Pion Electromagnetic Form Factor from Coulomb Interference*

GEOFFREY B. WEST
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York†
and
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California
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Using recent 24-MeV $\pi^{\pm}$-α elastic scattering data we critically analyze the Hofstadter-Sternheim proposal for measuring the pion electromagnetic form factor. A phenomenological nonlocal potential, whose parameters are fixed directly from the data, is used to describe the nuclear interaction. Using these parameters, a type of distorted-wave Born approximation is then employed to calculate the Coulomb contributions. We conclude that (i) nuclear distortion effects on the Coulomb contribution are large; (ii) the fits are not very sensitive to the pion radius and from the data only a conservative upper limit of $\sim$1.5 F can be reasonably given; (iii) experimental statistics need to be greatly improved before a more definitive result can be obtained; (iv) sensitivity to the model parameters seems to be smallest in the region of backward scattering ($\theta_{\text{lab}}\gtrsim 100°$), where sensitivity to the pion radius is greatest; and (v) the fruitfulness of going to much higher energies (>100 MeV) is probably quite limited. Some suggestions for present and future experimental and theoretical work are briefly discussed.

I. INTRODUCTION

ONE of the more striking successes of the dispersion-theory approach to strong-interaction physics is the insight it gives into the nature of the electromagnetic (em) form factors of the nucleons.[1] Via the dispersion relations one can directly relate the nucleon form factors to the integrals over the pion-nucleon (π-n) scattering amplitudes folded into the form factor of the pion, F_π. Quantitative ignorance of F_π has not yet allowed the direct verification of such a relationship, and further progress can only be made by subjecting F_π itself to a dispersion treatment, thereby relating it to π-π amplitudes. This approach culminated in the early 1960's with the work of Frazer and Fulco,[2] who concluded that one could fit the known data only if there existed strong resonances in the π-π system such as those earlier suggested by Nambu.[2] The existence of these resonances is now well substantiated, and nowadays one commonly parametrizes F_π by the spectral representation

$$F_\pi \sim (1-q^2/m_\rho^2)^{-1}, \tag{1}$$

where m_ρ is the mass of the 2π resonant state, the ρ meson, and q is the usual 4-momentum transferred to the pion.[3] A good measurement of F_π would certainly be welcome supporting evidence towards a fuller understanding of the em structure of the hadrons. This paper will be devoted to a critical study of a new method proposed by Hofstadter for measuring F_π.[4,5]

Before discussing this method, let us briefly review and discuss previous methods used to obtain F_π. Basically there are only two:

(i) The "direct" method: Pions are scattered from atomic electrons. Since the massive pion cannot transfer very much momentum to the relatively light electron, this method is limited to only very low q^2 values.[6] In an experiment performed at Princeton, Cassel *et al.*[7] have been able to deduce an upper bound for the root-mean-square pion radius of $\sim$3 F.

(ii) The "indirect" method: One attempts to isolate the pion-pole contribution (directly proportional to F_π) in the electroproduction of π's from protons.[8] The main drawback to this method is that the production process itself is not sufficiently well understood.[9] Before a reliable F_π can be deduced, the dominance of the N and N^* contributions, for instance, must be known quite accurately. Nevertheless, Berkelman *et al.*[10] have been able to deduce reasonable and consistent values of F_π by using a coincidence experiment in which the kinematics are arranged so as to maximize the pion-pole contribution. Gourdin[11] has recently suggested a program based upon a pure isobar model (successful in photoproduction studies) from which one might eventually hope to

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† Present address.

[1] For a comprehensive review of this work which includes a complete list of references, see S. D. Drell and F. Zachariasen, *Electromagnetic Structure of Nucleons* (Oxford University Press, New York, 1961).

[2] W. Frazer and J. Fulco, Phys. Rev. **115**, 1763 (1959); **117**, 1609 (1960); Y. Nambu, *ibid.* **106**, 1366 (1957).

[3] In this paper we use the metric in which the product of two 4-vectors a and b is given by $a\cdot b=a^0b_0-\mathbf{a}\cdot\mathbf{b}$. In particular, $q^2=q^{02}-\mathbf{q}^2$. Furthermore, we use an unrationalized system of emu, so that the fine-structure constant $(=1/137)=e^2$; we shall always set $\hbar=c=1$.

