

## Temperature Pulses in Dilute Liquid He<sup>3</sup>-He<sup>4</sup> Mixtures near 0°K\*

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The speed of second sound in dilute liquid He<sup>3</sup>-He<sup>4</sup> mixtures in concentrations up to 0.35% He<sup>3</sup> has been measured from 0.2 to 1°K at different pressures up to 20 atm. A pulsed time-of-flight technique was used. The second-sound values have been analyzed on Pomeranchuk's model, and the effective mass of the He<sup>3</sup> excitations has been obtained at different pressures, as well as different values of the roton parameters  $\Delta$  and  $\mu$ . The ratios of the effective mass of He<sup>3</sup> to the mass of a He<sup>3</sup> atom were found to be 2.55 at 46 cm Hg, 2.67 at 5.4 atm, 2.84 at 11.0 atm, and 3.03 at 19.8 atm. At the lowest temperature and especially at high pressures, the observations were complicated by poor coupling between phonons and He<sup>3</sup> excitations in the liquid, giving rise to two received pulses. From these measurements an estimate was made of a quantity  $\lambda^*$  that is thought to be closely related to the mean free path of phonons scattered by He<sup>3</sup> atoms. It was found that  $\lambda^* \propto T^{-4}$ .

### INTRODUCTION

IT is well known that temperature variations in liquid helium II obey a wave equation. Such behavior, called second sound, has been found not only in pure He<sup>4</sup> but also for solutions of He<sup>3</sup> in He<sup>4</sup>, as predicted by Pomeranchuk<sup>1</sup> and observed experimentally by Lynton and Fairbank,<sup>2</sup> King and Fairbank,<sup>3</sup> Elliott and Fairbank,<sup>4</sup> Sandiford and Fairbank,<sup>5</sup> and Niels-Hakkenberg, Meermans, and Kramers.<sup>6</sup> The present work was a continuation of the work by King and Fairbank, being concerned with dilute solutions in the temperature region below 1°K, particularly in the range where Pomeranchuk's analysis enables the velocity of second sound to be readily calculated in terms of the effective mass of He<sup>3</sup>.

The work to be discussed here extended the measurements to just below 20-atm pressure, giving the effective mass as a function of pressure. The measurements also enabled an estimate to be made of the phonon-He<sup>3</sup> mean free path and of the roton parameters  $\Delta$  and  $\mu$ .

### EXPERIMENTAL DETAILS

The experimental method was very similar to that used in the earlier work.<sup>3</sup> Temperatures below 1°K

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<sup>1</sup> I. Pomeranchuk, *Zh. Eksperim. i Teor. Fiz.* **34**, 33 (1949).

<sup>2</sup> E. A. Lynton and H. A. Fairbank, *Phys. Rev.* **80**, 1043 (1950).

<sup>3</sup> J. C. King and H. A. Fairbank, *Phys. Rev.* **93**, 21 (1954).

<sup>4</sup> S. D. Elliott and H. A. Fairbank, in *Proceedings of the Fifth International Conference on Low Temperature Physics and Chemistry* (University of Wisconsin Press, Madison, Wisconsin, 1958), p. 180.

<sup>5</sup> D. J. Sandiford and H. A. Fairbank, in *Proceedings of the Seventh International Conference on Low Temperature Physics* (University of Toronto Press, Toronto, Ontario, Canada, 1961), p. 641. This was a report on the results given in the present paper. The present paper gives a more comprehensive analysis of the results.

<sup>6</sup> C. G. Niels-Hakkenberg, L. Meermans, and H. C. Kramers,

were achieved by adiabatic demagnetization of a pill of about 50 g of compressed chromic methylamine alum crystals connected by means of a copper shank to the second-sound cavity, also of electrolytic copper. This is shown in Fig. 1. The pill and cavity were kept from touching the exchange-gas can by two thin stainless-steel spacers at the top and bottom, and the whole was suspended in place by a cupro-nickel fill line to the cavity, as shown. The vacuum seal between the exchange-gas can and the top flange was effected by an O ring of lead-indium alloy wire about 0.015 in. in diameter. Through the top flange passed the exchange-gas pumping line, the fill line to the cavity, and the electrical leads. These lead-ins were made with Stupakoff Kovar glass seals. The exchange-gas can and its

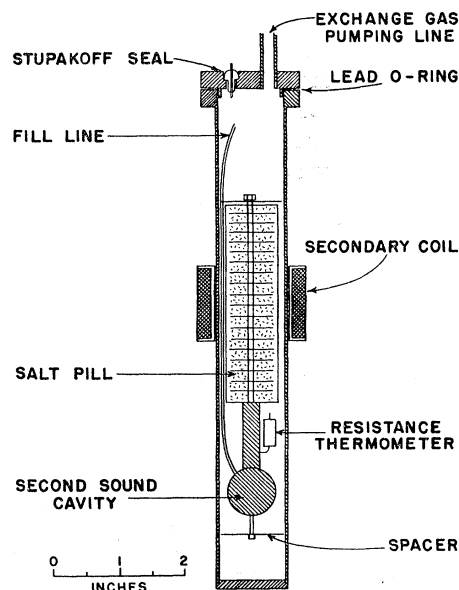


FIG. 1. Exchange-gas can containing second-sound cavity and paramagnetic salt pill. The apparatus shown is immersed in liquid helium at about 1°K.

in *Proceedings of the Eighth International Conference on Low Temperature Physics* (Butterworths Scientific Publications Ltd., London, England, 1963), p. 45.

contents were surrounded by liquid  $\text{He}^4$  at about  $1^\circ\text{K}$ .

