carrier density was determined to be $\sim 1.0\times 10^{16}$ cm^{-3} at 80°K.

The elastoresistance data in the extrinsic region can be explained by either of two valence-band models: a many-valley model with energy maxima along $\langle 111\rangle$ directions in momentum space,[11] or a nearly degenerate band located at the Brillouin zone center. However, magnetoresistance measurements by Umeda[7] indicate that the valleys, if any, are located in the $\langle 100\rangle$ directions. The situation is similar to that of p-type GaSb.[12] We conclude that the valence band structure is similar to that of germanium.

In the intrinsic range ($10^3/T<6.5$), the elastoresistance components have a symmetry characteristic of a many-valley conduction band with minima in the $\langle 100\rangle$ directions. This is in agreement with piezoresistance measurements reported by Crossman and Temple.[8]

Long[13] has measured the pressure coefficient of resistivity in Mg_2Sn at 300°K. His value for dE_g/dp was $+5\times 10^{-6}$ eV bar^{-1}. Lynch[14] has repeated Long's experiment and obtains a value of 3×10^{-6} eV bar^{-1}. Our data gives 3.3×10^{-6} eV bar^{-1}. When the values of π_{12} were corrected for contact resistance (by a graphic procedure), we found $dE_g/dp=2.1\times 10^{-6}$ eV bar^{-1}.

ACKNOWLEDGMENTS

We wish to thank D. A. Marshall for constructing the crystal growing facility and Professor G. C. Danielson for advice in crystal-growing procedures.

---

[11] C. Herring, Bell System Tech. J. 34, 237 (1955).

[12] O. N. Tufte and E. L. Stelzer, Phys. Rev. 133, A1450 (1964).

[13] D. Long, Phys. Rev. 101, 1256 (1956).

[14] D. W. Lynch (private communication).

PHYSICAL REVIEW VOLUME 162, NUMBER 3 15 OCTOBER 1967

Electron Mobility in GaSb at 77°K*

R. D. Baxter, F. J. Reid, and A. C. Beer
Battelle Memorial Institute, Columbus Laboratories, Columbus, Ohio

(Received 26 May 1967)

Electron mobility (more specifically, the product $R\sigma$ of the Hall coefficient and the conductivity) has been measured in n-type GaSb at 77°K as a function of carrier concentration and for several densities of compensating acceptors. Throughout the region where conduction is confined essentially to the principal conduction-band minimum, a good quantitative fit to the data is obtained by consideration of ionized-impurity scattering only, under conditions of compensation. Evidence suggests the compensating centers to be doubly ionized and of a density equal to that of the residual acceptors existing in the initial p-type material before it was doped to n type. Such results are also quantitatively consistent with findings of ion-pairing studies involving lithium. At higher carrier concentrations, where conduction in the [111] subsidiary band becomes important, theoretical predictions are rendered more difficult by uncertainties in parameters characterizing the [111] band, as well as other complicating effects such as screening and the scattering of [000] electrons upon [111] electrons. Semiquantitative agreement, however, tends to favor, for ΔE and $m_1{}^d/m_0{}^d$, the lower values that were previously published.

INTRODUCTION

The electrical properties of n-type GaSb have been studied by numerous investigators over the past few years. Many of the findings have been explained by considering the effects of relatively low-lying ($\sim$0.08 eV, at room temperature) subsidiary conduction-band minima, as originally postulated by Sagar.[1] Bate,[2] for instance, has accounted for the concentration dependence of mobility in Se-doped GaSb by considering a discrete Se donor level associated with the [111] minima and lying above the principal conduction-band edge. Also, Robinson and Rodriguez[3,4] have calculated the effects of population of the subsidiary minima on the screening parameter for ionized-impurity scattering. Their results are consistent with the abrupt increases with carrier concentration observed in both the Hall mobility and the amplitude of Shubnikov–de Haas oscillations at 4.2°K for electron concentrations greater than 1.25×10^{18} cm^{-3}.

There are, however, other peculiarities in the electrical properties of n-type GaSb which occur at lower doping levels, and which do not appear to be the result of two-band conduction. For example, the mobility μ at low temperatures is observed to increase monotonically with increasing electron concentration n for Te-doped GaSb.[5] Also, the diffusion of lithium into Te-doped material has resulted in significantly increased

---

* Work supported by the U. S. Air Force Office of Scientific Research, Grant No. 525-66.

[1] A. Sagar, Phys. Rev. 117, 93 (1960).

[2] R. T. Bate, J. Appl. Phys. 33, 26 (1962).

[3] J. E. Robinson and S. Rodriguez, Phys. Rev. 135, A779 (1964).

[4] J. E. Robinson and S. Rodriguez, Phys. Rev. 137, A663 (1965).

[5] A. J. Strauss, Phys. Rev. 121, 1087 (1961).

electron mobilities coupled with modest increases in electron concentration.[6,7]

Long and Hager,[8] in their investigation of near-resonance scattering in GaSb at low temperatures, pointed out the basic importance of compensation in GaSb in regard to the observed increase in μ with n. This behavior was in contrast to that found in studies of n-type specimens of GaAs and InAs, where compensation was significantly lower.[9] In our case, where mobility measurements were made at 77°K, resonance phenomena are not thought to be of controlling significance, since phase shifts higher than the zeroth order are evidently important. On the other hand, the influence of compensation is very real. A further consideration is the apparent importance of repulsive centers, which were found in lithium-doping experiments to be effective scattering centers.

To illustrate the effect of compensation on Coulomb scattering, we note that the well-known Brooks-Herring[10] treatment leads, for example, to an expression for the mobility in the form $\mu_I \propto f(n)/N_I$, where $f(n)$ is in general an increasing function of carrier concentration n, and where the total ionized impurity density is given by

$$N_I = N_D + N_A\,, \tag{1}$$

or, in the case of n-type samples ($n = N_D - N_A$),

$$N_I = 2N_A + n\,. \tag{2}$$

In the absence of compensation ($N_I = n$), the factor $1/N_I$ varies as $1/n$, and the mobility μ_I decreases with increasing carrier concentration. If, however, a relatively large and constant density of compensating centers N_A is present (as is the case with most specimens of n-type GaSb), the term $N_I (= 2N_A + n)$ will vary much more slowly with n. For $n \ll 2N_A$, N_I is essentially independent of n, and the variation of mobility is controlled entirely through $f(n)$ so that a mobility increasing with n is predicted.

Thus we regard the observation of a monotonic variation of mobility with carrier concentration, in the case of n-type GaSb at 77°K, as evidence for the dominance of impurity scattering under conditions of high compensation. Such a mechanism, our analysis shows, is able to account both for the concentration dependence of μ at 77°K in n-type GaSb prepared by tellurium doping, and for the incremental mobility observed in such specimens after the introduction of lithium by diffusion.

CALCULATION OF IONIZED-IMPURITY MOBILITY

A. Theoretical Considerations

Bate *et al.*[11] have calculated the ionized-impurity mobility for a nonparabolic band, using Kane's theory[12] to express the dependence of conduction-band energy on wave vector **k**. Their result, obtained for InSb, makes use of the approximations $\Delta \gg kP, E_g$, where Δ is the spin-orbit splitting, E_g is the energy gap, and P is the momentum matrix element.