[4] R. Hofstadter (private communication).

[5] M. M. Sternheim and R. Hofstadter, Nuovo Cimento **38**, 1854 (1965).

[6] See, however, B. Maglić and G. Costa, in *Proceedings of the Twelfth International Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965), p. 930.

[7] D. Cassel (private communication).

[8] W. R. Frazer, Phys. Rev. **115**, 1763 (1959); F. Hadjioannou, Ph.D. thesis, Stanford University, 1961 (unpublished).

[9] Two of the most recent investigations into the electroproduction process are N. Zagury, Phys. Rev. **145**, 1112 (1966); F. A. Behrends *et al.*, Cern report (unpublished).

[10] C. W. Akerlof *et al.*, Phys. Rev. Letters **16**, 147 (1966); K. Berkelman (private communication).

[11] M. Gourdin (private communication); see also J. P. Loubaton, Nuovo Cimento **39**, 591 (1965).

obtain reliable and model-independent values for the neutron charge form factor as well as for F_π.

Hofstadter's idea is also an "indirect" method (i.e., the F_π vertex is not the dominant contribution); however, by using the charge independence of nuclear forces one hopes to minimize the model dependence of the strong interactions. The idea is to scatter positively and negatively charged pions elastically from the isoscalar nucleus He^4, the α particle. The complete scattering amplitude can presumably be written as

$$f_{\text{tot}} = f_N \pm \mathfrak{F}_c, \tag{2}$$

where f_N is the (dominant) scattering amplitude for purely nuclear scattering and $\mathfrak{F}_c$ is some Coulomb correction amplitude whose sign, to first order in the fine-structure constant α, should depend only upon the sign of the pionic charge. The corresponding cross section is

$$\begin{aligned}\sigma_{\text{tot}}^{\pm} &= |f_N \pm \mathfrak{F}_c|^2 \\ &= |f_N|^2 \pm 2\,\text{Re} f_N^+ \mathfrak{F}_c + |\mathfrak{F}_c|^2.\end{aligned} \tag{3}$$

Hence to first order in α, the difference, defined by

$$\Delta\sigma \equiv (\sigma_{\text{tot}}^+ - \sigma_{\text{tot}}^-) = 4\,\text{Re} f_N^+ \mathfrak{F}_c, \tag{4}$$

is expected to be sensitive to F_π, while from the average

$$\sigma_N \equiv \tfrac{1}{2}(\sigma_{\text{tot}}^+ + \sigma_{\text{tot}}^-) = |f_N|^2 \tag{5}$$

one can deduce f_N and make the determination of F_π model-independent. Since f_N has the characteristic "diffraction minimum" at about 80° [in the center-of-mass (c.m. system)], one might hope that in this region $\mathfrak{F}_c$ would be sufficiently large for a good determination of F_π to be made. Data from a recent experiment at Rochester by Nordberg and Kinsey[12] have recently been published, and we shall use them as an example to illustrate the method one employs to extract F_π. Since the data are only at eight different c.m. angles and the statistics are poor, it is doubtful that we can obtain a definitive result for F_π. Rather, we view the work as a preliminary study to investigate just which aspects of the experiment need the most improvement, e.g., how much better do the statistics have to be? Or, is the diffraction minimum the best place to look?, etc.

Prior to the Rochester experiment, Sternheim and Hofstadter[5] had investigated the feasibility of the method and had concluded that, indeed, with presently available π beams the idea could be made to work. They calculated f_N from an approximate multiple-scattering formalism and replaced $\mathfrak{F}_c$ by $f_c^{(B)}$, the pure Born Coulomb amplitude:

$$f_c^{(B)} \equiv F_\pi(q^2) F_\alpha(q^2)/q^2, \tag{6}$$

where $F_\alpha(q^2)$ is the form factor of the α particle assumed known from e-α experiments. Schiff[13] pointed out that the use of $f_c^{(B)}$ may be an inconsistent approximation in that there could be large nuclear contributions to $\mathfrak{F}_c$. He therefore developed a type of distorted partial-wave Born expansion in which all nuclear effects are correctly taken into account to first order in α. Unfortunately, his result suffered from the occurrence of logarithmic divergences so characteristic of Coulomb problems.