The second-sound measurements were made with two different cavities. The first cavity (the "horizontal" cavity) shown in Fig. 2 allowed the temperature pulses to propagate in a horizontal direction through a distance of  $4.98 \pm 0.02$  mm. The second cavity (the "vertical" cavity) was similar in construction but rotated through  $90^\circ$  and had a vertical second-sound channel  $9.50 \pm 0.05$  mm long. Both cavities had readily demonstrable transducers which were vacuum sealed to the cavity with lead-indium O rings. Small Stupakoff Kovar glass seals for the electrical lead-ins were also used as mechanical supports for the I.R.C.<sup>7</sup> carbon-resistor boards whose purposes were to generate and detect the temperature changes. The resistor boards ( $500\Omega/\square$ ) were silver painted to recessed screws passing through the Kovar glass seals. The carbon covering of the resistor boards was scratched through so that only a rectangular, uniform-current region of the boards would be active in the measurements.

The temperature  $T$  was measured initially by means of the susceptibility of the salt pill using a standard ballistic galvanometer technique, after calibration above  $1^\circ\text{K}$  with the temperature of the  $\text{He}^4$  bath. With no helium in the cavity and exchange gas pumped out, the heat leak to the cavity, when it was at low temperatures, was about 12 erg/sec. Calculation and experiment showed this to be due to conduction down the fill line (0.030-in. o.d., 0.003-in. i.d.). The resistance  $R$  of an I.R.C. 270- $\Omega$  resistor soldered to the cavity was monitored during this run with no helium, and the resistance was later used to give the temperature of the cavity during the experimental runs. The resistances were measured with a low-power sensitive ac bridge operating at 1 kc/sec. The salt pill and cavity were not in thermal equilibrium at the lowest temperatures. There was a temperature discontinuity between the copper shank of the salt pill and the chromic methylamine alum crystals<sup>8</sup> that has been found in a similar experiment in this laboratory<sup>9</sup> to be given by  $10^{-4}QT^{-2}$   $^\circ\text{K}$ , where  $Q$  is expressed in ergs/sec. Thus after the measured magnetic temperature of the salt was converted to its absolute temperature, there was a further correction to find the temperature of the cavity and hence the  $R$ - $T$  curve. However, the I.R.C. resistor and the cavity were in good thermal contact at all times and the resistor temperature could be used when helium was in the cavity. For each cooling down, the resistor was calibrated anew, since it was found that slight changes in the  $R$ - $T$  relation occurred after the system was warmed up to room temperature.

Measurements were taken in solutions of 0.07, 0.20, and 0.35%  $\text{He}^3$ . The  $\text{He}^3$ - $\text{He}^4$  mixtures were made from

<sup>7</sup> International Resistor Company.

<sup>8</sup> E. Mendoza, in *Experimental Cryophysics*, edited by F. E. Hoare, L. C. Jackson, and N. Kurti (Butterworths Scientific Publications Ltd., London, 1961), p. 165.

<sup>9</sup> M. Strongin, Ph.D. thesis, Yale University, 1961 (unpublished).

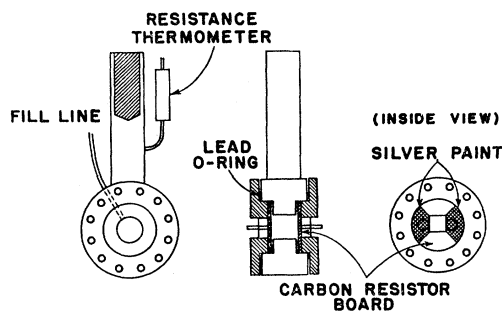


FIG. 2. Diagram of the horizontal second-sound cavity.

pure  $\text{He}^4$  and  $\text{He}^3$ , mixed in a glass gas-handling system with two mercury Toepler pumps. The concentrations were also checked in some instances with mass-spectrographic techniques. For measurements above 1-atm pressure the gas was transferred to the cavity by means of a stainless-steel Toepler pump using a system similar to that used previously by Walker.<sup>10</sup>

The method for measuring second-sound velocities in this low-temperature range was proscribed by two special considerations. First, the total power fed into the refrigerated sample must be modest to avoid too rapid warm-up. Second, steady heat flow through the  $\text{He}^3$ - $\text{He}^4$  sample must be avoided to prevent the inhomogeneous "heat flush" of  $\text{He}^3$  to the colder parts of the cavity. Consequently, a single shot pulse method was used in which the receiver current flowed for the smallest necessary amount of time.

A schematic diagram of the circuit used for the second sound measurements is shown in Fig. 3. The sequence of operations was controlled by the control box which when activated manually sent a 50-msec rectangular pulse to the current supply, tripping a millisecond relay which in turn fed a current pulse into the receiver  $R$ . This pulse rose with a time constant of

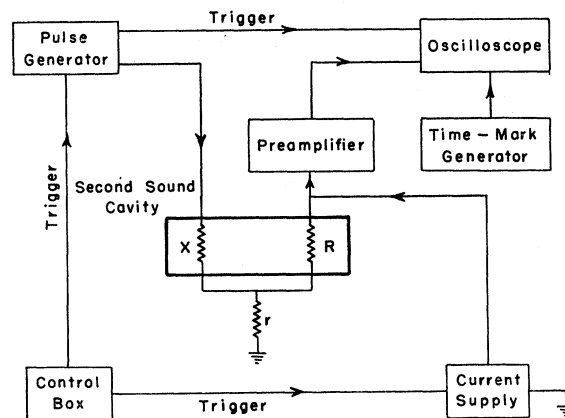


FIG. 3. Electronic block diagram for temperature pulse measurements.

<sup>10</sup> E. J. Walker, Ph.D. thesis, Yale University, 1960 (unpublished).