In the case of GaSb, Δ is of the same order[13] as E_g, and therefore it is necessary to examine the nature of the solution to Kane's secular equation for the more general condition $E_g \gg kP$. To obtain energy relations for the principal conduction band, the secular equation is put in the form

$$x(x+E_g)(x+E_g+\Delta) = k^2P^2(x+E_g+\tfrac{2}{3}\Delta)\,, \tag{3}$$

where Kane's notation is followed, except that

$$x \equiv E' - E_g\,.$$

The parameter x is related to the energy ϵ measured from the band edge, as follows:

$$x = \epsilon - \hbar^2k^2/2m\,,$$

where m is the free-electron mass. Since $x \ll E_g$, an approximate solution to (3) is obtained by neglecting the x^3 term. As usual, the matrix element P is expressed in terms of the effective mass at the band edge m_0^*, namely,

$$\hbar^2/m_0^*[\partial^2\epsilon/\partial k^2]_{k=0}\,,$$

which yields

$$P^2 = \hbar^2\left(\frac{1}{m_0^*} - \frac{1}{m}\right)\frac{E_g(E_g+\Delta)}{2(E_g+\frac{2}{3}\Delta)}\,. \tag{4}$$

Because of the small effective mass in the principal conduction band of GaSb, the term $1/m$ can be neglected as compared to $1/m_0^*$, so that the approximate solution to Eq. (3) becomes

$$\epsilon = \frac{E_g(E_g+\Delta)}{2(2E_g+\Delta)}\left\{-1 + \frac{\hbar^2k^2}{2m_0^*}\frac{1}{E_g+2\Delta/3} + \left[\left(1 - \frac{\hbar^2k^2}{2m_0^*}\frac{1}{E_g+\frac{2}{3}\Delta}\right)^2 + \frac{2\hbar^2k^2}{m_0^*}\frac{2E_g+\Delta}{E_g(E_g+\Delta)}\right]^{1/2}\right\}\,. \tag{5}$$

If the above expression is expanded in powers of k through k^4 terms, or if (3) is solved directly by power

---

[6] T. O. Yep and W. M. Becker, J. Appl. Phys. **37**, 456 (1966).

[7] R. D. Baxter and F. J. Reid, Bull. Am. Phys. Soc. **10**, 599 (1965).

[8] D. Long and R. J. Hager, J. Appl. Phys. **36**, 3436 (1965).

[9] D. Long, J. D. Zook, P. W. Chapman, and O. N. Tufte, Solid State Commun. **2**, 191 (1964).

[10] H. Brooks, in *Advances in Electronics and Electron Physics* (Academic Press Inc., New York, 1955), Vol. 7, p. 156.

[11] R. T. Bate, R. D. Baxter, F. J. Reid, and A. C. Beer, J. Phys. Chem. Solids **26**, 1205 (1965).

[12] E. O. Kane, J. Phys. Chem. Solids **1**, 249 (1957); in *Semiconductors and Semimetals*, edited by R. K. Willardson and A. C. Beer (Academic Press Inc., New York, 1966), Vol. 1, p. 75.

[13] H. Ehrenreich, J. Appl. Phys. **32**, 2155 (1961).

series, the result is[14]

$$\epsilon = \hbar^2k^2/2m_0^* - f(\lambda)\hbar^4k^4/4m_0^{*2}E_g\,, \tag{6}$$

where

$$f(\lambda) \equiv \frac{2+\lambda}{1+\lambda}\,\frac{1}{1+\frac{2}{3}\lambda}\,, \quad \lambda \equiv \frac{\Delta}{E_g}\,. \tag{7}$$

For evaluating transport integrals, it is useful to express k^2 in terms of energy. For this purpose, Eq. (5) yields directly

$$\frac{\hbar^2k^2}{2m_0^*} = \frac{\epsilon(1+\epsilon/\mathcal{E}_g)}{1+\epsilon/(E_g+\frac{2}{3}\Delta)}\,, \quad \text{where} \quad \mathcal{E}_g = \frac{E_g(E_g+\Delta)}{2E_g+\Delta}\,. \tag{8}$$

Neglecting, again, terms above $(\epsilon/E_g)^2$, we obtain

$$\hbar^2k^2/2m_0^* = \epsilon[1+f(\lambda)\epsilon/E_g] = \epsilon[1+\epsilon/E_g']\,, \tag{9}$$

where $E_g' \equiv E_g/f(\lambda)$. It is interesting to note from (7) that for GaAs ($\Delta \ll E_g$) as well as for InSb($\Delta \gg E_g$), $f(\lambda)$ approaches unity, so that (5), (6), and (9) are approximated for such cases by the expressions used previously for InSb.[11] For the more general case where Δ is arbitrary, an inspection of (6) or (9) shows that we can still use the relationship developed for InSb except that E_g is to be replaced by $E_g/f(\lambda)$. In the case of GaSb, where $\lambda \sim 1$, this amounts to a value of 0.9 for $f(\lambda)$. Thus, by taking account of this small factor, we can apply directly to GaSb the expression derived in Ref. 11 for InSb, so long as the conduction is confined predominantly to a single band—as is the case for sufficiently low temperatures or electron concentrations. Such a situation prevails for much of the data to be analyzed.

When, however, electron concentrations exceed about 8×10^{17} cm^{-3}, the proximity of the [111] conduction band to the [000] minimum causes population of the subsidiary minima, even at low temperatures. It then becomes necessary, when calculating the impurity-scattering mobility in the [000] conduction band, to include the effect of screening by electrons in the [111] minima. This may be accomplished by applying the method of Dingle[15] to the two-band case.

We have, after Dingle, that

$$\frac{1}{a^2} = \frac{4\pi e^2}{\kappa KT}\frac{\partial n}{\partial \eta}\,, \tag{10}$$

where η is the reduced Fermi energy ($=\zeta/KT$) measured in all cases from the bottom of the [000] band, KT is the usual thermal energy, and κ is the dielectric constant. The electron concentration for the nonparabolic [000] band, follows from Eq. (B12) of Ref. 11, namely,

$$n_{[000]} = \frac{1}{2\pi^2}\left(\frac{2m_0^*KT}{\hbar^2}\right)^{3/2}\left[F_{1/2}(\eta) + \frac{5}{2}\frac{KT}{E_g'}F_{3/2}(\eta) + \frac{7}{8}\left(\frac{KT}{E_g'}\right)^2 F_{5/2}(\eta) + \cdots\right], \quad E_g' \gg \zeta, KT. \tag{11}$$

For our case, $E_g' \equiv E_g/f(\lambda) \cong 1.1E_g$; and the F_j are the tabulated Fermi-Dirac functions[16] defined by

$$F_j(\eta) = (KT)^{-(j+1)}\int_0^\infty \epsilon^j f_0(\epsilon)\,d\epsilon\,.$$