Bell and, independently, Yennie and Gross first showed how to make Schiff's result convergent to first order.[14] In attempting to generalize their procedure to all orders, Antoine[15] could only show that the divergences cancelled for a limited subclass of terms. We have investigated this problem in some detail in a previous paper[16] and find that the scattering amplitude can be written in an explicitly convergent form whose first order iteration is also convergent. We can express the result in the form

$$\begin{aligned}f_{\text{tot}} = f_N - g\tilde{f}_c^{(B)} - g\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)\int_0^\infty V(r) \\ \times[e^{2i\delta_l}R_l^2 - j_l^2(kr) - ((e^{2i\delta_l}-1)/2k^2r^2)]r^2dr,\end{aligned} \tag{7}$$

where R_l is the radial wave function for scattering from the nuclear potential U alone and δ_l is the corresponding phase shift.[17] The pion wave number is k and the c.m. scattering angle is θ. Hence $q = 2k\sin\frac{1}{2}\theta$. The Coulomb potential $V(r)$ is related to the form factors via

$$V(r) = (2/\pi)\int_0^\infty F_\pi(q^2)F_\alpha(q^2)j_0(qr)dq. \tag{8}$$

Its strength is represented by

$$g \equiv 2m\alpha Z,$$

where m is the reduced mass of the π and Z is the nuclear charge. An important gauge of the Coulomb strength is given by the quantity

$$n \equiv g/2k = m\alpha Z/k.$$

We have also introduced the amplitude

$$\tilde{f}_c^{(B)} \equiv \int_0^\infty V(r)e^{2in\,\ln 2kr}j_0(qr)\,dr. \tag{9}$$

In Ref. 16 we have shown that, provided one does not look near $\theta \sim 0°$, $\tilde{f}_c^{(B)}$ can be replaced by $f_c^{(B)}$. The above form for f_{tot} is not unique, this particular one being chosen purely for reasons of convenience. As with

[12] M. E. Nordberg and K. F. Kinsey, Phys. Letters **20**, 692 (1966).

[13] L. I. Schiff, Progr. Theoret. Phys. (Kyoto) Suppl., Extra No. 400 (1965).

[14] L. I. Schiff, addendum to Ref. 13, Stanford report No. ITP-244 (unpublished).

[15] J. P. Antoine, Nuovo Cimento 44, 1068 (1966).

[16] G. B. West, J. Math. Phys. **8**, 942 (1967). The reader is referred to this paper for a fuller discussion of the divergence problem.

[17] In a recent paper [Nuovo Cimento 47, 49 (1967)], M. Ericson has set up the problem using the conventional Coulomb-wave approach.

Schiff's expression, only terms for which $\delta_l \neq 0$ occur. This assures rapid convergence of the l sum. Furthermore, the integrals are in a highly convenient form for numerical integration; in contrast to this it is to be noted that Schiff's original result with a cutoff would encounter problems with consistent convergence when integrated numerically.

In treating the purely nuclear part of the potential, we shall follow more closely the method of Schiff than that of Hofstadter and Sternheim. We simulate the strong interactions by some phenomenological potential whose parameters are fixed by fitting the average cross section σ_N to the data. The point is that an attempt to calculate f_N from π-n amplitudes and α-particle wave functions only serves to complicate the analysis at this stage and should be treated as an independent problem. In choosing a form for $U(r)$ we shall be strongly guided by the results of previous calculations on π-nuclear scattering.

In Sec. II we present a model from which we calculate the nuclear and Coulomb contributions. Throughout, our emphasis will be on the simplicity and tractability of the model rather than the "best" or "most realistic." In Sec. III we apply our calculations to the 24-MeV data of Nordberg and Kinsey. We conclude that, contrary to their claims, (at best) only a modest upper limit for r_π can be given without greatly improving the data. However, we are able to indicate an empirical method by which the results could possibly be made model-independent. This involves taking good data in the $\theta \gtrsim 100°$ region rather than near the minimum, $\theta \sim 80°$, where one expects the Coulomb contribution to be maximized.