5 msec. The low-frequency pass of the preamplifier was about 10 kc/sec. Thus, the preamplifier did not saturate due to the pulse of several hundred volts applied across it, and after about 20 msec dc conditions effectively prevailed. Ten milliseconds before the end of this pulse, the control box sent a trigger pulse to the pulse generator which in turn triggered the oscilloscope time base. After a delay of about 100  $\mu$ sec (this could be varied) a pulse was fed to the transmitting resistor  $X$ . This latter pulse was from 1 to 50  $\mu$ sec long and appeared on the oscilloscope screen because of the voltage drop across the resistance  $r$  of the ground lead. The power dissipation in  $X$  caused a rise in the temperature of the surrounding helium. When the temperature pulse arrived at the receiver  $R$  its resistance was modulated giving a corresponding voltage across it. This was displayed on the oscilloscope screen together with marker pips from a time-mark generator; the traces were photographed for later analysis.

### EXPERIMENTAL RESULTS

The first second-sound measurements were made with pure  $\text{He}^4$  in the horizontal cavity in order to determine its effective length above 1°K, where the speed of second sound  $u_2$  is accurately known. The cavity was then taken to 0.2°K and broad temperature pulses were received characteristic of phonon propagation with long mean free paths in which scattering occurs mainly at the walls of the container (Fig. 4). As the temperature was raised the dispersive character of the receiver pulse diminished, and it increasingly took on the character of wave propagation where the mean free paths of the excitations—phonons and rotons—are short. Such phenomena have frequently been observed before.<sup>3,11-13</sup>

A  $\text{He}^3$ - $\text{He}^4$  mixture was then condensed into the cavity and measurements made down to 0.2°K. In general the observations of King and Fairbank were confirmed: The values of  $u_2$  were much lower than in the pure  $\text{He}^4$  and little dispersion occurred except close to 0.2°K. As Fig. 5(e) shows when the temperature is as high as 1°K the received pulses had a more definite rising edge in the mixtures than in pure  $\text{He}^4$  [see Fig. 4(e)]. This effect can be readily understood as due to the reduction in mean free path of the excitations by the  $\text{He}^3$  atoms. The results in the mixtures confirm that no significant amount of the broadening of the leading edges of the received pulses in Fig. 4 arise from time constants of the receiver and resistor.

#### The Two-Pulse Effect

At pressures higher than those at which these preliminary experiments were carried out, the results for

<sup>11</sup> D. V. Osborne, Proc. Phys. Soc. (London) **A64**, 114, (1951); Nature **162**, 213 (1948).

<sup>12</sup> D. de Klerk, R. P. Hudson, and J. R. Pellam, Phys. Rev. **93**, 28 (1954).

<sup>13</sup> H. C. Kramers, Tineke van Peski-Tinbergen, J. Wiebes, F. A. W. van den Burg, and J. C. Gorter, Physica **20**, 743 (1954).

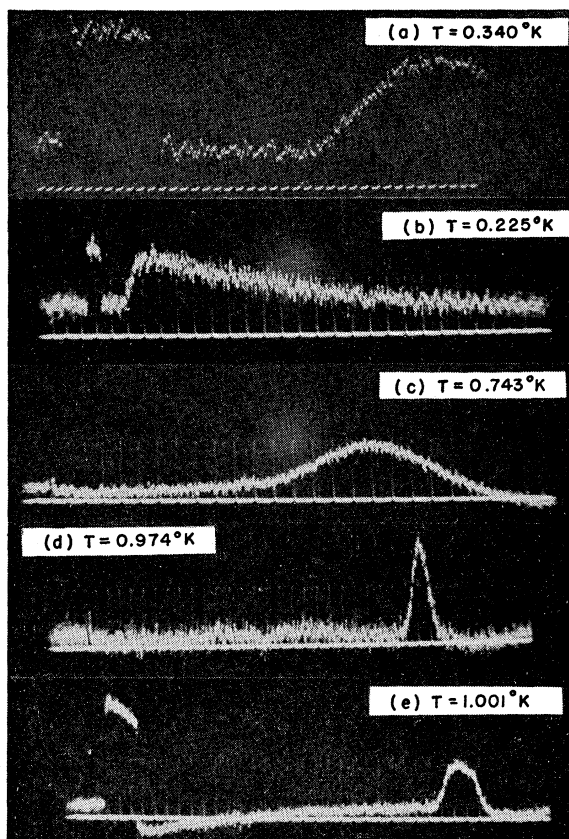


FIG. 4. Oscilloscope traces of the temperature pulses at the receiver in a run with the horizontal cavity in pure  $\text{He}^4$ . The pressure was 47 cm Hg. The first pulse at the left of each trace is a measure of the current flowing in the transmitting resistor as mentioned in the text. The marker pips are 1  $\mu$ sec apart in (a) and 10  $\mu$ sec apart in the other traces.

the mixtures became more complicated. At the highest pressures the complications are most marked and can be seen in Figs. 5(a)–5(d). In Fig. 5(a) it is seen that the input pulse gave rise to two received pulses: a fast pulse whose leading edge arrived with a speed corresponding to the phonon speed at 19.8 atm; and a slower pulse travelling at about the expected speed of second sound in mixtures. At temperatures lower than 0.2°K, the second pulse could not be observed for this concentration of 0.35%. The first pulse was exactly similar to a diffusion pulse of phonons in pure  $\text{He}^4$  (the decay of the pulse was determined by the low-frequency pass of the preamplifier). As the temperature increased, the fast pulse decreased in amplitude and the second pulse increased until only the second, slower pulse remained. The second pulse displayed echoes, as in Fig. 5(d), characteristic of wave motion where the pulse bounces off the receiving resistor, returns to the transmitter, and is again reflected back to the receiving resistor, and so on. It is significant that successive echoes do not noticeably spread out but only decrease in amplitude. As found by Fairbank and King,<sup>3</sup> at about 0.4°K the

received pulse had a good shape and did not display much dispersion.