Taking the [111] band to be parabolic,[17]

$$n_{[111]} = \frac{1}{2\pi^2}\left(\frac{2m_1^*KT}{\hbar^2}\right)^{3/2} \nu F_{1/2}\left(\eta - \frac{\Delta E}{KT}\right), \tag{12}$$

where ν is the number of equivalent minima, and ΔE is the energy separation between the [111] and [000] band edges. The total electron concentration n is then given by

$$n = n_{[000]} + n_{[111]} = \frac{1}{2\pi^2}\left(\frac{2m_0^*KT}{\hbar^2}\right)^{3/2} \times\left[F_{1/2}(\eta) + \frac{5}{2}\left(\frac{KT}{E_g'}\right)F_{3/2}(\eta) + \frac{7}{8}\left(\frac{KT}{E_g'}\right)^2 F_{5/2}(\eta) + \cdots + \nu\left(\frac{m_1^*}{m_0^*}\right)^{3/2} F_{1/2}\left(\eta - \frac{\Delta E}{KT}\right)\right]. \tag{13}$$

In view of the property of the Fermi-Dirac functions, namely,

$$\partial F_p(\eta)/\partial\eta = pF_{p-1}(\eta)\,, \quad p>0\,, \tag{14}$$

we obtain from (10) and (13),

$$\frac{1}{a^2} = \frac{4\pi e^2}{\kappa KT}\left(\frac{1}{2\pi^2}\right)\left(\frac{2m_0^*KT}{\hbar^2}\right)^{3/2}\left[\tfrac{1}{2}F_{-1/2}(\eta) + \frac{15}{4}\left(\frac{KT}{E_g'}\right) \times F_{1/2}(\eta) + \frac{35}{16}\left(\frac{KT}{E_g'}\right)^2 F_{3/2}(\eta) + \cdots + \frac{\nu}{2}\left(\frac{m_1^*}{m_0^*}\right)^{3/2} F_{-1/2}\left(\eta - \frac{\Delta E}{KT}\right)\right]. \tag{15}$$

In terms of the total electron concentration n, the ex-

---

[14] The expression is identical with that used by Cardona for InAs after his second-order correction is applied. See, M. Cardona, Phys. Rev. **121**, 752 (1961).

[15] R. B. Dingle, Phil. Mag. **46**, 831 (1955).

[16] J. McDougall and E. C. Stoner, Phil. Trans. Roy. Soc. **A237**, 350 (1938); A. C. Beer, M. N. Chase, and P. F. Choquard, Helv. Phys. Acta **28**, 529 (1955).

[17] Inasmuch as the [111] minima are spheroidal rather than spherical, the m_1^* in the equation should more precisely be designated as the density-of-states mass. This means that in the calculation of the mobility in the subsidiary band, it will be necessary to take account of the eccentricity of the [111] energy surfaces. This calculation is discussed later.

pression for a assumes the form

$$a^2=\frac{\kappa KT}{4\pi e^2}\frac{2}{n}\frac{F_{1/2}(\eta)}{F_{-1/2}(\eta)}\left[1+\frac{5}{2}\frac{KT}{E_g'}\frac{F_{3/2}(\eta)}{F_{1/2}(\eta)}+\frac{7}{8}\left(\frac{KT}{E_g'}\right)^2\frac{F_{5/2}(\eta)}{F_{1/2}(\eta)}+\cdots+\nu\left(\frac{m_1^*}{m_0^*}\right)^{3/2}\frac{F_{1/2}(\eta-\Delta E/KT)}{F_{1/2}(\eta)}\right]$$
$$\times\left[1+\frac{15}{2}\frac{KT}{E_g'}\frac{F_{1/2}(\eta)}{F_{-1/2}(\eta)}+\frac{35}{8}\left(\frac{KT}{E_g'}\right)^2\frac{F_{3/2}(\eta)}{F_{-1/2}(\eta)}+\cdots+\nu\left(\frac{m_1^*}{m_0^*}\right)^{3/2}\frac{F_{-1/2}(\eta-\Delta E/KT)}{F_{-1/2}(\eta)}\right]^{-1}. \tag{16}$$

For $\eta\ll\Delta E/KT$, this reduces to Bate's Equation (A5). For parabolic bands and strong degeneracy, i.e., $\eta\gg1$ and $\eta\gg\Delta E/KT$, Eq. (15) reduces to

$$\frac{1}{a^2}=\frac{4e^2}{\pi\kappa h^2}\left\{m_0^*\left(\frac{2m_0^*\zeta}{h^2}\right)^{1/2}+\nu m_1^*\left(\frac{2m_1^*}{h^2(\zeta-\Delta E)}\right)^{1/2}\right\} \tag{17}$$

which is identical with the Fermi-Thomas screening constant k_s^2, derived for a many-valley semiconductor by Robinson and Rodriguez.[3]

With the expression for a^2 given by Eq. (16), the ionized-impurity mobility becomes[11]

$$\mu_I=\frac{2^{3/2}\kappa^2(KT)^{3/2}}{\pi e^3N_Im_0^{*1/2}}\frac{F_2(\eta)}{F_{1/2}(\eta)}\frac{\Phi(\bar{x})}{X(\bar{x})},\quad E_g'\gg\zeta,\ KT, \tag{18}$$

where $\eta=\zeta/KT$ and $\bar{x}$ is the solution of the equation

$$\eta=\bar{x}-\ln\frac{\bar{x}+3}{\bar{x}-3}. \tag{19}$$

The correction factor for nonparabolicity is given by

$$\Phi(\bar{x})=\frac{(1+\bar{x}KT/E_g')^3}{(1+2\bar{x}KT/E_g')^2}\left[1+\frac{5}{2}\left(\frac{KT}{E_g'}\right)\frac{F_{3/2}(\eta)}{F_{1/2}(\eta)}+\frac{7}{8}\left(\frac{KT}{E_g'}\right)^2\frac{F_{5/2}(\eta)}{F_{1/2}(\eta)}+\cdots\right]^{-1}, \tag{20}$$

and the other slowly varying factor by

$$X(\bar{x})=\ln(1+\beta\bar{x})-\frac{\beta\bar{x}}{(1+\beta\bar{x})}, \tag{21}$$

where $\beta\bar{x}\equiv(2\bar{k}a)^2$, i.e., $\beta\equiv(2\bar{k}a)^2KT/\bar{\epsilon}$. Thus from Eqs. (9) and (16),

$$\beta\approx\frac{4(KT)^2\kappa m_0^*}{\pi e^2nh^2}\frac{F_{1/2}(\eta)}{F_{-1/2}(\eta)}\left[1+\frac{KT}{E_g'}\bar{x}\right]$$
$$\times\left[\frac{1+\frac{5}{2}(KT/E_g')F_{3/2}(\eta)/F_{1/2}(\eta)+\nu(m_1^*/m_0^*)^{3/2}F_{1/2}(\eta-\Delta E/KT)/F_{1/2}(\eta)}{1+(15/2)(KT/E_g')F_{1/2}(\eta)/F_{-1/2}(\eta)+\nu(m_1^*/m_0^*)^{3/2}F_{-1/2}(\eta-\Delta E/KT)/F_{-1/2}(\eta)}\right]. \tag{22}$$

B. Choice of Parameters

For the [000] band-edge effective mass m_0^*, we chose the value $0.040m$ obtained by Yep and Becker[18] on the basis of Shubnikov-de Haas measurements. The static dielectric constant κ was taken to be 14, and 0.80 eV was used as the energy gap E_g at 77°K.[19] It was not considered necessary to take account of the 10% difference between E_g and E_g' inasmuch as these parameters enter only into the higher-order terms which correct for the nonparabolicity.

