langle\mathbf{K}'1\nu'|\tilde{T}(W)|\mathbf{K}1\nu\rangle$ is just the nuclear part of the deuteron elastic-scattering matrix. The object $\tilde{B}(W)$ plays the role of the deuteron optical potential: when it is substituted into the "Lippmann-Schwinger" Eq. (3.10) it gives the correct (Coulomb-modified) elastic scattering matrix. $\tilde{B}(W)$ differs from the traditional deuteron optical potential in that it can, in general, produce transitions between $S=1$ and $S=0$

[12] This term is defined in Ref. 7, and in several of the references contained therein.

[13] L. D. Faddeev, *Mathematical Aspects of the Three-Body Problem in the Quantum Scattering Theory* (Israel Program for Scientific Translations, Jerusalem, 1965).

states, even when V_n and V_p are identical, as in reactions on $T_3=0$ nuclei. The importance of this type of transition will be discussed in the next section.

Finally, we see by inspection of Eqs. (3.5), (3.6), (3.8), and (3.9) that Φ_i^+ may be expressed directly in terms of $\tilde{T}(W)$ and the various potentials:

$$|\Phi_i^+\rangle=\sum_{S''\nu''}\int d\mathbf{K}''\Big\{\int d\mathbf{q}''[\mathbf{1}+(E+i\eta-H_0-V_n-V_p-U_p)^{-1}(V_n+V_p+U_p-\tilde{U}_d)]$$
$$\times|\phi_{\mathbf{K}''}{}^c\mathbf{q}''S''\nu''\rangle\frac{\langle\mathbf{q}''|v_{S''}\rangle}{E+i\eta-(2M)^{-1}K''^2-m^{-1}q''^2}\Big\}$$
$$\times\{\delta(\mathbf{K}''-\mathbf{K})\delta_{S''1}\delta_{\nu''\nu}-\tau_{S''}(E+i\eta-(2M)^{-1}K''^2)\langle\mathbf{K}''S''\nu''|\tilde{T}(E+i\eta)|\mathbf{K}1\nu\rangle\}. \quad (3.17)$$

IV. ISOTOPIC-SPIN VIOLATION

We now apply the considerations of the previous section to targets with $N=Z$, for which $V_n{}^{\text{opt.}}\equiv V_p{}^{\text{opt.}}$ (within small corrections for the finite charge-radius of the nucleus). The Coulomb-corrected matrix element, $\langle\mathbf{K}'S'\nu'|\tilde{B}(W)|\mathbf{K}S\nu\rangle$, of the deuteron "effective optical potential," in general can connect the $S=1$, $T=0$, "d" state with the $S=0$, $T=1$, "φ" state. How does this happen? First let us note that if the potentials V_n and V_p are purely central, the $d\rightarrow\varphi$ matrix element of $\tilde{B}$ will vanish, regardless of whether or not V_n and V_p are identical. That is, as Drachman[4] has correctly stated, the Coulomb force alone cannot produce the preferential spin flipping of one nucleon relative to the other which leads to $d\rightarrow\varphi$ transitions. The actual mechanism is more subtle. The nucleon-nucleus optical potential has been found to contain a spin-orbit term, with a strength of several MeV.[14] When one properly accounts for the effect of the p-C^{12} Coulomb repulsion to all orders, one finds that the proton wave function is excluded from the nuclear region, relative to the neutron wave function. This exclusion destroys the symmetry between the spin-orbit parts of the n-C^{12} and p-C^{12} forces, and leads to preferential spin flip. Anticipating the end result, we note that this mechanism can lead to relatively large isospin violation (as compared, say, with $M1$ transitions) because it is the strong nuclear $\mathbf{l}\cdot\mathbf{s}$ interaction which actually does the spin flipping; the role of the long-range repulsion in this process is merely to produce an effective asymmetry in the nuclear force.

The matrix elements of $\tilde{B}(W)$ have a simple graphical representation, indicated in Fig. 1(a). The "bubble" represents all graphs involving n, p, and c in which the n and p do not interact with each other; the first few terms are shown in Fig. 1(b). In order to treat the Coulomb interaction correctly, the proton intermediate states must be taken to be Coulomb-modified plane waves.

The lowest-order contribution (in V_n and V_p) to the $d\rightarrow\varphi$ matrix element of $\tilde{B}$ may be written down from Eq. (3.11) or from Fig. 1 by inspection:

$$\langle\mathbf{K}'00|\tilde{B}(W)|\mathbf{K}1\nu\rangle\simeq\langle\phi_{\mathbf{K}}{}^c;v_0|(W-H_0-U_p)^{-1}$$
$$\times[V_n{}^{\text{s.o.}}+V_p{}^{\text{s.o.}}](W-H_0-U_p)^{-1}|\phi_{\mathbf{K}}{}^c;v_1\nu\rangle. \quad (4.1)$$

In Ref. 8 it is shown that (4.1) is a reasonable representation of the off-diagonal matrix elements of $\tilde{B}(W)$. Since $V_n{}^{\text{s.o.}}$ and $V_p{}^{\text{s.o.}}$ (the spin-orbit parts of the nucleon optical potentials) are small, we may neglect terms of order $V_{n,p}{}^{\text{s.o.}}(W-H_0-U_p)^{-1}(U_p-\tilde{U}_d)$ (note that $U_p-\tilde{U}_d$ is a weak, short-ranged potential under

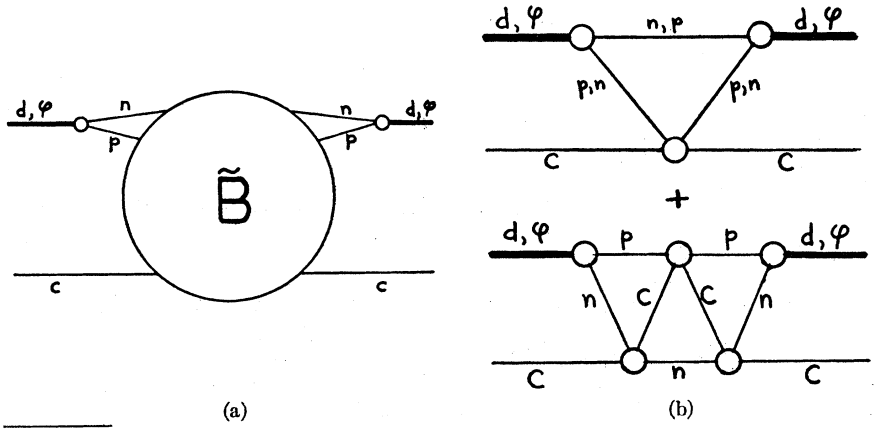

FIG. 1. (a) Graphical representation of a general matrix element of $\tilde{B}(W)$; (b) the first two terms in the perturbative expansion of $\tilde{B}(W)$.