Two possible explanations were devised to explain the low-temperature behavior.

(1) A phase separation might occur causing the mixture to split into two layers: one of pure He<sup>4</sup> and the other a He<sup>3</sup>-He<sup>4</sup> mixture rather richer in He<sup>3</sup> than before. Since the cavity was horizontal this arrangement would allow the independent propagation of two pulses, one in each phase, apart from some transmission across the interface.

(2) The two pulses might be propagated in a single phase by uncoupled excitations of the liquid mixture. In this low-temperature range, one expects second sound to take place almost entirely by means of the He<sup>3</sup> excitations which account for nearly all of the internal energy of the liquid. Thus, since it seems reasonable to suppose that phonons are generated at the transmitting heater, those phonons must excite He<sup>3</sup> atoms for their energy to be transmitted as second sound. However, should the mean free path of a phonon scattered by a He<sup>3</sup> atom be sufficiently large, some phonons will arrive at the receiver without scattering. Thus one might expect one pulse due to phonons arriving, and another, slower pulse due to the He<sup>3</sup> excitations.

A crucial test of the two suggestions, and one which would have enabled precise measurements to be made of a phase separation, was given by the use of a vertical cavity. In measurements with this cavity, if suggestion (1) is correct, the two phases which separate out will lie one above the other. The temperature pulse will move first through one phase and then through the other phase. The fastest pulse (granting the possibility of other pulses due to reflection at the interface) will travel with an apparent speed whose value is between the two speeds found with the horizontal cavity. If suggestion (2) is correct, the orientation of the cavity will have no effect whatsoever.

Experiments with the vertical cavity found that this latter was indeed the case. The leading edge of the fast pulse always arrived at a time corresponding to the phonon speed. (The traces of Fig. 5 were, in fact, taken with the vertical cavity.) Because of the occurrence of long mean free paths for the scattering of phonons by He<sup>3</sup> atoms shown by these experiments, and the consequent broadening of the second-sound pulse, measurements of  $u_2$  were made by determining the transit time between echoes, which were of similar shape as mentioned previously. Since these pulses are apparently obeying a wave equation, it follows that He<sup>3</sup>-He<sup>3</sup> scattering has a small mean free path, as expected from simple kinetic theory.

A qualitative estimate of the phonon-He<sup>3</sup> mean free path was made in the following way. For each concentration  $x$  and cavity length  $L$  at a certain pressure, the temperature was noted at which the phonon and second-sound pulses were equal in amplitude. Under these

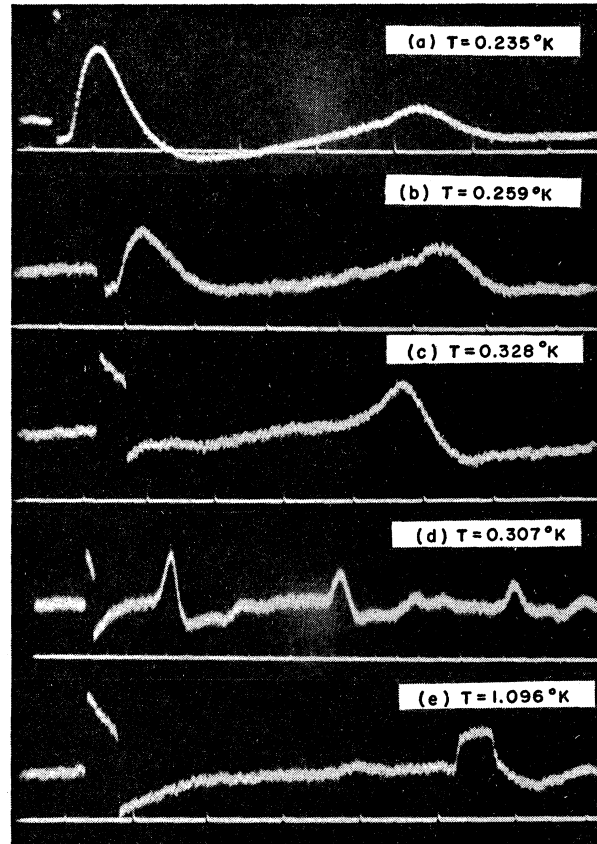


FIG. 5. Oscilloscope traces obtained with a 0.35% He<sup>3</sup>-He mixture in the vertical cavity. For (a), (b), (c), and (e) the pressure was 19.8 atm; for (d) 5.4 atm. The marker pips are 100  $\mu$ sec apart.

conditions, a certain fraction of the energy of the phonons created at the transmitter was given up to the He<sup>3</sup> excitations in the form of second sound. When the above criterion is satisfied we take, as a crude approximation, the length  $L$  to be proportional to the mean free path  $\lambda$ . Since the mean free path is proportional to  $x^{-1}$ , the results for different concentrations may be related to one another by normalizing to a constant concentration.

We define a quantity  $\lambda^*$  such that  $\lambda^* = L$  for those values of concentration, temperature, and pressure for which the above criterion is satisfied. Here  $\lambda^*$  is an estimate of a number which is proportional to the mean free path,  $\lambda$ .

On this basis Fig. 6 has been obtained. This gives  $\lambda^*$  as a function of temperature, normalized to a 0.1% concentration, for various pressures. Nearly all the data have been obtained at about 20 atm and here  $\lambda^* \propto T^{-4}$ . Khalatnikov and Zharkov<sup>14</sup> have calculated that  $\lambda \propto T^{-n}$  where  $4 < n < 5$ ; however, experiments of Niels-

<sup>14</sup> I. M. Khalatnikov and V. N. Zharkov, Zh. Eksperim. i Teor. Fiz. 32, 1108 (1957) [English transl.: Soviet Phys.—JETP 5, 905 (1957)].