In regard to the parameters specifying the [111] minima, there is a considerable uncertainty. For example, the separation ΔE of the [111] edge from the [000] edge was originally estimated by Sagar[1] to be 0.08 eV at room temperature. Later work[20] yielded the value of 0.08 at 4.2°K, while some recent studies of

---

[18] T. O. Yep and W. M. Becker, Phys. Rev. **144**, 741 (1966).

[19] E. J. Johnson, I. Filinski, and H. Y. Fan, in *Report of the International Conference on the Physics Semiconductors, Exeter, 1962* (The Institute of Physics and The Physical Society, London, 1962), p. 375.

[20] W. M. Becker, A. K. Ramdas, and H. Y. Fan, J. Appl. Phys. Suppl. **32**, 2094 (1961).

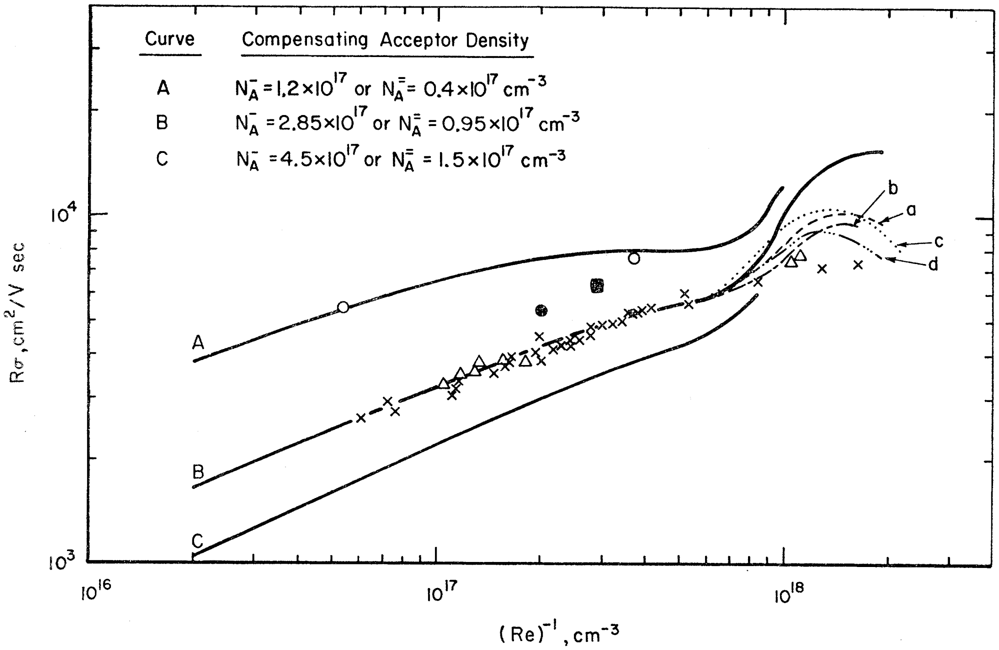

FIG. 1. Experimental values at 77°K of $R\sigma$ as a function of $1/Re$ for n-type GaSb. The ×'s represent measurements on a group of samples having approximately the same residual acceptor concentration N_A. The other data (○, ■, and ●) are for specimens having lower values of N_A, with the Δ's representing unpublished measurements by Becker. The lines are theoretical predictions, as discussed in the text, for several values of compensating acceptor densities—either singly or doubly ionized. For $(\mathrm{Re})^{-1} > 6\times10^{17}$ cm^{-3}, where the [111] band effects become noticeable, the following conditions prevail: (I.)—Ionized-impurity scattering only, with $\Delta E = 0.095$ eV, $m_1^d/m_0^d = (17.3)/4^{2/3} = 6.85$; II. ionized-impurity scattering with μ_{000} corrected for scattering of [000] band electrons on [111] band electrons. (a) ---- $\Delta E = 0.095$ eV, $m_1^d/m_0^d = (17.3)/4^{2/3} = 6.85$; (b) ——— $\Delta E = 0.095$ eV, $m_1^d/m_0^d = 12/4^{2/3} = 4.78$; (c) · · · · $\Delta E = 0.086$ eV, $m_1^d/m_0^d = (17.3)/4^{2/3} = 6.85$; (d) · · · – · · · $\Delta E = 0.086$ eV, $m_1^d/m_0^d = 12/4^{2/3} = 4.78$.

oscillatory effects in Hall coefficient and magnetoresistance indicate values as high as 0.095 eV.[21] Furthermore, the piezoresistance studies of Sagar suggested a negative temperature coefficient for ΔE, while Cardona[22] and Pillar[23] obtained the opposite result. Recent findings of Harland and Woolley[24] from the temperature dependence of Hall effect and transverse magnetoresistance yield $\Delta E = 0.084$ eV at 4.2°K, with $d(\Delta E)/dT = +0.08\times10^{-4}$ eV/°C. At 77°K, their value for ΔE is 0.088 eV.

For the density-of-states mass ratio for each minimum, m_1^d/m_0^d, the results of Becker, Ramdas, and Fan[20] yielded a value of $(17.3)/4^{2/3}$ or 6.85 at 4.2°K, when account is taken of the number of extrema. The quotient appears above inasmuch as our definition of m^d (see Appendix A), which applies only to a single minimum, does not include the factor $4^{2/3}$. Piller's measurements, interpreted likewise, give a ratio of $(14.3)/4^{2/3}$ or 5.7 Harland and Woolley's data yield a ratio between 8 and 9—depending upon the position of the Fermi level above the [000] band edge.

Because of the number of parameters necessary to characterize the [111] band and the present uncertainty in their values, it is not worthwhile to spend much effort in curve fitting in the region where contributions from both bands are significant. Our procedure rather will be to calculate several curves which will give an indication of the effect of variations in the parameters in question. In particular, values of 0.095 and 0.086 eV were chosen for ΔE, with density-of-states mass ratios for each valley of $(17.3)/4^{2/3}$ and also $12/4^{2/3}$. Correction was made for the ellipsoidal nature of the [111] minima (see Appendix A) assuming that $m_l/m_t = 8.6$, as indicated by Piller's results.[23]

EXPERIMENTAL

All GaSb crystals used in this study were grown by the Czochralski technique, starting with high-purity (99.9999%) gallium and antimony. In the case of crystals grown from stoichiometric melts, sufficient tellurium was added to produce n-type crystals with excess donor concentrations covering the range 6×10^{16} cm^{-3} to greater than 2×10^{18} cm^{-3}. Te-doped crystals were also grown from nonstoichiometric melts of initial composition $\mathrm{Ga_{0.32}Sb_{0.68}}$. Excess donor concentrations for these crystals were in the range 1.3×10^{16} to 3.6×10^{17} cm^{-3}.

Samples for Hall-coefficient and resistivity measurements were taken from slices cut perpendicular to the

---

[21] T. O. Yep and W. M. Becker, J. Phys. Soc. Japan Suppl. **21**, 366 (1966).

[22] M. Cardona, J. Phys. Chem. Solids **17**, 336 (1961).