II. CALCULATION

One of the first successful optical-model potentials used to describe π-nuclear scattering was that derived by Kisslinger.[18] He used a simple impulse approximation to obtain the following nonlocal potential:

$$\begin{aligned} U\psi &= C_0 k^2 \rho\psi - C_1 \nabla \cdot (\rho \nabla \psi) \\ &= C_0 k^2 \rho\psi - C_1 \nabla^2 \psi - C_1 \nabla\rho \cdot \nabla\psi . \end{aligned} \tag{10}$$

Here $\rho(\mathbf{r})$ is the nuclear density function normalized to unity at the origin and the C_Λ ($\Lambda=0, 1$) are complex parameters related to the S- and P-wave π-n amplitudes. The gradient term, which has its origins in the important P-wave ($\Lambda=1$) π-n amplitude, has been shown by Baker *et al.*[19] to be indispensable for fitting the data for a whole series of light nuclei. With the recent renewed interest in π-mesonic atoms, more sophisticated calculations have been performed. In particular, Ericson and Ericson[20] have used the Watson formalism to derive, again in terms of π-n scattering amplitudes, a rather complicated expression for $U(r)$. However, their result does not differ in its essential characteristics from that of Kisslinger. These characteristics are well illustrated by the three terms in Eq. (10) and can be summarized as follows:

(a) A repulsive interaction term corresponding to the S-wave ($\Lambda=0$) π-n amplitude.

(b) An effective-mass term which is always attractive.

(c) A term sensitive to the nuclear surface effects (these will be of particular importance in light nuclei).

We therefore take Eq. (10) as the basis for our nuclear model and choose the simplest possible form for $\rho(\mathbf{r})$;

$$\begin{aligned} \rho(\mathbf{r}) &= 1 \text{ for } r<b \\ &= 0 \text{ otherwise}. \end{aligned} \tag{11}$$

We shall treat the (complex) C_Λ and the (real) b as phenomenological parameters.

The Schrödinger equation is

$$(1+C_1\rho)(\nabla^2+k^2)\psi = (C_0+C_1)k^2\rho\psi - C_1\nabla\rho\cdot\nabla\psi . \tag{12}$$

Baker *et al.* have pointed out that the effective-mass term $(1+C_1\rho)$ can introduce an unphysical singularity into the potential and suggest replacing $(1+C_1\rho)^{-1}$ by $(1-C_1\rho)$. We shall follow this procedure. Upon performing a partial-wave separation, we obtain the usual radial equation with an effective potential term

$$(1-C_1\rho)[(C_0+C_1)k^2\rho R_l - C_1\rho' R_l'] . \tag{13}$$

With our chosen form for ρ, the ρ' term behaves like a surface δ function, $\delta(r-b)$, and we now show that the only effect that this introduces is to modify the boundary condition for R_l' at $r=b$. We first consider $\rho(r)$ to be parametrized by some $\lambda(>0)$ such that for any finite λ, $\rho(r,\lambda)$ and its first r derivative are everywhere continuous. Furthermore, we demand that for $\lambda \to 0$, ρ reduce to the step function, Eq. (11). We now divide the radial Schrödinger equation throughout by R_l and perform the operation

$$\lim_{\epsilon\to 0+} \int_{b-\epsilon}^{b+\epsilon} dr .$$

One easily obtains

$$\begin{aligned} \ln R_l'(b+\epsilon)/R_l'(b-\epsilon) &= -\int_{b-\epsilon}^{b+\epsilon} (1-C_1\rho)C_1 \frac{d\rho}{dr} dr \\ &= C_1[\rho - \tfrac{1}{2}C_1\rho^2]_{b-\epsilon}^{b+\epsilon} . \end{aligned}$$

Taking $\lim_{\lambda\to 0^+}$ followed by $\lim_{\epsilon\to 0^+}$, we find that,

$$R_l'(b+)/R_l'(b-) = e^{C_1(1-\frac{1}{2}C_1)} \equiv A . \tag{14}$$

(Note that for a regular optical potential $C_1=0$, and the above reduces to the usual continuity condition for the first derivative.) The solutions are now easy to

[18] L. S. Kisslinger, Phys. Rev. **98**, 761 (1955).

[19] R. M. Edelstein, W. F. Baker, and J. Rainwater, Phys. Rev. **122**, 252 (1961), and references quoted therein.