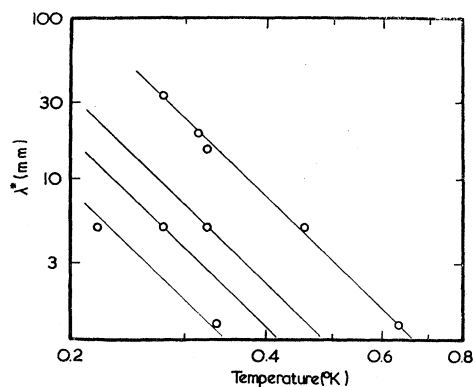


FIG. 6.  $\lambda^*$  as a function of temperature normalized to 0.1%  $\text{He}^3$  concentration for various pressures.  $\lambda^*$  is a measure of the phonon- $\text{He}^3$  mean free path.

Hakkenberg *et al.*<sup>6</sup> on the interpretations of the shape of phonon pulses in mixtures have given  $\lambda$  approximately varying as  $T^{-3}$ . The following simple argument gives a  $T^{-4}$  dependence.

When the temperature of the transmitting resistor board is raised relative to the constant helium temperature  $T$ , phonons diffuse from it into the helium that are characteristic of the temperature of the resistor board.<sup>15</sup> These phonons are peaked around a certain frequency  $\nu$ , where  $h\nu(kT)^{-1}$  is a constant of order unity. Since the phonon wavelengths are rather larger than the size of the  $\text{He}^3$  atoms the phonons are scattered by the  $\text{He}^3$  atoms according to Rayleigh's law. Thus we expect  $\lambda \propto c^5 T^{-4} (x\rho)^{-1}$ . This expression gives a pressure dependence through the variation of

$c$  and  $\rho$  with pressure. In going from low pressures to 20 atm  $c^5$  changes by a factor 6.6, whereas  $c^5\rho^{-1}$  changes by a factor 5.7. In view of the crudeness of this model, this change can be interpreted as good agreement with the observed change in  $\lambda^*$  of a factor of 10.

### Second Sound

The measurements of second sound were made by observations on the reflected echoes of the heat pulse. Measurements of  $u_2$  on a 0.35% concentration at various pressures are shown in Fig. 7 and given in the first two columns of Table I. The results were analyzed using Pomeranchuk's formulas<sup>1</sup> for  $u_2$ :

$$u_2^2 = \frac{\rho_s T}{\rho_n C} \left[ \left( S_4 + \frac{kx}{m_4} \right)^2 + \frac{kCx}{m_4} \right], \quad (1)$$

where

$$C = C_4 + \frac{3}{8}xR$$

and

$$\rho_n = \rho_{n,4} + x(m_3^*/m_4)\rho.$$

The various symbols are defined in Table II. At low temperatures where the vastly predominant excitations in equilibrium are the  $\text{He}^3$  atoms, the expression for  $u_2$  may be written

$$u_2^2 = \frac{5 kT}{3 m_3^*}, \quad (2)$$

which is just the speed of first sound in an ideal classical monatomic gas of particles of mass  $m_3^*$ . Taking into account Fermi statistics, Eq. (2) becomes with first-

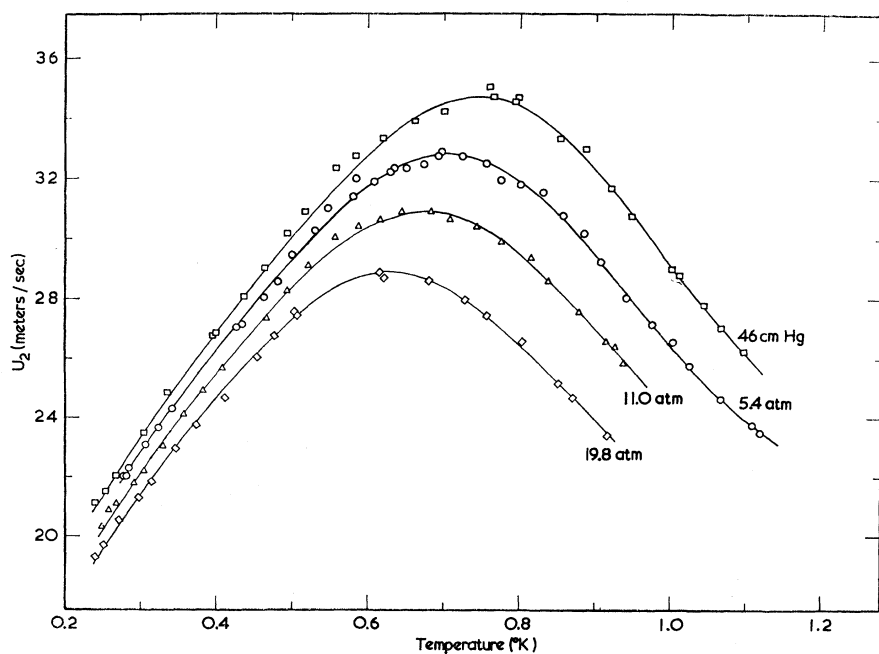


FIG. 7. The speed of second sound as a function of temperature for various pressures in a 0.35%  $\text{He}^3$ - $\text{He}^4$  mixture.

<sup>15</sup> I. M. Khalatnikov, *Zh. Eksperim. i Teor. Fiz.* **22**, 687 (1952).

TABLE I. Speed of second sound in 0.35% mixture at the indicated pressures.  $T$  is given in °K. The measured speed  $u_2$  and the calculated speed  $u_2'$  are in m/sec.