[23] H. Piller, J. Phys. Chem. Solids **24**, 425 (1963).

[24] H. B. Harland and J. C. Woolley, Can. J. Phys. **44**, 2715 (1966).

crystal growth direction. The samples were either bridge-shaped or of the Van der Pauw type, depending on the crystal diameter. Sample homogeneity was checked by taking duplicate Hall and resistivity measurements on various combinations of bridge arms or by successively permuting the contact arrangement on the Van der Pauw specimens. For the samples reported here, deviations in these measurements were less than ±5%. All measurements were obtained using conventional dc potentiometric techniques, taking the usual precautions to minimize the effects of thermal gradients and Hall probe misalignment.

RESULTS AND DISCUSSION

Figure 1 shows the variation of $R\sigma$ with $1/Re$ for a number of Te-doped n-type GaSb samples at 77°K. Where conduction is confined essentially to the principal band, the quantity $R\sigma$ is, of course, the Hall mobility, μ_{000} for that band.[25] Likewise, under these conditions, the quantity $1/Re$ represents the carrier density n. When the subsidiary band effects become noticeable—in our case for $1/Re > 6\times10^{17}$ cm^{-3}—the quantity $R\sigma$ is a function of n_{000}, n_{111}, μ_{000}, and μ_{111}, as given by the usual two-band equations.[26] The data points represented by x's indicate measured values of R and σ for single-crystal samples grown from stoichiometric melts. Growth of undoped GaSb under similar conditions invariably results in p-type material having room-temperature hole concentrations in the range $(1\text{–}2)\times10^{17}$ cm^{-3}. Furthermore, this value remains essentially constant along the direction of growth, indicating an effective segregation coefficient of approximately unity for the residual acceptors. It is reasonable to expect then that the n-type samples in question all have about the same concentration of compensating residual acceptors. The circles in Fig. 1, to be discussed later, are measurements on specimens produced under different growth conditions and presumably possessing lower densities of compensating acceptors.

The curves in Fig. 1 are theoretical values of $R\sigma$. These assume ionized-impurity scattering to be predominant at 77°K so that the mobility follows from Eq. (18), with N_I given by relation (2) and the values of the band parameters quoted in the previous section. Where only single-band effects need be considered, μ_I is assumed identical to $R\sigma$; otherwise the two-band equations are used to determine $R\sigma$ from the calculated value of μ and n for each band. The calculations of $\mu_{111}{}^I/\mu_{000}{}^I$ yielded ratios ranging from 0.07 to 0.1 as $1/Re$ varied from 6.0×10^{17} to 1.2×10^{18} cm^{-3} for the case where $\Delta E=0.086$ eV and $m_1{}^d/m_0{}^d=4.78$.

It is seen that the data points for the samples grown from stoichiometric melts and the curve calculated for $N_A=2.85\times10^{17}$ cm^{-3} are in good agreement for carrier concentrations less than about 8×10^{17} cm^{-3}. At higher carrier concentrations, where the effects of the [111] band become increasingly important, noticeable uncertainty exists. The uncertainties in values of parameters specifying the subsidiary band were pointed out in the previous section. In addition, it appears that scattering mechanisms other than those by ionized impurities should be given some weight. Any process, however, which merely reduces μ_{111} without lowering μ_{000} would cause an actual increase[27] in the predicted value of $R\sigma$—a situation not in agreement with observation. This tends to rule out the possible importance of an admixture of polar scattering which, because of the much higher value of $m_1{}^*$ compared to $m_0{}^*$, could be relatively more important in the [111] band[28] and therefore might reduce μ_{111} somewhat below the value given by $\mu_{111}{}^I$.

Other considerations are the interband scattering effects. Since, however, both the ionized-impurity scattering and the polar scattering emphasize small crystal momentum transfer,[29] it is likely that we can neglect the *interband* effects as compared to the *intraband* processes. Estimates of the ratios of scattering frequencies for the interband and intraband transitions via the ionized impurities have been made[30] and are presented in Appendix B. The ratio is the order of 1% for the largest ionized-impurity concentrations reported here.

One other mechanism which was examined was a type of electron-electron scattering, whereby charge carriers in the [000] band are scattered by the electrons populating the [111] band. Inasmuch as the latter class of electrons possess substantially larger masses and are therefore relatively immobile, an estimate of the effect of such scattering was made by treating the process as electron-ion scattering. The correction turns out to be significant, as is indicated in Fig. 1.

Because of the uncertainties in the values of the parameters specifying the [111] band as well as the greater complexity of the scattering, it does not appear worthwhile to infer much from attempts to fit the data by the theoretical curves in the region where two-band

---

[25] For GaSb of the carrier densities studied here, the Hall coefficient factor r in the expression $R=r/ne$ is approximately unity for the principal band. Therefore we do not distinguish between Hall mobility and conductivity mobility in the [000] band.

[26] See, for example, A. C. Beer, *Galvanomagnetic Effects in Semiconductors* (Academic Press Inc., New York, 1963), Chap. VI.

[27] This is because in the two-band expression,

$$R\sigma=\mu_{000}{}^H[1+(n_1/n_0)(\mu_{111}\mu_{111}{}^H/\mu_{000}\mu_{000}{}^H)]/[1+(n_1/n_0)(\mu_{111}/\mu_{000})],$$

the mobility ratios are sufficiently small that the denominator determines the dependence of $R\sigma$ on μ_{111}/μ_{000}.

[28] An approximate expression for the polar mobility due to Hilsum [C. Hilsum, Proc. Phys. Soc. (London) **76**, 414 (1960)] gives $\mu_{\text{polar}}\propto m^{*-3/2}$. Inasmuch as $\mu_I\propto m^{*-1/2}$, it follows that μ_{polar} will be relatively more important (i.e., it will be *lower* in magnitude) in the band with larger m^*.

[29] Except for a screening factor, the square of the interaction potential matrix element is given by $|U(\mathbf{k},\mathbf{k}')|^2\propto1/|\mathbf{k}-\mathbf{k}'|^2$. In the intraband case, the vector $\mathbf{k}'$ is measured from the band minimum and is relatively small, while in the interband situation it involves a good fraction of a reciprocal lattice vector.

[30] I. Adawi (unpublished).

effects predominate. In addition, the experimental data in this region are relatively sparse. We shall therefore confine ourselves to the qualitative observation that the results tend to favor the lower values for ΔE and m_l/m_0.

On the other hand, the agreement between theory and experiment at lower electron concentrations could be considered quite satisfying if it were possible to account for the fact that the acceptor density ($N_A=2.85\times10^{17}$ cm^{-3}) required to fit the mobility data is a factor of 2 to 3 higher than expectations based on the hole concentrations observed in undoped GaSb. Even though the n-type samples discussed here are rather heavily doped, the seemingly high estimate obtained for N_A does not appear to be the result of failure of the Born approximation. Blatt's[31] work indicates that when the Born approximation fails, the scattering cross sections are overestimated as compared to the results of a partial-wave treatment. This implies that even larger ionized-impurity contents and hence larger N_A values would be required to fit the data using a partial-wave treatment.