[14] M. A. Preston, *Physics of the Nucleus* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1962), p. 547.

these circumstances) to obtain

$$\langle \mathbf{K}'00|\tilde{B}(W)|\mathbf{K}1\nu\rangle \simeq \sum_{\sigma_n'\sigma_p'}\sum_{\sigma_n\sigma_p}\langle 00|\tfrac{1}{2}\tfrac{1}{2};\sigma_n'\sigma_p'\rangle\Big\{\int d\mathbf{x}\frac{v_0^*(\mathbf{x}+\frac{1}{2}\mathbf{K}')}{W-(2M)^{-1}K'^2-m^{-1}(\mathbf{x}+\frac{1}{2}\mathbf{K}')^2}$$
$$\times[\langle\xi\mathbf{x}+\mathbf{K}',\sigma_n'|V_n^{\text{s.o.}}|\xi\mathbf{x}+\mathbf{K},\sigma_n\rangle\delta_{\sigma p'\sigma p}+\langle\xi\mathbf{x}+\mathbf{K}',\sigma_p'|\tilde{V}_p^{\text{s.o.}}|\xi\mathbf{x}+\mathbf{K},\sigma_p\rangle\delta_{\sigma n'\sigma n}]$$
$$\times\frac{v_1(\mathbf{x}+\frac{1}{2}\mathbf{K})}{W-(2M)^{-1}K^2-m^{-1}(\mathbf{x}+\frac{1}{2}\mathbf{K})^2}\Big\}\langle\tfrac{1}{2}\tfrac{1}{2};\sigma_n\sigma_p|1\nu\rangle. \quad (4.2)$$

In (4.2) the matrix elements of $V_p^{\text{s.o.}}$ in the basis of Coulomb wave functions for the proton alone [i.e., eigenfunctions of H_0+U_p] have been abbreviated $\tilde{V}_p^{\text{s.o.}}$; the constant ξ is just $A/(A+1)$, where A is the nucleon number of the target. We require the matrix element of the spin-orbit potential. It turns out to be

$$\langle \mathbf{k}_n'00|V_n^{\text{s.o.}}|\mathbf{k}_n1\nu\rangle=(\mathbf{k}_n'\times\mathbf{k}_n)\cdot\langle 00|\mathbf{s}_n|1\nu\rangle f(|\mathbf{k}_n'-\mathbf{k}_n|), \quad (4.3)$$

where

$$f(Q)=i(2\pi^2Q)^{-1}\int_0^\infty dr\, r^3V_n^{\text{s.o.}}(r)j_1(Qr). \quad (4.4)$$

We use the local plane-wave approximation[15] for the Coulomb wave functions to obtain

$$\langle \mathbf{k}_p'00|\tilde{V}_p^{\text{s.o.}}|\mathbf{k}_p1\nu\rangle\simeq c_0(k_p')c_0(k_p)(\mathbf{k}_p'\times\mathbf{k}_p)\cdot\langle 00|\mathbf{s}_p|1\nu\rangle f(|\mathbf{k}_p'-\mathbf{k}_p|), \quad (4.5)$$

where $c_0(k)$ is the appropriate Coulomb-barrier penetration factor.[16–18] Noting that $(\mathbf{x}+\frac{1}{2}\mathbf{K})^2$ and $(\mathbf{x}+\frac{1}{2}\mathbf{K}')^2$ are restricted to be small when (4.2) is on or near the energy shell, we get

$$\langle \mathbf{K}'00|\tilde{B}(W)|\mathbf{K}1\nu\rangle\simeq f(|\mathbf{K}'-\mathbf{K}|)\int d\mathbf{x}\frac{v_0^*(\mathbf{x}+\frac{1}{2}\mathbf{K}')v_1(\mathbf{x}+\frac{1}{2}\mathbf{K})}{[W-(2M)^{-1}K'^2-m^{-1}(\mathbf{x}+\frac{1}{2}\mathbf{K}')^2][W-(2M)^{-1}K^2-m^{-1}(\mathbf{x}+\frac{1}{2}\mathbf{K})^2]}$$
$$\times[(\mathbf{x}+\mathbf{K}')\times(\mathbf{x}+\mathbf{K})\cdot\langle\tfrac{1}{2}\tfrac{1}{2};00|\{\mathbf{s}_n+c_0(\tfrac{1}{2}K')c_0(\tfrac{1}{2}K)\mathbf{s}_p\}|\tfrac{1}{2}\tfrac{1}{2};1\nu\rangle]. \quad (4.6)$$

In evaluating the vectorial dependence of the integral, we use the fact that $\mathbf{a}\times\mathbf{a}\equiv 0$; letting $\mathbf{x}=\mathbf{q}-\frac{1}{2}\mathbf{K}$, we obtain an expression of the form

$$\int d\mathbf{q}(\mathbf{K}'-\mathbf{K})\times(\mathbf{q}+\tfrac{1}{2}\mathbf{K})\psi_1(|\mathbf{q}+\tfrac{1}{2}(\mathbf{K}'-\mathbf{K})|)\psi_2(q)$$
$$=\tfrac{1}{2}(\mathbf{K}'\times\mathbf{K})\int d\mathbf{q}\,\psi_1(|\mathbf{q}+\tfrac{1}{2}(\mathbf{K}'-\mathbf{K})|)\psi_2(\mathbf{q})+\mathbf{Q}\times\int d\mathbf{q}\,\mathbf{q}\psi_1(|\mathbf{q}+\tfrac{1}{2}\mathbf{Q}|)\psi_2(q).$$