$T$	$u_2$	$u_2'$	$T$	$u_2$	$u_2'$
46 cm Hg:					
0.241	21.10	20.89	0.700	34.23	34.52
0.254	21.47	21.45	0.759	35.05	34.70
0.268	22.03	22.03	0.764	34.70	34.69
0.304	23.43	23.47	0.793	34.54	34.48
0.336	24.84	24.67	0.797	34.67	34.44
0.395	26.76	26.75	0.853	33.33	33.52
0.400	26.85	26.84	0.886	32.99	32.72
0.437	28.04	28.06	0.920	31.67	31.73
0.465	29.01	28.95	0.947	30.74	30.87
0.494	30.16	29.84	1.000	29.01	29.11
0.517	30.89	30.53	1.010	28.79	28.78
0.558	32.34	31.69	1.043	27.78	27.05
0.583	32.76	32.36	1.065	27.06	27.05
0.620	33.33	33.24	1.096	26.24	26.19
0.662	33.93	34.04			
5.4 atmospheres:					
0.277	22.03	21.89	0.673	32.48	32.70
0.282	22.03	22.09	0.693	32.76	32.78
0.285	22.29	22.20	0.697	32.90	32.79
0.307	23.10	23.05	0.724	32.76	32.75
0.324	23.68	23.68	0.756	32.48	32.52
0.342	24.28	24.32	0.775	31.93	32.29
0.427	27.05	27.09	0.800	31.80	31.89
0.434	27.14	27.31	0.830	31.28	31.28
0.463	28.04	28.20	0.856	30.77	30.64
0.481	28.57	28.74	0.884	30.16	29.88
0.500	29.46	29.29	0.905	29.23	29.26
0.530	30.28	30.12	0.904	28.04	28.20
0.547	31.02	30.56	0.973	27.14	27.19
0.581	31.40	31.36	1.002	26.57	26.33
0.584	31.99	31.42	1.024	25.76	25.71
0.608	31.93	31.89	1.107	23.75	23.77
0.630	32.20	32.25	1.065	24.67	24.67
0.635	32.34	32.32	1.118	23.51	23.56
0.650	32.34	32.50			
11.0 atmospheres:					
0.250	20.32	20.16	0.587	30.40	30.25
0.259	20.88	20.52	0.616	30.64	30.64
0.269	21.11	20.92	0.645	30.89	30.88
0.292	21.78	21.79	0.683	30.89	30.94
0.305	22.22	22.27	0.708	30.64	30.82
0.330	23.03	23.17	0.743	30.40	30.47
0.358	24.20	24.13	0.775	29.80	29.98
0.383	24.92	24.96	0.815	29.34	29.16
0.409	25.68	25.69	0.837	28.57	28.63
0.467	27.34	27.43	0.878	27.54	27.56
0.494	28.25	28.18	0.913	26.57	26.59
0.521	29.12	28.89	0.926	26.39	26.23
0.557	30.04	29.70	0.937	25.85	25.93
19.8 atmospheres:					
0.240	19.29	19.13	0.507	27.44	27.46
0.251	19.69	19.56	0.615	28.90	28.86
0.272	20.54	20.36	0.621	28.69	28.88
0.298	21.29	21.31	0.681	28.57	28.60
0.315	21.84	21.91	0.728	27.94	27.96
0.347	22.96	23.00	0.757	27.44	27.41
0.374	23.75	23.88	0.768	27.24	27.17
0.412	24.67	24.95	0.803	26.57	26.36
0.455	26.03	26.18	0.851	25.16	25.15
0.477	26.76	26.75	0.870	24.67	24.66
0.503	27.54	27.37	0.916	23.38	23.52

order correction<sup>16</sup>

$$u_2^2 = -\frac{5kT}{3m_3^*} \left[ 1 + \frac{N}{32V} \frac{h^3}{(m_3^* \pi kT)^{3/2}} \right]. \quad (3)$$

<sup>16</sup> In a preliminary report on this work (Ref. 5), the numerical

TABLE II. Symbols used in the text.

$u_2$	Speed of second sound
$\rho$	Density of liquid helium
$\rho_s$	Density of superfluid component
$\rho_n$	Density of normal component
$c$	Speed of first sound
$T$	Absolute temperature
$S$	Entropy per gram of helium
$C$	Specific heat per gram of helium
$m_3^*$	Effective mass of He <sup>3</sup> excitations
$m_4$	Mass of He <sup>4</sup> atom; the suffix 3(4) refers to pure He <sup>3</sup> (He <sup>4</sup> )
$k$	Boltzmann's constant
$R$	Gas constant
$x$	Molar concentration of He <sup>3</sup> in He <sup>4</sup>
$N$	Number of He <sup>3</sup> atoms
$V$	Volume
$\Delta$	Roton energy parameter
$p_0$	Roton momentum parameter
$\mu$	Roton effective mass

This holds so long as the temperature is not so low that the correction term becomes comparable with unity. This formula has been used to calculate the effective mass of He<sup>3</sup> at various pressures and temperatures below 0.4°K. The data give temperature-independent effective masses whose values are given in Table III for the different pressures and shown in Fig. 8 as a function of density. Also given in Fig. 8 are the values of  $m_3^*/m_3$  at low pressures found by other second-sound workers<sup>5</sup> and from recent thermal and magnetic measurements by Anderson *et al.*,<sup>17</sup> extrapolated to zero concentration.<sup>18</sup>

An estimate of the effective mass of a He<sup>3</sup> atom in He<sup>4</sup> at low pressure has been made from first principles

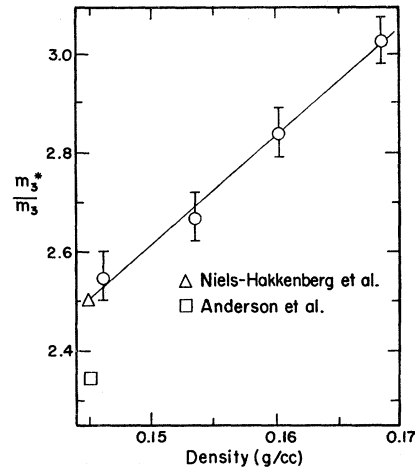


FIG. 8. The He<sup>3</sup> effective mass relative to the He<sup>3</sup> atomic mass as a function of liquid density.

denominator of the term in brackets was erroneously given as 2 rather than 32. In addition the temperature scale was uncorrected for the Kapitza jump mentioned earlier. The values of  $m_3^*$  have been recalculated; they are now lower in value than those of the report and are temperature-independent.