Nor does it appear that either lattice or electron-electron scattering would appreciably affect the results at the temperature and impurity contents considered here. Calculated values of μ_{polar} and $\mu_{\text{acoustical}}$ are both greater than 10^5 cm^2/V sec at 77°K, and because of the combined effects of degeneracy and compensation, the influence of electron-electron scattering will be relatively small.[32,33] One possibility is that the actual acceptor concentration is in fact larger than estimates obtained from room-temperature Hall-effect measurements on p-type GaSb. It has been suggested, for instance, that incorporation of the donor dopant (Te) is accompanied by the formation of acceptor states.[19] According to the mobility analysis, however, these additional acceptors must occur at a constant density such that the total acceptor concentration remains fixed at about 3.0×10^{17} cm^{-3}. This would require that the incorporation of Te introduce between 1 and 2×10^{17} cm^{-3} acceptors, independent of the Te content. A more likely possibility is that there exists a deep-lying acceptor level which is not appreciably ionized at room temperature in p-type material but which is present in constant density in all the samples designated by ×'s in Fig. 1. Such a level would, of course, be completely ionized in n-type material and could account for the apparently high estimates for N_A obtained from the mobility analysis. A variation of this suggestion, which is consistent with luminescence[34] and Hall-effect[35] data, is that the residual acceptors be double acceptors. Assuming a z^2 scattering law, each doubly charged center would scatter as effectively as four singly charged centers. Thus, to express the "effective" density of charged centers, we must replace Eq. (1) by

$$N_I^*=N_D+4N_A^=,\tag{23}$$

where each donor is assumed to be only single ionized. Since the carrier density, in this case, is given by

$$n=N_D-2N_A^=,\tag{24}$$

we obtain, in place of Eq. (2), the following expression

$$N_I^*=6N_A^=+n.\tag{25}$$

A comparison of Eqs. (2) and (25) shows that in evaluating N_A from mobility data, we must account for a factor of 3 if the acceptors are doubly ionized. Under such conditions the calculated curve in Fig. 1 would correspond to an acceptor density of 9.5×10^{16} cm^{-3}—a value in good agreement with other estimates for the residual acceptor concentration.

With regard to the experimental data in Fig. 1, the circles lying above the calculated curve B represent results of measurements on samples taken from Te-doped ingots grown from antimony-rich melts. It has been demonstrated that growth of undoped GaSb from Sb-rich melts results in a reduction in the residual acceptor density,[36,37] and that the larger the Sb-to-Ga ratios in the melt, the greater the reduction in N_A. The samples in question (i.e., those for which the data points are designated by circles) were not all obtained from the same melt composition so that the concentration dependence of mobility exhibited by these samples is not characteristic of the behavior obtained under conditions of constant acceptor density. The samples corresponding to the largest Sb-to-Ga ratios are those with electron concentrations of 5.4×10^{16} and 3.7×10^{17} cm^{-3}, respectively. These two samples should contain about the lowest density of compensating centers of any n-type GaSb prepared to date. It should be noted that in spite of this, the mobility is an increasing function of carrier concentration so that the effects of compensation are still significant. Analysis of these two samples, assuming singly charged acceptors, yields estimates for N_A^- of 1.2×10^{17} cm^{-3} for each sample. If doubly charged centers are assumed, the estimate for $N_A^=$ is 4.0×10^{16} cm^{-3}, again in better agreement with the hole concentrations observed in p-type (undoped) GaSb grown from similar melt compositions.[37]

It appears that within the framework of the present mobility calculation, it is possible to account for both the magnitude and concentration dependence of electron mobility in Te-doped GaSb grown from both stoichiometric and antimony-rich melts if the residual acceptors are assumed to be doubly charged. It can also be shown that this assumption is consistent with observations

---

[31] F. J. Blatt, J. Phys. Chem. Solids 1, 262 (1957).

[32] J. Appel, Phys. Rev. **122**, 1760 (1961).

[33] I. M. Dykman and P. M. Tomchuk, Fiz. Tverd. Tela 4, 1082 (1962) [English transl.: Soviet Phys.—Solid State 4, 798 (1962)].

[34] A. Mooradian, Ph.D. thesis, Purdue University, 1966 (unpublished).

[35] C. S. Fuller and K. B. Wolfstirn, J. Appl. Phys. 36, 3613 (1965).

[36] D. Effer and P. J. Etter, J. Phys. Chem. Solids **25**, 451 (1964).

[37] F. J. Reid, R. D. Baxter, and S. E. Miller, J. Electrochem. Soc. **113**, 713 (1965).

TABLE I. Characteristics of specimens before (n^i,μ^i) and after (n^f,μ^f) treatment with lithium, including mobility (μ') in untreated specimens of carrier density n^f.

$n^i\times10^{-17}$ (cm^{-3})	μ^i (cm^2/V sec)	$\Delta n^f\times10^{-17}$ (cm^{-3})	μ^f (cm^2/V sec)	μ' (cm^2/V sec)	$\Delta n\times10^{-17}$ (cm^{-3})	$-\Delta N_I^*\times10^{-17}$ (cm^{-3})	$\frac{-\Delta N_I^*}{\Delta n}$
1.09	3100	1.68	5200	3900	0.59	1.66	2.8
1.13	3150	2.04	6000	4200	0.91	1.80	2.0
1.26	2900	1.96	5200	4000	0.70	2.3	3.3
1.93	3800	2.82	8000	4700	0.89	3.2	3.6
2.31	3900	3.17	6800	4900	0.86	2.7	3.1
2.78	4550	3.65	8100	5200	0.87	2.9	3.3

on the effects of lithium on the electrical properties of Te-doped n-type GaSb, and offers a plausible explanation for the behavior. Yep and Becker[6] have described these effects in a recent publication. In particular, they found that diffusion of Li into initially n-type GaSb with electron concentration less than about 7×10^{17} cm^{-3}, led to an increase in electron concentration and a relatively larger increase in Hall mobility at low temperatures. They arrive also at the conclusion that the observed effects are caused by a reduction in Coulomb scattering through an interaction between Li donors and compensating acceptors.

We have observed similar increases in values of electron concentration and Hall mobility at 300 and 77°K upon diffusion of Li at 400°C into Te-doped n-type GaSb grown from stoichiometric melts. These results are summarized in Table I, which gives the pre- and post-diffusion characteristics (n^i, μ^i and n^f, μ^f, respectively), and the mobility values (μ') for as-grown specimens which do not contain lithium but which have the same carrier concentration (n^f) as the GaSb: Li specimens. Since $\mu^i<\mu'<\mu^f$, it is clear that these results cannot be explained if the introduction of lithium merely increases the ionized donor content by an amount $\Delta N_D=\Delta n=n^f-n^i$, for if this were the case one would expect μ^f and μ' to be equal. These observations again suggest that lithium effects a decrease in the density or charge state of compensating acceptors thereby causing a reduction in Coulomb scattering—in agreement with the conclusions reached by Yep and Becker, and consistent also with the data in Fig. 1 for crystals grown from Sb-rich melts.