By symmetry, the second term vanishes, and so we get

$$\langle \mathbf{K}'00|\tilde{B}(W)|\mathbf{K}|\nu\rangle\simeq\tfrac{1}{2}f(|\mathbf{K}'-\mathbf{K}|)F_{01}(|\mathbf{K}'-\mathbf{K}|,K'^2,K^2;W)$$
$$\times[(\mathbf{K}'\times\mathbf{K})\cdot\langle\tfrac{1}{2}\tfrac{1}{2};00|\{\mathbf{s}_n+c_0(\tfrac{1}{2}K')c_0(\tfrac{1}{2}K)\mathbf{s}_p\}|\tfrac{1}{2}\tfrac{1}{2};1\nu\rangle], \quad (4.7)$$

where

$$F_{01}(Q,K'^2,K^2;W)=\int d\mathbf{q}\frac{v_0^*(|\mathbf{q}+\frac{1}{2}\mathbf{Q}|)v_1(q)}{[W-(2M)^{-1}K'^2-m^{-1}(\mathbf{q}+\frac{1}{2}\mathbf{Q})^2][W-(2M)^{-1}K^2-m^{-1}q^2]}. \quad (4.8)$$

Since $\mathbf{S}=\mathbf{s}_n+\mathbf{s}_p$ is diagonal, we have, finally,

$$\langle \mathbf{K}'00|\tilde{B}(W)|\mathbf{K}1\nu\rangle\simeq-\tfrac{1}{2}f(|\mathbf{K}'-\mathbf{K}|)F_{01}(|\mathbf{K}'-\mathbf{K}|,K'^2,K^2;W)$$
$$\times(1-c_0(\tfrac{1}{2}K')c_0(\tfrac{1}{2}K))[(\mathbf{K}'\times\mathbf{K})\cdot\langle\tfrac{1}{2}\tfrac{1}{2};00|\mathbf{s}_p|\tfrac{1}{2}\tfrac{1}{2};1\nu\rangle]. \quad (4.9)$$

We see that (4.9) contains the approximate "asymmetry factor" $[1-c_0(K'/2)\cdot c_0(K/2)]$; this factor vanishes if the Coulomb interaction is turned off, or if K' and $K\to\infty$.

The (d,α) reaction matrix element introduced in Sec. II may now be calculated. The Born term,

$$\langle \mathbf{k}_\alpha' I_f M_f|V_f|d;\phi_{\mathbf{K}^c}\nu\rangle,$$

[15] L. Schulman, Phys. Rev. **156**, 1129 (1967). Schulman gives a number of references to earlier uses of this approximation.
[16] See, e.g., A. Messiah, *Quantum Mechanics* (John Wiley & Sons, Inc., New York, 1961), Vol. 1, p. 486.
[17] R. Aaron, R. D. Amado, and Y. Y. Yam, Phys. Rev. **140**, B1291 (1965).
[18] Because of the cutoff at backward angles, it is only appropriate to integrate from 0 to $\pi/2$.

typically has a pole in the momentum transfer $|\mathbf{k}_\alpha'-\mathbf{K}|$ near the physical region, and so has a strong angular dependence. [This term is responsible for the strong forward peaking of direct reactions.] In contrast, the next term obtained from the expression (3.17) is

$$\langle \mathbf{k}_\alpha' I_f M_f | V_f(E+i\eta-H_0-V_n-V_p-U_p)^{-1}(V_n+V_p)|d;\phi_{\mathbf{K}}{}^c\nu\rangle,$$

which has much weaker angular variation. (The same remarks apply if the initial state contains the φ rather than the d.) The dominant part of the (d,α) amplitude, at least at forward angles, may thus be written

$$\sum_{S''\nu''}\int d\mathbf{K}''\langle \mathbf{k}_\alpha' I_f M_f|V_f(E+i\eta-H_0-\bar{U}_d)^{-1}|\phi_{\mathbf{K}}{}^c v_{S''\nu''}\rangle \times\{\delta(\mathbf{K}''-\mathbf{K})\delta_{S''1}\delta_{\nu''\nu}-\tau_{S''}(E+i\eta-(2M)^{-1}K''^2)\langle \mathbf{K}''S''\nu''|\tilde{T}(E+i\eta)|\mathbf{K}1\nu\rangle\}. \quad (4.10)$$

In order to calculate this matrix element, one must know the effective d and φ single-particle wave functions in C^{12}, the interaction V_f and the $\alpha\to 2d$ and $\alpha\to 2\varphi$ vertex functions, as well as the empirical n-C^{12} and d-C^{12} scattering matrices. These objects must then be put together in a lengthy and complicated calculation. Both because the various pieces are not really well known, and because the problem does not seem to merit such a detailed study, only a rough estimate of the ratio of the forbidden to the allowed cross section will be given here. We take $V_n{}^{\text{s.o.}}(r)$ to have the usual form

$$V_n{}^{\text{s.o.}}(r)=-\frac{(1.4)^2}{Dr}V_{\text{s.o.}}\exp[(r-R)/D]\{1+\exp[(r-R)/D]\}^{-2}, \quad (4.11)$$

where $K_{\text{s.o.}}\sim 10$ MeV, $R\sim 2.3$ F, $D\sim 0.65$ F. Substituting (4.11) into (4.4), we find

$$f(Q)=-i(6\pi^2)^{-1}(1.4)^2K_{\text{s.o.}}R^3\bar{f}(Q), \quad (4.12)$$

where $\bar{f}(Q)$ is a dimensionless quantity which falls off, for large Q, faster than Q^{-2}, and $\bar{f}(0)\simeq 1$. We now calculate the (allowed) amplitude for $C^{12}+d\to B^{10}(T=0,\ J^\pi=1^+)+\alpha$,