<sup>17</sup> A. C. Anderson, D. O. Edwards, W. R. Roach, R. E. Sarwinski, and J. C. Wheatley, *Phys. Rev. Letters*, **17**, 367 (1966).

<sup>18</sup> J. Bardeen, G. Baym, and D. Pines, *Phys. Rev. Letters* **17**, 372 (1966); *Phys. Rev.* **156**, 207 (1967).

TABLE III. A summary of roton parameters obtained from these and other measurements.

Pressure	$m_3^*/m_3$	$p_0/\hbar(A^{-1})$	$\Delta/k(^{\circ}K)$	$\mu/m_4$	Reference
vapor		$1.92\pm 0.03$	$8.70\pm 0.1$	$0.20\pm 0.02$	a
vapor		1.92	8.65	0.16	b
vapor		$1.91\pm 0.01$	$8.65\pm 0.1$	0.16	c
vapor		$1.91\pm 0.01$	$8.90\pm 0.15$	$0.29\pm 0.08$	d
0 atm		1.7	8.9, 8.7	0.5	e
46 cm Hg	$2.55\pm 0.05$	1.92	$8.90\pm 0.3$	$0.40\pm 0.2$	This paper
5 atm		$1.95\pm 0.02$	$8.45\pm 0.15$	$0.21\pm 0.06$	d
5 atm		$1.92\pm 0.03$	$8.25\pm 0.10$		a
5 atm		2.0	8.2, 8.5	0.15	e
5.4 atm	$2.67\pm 0.05$	1.96	$8.20\pm 0.3$	$0.14\pm 0.10$	This paper
10 atm		$2.00\pm 0.03$	$8.00\pm 0.15$	$0.16\pm 0.05$	d
10 atm		$1.93\pm 0.03$	$7.90\pm 0.10$		a
10 atm		2.2	7.8, 8.1	0.08	e
				+0.06	
11.0 atm	$2.84\pm 0.05$	1.99	$7.50\pm 0.3$	$0.06-0.03$	This paper
				+0.03	
19.8 atm	$3.03\pm 0.05$	2.02	$6.70\pm 0.3$	$0.06-0.02$	This paper
20 atm		$2.07\pm 0.04$	$7.25\pm 0.15$	$0.10\pm 0.03$	d
20 atm		$1.95\pm 0.04$	$7.50\pm 0.10$		a
20 atm		2.2	7.2, 7.5	0.06	e
25.3 atm		2.05	7.0		f

<sup>a</sup> R. L. Mills, Ann. Phys. (N.Y.) 35, 410 (1965).

<sup>b</sup> See Ref. 22.

<sup>c</sup> D. G. Henshaw and A. B. D. Woods, Phys. Rev. 121, 1266 (1961).

<sup>d</sup> See Ref. 23.

<sup>e</sup> C. Boghosian and H. Meyer, Phys. Rev. 152, 200 (1966). The first value of  $\Delta$  given in the table for this reference is calculated from entropy measurements and the second is derived from the density of the normal component.

<sup>f</sup> D. G. Henshaw and A. B. D. Woods, in *Proceedings of the Seventh International Conference on Low Temperature Physics* (University of Toronto Press, Toronto, Ontario, Canada, 1961), p. 539.

by Feynman.<sup>19</sup> Feynman constructed a possible wave function to describe the flow of a He<sup>3</sup> atom through He<sup>4</sup> at 0°K and obtained an estimate for  $m_3^*/m_3$  equal to 1.9. More recent theories<sup>18,20</sup> do not give a value for  $m_3^*$  based solely on microscopic considerations but instead give a variation of effective mass with concentration based on various experimental results.<sup>17</sup> The effective mass at zero concentration for both theories<sup>18,20</sup> is about  $2.34m_3$ , which differs from Feynman's value and from the experimental results obtained from second sound at low concentration ( $2.5m_3$ ). However, it may be significant that the second-sound values were obtained in the temperature range where the He<sub>3</sub> gas was nondegenerate whereas the value  $2.34m_3$  refers to experiments in the degenerate region.

Above 0.4°K the experimental values of  $u_2$  were compared with those obtained from Eq. (1) using the effective-mass values of Table III, and values  $C_4$ ,  $S_4$ , and  $\rho_{n,4}$  given by the Landau expressions in terms of phonon and roton parameters.<sup>21</sup> The procedure adopted was to assume the phonon speed<sup>22</sup> and roton parameters constant with temperature in the temperature range considered (below 1.1°K). For a fixed value of  $c$  and  $p_0$ ,  $\Delta$  and  $\mu$  were allowed to vary to find the best mean square fit to the experimental results. These gave best values of  $\Delta$  and  $\mu$  shown in Table III.