It is well established that lithium interacts with the residual acceptors in initially undoped p-type GaSb,[38–40] and it seems reasonable to assume that the same process occurs here, namely that ion pairs or complexes form between lithium donors and residual acceptors. In order to examine the effects of such an interaction on the properties on n-type material, we shall assume only that one or more lithium atoms interacts with an acceptor center initially in charge state z thereby decreasing the charge state on the center to $z-1$. Initially the electron concentration and "effective" ionized-impurity density are given by

$$n^i=N_D-zN_A\,, \tag{26}$$

$$N_I^{*i}=N_D+z^2N_A\,. \tag{27}$$

If α denotes the fraction of total acceptors undergoing an interaction, there are, after lithium diffusion, αN_A centers in charge state $(z-1)$ and $(1-\alpha)N_A$ centers remaining in charge state z.

Under these conditions,

$$n^f=N_D-\alpha(z-1)N_A-(1-\alpha)zN_A\,, \tag{28}$$

$$N_I^{*f}=N_D+\alpha(z-1)^2N_A+(1-\alpha)z^2N_A\,. \tag{29}$$

Equations (26)–(29) lead to the result that

$$\frac{-\Delta N_I^*}{\Delta n}=\frac{N_I^{*i}-N_I^{*f}}{n^f-n^i}=2z-1\begin{cases}=1 \text{ if } z=1\\ =3 \text{ if } z=2\\ =5 \text{ if } z=3\,.\end{cases} \tag{30}$$

According to Eq. (30), the ratio $-\Delta N_I^*/\Delta n$ depends only on the initial charge state of the compensating centers, and is greater than unity only for multiply charged centers.

Returning to Table I, we note that the last column gives values for the ratio $-\Delta N_I^*/\Delta n$, where ΔN_I^* is obtained from a mobility analysis using Eq. (18), and $\Delta n=n^f-n^i$ is the measured change in electron concentration. It may be seen that these ratios are all greater than unity and, in fact, tend to cluster around the value $-\Delta N_I^*/\Delta n=3$ predicted by Eq. (30) for doubly charged centers.

The observed changes in electron concentration are also consistent with the proposed model. From Eqs. (26) and (28), $\Delta n=\alpha N_A\leq N_A$. In view of our previous discussions we would expect Δn to be independent of Te content and α close to unity so that Δn values approaching but never exceeding 0.95×10^{17} cm^{-3} are anticipated. Although there are some variations among the samples listed in Table I, there is no clear-cut dependence of Δn on n, and the values are mostly close to but less than 0.95×10^{17} cm^{-3}.

Thus, the mobility analysis presented here not only accounts for the observed concentration dependence of electron mobility in Te-doped GaSb at 77°K and for the increased mobilities observed in samples which have

---

[38] H. J. Hrostowski and C. S. Fuller, J. Phys. Chem. Solids **4**, 155 (1958).

[39] R. D. Baxter, R. T. Bate, and F. J. Reid, J. Phys. Chem. Solids **26**, 41 (1965).

[40] M. H. Van Maaren, J. Phys. Chem. Solids **27**, 472 (1966).

been grown from Sb-rich melts, but also leads to a plausible interpretation of the results of lithium additions. Furthermore, the conclusions regarding the density and charge state of the residual acceptors resulting from our analysis of Te-doped n-type samples are found to be consistent with expectations based upon the reported characteristics of undoped p-type GaSb.

The present results also indicate that the importance of impurity scattering, and particularly the effects of compensation, on the room-temperature properties of Te-doped GaSb have been severely underestimated. Ehrenreich's[13] theoretical treatment of the electron mobility, taking into account the combined effects of polar scattering, conduction-band nonparabolicity, and intervalley scattering, results in a lattice mobility at 300°K of about 14 000 cm^2/V sec. His analysis of Sagar's[1] Sample 1B yielded an ionized-impurity mobility of 37 000 cm^2/V sec, suggesting that lattice scattering was dominant, and leading to about a factor-of-3 discrepancy between predicted (14 000 cm^2/V sec) and observed ($\sim$4000 cm^2/V sec) values of room-temperature mobility. However, Ehrenreich's estimate of the impurity mobility did not allow for compensation in the samples. Our investigations indicate that in essentially all n-type samples which have been produced to date the effects of compensation are considerable. If, therefore, we assume compensation by 1×10^{17} cm^{-3} doubly charged acceptors, we obtain an impurity mobility at 300°K at about 6500 cm^2/V sec for samples similar to Sagar's IB. Combined reciprocally with a lattice mobility of 14 000 cm^2/V sec, this results in a total mobility in the [000] minimum of about 4400 cm^2/V sec. Correcting this for multiband effects, we calculate $R\sigma=3500$ cm^2/V sec at 300°K, in good agreement with experiment.

ACKNOWLEDGMENTS

We wish to express our thanks to Dr. I. Adawi for helpful contributions and to Professor W. M. Becker who sent us reports of his work in advance of publication. Comments on the manuscript by Dr. E. M. Baroody are also appreciated.

APPENDIX A

A correction can be made very simply for the effect of eccentricity of the [111] minima on the mobility in that band if we assume τ to be isotropic over an energy ellipsoid. This assumption is admittedly not very satisfying,[41] but it has nevertheless proved fairly satisfactory in the case of n-type germanium,[42] where the [111] spheroids are apparently substantially more eccentric ($m_l/m_t\sim19$) than in GaSb.

Under the above assumption, the conductivity mobility for sets of [111] spheroids having cubic symmetry is given by[43]

$$\mu=\tfrac{1}{3}[2/m_t+1/m_l]e\langle\tau\rangle,$$

where the symbols $\langle\ \ \rangle$ represent the usual average over energy. This average is defined in the case of classical statistics by

$$\langle g(x)\rangle\equiv\frac{4}{3\sqrt{\pi}}\int_0^\infty g(x)x^{3/2}e^{-x}dx,\quad x\equiv\epsilon/KT,$$

and by an equally familiar expression in the case of Fermi-Dirac statistics.[44] Expressing the ratio of longitudinal and transverse masses by the anisotropy ratio ($m_l/m_t\equiv\mathrm{K}$), and introducing the density of states mass

$$m_d\equiv(m_1m_2m_3)^{1/2}=m_t^{2/3}m_l^{1/3},$$

we obtain

$$\mu=\frac{e\langle\tau\rangle}{m_d}\frac{2\mathrm{K}+1}{3\mathrm{K}^{2/3}}=\frac{2\mathrm{K}+1}{3\mathrm{K}^{2/3}}\mu_s,$$

where μ_s is the conductivity mobility for spherical surfaces ($\mathrm{K}=1$).

In a similar fashion, one may show[45]

$$R=\frac{\sigma_{123}}{\sigma_0^2}=\frac{3\mathrm{K}(\mathrm{K}+2)}{(2\mathrm{K}+1)^2}R_s,$$

$$\mu^H=\frac{\mathrm{K}(\mathrm{K}+2)}{\mathrm{K}^{2/3}(2\mathrm{K}+1)}\mu_s{}^H.$$

For the case where $\mathrm{K}=8.6$, the above equations yield

$$\mu=1.44\mu_s,\quad R=0.826R_s,\quad \mu^H=1.193\mu_s{}^H.$$

APPENDIX B

A comparison of the scattering rates for interband and intraband processes for a screened Coulomb potential has been made by Adawi,[30] and the details are presented below.