[20] M. Ericson and T. Ericson, Ann. Phys. (N. Y.) **36**, 323 (1966).

obtain. We have

$$r>b:\quad R_l(r)=\cos\delta_l j_l(kr)-\sin\delta_l n_l(kr)\,, \tag{15}$$

$$r<b:\quad R_l(r)=B_l j_l(\alpha r)\,, \tag{16}$$

where

$$\alpha^2=k^2[1-(1-C_1)(C_0+C_1)].$$

The constants B_l and the phase shifts δ_l can be obtained by imposing the boundary conditions at $r=b$; e.g., one finds

$$\tan\delta_l=\frac{kj_l'(kb)-\gamma_l j_l(kb)}{kn_l'(kb)-\gamma_l n_l(kb)}\,, \tag{17}$$

where

$$\gamma_l=\alpha A[j_l'(\alpha b)/j_l(\alpha b)].$$

In order to calculate the Coulomb contribution, we require an explicit form for $V(r)$ which in turn requires explicit forms for F_π and F_α. It turns out to be convenient to take the form of Eq. (1) for F_π,[21] which readily allows us to compare our results with other estimates, and to choose for F_α

$$F_\alpha(q^2)=3j_1(qb)/qb\,. \tag{18}$$

This is the Fourier transform of Eq. (11) and so in some sense we are treating the α particle consistently. However, in choosing this form for F_α we were again guided more by tractability, for in this case one can give a comparatively simple analytic form for $V(r)$:

$$V(r)=\frac{3}{2b}-\frac{r^2}{2b^3}-\frac{3a}{(ab)^3}+\frac{3e^{-ab}(1+ab)}{(ab)^3}\frac{\sinh(ar)}{r}\,,\quad r<b$$

$$=\frac{1}{r}\left[1-\frac{3e^{-ar}}{(ab)^3}\{ab\cosh(ab)-\sinh(ab)\}\right],\quad r>b\,. \tag{19}$$

Note that it is not necessary to assume that the b used here be the same b as was used in describing the optical-model radius.

III. NUMERICAL CALCULATIONS AND RESULTS

In this section we present the results of our analysis of the recent 24-MeV experiment performed by Nordberg and Kinsey.

The data are shown in Table I. The errors quoted in columns 2 and 4 ($S_\pm$, say) are those of the experimenters. We have estimated corresponding errors in σ_N and $\Delta\sigma$ by taking the optimistic viewpoint that the S's represent standard deviations resulting from independent observations. In that case the errors in σ_N and $\Delta\sigma$ are each

$$\tfrac{1}{2}(S_+^2+S_-^2)^{1/2}.$$

Unfortunately, data were taken at only eight different angles. As we shall see shortly, further data with a little better statistics would have been invaluable. Nordberg and Kinsey claim $r_\pi=1.8\pm0.8$ F. However, they make essentially no remarks concerning how these numbers were obtained, so it will be difficult to make a detailed comparison of their results with ours.

We have evaluated σ_N for a large number of sets of the five parameters C_0, C_1, and b. The mean-square deviation defined by

$$S^2=\frac{1}{8}\sum_{i=1}^{8}\left[\frac{\sigma_N^{(\mathrm{exp})}(\theta_i)-\sigma_N^{(\mathrm{theor})}(\theta_i)}{\sigma_N^{(\mathrm{exp})}(\theta_i)}\right]^2 \tag{20}$$

was minimized to find the "best" fit to the experimental data. The parameters were varied in steps of not less than 0.05 over a large region: b from 1.5 to 2.8; ReC_1 from -0.5 to -1.5; ImC_1 from -0.5 to $+0.1$; ReC_0 from 0.1 to 0.8; ImC_0 from -0.8 to $+0.1$.

There were several sets of parameters which seemed to fit the data extremely well ($S\sim5\%$). The distinction among them seemed to depend only upon how closely they came to fitting the eighth (151.9°) point. In Table II, we have listed three such representative sets ($1\rightarrow3$). For comparison, we have included one set, numbered 4, which has $S\sim10\%$. This is representative of sets which fit the 100°–140° region equally as well as the first sets but have greater difficulties with the

TABLE I. Differential scattering cross section $d\sigma/d\Omega$ for $\pi^\pm$-α elastic scattering (c.m.).