$$\langle \mathbf{k}_\alpha' 1M_f|a^+(E)|\mathbf{K}1\nu\rangle\simeq\int d\mathbf{K}''\langle \mathbf{k}_\alpha' 1M_f|V_f(E+i\eta-H_0-\bar{U}_d)^{-1}|\phi_{\mathbf{K}''}{}^c v_{1\nu}\rangle\Phi_{\mathbf{K}}{}^d(\mathbf{K}''), \quad (4.13)$$

and the amplitude for the forbidden reaction $C^{12}+d\to B^{10}(T=1,\ J^\pi=0^+)+\alpha$

$$\langle \mathbf{k}_\alpha'|b^+(E)|\mathbf{K}1\nu\rangle\simeq -i\frac{(1.4)^2}{2}V_{\text{s.o.}}R^3(6\pi^2)^{-1}2M[1-c_0{}^2(K/2)]\int d\mathbf{K}'''\int d\mathbf{K}'' \times\left\{\langle \mathbf{k}_\alpha'|V_f(E+i\eta-H_0-\bar{U}_d)^{-1}|\phi_{\mathbf{K}''}{}^c v_0\rangle[(\mathbf{K}''\times\mathbf{K}''')\cdot\langle 00|\mathbf{s}_p|1\nu\rangle]\frac{\bar{f}(|\mathbf{K}''-\mathbf{K}'''|)}{(5/3)k_\alpha{}'^2+i\eta-K''^2}\Phi_{\mathbf{K}}{}^d(\mathbf{K}''')\right\}. \quad (4.14)$$

Several approximations have been made in obtaining (4.13) and (4.14) from (4.10). $\Phi_{\mathbf{K}}{}^d(\mathbf{K}'')$ is the purely diagonal part of the deuteron optical wave function,

$$\delta_{\nu''\nu}[\delta(\mathbf{K}''-\mathbf{K})-\tau_1(E+i\eta-(2M)^{-2}K''^2) \times\langle \mathbf{K}''1\nu''|\tilde{T}^+(E)|\mathbf{K}1\nu\rangle].$$

Clearly the off-diagonal elements of $\tilde{T}$ do not contribute very much and may be neglected. The matrix element $\langle \mathbf{K}'00|\tilde{T}(E+i\eta)|\mathbf{K}1\nu\rangle$ was approximated by $\int d\mathbf{K}''\langle \mathbf{K}''00|\tilde{B}(E+i\eta)|\mathbf{K}''1\nu\rangle\Phi_{\mathbf{K}}{}^d(\mathbf{K}'')$, and the behavior of $\bar{f}(Q)$ and of $\Phi_{\mathbf{K}}{}^d(\mathbf{K}'')$ allowed F_{01} to be replaced by its on-shell $Q=0$ value, namely unity. Finally, $\tau_0(z)$ was replaced by $-z^{-1}$ in (4.10), and the Q value of the forbidden reaction was neglected because of its smallness.

To compare the T-allowed amplitude $a^+(E)$ with the T-forbidden amplitude, we assume

$$\langle \mathbf{k}_\alpha' 1M_f|V_f(E+i\eta-H_0-\bar{U}_d)^{-1}|\phi_{\mathbf{K}}{}^c v_{1\nu}\rangle \simeq\delta_{M_f\nu}\langle \mathbf{k}_\alpha'|V_f(E+i\eta-H_0-\bar{U}_d)^{-1}|\phi_{\mathbf{K}}{}^c v_0\rangle \equiv\delta_{M_f\nu}\langle \mathbf{k}_\alpha'|b_{\text{Born}}{}^+(E)|\mathbf{K}\rangle. \quad (4.15)$$

Equation (4.15) states that the diagonal part of the Born amplitude for $d+C^{-2}\to\alpha+B^{10}(T=0,\ J^\pi=1^+)$ is approximately the same as that for $\varphi+C^{12}\to\alpha+B^{10}$ $(T=1,\ J^\pi=0^+)$. This is reasonable since the C^{12} ground state has the same $SU(4)$ symmetry as the α particle, and the final states are fairly close in energy. Invoking rotational symmetry and noting that $\bar{f}(Q)$ is a rapidly decreasing function of Q and that $\Phi_{\mathbf{K}}{}^d(\mathbf{K}')$ is strongly

peaked at $\mathbf{K}'=\mathbf{K}$, we approximate Eq. (4.14) by

$$\langle \mathbf{k}_\alpha' | b^+(E) | \mathbf{K}1\nu\rangle \simeq -i(1.4)^2(12\pi^2)^{-1}V_{\text{s.o.}}R^3 2M \times[1-c_0^2(K/2)]I(k_\alpha',K)(5/3)^{1/2} \times[(\mathbf{k}_\alpha'\times\mathbf{K})\cdot\langle 00|s_p|1\nu\rangle]\langle \mathbf{k}_\alpha'|b_{\text{Born}}{}^+(E)|\Phi_{\mathbf{K}}{}^d\rangle. \quad (4.16)$$

The function $I(k_\alpha',K)$ in (4.16) is just $(5/3)^{1/2}\int d\mathbf{Q}\tilde{f}(Q) \times[(5/3)k_\alpha'^2+i\eta-(\mathbf{K}+\mathbf{Q})^2]^{-1}$; a reasonable form for $\tilde{f}(Q)$ which lets us evaluate the integral is $\beta^4(Q^2+\beta^2)^{-2}$, where $\beta R\simeq 1$. Using this form and recalling that $k_\alpha'\gg\beta$, we find $I(k_\alpha',K)\simeq -\beta^2\pi^2 i/2k_\alpha'$. The ratio of the forbidden to the allowed amplitude at $\theta=30°$ is then

$$\begin{aligned}\mathfrak{R}&\simeq|\langle \mathbf{k}_\alpha'|b^+(E)|\mathbf{K}1\nu\rangle|/|\langle \mathbf{k}_\alpha'|b_{\text{Born}}{}^+(E)|\Phi_{\mathbf{K}}{}^d\rangle| \\ &\simeq(5/3)^{1/2}[(1.4)^2/24]|V_{\text{s.o.}}|2M\beta^2R^3K \\ &\qquad\times[1-c_0^2(K/2)](\sin 30°)/2\sqrt{3} \\ &\simeq 0.03, \end{aligned} \quad (4.17)$$