The  $p_0$  values for this procedure were chosen in the following way. At the vapor pressure the neutron-

scattering data of Yarnell, Arnold, Bendt, and Kerr<sup>23</sup> gives  $p/\hbar$  equal to  $1.92 \text{ \AA}^{-1}$ . This is equal, within experimental error, to the value deduced by Van den Meijenberg, Taconis, and de Bruyn Ouboter<sup>24</sup> from entropy measurements of He II. Thus there is agreement at this pressure. We have assumed that  $p_0 \propto \rho^{1/3}$  for other pressures (and densities). This is equivalent to assuming that the roton wavelength ( $\hbar/p_0$ ) is proportional to the mean distance between He<sup>4</sup> atoms. This assumption relies on the demonstration that the minimum in the elementary excitation curve (i.e., roton region) corresponds to the first maximum in the liquid-structure factor.<sup>25</sup>

In fact the fit did not give very definite values for  $\Delta$  and  $\mu$ , there being a range of correlated values for which a similar fit could be obtained. This correlation is such that an increase in  $\Delta$  gives a fit with increase in  $\mu$ . For comparison, Table III also gives values of the roton parameters obtained by other workers.

The third column of Table I gives calculated values of  $u_2$  using the roton parameters and effective masses for He<sup>3</sup> in Table III. The calculated values are shown in Fig. 7 as the smooth curves.

#### ACKNOWLEDGMENTS

The authors wish to thank their colleagues Myron Strongin, David Caplin, and George Zimmerman for experimental assistance.

<sup>19</sup> J. L. Yarnell, G. P. Arnold, P. J. Bendt, and E. C. Kerr, Phys. Rev. 113, 1379 (1959).

<sup>20</sup> V. J. Emery, Phys. Rev. 148, 138 (1966), and (to be published).

<sup>21</sup> L. D. Landau, J. Phys. USSR 5, 71 (1941).

<sup>22</sup> J. H. Vignos and H. A. Fairbank, Phys. Rev. 147, 185 (1966).

<sup>23</sup> C. J. N. Van den Meijdenberg, K. W. Taconis, and R. de Bruyn Ouboter, Physica 27, 197 (1961).

<sup>25</sup> R. P. Feynman and Michael Cohen, Phys. Rev. 5, 1189 (1956).

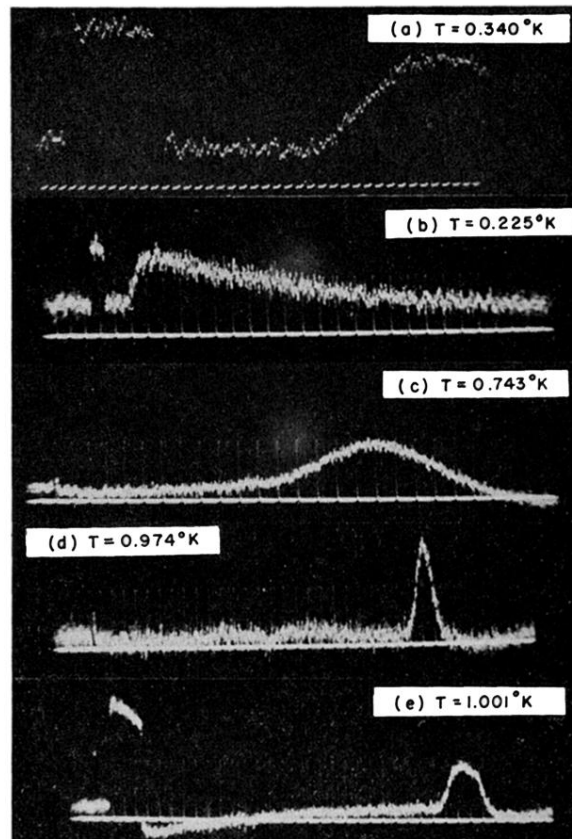


FIG. 4. Oscilloscope traces of the temperature pulses at the receiver in a run with the horizontal cavity in pure  $\text{He}^4$ . The pressure was 47 cm Hg. The first pulse at the left of each trace is a measure of the current flowing in the transmitting resistor as mentioned in the text. The marker pips are  $1\ \mu\text{sec}$  apart in (a) and  $10\ \mu\text{sec}$  apart in the other traces.



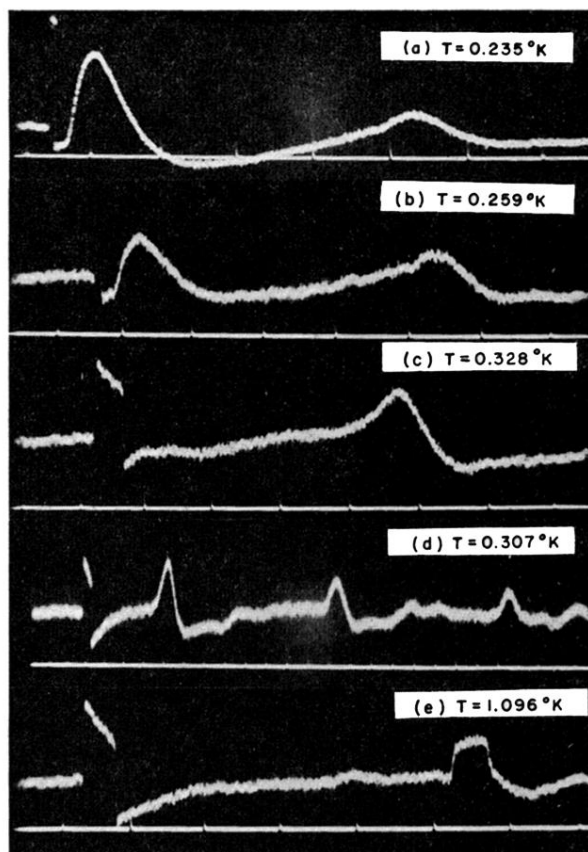


FIG. 5. Oscilloscope traces obtained with a 0.35%  $\text{He}^3$ -He mixture in the vertical cavity. For (a), (b), (c), and (e) the pressure was 19.8 atm; for (d) 5.4 atm. The marker pips are 100  $\mu\text{sec}$  apart.