$\theta_{\mathrm{c.m.}}$ (deg)	$\frac{d\sigma^+}{d\Omega}$ (mb)	$\Delta\sigma$ $[=\frac{1}{2}(\sigma^+-\sigma^-)]$	$\frac{d\sigma^-}{d\Omega}$ (mb)	$\left[\frac{d\sigma}{d\Omega}\right]_N=\frac{1}{2}(\sigma^++\sigma^-)$
51.6	0.27±0.05	−0.26	0.79±0.11	0.53±0.06
61.8	0.38±0.06	+0.01(5)	0.35±0.08	0.36±0.05
76.9	0.44±0.05	+0.06	0.12±0.05	0.28±0.04
92.0	0.73±0.05	+0.20	0.33±0.12	0.53±0.07
107.0	1.04±0.08	+0.14(5)	0.75±0.10	0.89±0.06
121.8	1.53±0.15	+0.10	1.33±0.13	1.43±0.10
139.3	2.33±0.16	+0.29	1.75±0.27	2.04±0.16
150.9	2.48±0.12	−0.13(5)	2.75±0.22	2.61±0.13

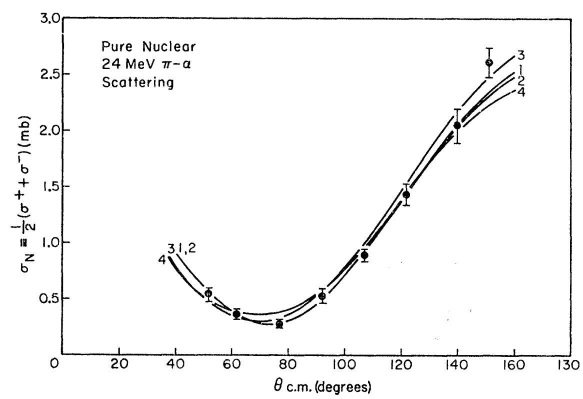

FIG. 1. Sensitivity of the theoretical $\Delta\sigma$ to the nuclear-model parameters for a fixed pion radius r_π of 1.25 F. Note that the differences in the region 100° are actually exaggerated in this figure.

[21] For reasons of clarity we replace m_ρ of Eq. (1) by a variable parameter a. The corresponding root-mean-square radius for the pion is $r_\pi=(\sqrt{6})/a$. For $m_\rho\sim750$ MeV, one finds that $r_\pi\sim0.6$ F.

TABLE II. Four sets of parameters which give good fits to the data. The division of each of the four columns indicates Re and Im part of the parameters listed on the far left. The number of decimal points retained in the δ_l is not supposed to be significant.

	1		2		3		4	
Radius b (F)	2.08		2.05		2.10		2.10	
C_1	−0.90	−0.25	−0.90	−0.30	−0.90	−0.25	−0.80	−0.25
C_0	0.50	−0.55	0.60	−0.55	0.50	−0.50	0.60	−0.40
δ_0 (rad)	−0.0747	0.0442	−0.0766	0.0416	−0.0769	0.0404	−0.0856	0.0390
δ_1	0.0609	0.0207	0.0589	0.0229	0.0622	0.0207	0.0542	0.0217
δ_2	0.0045	0.0013	0.0043	0.0014	0.0047	0.0013	0.0042	0.0014
Mean-sq. dev. S(%)	5.1		5.3		6.0		11.0	

eighth point and were not quite so good in the region near the minimum. These results are summarized in Fig. 1.[22]

Using several such sets of parameters, we have calculated the Coulomb interference term $\Delta\sigma$. The charge radius of the α was always taken to be that derived from the optical-model fits. These values of b ($\sim$2.08 F) are in remarkably good agreement with those derived from electron-scattering experiments.[23] Figure 2 shows a plot of our calculated $\Delta\sigma$ for various values of the pion radius: The nuclear parameters used were those which gave the best fit to σ_N (set I). For comparison one plot is shown in which $\Delta\sigma$ was calculated using $f_c^{(B)}$ only. The plots make it abundantly clear that we can derive very little definitive information about r_π from this particular experiment. The most one could possibly claim is that $a \gtrsim 1.5$ F^{-1}. This would imply $r_\pi \lesssim 1.5$ F.