which leads to a ratio of forbidden to allowed cross sections of $\sim 10^{-3}$. It should be emphasized that because of all the uncertainties involved in calculating the ratio $\mathfrak{R}$, the cross-section ratio $\mathfrak{R}^2$ is only an order-of-magnitude figure. (The fact that it agrees reasonably well with the observed cross-section ratio is probably an example of the well-known phenomenon of fortuitous cancellation of errors.) It is much more significant to compare the amplitude (4.14) with that given by the $M1$ mechanism suggested by MEA. This ratio is probably must less uncertain since the functional forms of the amplitudes are nearly identical (see Appendix A). The appropriate ratio turns out be be

$$\begin{aligned} r&=|\text{amplitude for final } M1 \text{ transition}|/ \\ &\qquad\qquad |\text{spin-flip amplitude}| \\ &=[5.7e^2R^2(mc^2\pi^2)^{-1}][2M(1.4)^2(6\pi^2)^{-1}|V_{\text{s.o.}}|R^3 \\ &\qquad\qquad\times\{1-c_0^2(K/2)\}]^{-1} \\ &\simeq 10^{-2}. \end{aligned} \quad (4.18)$$

Thus, even though the $M1$ mechanism undoubtedly occurs and contributes, it will be far too small to explain the data, or to compete with the semidirect $d\rightarrow\varphi$ transition mechanism described in this paper.

As further evidence that it is the asymmetric spin-flip rather than $M1$ transitions which produces the isotopic-spin violation, we note that the $M1$-produced amplitude is strongly forward-peaked (the pure $M1$ interaction is weakly singular at $\theta=0°$). On the other hand, the expression (4.14) vanishes at $\theta=0°$. (This is typical of short-ranged spin-orbit interactions.) An examination of the T-violating differential cross sections of MEA (which are small near $\theta=0°$, and peaked at $\theta\sim 20°$) convincingly excludes the $M1$ mechanism on these qualitative grounds alone, exclusive of the estimate (4.18) of the ratio of the cross sections of the two mechanisms.

It is philosophically unsatisfying to explain an experimental anomaly by means of a new mechanism whose sole observable manifestation is the very effect which it was invoked to explain. This consideration instigated a search by me for an independent way to test the hypothesis of preliminary $d\rightarrow\varphi$ transitions. If this mechanism is indeed responsible for the observations of MEA, it also implies that there will be $S=1\rightarrow S=0$ transitions in the breakup reaction, $C^{12}(d,np)C^{12}$. Ideally, one would like to be able to examine the relative spin orientations of the nucleons in the final state. This is not yet a technologically feasible experiment; fortunately there is a much simpler experiment one can do which may be sensitive to the presence of the $T=1$, φ "state" in breakup. The idea is to detect final states in which the neutron and proton are at small relative momentum. Under these conditions, the breakup differential cross section is given by[19]

$$\frac{d\sigma_{\text{b.u.}}}{d\Omega}\simeq 9q_0{}^3\left[\frac{d\sigma_{\text{el}}}{d\Omega}+R\frac{d\sigma_{\text{q.e.}}}{d\Omega}\right]. \quad (4.19)$$

In (4.19), $d\sigma_{\text{el}}/d\Omega$ is the elastic differential cross section [i.e., for $C^{12}(d,d)C^{12}$] with the final momentum slightly off the energy shell, $(d\sigma_{\text{q.e.}}/d\Omega)$ is the "differential cross section" for the "quasielastic" reaction $C^{12}(d,\varphi)C^{12}$, in which the neutron and proton come off with zero relative momentum; and q_0 is the maximum n-p relative momentum accepted by the detection system (in inverse fermis). The number R is essentially the square of the ratio of the singlet-to-triplet n-p scattering lengths (it is actually slightly less than this ratio) and has a value of 14 or 15. From the size of the isospin-violating (d,α) cross section, one might expect $\sigma_{\text{q.e.}}$ to be ~ 1–5% of σ_{el}. [One should note that Eq. (4.19) is dimensionally correct when q_0 is given in units of inverse fermis; see Eq. (B19) for details.]

Now, $d\sigma_{\text{el}}/d\Omega$ typically is forward-peaked, and has sharp diffraction minima. To attempt to detect the presence of φ states, one would select breakup events with small q_0, and measure $d\sigma_{\text{b.u.}}/d\Omega$ as a function of angle. Since $d\sigma_{\text{q.e.}}/d\Omega$ vanishes at $\theta=0°$, and $d\sigma_{\text{el}}/d\Omega$ is maximum there, the two angular distributions will be out of phase. The factor $R\sim 14$–15 also is highly favorable. Thus, if $d\sigma_{\text{q.e.}}/d\Omega$ is present at all, the differential cross section (4.19) should be much smoother than $d\sigma_{\text{el}}/d\Omega$ since the term $R(d\sigma_{\text{q.e.}}/d\Omega)$ should "fill in the dips" in $d\sigma_{\text{el}}/d\Omega$.

Despite the rather small expected value for the cross section $d\sigma_{\text{b.u.}}/d\Omega$, measured under these conditions, such an experiment is probably feasible at the present level of experimental technology. A search for these phenomena is now being initiated at the University of Pennsylvania tandem accelerator facility.[20]

[19] See Appendix B, and Ref. 7, as well as P. M. Fishbane and J. V. Noble, Phys. Rev. (to be published).
[20] R. Zurmuhle (private communication).

V. SUMMARY AND CONCLUSIONS

In this paper, a new mechanism for producing isotopic-spin violations was introduced to account for the observations of MEA. This mechanism has been shown to be a far more likely candidate than those proposed by MEA, in that it has the right magnitude and the right angular behavior to explain the data. A direct experimental test of this hypothesis was proposed.