For an energy-conserving process with initial state having wave vector[46] $\mathbf{k}$ and the final state $\mathbf{k}'$, the scattering frequency is conventionally given by

$$\left(\frac{1}{\tau}\right)=-\left(\frac{\partial f}{\partial t}\right)_{\text{coll}}\Big/f_1(\mathbf{k})$$
$$=\frac{V}{(2\pi)^3}\int\frac{2\pi}{\hbar}|U_{\mathbf{kk}'}|^2\left[1-\frac{f_1(\mathbf{k}')}{f_1(\mathbf{k})}\right]\times\delta[E(\mathbf{k})-E(\mathbf{k}')]d^3k',\quad\text{(B1)}$$

---

[41] C. Herring, Bell Syst. Tech. J. 34, 237 (1955).

[42] E. G. S. Paige, in *Progress in Semiconductors*, edited by A. F. Gibson and R. E. Burgess (John Wiley & Sons, Inc., New York, 1964), Vol. 8, p. 108.

[43] This result follows directly from Herring's equations (Ref. 41), when account is taken of the different definition of the average.

[44] See, for example, A. C. Beer, *Galvanomagnetic Effects in Semiconductors* (Academic Press Inc., New York, 1963), pp. 94, 123.

[45] Reference 44, pp. 222–228.

[46] The origin for both $\mathbf{k}$ and $\mathbf{k}'$ is taken to be the [000] extremum.

where the transition rate from state **k** to **k**′ is $(2\pi/\hbar)\times|U_{\mathbf{kk}'}|^2\delta[E(\mathbf{k})-E(\mathbf{k}')]$, and the volume enclosing the *total number* of charge carriers is V. The quantity $f_1(\mathbf{k})[\equiv f(\mathbf{k})-f_0(\mathbf{k})]$ is the deviation in the distribution function from equilibrium.

The interaction matrix element is given approximately by

$$|U_{\mathbf{kk}'}|^2=N_I^{(T)}\left|\int\frac{e^{-i\mathbf{k}'\cdot\mathbf{r}}}{(\sqrt{V})}\frac{e^2}{\kappa r}e^{-r/a}\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{(\sqrt{V})}d^3r\right|^2 \tag{B2}$$

$$=\frac{N_I^{(T)}}{V^2}\left|\frac{4\pi e^2}{\kappa}\frac{1}{a^{-2}+|\mathbf{k}'-\mathbf{k}|^2}\right|^2, \tag{B3}$$

where $N_I^{(T)}$ is the *total* ionized-impurity content and a is the Thomas-Fermi screening length.

For interband transitions where **k** and **k**′ are in different bands, namely, the [000] and [111] band we have

$$\mathbf{k}'=\mathbf{K}_0+\Delta\mathbf{k}', \tag{B4}$$

$$\mathbf{k}'\approx\mathbf{k}+\mathbf{K}_0,\quad |\mathbf{k}|\ll|\mathbf{K}_0|, \tag{B5}$$

$$d^3k'=(\Delta k')^2d(\Delta k')d\Omega'=\frac{m_1^*}{\hbar^2}\left(\frac{2m_1^*}{\hbar^2}[E(\mathbf{k}')-E_0']\right)^{1/2}\times dE(\mathbf{k}')d\Omega'. \tag{B6}$$

In the above, $\Delta\mathbf{k}'$ is the wave vector measured from the [111] minimum and E_0' is the energy of the [111] band edge; $\mathbf{K}_0$ is a vector in reciprocal lattice space connecting the [000] to a [111] minimum. For simplicity, the energy surfaces are assumed to be spherical.

In view of approximation (B5), Eq. (B3) becomes, for the interband process,

$$|U_{\mathbf{kk}'}|^2\approx\frac{N_I^{(T)}}{V^2}\left[\frac{4\pi e^2}{\kappa}\frac{1}{a^{-2}+K_0^2}\right]^2. \tag{B7}$$

Since the above expression is independent of **k**′, the integration of the first term in the integrand in Eq. (B1) is trivial and the second term yields zero since $f_1(\mathbf{k}')$ is an odd function of **k**′. The result is

$$\left(\frac{1}{\tau}\right)_{0\to1}=\frac{\nu N_I}{(2\pi)^3}\left(\frac{4\pi e^2}{\kappa}\right)^2\left(\frac{1}{a^{-2}+K_0^2}\right)^2\frac{2\pi}{\hbar}\frac{4\pi m_1^*}{\hbar^2}\times\left(\frac{2m_1^*}{\hbar^2}[E(\mathbf{k})-E_0']\right)^{1/2}, \tag{B8}$$

where ν is the number of equivalent minima in the [111] band and N_I is the ionized-impurity *density*.

In the case of the intraband Coulomb scattering the matrix element, Eq. (B3), becomes

$$|U_{\mathbf{kk}'}|^2=\frac{N_I^{(T)}}{V^2}\left|\frac{4\pi e^2}{\kappa}\frac{1}{a^{-2}+2k^2(1-\cos\theta)}\right|^2,\quad k=k', \tag{B9}$$

where θ is the angle between **k** and **k**′, and the equality of $|\mathbf{k}|$ and $|\mathbf{k}'|$ results from the conservation of energy. The expression (B1) now involves a $\cos\theta$ in the denominator from (B9) and $a(1-\cos\theta)$ in the numerator from the factor $[1-f_1(\mathbf{k}')/f_1(\mathbf{k})]$. Integration over angles and the δ function of energy yields the familiar Brooks-Herring result, namely

$$\left(\frac{1}{\tau}\right)_{0\to0}=N_I\frac{2\pi}{\hbar}\frac{e^4}{\kappa^2}\frac{m_0^*}{\hbar^2k^3}\left[\ln(1+4k^2a^2)-\frac{4k^2a^2}{1+4k^2a^2}\right]. \tag{B10}$$

A comparison of scattering times at the Fermi level yields

$$\frac{(1/\tau)_{0\to1}}{(1/\tau)_{0\to0}}=\frac{32\nu_1m_1^*(m_1^*m_0^*)^{1/2}\zeta[\zeta(\zeta-E_0')]^{1/2}}{\hbar^2(a^{-2}+K_0^2)X(\zeta)},$$

where $X(\zeta)$ is the slowly varying logarithmic term which was given in Eq. (21), and the screening length a is the expression in Eq. (16). The magnitude of $\mathbf{K}_0$ is the order of the radius of the Brillouin zone (i.e., $K_0\sim\pi/a_L$, where a_L is the lattice constant).

For GaSb we took $m_0^*=0.04m$, $m_1^*=0.19m$, $\nu_1=4$, and $E_0'=0.086$ eV above the [000] band edge. The lattice constant[47] a_L is 6×10^{-8} cm, and for the largest impurity density we had $\zeta=15KT$, $X(\zeta)=0.3$ and $a\approx1.5\times10^{-7}$ cm. These parameters give the result

$$\frac{(1/\tau)_{0\to1}}{(1/\tau)_{0\to0}}<0.01$$

at the largest impurity concentration studied.

---

[47] G. Giesecke, in *Semiconductors and Semimetals*, edited by R. K. Willardson and A. C. Beer (Academic Press, Inc. New York, 1966), Vol. 2, p. 70.