The following observations concerning Fig. 2 are worth noting:

(i) Effects due to nuclear distortions on the Coulomb scattering are large. Unfortunately, because of the poor statistics in the seventh and eighth points, it is not possible to conclude that the Born term is definitely insufficient. In fact, the Born term probably fits the data better than our theory. We would like to argue that the seventh point is anomalous and suggest that it be ignored. The large effect of the nuclear-distortion terms can be understood when it is realized that they introduce imaginary contributions into $\mathfrak{F}_c$ which are enhanced by interference with the large imaginary part of f_N. The pure Born Coulomb term $f_c^{(B)}$ is real and can therefore interfere only with the smaller real part of f_N.

(ii) Even if statistics are greatly improved, it is still doubtful that at this energy the experiment can give a definitive result. The sensitivity of $\Delta\sigma$ to r_π in the region of ρ dominance ($r_\pi \sim 0.6$ F) is clearly extremely poor.

(iii) What sensitivity there is seems to occur near the region of the minimum *and continues* through all backward-scattering angles.

It is this last observation that leads us to the only encouraging aspect of this analysis. The curves in Fig. 2 were obtained using the one best-fit set of parameters. Now observe Fig. 3. The plots here were obtained

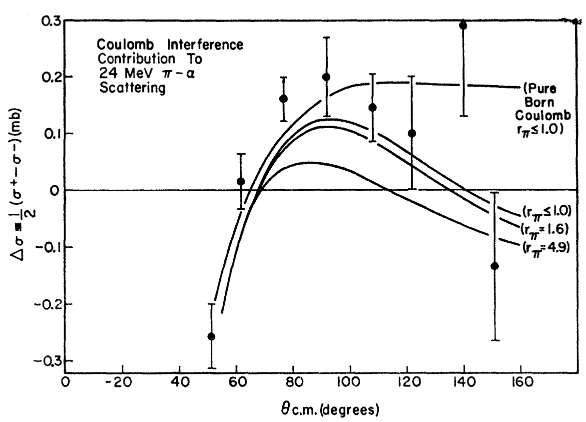

FIG. 2. Variation of the first-order Coulomb contribution with the pion radius r_π (expressed in fermis). The "best" fit set of nuclear parameters from Fig. 1 were used (set 1 of Table II).

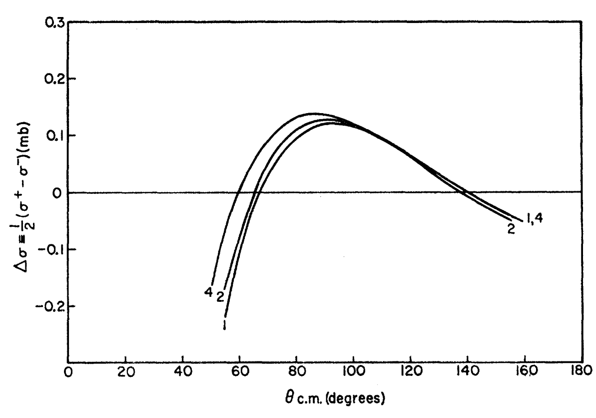

FIG. 3. Representative sample of some of the best fits to the purely nuclear contribution. The points are from the experimental work of Nordberg and Kinsey (Ref. 12). For key to numbering of curves, see Table II.

[22] Note that near the diffraction dip ($\sim$50°) the strong and Coulomb amplitudes are comparable. However, the analysis in terms of $\Delta\sigma$ and σ_N [Eqs. (4) and (5)] still goes through since it is f_N which becomes small there, rather than $\mathfrak{F}_c$ which becomes anomalously large; in other words corrections are still of order α (i.e., $\lesssim$1%).

[23] H. Frank, D. Haas, and H. Prange, Phys. Letters 19, 391 (1965).