Zafiratos *et al.*[21] have reported the nonobservation of the $J^\pi=0^+$, $T=1$ state in N^{14} in a study of the C^{12}-$(\alpha,d)N^{14}$ reaction. On the other hand, Jobst[22] has seen the $T=1$ state in N^{14} using the $O^{16}(d,\alpha)N^{14}$ reaction. The cross sections should be of the same order of magnitude; however Jobst's statistics seem to be considerably better than those of Zafiratos *et al.*, as far as was indicated by their respective articles. The upper limit of 0.27% of the ground-state cross section given by Zafiratos *et al.* is probably slightly low; i.e., they may have overestimated the sensitivity of their experiment. On the other hand, both $C^{12}(d,\alpha)B^{10}$ and $O^{16}(d,\alpha)N^{14}$ have small Q values, whereas $C^{12}(\alpha,d)N^{14}$ is quite endoergic (by about 13 MeV). In view of the sensitivity of the T-forbidden cross section to such effects, it would not be surprising to find that the T-forbidden (α,d) reaction leading to the $T=1$ state in N^{14} has a cross section lower than the upper limit given by these authors.

Finally, we mention that Jobst's data[22] were not taken at sufficiently high energies to show the forward-peaking effect found by MEA. By analogy with the $C^{12}(d,\alpha)B^{10}$ reaction, one would expect this effect to appear above 12 MeV.

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APPENDIX A: ESTIMATE OF THE MAGNETIC-DIPOLE TRANSITION MATRIX ELEMENT

The interaction energy of a magnetic dipole with an applied magnetic field is just

$$H_I=-\mathfrak{y}\cdot\mathbf{B}. \tag{A1}$$

For nucleons in a nucleus, with definite orbital angular momentum and total angular momentum,

$$\mathfrak{y}_{n,p}(l,j)=\mu_{n,p}(l)(e\hbar/2mc)\mathbf{j}. \tag{A2}$$

The shell-model picture of the $T=0$, $J^\pi=1^+$ states of B^{10} gives the basic configuration of a neutron and a proton hole in the $p_{3/2}$ shell coupled to total J; thus, $\mu_n=-1.91$ and $\mu_p=3.79$ in the $l=1$, $j=\frac{3}{2}$ state.[23] The magnetic field of an α particle with orbital angular momentum $\mathbf{L}_\alpha$ is[24] $\mathbf{B}=(Q\hbar/M_rcR^3)\mathbf{L}_\alpha$, where R is the distance from the α particle, $Q=2e$, and M_r is the α-particle reduced mass. Thus, if $\mathbf{J}_N=\mathbf{j}_n+\mathbf{j}_p$, the interaction matrix element becomes (note $\mathbf{J}_N$ is diagonal and so cannot connect the 1^+ with the 0^+ state[25])

$$\langle\mathbf{k}_\alpha',J_N=0|H_I|\mathbf{k}_\alpha,J_N=1,J_z\rangle$$
$$=-\left(\frac{5.7e^2\hbar^2}{8mM_rc^2\pi^3}\right)\int d^3R\{R^{-3}\exp(-i\mathbf{k}_\alpha'\cdot\mathbf{R})$$
$$\times\mathbf{L}_\alpha\exp(i\mathbf{k}_\alpha\cdot\mathbf{R})\}\cdot\langle 00|\mathbf{j}_p|J_N=1,J_z\rangle,$$

i.e.,

$$\langle\mathbf{k}_\alpha',J_N'=0|H_I|\mathbf{k}_\alpha,J_N=1,J_z\rangle$$
$$=-\frac{5.7e^2\hbar^2i}{mc^2M_r2\pi^2}\left[\frac{\mathbf{k}_\alpha'\times\mathbf{k}_\alpha}{|\mathbf{k}_\alpha'-\mathbf{k}_\alpha|^2}\cdot\langle 00|\mathbf{j}_p|1,J_z\rangle\right]. \tag{A3}$$

Neglecting the final-state Coulomb interaction, and calling the spacing between the two levels

$$\Delta E=(\hbar^2/2M_r)\delta^2,$$

we get the over-all transition matrix element

$$\langle\mathbf{k}_\alpha',J_N'=0|H_IG_f^+(E)V_f|d;\mathbf{K}\nu\rangle\simeq\frac{5.7e^2i}{\pi^2mc^2}$$
$$\times\int\frac{d\mathbf{k}''[(\mathbf{k}_\alpha'\times\mathbf{k}'')\cdot\langle 00|\mathbf{j}_p|1\nu\rangle]\langle\mathbf{k}''|b_{\text{Born}}^+(E)|\mathbf{K}\rangle}{(\mathbf{k}_\alpha'-\mathbf{k}'')^2(k_\alpha'^2+i\eta\pm\delta^2-k''^2)}. \tag{A4}$$

To put (A4) in a form which may be compared with (4.14) and which leads to (4.18), we note that:

(1) Equation (A4) is not a singular integral.

(2) The limit of $\delta^2\to 0$ is finite and bounded in (A4). Thus, we identify $[R^2(\mathbf{k}_\alpha'-\mathbf{k}'')^2]^{-1}$ with the same kind of dimensionless function of momentum transfer as $\tilde{f}(Q)$ in (4.12), so that the expressions (A4) and (4.14) are seen to be virtually identical in form, and simply have different over-all coupling constants in front of the integrals. [Of course, the range R appearing above reflects the structure of b_{Born}^+, but will be about the same as the range appearing in (4.12).]

APPENDIX B: ESTIMATING THE (d,np) CROSS SECTION

Let us denote the elastic (d,d) amplitude (for scattering from a spin-0 target) by $\langle\mathbf{K}'\nu'|A^+(E)|\mathbf{K}\nu\rangle$, where $\mathbf{K}$ and $\mathbf{K}'$ are the c.m. momentum of the deuteron before and after scattering, and ν and ν' are the z projec-

[21] C. D. Zafiratos, J. S. Lilley, and F. W. Slee, Phys. Rev. **154** 887 (1967).

[22] J. Jobst, Bull. Am. Phys. Soc. **10**, 462 (1965); Ph.D. thesis, University of Wisconsin, 1966 (unpublished).

[23] See Ref. 5, p. 125.

[24] J. D. Jackson, *Classical Electrodynamics* (John Wiley & Sons, Inc., New York, 1962), p. 134.

[25] A. de-Shalit and I. Talmi, *Nuclear Shell Theory* (Academic Press Inc., New York, 1963), p. 117.

tions of its spin. The differential cross section for elastic deuteron scattering may be written

$$\frac{d\sigma_{\rm el}}{d\Omega}=NK^{-1}\int_0^\infty dK'K'^2\delta(K'^2-K^2)\times\tfrac{1}{3}\sum_{\nu'=-1}^{1}\sum_{\nu=-1}^{1}|\langle\mathbf{K}'\nu'|A^+(E)|\mathbf{K}\nu\rangle|^2, \quad \text{(B1)}$$

where N is just a constant. We may denote the breakup amplitude by $\langle\mathbf{K}'\mathbf{q}'S'\nu'|B^+(E)|\mathbf{K}\nu\rangle$ where $\mathbf{K}$ and ν retain their former meanings, and $\mathbf{q}'$ is the relative n-p momentum in the final state, and S' is their total spin, which may be 0 or 1. In terms of the amplitude B, the breakup cross section may be written

$$\Delta\sigma_{\rm b.u.}=NK^{-1}\int d\mathbf{K}'\int d\mathbf{q}'\times\delta\left(K'^2+\frac{2M}{m}q'^2+\frac{2M}{m}\kappa_d^2-K^2\right)\times\tfrac{1}{3}\sum_{S'=0}^{1}\sum_{\nu'=-S'}^{S}\sum_{\nu=-1}^{1}|\langle\mathbf{K}'\mathbf{q}'S'\nu'|B^+(E)|\mathbf{K}\nu\rangle|^2, \quad \text{(B2)}$$

where the integrations are over the range of observed final states and where $\kappa_d^2=(m/\hbar^2)\epsilon_d$. Now suppose we restrict the range of final q' we observe to

$$0\leqslant q'\leqslant q_0<q_{\rm max},$$

where

$$q_{\rm max}^2=\frac{mK^2}{2M}-\kappa_d^2. \quad \text{(B3)}$$

Then for sufficiently small q_0, the final state has no dependence on the direction of $\hat{q}'$ (i.e., the n and p can be only in relative s states) so we may integrate over $\hat{q}'$ and recover a factor of 4π. We now assume that a small range of solid angles $d\hat{K}'\equiv d\Omega$ is observed, and perform the K' integration to get

$$\frac{d\sigma_{\rm b.u.}}{d\Omega}=2\pi NK^{-1}\int_0^{q_0}dq'q'^2K_0(q')\times\tfrac{1}{3}\sum_{S'=0}^{1}\sum_{\nu'=-S'}^{S'}\sum_{\nu=-1}^{1}|\langle K_0(q')\hat{K}',q',S'\nu'|B^+(E)|\mathbf{K}\nu\rangle|^2, \quad \text{(B4)}$$

where

$$K_0^2(q')=(2M/m)(q_{\rm max}^2-q'^2).$$

We now wish to express the breakup amplitude (with q' small) in terms of the elastic (d,d) and "quasi-elastic" (d,φ) amplitudes. We note that the elastic-scattering operator is given by[7,19]

$$\mathfrak{A}(W)=V_n+V_p+(V_n+V_p)(W-H)^{-1}(V_n+V_p) \quad \text{(B5)}$$

and the breakup operator by[7,19]

$$\mathfrak{B}(W)=V_n+V_p+(V_n+V_p+V_{np})(W-H)^{-1}\times(V_n+V_p). \quad \text{(B6)}$$

We may obviously write

$$\mathfrak{B}(W)=\mathfrak{A}(W)+V_{np}(W-H)^{-1}(V_n+V_p)=\mathfrak{A}(W)+V_{np}(W-H_0)^{-1}\mathfrak{B}(W). \quad \text{(B7)}$$

But (B7) may be solved to get

$$\mathfrak{B}(W)=\mathfrak{A}(W)+t_{np}(W)G_0(W)\mathfrak{A}(W). \quad \text{(B8)}$$

We note that (on the energy shell)

$$\langle\mathbf{K}'S'\nu'|A^+(E)|\mathbf{K}\nu\rangle=\int d\mathbf{q}'\int d\mathbf{q}\frac{v_{S'}^*(q')}{E+i\eta-(2M)^{-1}K'^2-m^{-1}q'^2}\times\langle\mathbf{K}'\mathbf{q}'S'\nu'|\mathfrak{A}(E+i\eta)|\mathbf{K}\mathbf{q}1\nu\rangle\psi_d(q). \quad \text{(B9)}$$

[With $S'=1$, $K'=K$, we get just $\langle\mathbf{K}'\nu'|A^+(\mathrm{E})|\mathbf{K}\nu\rangle$.] We also see that

$$\langle\mathbf{K}'\mathbf{q}'S'\nu'|B^+(E)|\mathbf{K}\nu\rangle=\int\langle\mathbf{K}'\mathbf{q}'S'\nu'|\mathfrak{B}(E+i\eta)|\mathbf{K}\mathbf{q}1\nu\rangle\psi_d(q)d\mathbf{q}. \quad \text{(B10)}$$

Therefore from (B8) and using (3.12)

$$\langle\mathbf{K}'\mathbf{q}'S'\nu'|B^+(E)|\mathbf{K}\nu\rangle|_{q'=0}=\int d\mathbf{q}\langle\mathbf{K}'0S'\nu'|\mathfrak{A}(E+i\eta)|\mathbf{K}\mathbf{q}1\nu\rangle\psi_d(q)-v_{S'}(0)\tau_{S'}(E+i\eta-(2M)^{-1}K'^2)\times\langle\mathbf{K}'S'\nu'|A^+(E)|\mathbf{K}\nu\rangle. \quad \text{(B11)}$$