[24] We choose this value of r_π, rather than a more realistic one, because for this value there will still be some sensitivity to changes in a.

keeping a constant ($r_\pi \sim 1.25$ F)[24] and varying the nuclear parameters. Several sets were used, and it was found that all those with $S \sim 5\%$ fell between curves 1 and 2, while those with $S \sim 10\%$ lay on a curve such as 4. The important point to note is that beyond $\sim 95°$ it is almost impossible to discriminate among any of these curves (the differences are actually slightly exaggerated in the diagram). In other words, in the backward-scattering angles where $\Delta\sigma$ is most sensitive to r_π it is apparently least sensitive to the variations in the nuclear model. This strongly suggests that, in order to obtain the most reliable and least model-dependent results, the experiment should be performed with the emphasis on getting good statistics at several points in the region 90°–180°. Repeating the calculations and obtaining similar conclusions using different shapes for both the nucleus and the pion would certainly lend weight to this recommendation. In the energy range considered (<100 MeV), the results are unlikely to be sensitive to the shape of the form factors. However, it is difficult to argue in an entirely convincing fashion concerning the sensitivity of the conclusions to the nuclear shape. The model dependence enters into the calculation only via the quantity R_l^2 in the region $r<b$ [see Eq. (7)]; for $r>b$, R_l is essentially model-independent and is given by Eq. (15). Presumably, if one could show that for our particular model the major contribution to the Coulomb integral came from outside the nucleus ($r>b$), one could then extrapolate the result to all reasonable models. Unfortunately, such a numerical calculation shows that contributions to the integral come about equally from inside and outside the nucleus and an extrapolation is not possible. However, it is to be strongly emphasized that the analysis is a self-consistent one (*all* of the phenomenological parameters are determined from the experimental data), and it would indeed be unreasonable for the Coulomb effects to be appreciably changed by introducing a change in the nuclear shape.

We conclude this paper with a brief discussion of some of the problems introduced by going to higher energies[25] where one might expect the sensitivity to F_π to be greatest:

(i) Suppose that we neglect the nuclear-distortion effects; we can then write

$$\Delta\sigma \propto F_\pi \sim (1-q^2/a^2)^{-1}.$$

Hence

$$d(\Delta\sigma)/\Delta\sigma \sim (2q^2/a^2)da/a.$$

At 24 MeV, $q^2/a^2 \sim 0.06$, which means that even a large change in a produces only a comparatively small change in $\Delta\sigma$. This, of course, is well illustrated by Fig. 2. The situation can presumably be improved by going to much higher q^2. In fact, at 100 MeV, q^2/a^2 can reach 0.5 near the position of the minimum. Unfortunately, however, $\Delta\sigma \propto 1/q^2$, and so, even though the sensitivity to a has increased by an order of magnitude, the absolute value of $\Delta\sigma$ may be too small to obtain reliable measurements. The only hope then is that the nuclear-distortion contributions do not decrease quite so drastically with q^2.

An analysis then should be undertaken to determine the optimum energy at which the sensitivity is still high and the interference term still measurable.

(ii) At higher energies inelastic processes assume far greater importance and, as a check on the fits, the total inelastic cross section should be accurately measured. Our phase shifts δ_l give a total inelastic cross section of ~ 70 mb. This is to be compared with the 13 mb estimated by Nordberg and Kinsey; they point out, however, that this number is probably a gross underestimation of the correct estimate.

(iii) At energies approaching 100 MeV, even if one still believes that a phenomenological potential can be used, one must begin to have serious doubts about the use of the purely nonrelativistic Schrödinger equation. To modify our previous approach without using manifestly covariant quantum mechanics is not straightforward and one runs into matters of principle. The problem is basically one of how we treat the two potentials U and V when we try to incorporate them into the Klein-Gordon equation. Are they to be considered as scalars or as the fourth component of a 4-vector, for instance? Depending upon how one proceeds, one can obtain somewhat different expressions for the Coulomb contribution and this will clearly affect one's interpretation of the F_π contribution.[26]

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[25] Such experiments are already underway: M. Block (private communication) at Argonne and K. Crowe (private communication) at Berkeley. Our investigation of Crowe's preliminary data has not yet proved very encouraging. Results of these analyses will be presented at a later time when the data have been more thoroughly analyzed.

[26] In Ref. 17, M. Ericson mentions a remark of S. M. Berman's to the effect that the potential approach ignores whole classes of diagrams which conceivably could contribute to the Coulomb contribution. Presumably, one expects these contributions to be larger at higher energies.