Now we note that (on the energy shell)

$$E=(2M)^{-1}K^2-\epsilon_d=(2M)^{-1}K'^2+m^{-1}q'^2, \quad \text{(B12)}$$

and so with $q'=0$,

$$\langle\mathbf{K}'S'\nu'|A^+(E)|\mathbf{K}\nu\rangle=-m\int d\mathbf{q}''\int d\mathbf{q}\frac{v_{S'}^*(q'')}{q''^2}\times\langle\mathbf{K}'\mathbf{q}''S'\nu'|\mathfrak{A}(E+i\eta)|\mathbf{K}\mathbf{q}1\nu\rangle\psi_d(q)$$

or

$$\langle\mathbf{K}'S'\nu'|A^+(E)|\mathbf{K}\nu\rangle\simeq\left(-m\int d\mathbf{q}''\frac{v_{S'}^*(q'')}{q''^2}\right)\times\int\langle\mathbf{K}'0S'\nu'|\mathfrak{A}(E+i\eta)|\mathbf{K}\mathbf{q}1\nu\rangle\psi_d(q)d\mathbf{q}. \quad \text{(B13)}$$

Thus we at last write

$$\langle \mathbf{K}'0S'\nu'|B^+(E)|\mathbf{K}\nu\rangle \simeq \xi_{S'}\langle \mathbf{K}'S'\nu'|A^+(E)|\mathbf{K}\nu\rangle, \quad \text{(B14)}$$

where

$$\xi_{S'} = -\left[4\pi m\int_0^\infty dq v_{S'}{}^*(q)\right]^{-1} - v_{S'}(0)\tau_{S'}(0). \quad \text{(B15)}$$

Suppose we choose the form

$$v_S(q) = c_1(q^2+\alpha_S{}^2)^{-1}; \quad \text{(B16)}$$

then comparing $t_{np}{}^S(E)$ with the appropriate effective-range formula, we find

$$\xi_{S'} = [2\pi^2 m v_{S'}(0)]^{-1}[a_0{}^{(S')} - (\alpha_{S'})^{-1}], \quad \text{(B17)}$$

where $a_0{}^{(S)}$ is the triplet or singlet n-p scattering length accordingly as $S=1$ or 0. The normalization constant c_1 is determined by the requirement that $\int d\mathbf{q}|\psi_d(q)|^2=1$, i.e., that

$$4\pi c_1{}^2\int_0^\infty dq\, q^2(q^2+\alpha_1{}^2)^{-2}(m^{-1}q^2+\epsilon_d)^{-2} = 1.$$

Putting (B14) into (B4), assuming $v_1(0)\simeq v_0(0)$, and using (B17), we obtain $[K_0(0)\simeq K]$

$$\frac{d\sigma_{\text{b.u.}}}{d\Omega} \simeq \frac{2\pi N}{3} q_0{}^3\left(\frac{\alpha^2}{2\pi^2 m c_1}\right)^2 |a_0{}^{(1)} - \alpha^{-1}|^2 \times\{\tfrac{1}{3}\sum_{\nu'\nu}|\langle \mathbf{K}'1\nu'|A^+(E)|\mathbf{K}\nu\rangle|^2 + R\tfrac{1}{3}\sum_{\nu}|\langle \mathbf{K}'00|A^+(E)|\mathbf{K}\nu\rangle|^2\}. \quad \text{(B18)}$$

Comparing (B18) with (B1), and defining the "quasi-elastic" (d,φ) differential cross section analogously with (B1) but using $\langle \mathbf{K}'00|A^+(E)|\mathbf{K}\nu\rangle$ we have

$$\frac{d\sigma_{\text{b.u.}}}{d\Omega} = \frac{4\pi}{3} q_0{}^3\left(\frac{\alpha^2}{2\pi^2 m c_1}\right)^2 |a_0{}^{(1)} - \alpha^{-1}|^2 \times\left\{\frac{d\sigma_{\text{el}}}{d\Omega}(\text{off-shell}) + R\frac{d\sigma_{\text{q.e.}}}{d\Omega}\right\}, \quad \text{(B19)}$$

where $R = |a_0{}^{(0)} - \alpha^{-1}|^2|a_0{}^{(1)} - \alpha^{-1}|^{-2} \simeq 15$. The parameter c_1 may be evaluated from the above normalization condition (note $\alpha \sim 5\kappa_d$), and inserted into (B19) to get the expression (4.19).

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Analysis of Recoil Angular and Range Distributions from Compound-Nucleus Reactions*

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A method is presented for calculating the observable kinematic properties of nuclei recoiling from compound-nucleus reactions. The computations are performed by specifying center-of-mass distribution functions in angle and speed which arise from the emission of particles, and carrying out the appropriate integrations over laboratory speeds and angles. The calculated angular distributions, range distributions, and range distributions with angular restrictions (collimated ranges) are directly comparable to the corresponding properties measured in recoil experiments. Results are given for a number of compound-nucleus reactions induced by heavy ions and α particles, where sufficiently detailed experimental data are available for comparison. Very good agreement was obtained by assuming isotropic particle emission in the center-of-mass system and using a Maxwellian distribution of recoil speeds. The single parameter in the Maxwellian function was fixed by relating it to the mean-square velocity of the recoiling nuclei, the latter being determined from experimental angular distribution data by means of a previous analysis. The effects of different forms of the center-of-mass angular and velocity distributions on the calculated angular and range distributions are discussed. We conclude that, in all the cases investigated, the recoil angular distributions are essentially isotropic in the center-of-mass system, with no more than a small anisotropy being allowed by the experimental data. Furthermore, velocity distributions which differ greatly from the Maxwellian form are not consistent with the observations. The laboratory velocity spectrum of recoils emitted within a small angular interval about 0° has been calculated for a specific reaction, and is found to resemble rather closely a recent experimental measurement of this distribution.

I. INTRODUCTION

THE use of recoil techniques in nuclear reaction studies can provide detailed information on the reaction mechanism.[1] Measurements of average ranges of recoiling product nuclei can be used to test for full linear momentum transfer from an incident beam and for the symmetry (about $\frac{1}{2}\pi$) of particle emission in the center-of-mass system. These criteria may be sufficient to indicate that the reaction under study proceeds via

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[1] B. G. Harvey, Ann. Rev. Nucl. Sci. **10**, 240 (1960); J. M. Alexander, in *Nuclear Chemistry*, edited by L. Yaffe (Academic Press Inc., New York, to be published).