$$3R_{12}=m_{N^*}[-\tfrac{1}{2}\,\mathrm{Re}(F_1{}^*F_{14})-2m_N\,\mathrm{Re}(F_1{}^*F_{15})-m_{N^*}\,\mathrm{Re}(F_2{}^*F_9)$$
$$-m_{N^*}m_N\,\mathrm{Re}(F_2{}^*F_{10})+4m_{N^*}{}^2m_N\,\mathrm{Re}(F_3{}^*F_{13})-\tfrac{1}{2}m_{N^*}\,\mathrm{Re}(F_5{}^*F_{10})-2m_{N^*}m_N\,\mathrm{Re}(F_5{}^*F_{11})$$
$$+m_{N^*}\,\mathrm{Re}(F_6{}^*F_{13})+m_{N^*}m_N\,\mathrm{Re}(F_6{}^*F_{14})-4m_{N^*}{}^2m_N\,\mathrm{Re}(F_7{}^*F_9)]+2(NN^*)[\mathrm{Re}(F_1{}^*F_{11})$$
$$+2m_{N^*}m_N\,\mathrm{Re}(F_2{}^*F_{15})-2m_{N^*}{}^2\,\mathrm{Re}(F_3{}^*F_9)+m_{N^*}\,\mathrm{Re}(F_5{}^*F_{15})+m_{N^*}\,\mathrm{Re}(F_6{}^*F_{10})$$
$$+2m_{N^*}m_N\,\mathrm{Re}(F_6{}^*F_{11})+2m_{N^*}{}^2\,\mathrm{Re}(F_7{}^*F_{13})]-4(NN^*)^2[\mathrm{Re}(F_2{}^*F_{11})+\mathrm{Re}(F_6{}^*F_{15})]$$
$$+4[(NN^*)^2-m_{N^*}{}^2m_N{}^2][2(NN^*)\,\mathrm{Re}(F_3{}^*F_{11})+m_{N^*}\,\mathrm{Re}(F_3{}^*F_{14})-2m_{N^*}m_N\,\mathrm{Re}(F_3{}^*F_{15})$$
$$+m_{N^*}\,\mathrm{Re}(F_7{}^*F_{10})+2m_{N^*}m_N\,\mathrm{Re}(F_7{}^*F_{11})+2(NN^*)\,\mathrm{Re}(F_7{}^*F_{15})].$$

X_{13}, X_{14}, and X_{15} are obtained from X_9+X_{10}, X_{11}, and X_{12}, respectively, by changing Re to iIm in the latter expressions. Here F_{11}, F_{12}, F_{15}, and F_{16} are redefined to be the $(F_{11}+F_4/m_l)$, $(F_{12}-F_4/m_l)$, $(F_{15}+F_8/m_l)$, and $(F_{16}-F_8/m_l)$, respectively, of Eq. (27).

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Spin-$\frac{3}{2}$ Polarization in Production of N^* by Neutrinos†

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The spin-$\frac{3}{2}$ polarization and projection operators are obtained. The polarization of the outgoing N^* resonance for the process $\nu_l+N\rightarrow N^*+l$ is treated and the explicit value of the density matrix of the N^* resonance is obtained in two special cases, using the explicit values of three form factors obtained by Berman and Veltman and by Albright and Liu.

I. INTRODUCTION

In the preceding paper,[1] the interaction Hamiltonian for the process $\nu_l+N\rightarrow N^*+l$ has been expressed in terms of 14 form factors in general and the polarization of the outgoing lepton was considered. If it is assumed that the interaction Hamiltonian is the current-current form and γ_5 symmetry holds for the lepton current, only eight form factors need to be considered. Berman and Veltman,[2] Albright and Liu,[3,4] and Furlan, Jengo, and Remiddi[5] obtained explicit values for these form factors using different methods.

In this paper the spin-$\frac{3}{2}$ polarization and projection operators are obtained using the spin-$\frac{3}{2}$ wave functions which were recently developed by Joos,[6] Weinberg,[7] and Weaver, Hammer, and Good.[8] The polarization effect of N^* for the process $\nu_l+N\rightarrow N^*+l$ is treated by using the explicit values of the form factors derived from N^* photoproduction and $SU(6)$ theory by Berman and Veltman[2] and by Albright and Liu,[3] respectively. Also, the explicit value of the density matrix is obtained in these cases. This density matrix was given by Shay, Song, and Good,[9] and will be useful in discussing the angular distribution of the decay of the N^* resonance into a nucleon and a pion as discussed by Gottfried and Jackson.[10]

Finally, some useful matrix properties are given in the Appendix.

II. PROJECTION OPERATORS

In the notation of Refs. 1 and 9, the plane-wave amplitude of a particle with mass is written in momentum space as

$$\psi_{\epsilon KP}=\begin{pmatrix}\chi\\ \varphi\end{pmatrix},\qquad(1)$$

which is normalized in the sense that

$$\sum_\alpha(\bar\psi_{\epsilon K})_\alpha(\psi_{\epsilon'K'})_\alpha=(\epsilon m^{2s}/E)\delta_{\epsilon\epsilon'}\delta_{KK'},\qquad(2)$$

where ϵ is $+1$ or -1 for positive or negative energy states, and s and K indicate half-integral spin and polarization state. Here the ψ's are at same physical momentum P', and $\bar\psi$ is defined by $\psi^\dagger\gamma_4$. The energy and polarization projection operators for arbitrary half-

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integral spin s can be expressed in terms of the plane-wave amplitudes as

$$(\Lambda_\epsilon)_{\alpha\beta}=\sum_K (\epsilon E/m^{2s})(\psi_{\epsilon K})_\alpha(\bar{\psi}_{\epsilon K})_\beta\,, \tag{3}$$

$$(P_K)_{\alpha\beta}=\sum_\epsilon (\epsilon E/m^{2s})(\psi_{\epsilon K})_\alpha(\bar{\psi}_{\epsilon K})_\beta\,, \tag{4}$$

$$(\Lambda_\epsilon P_K)_{\alpha\beta}=(\epsilon E/m^{2s})(\psi_{\epsilon K})_\alpha(\bar{\psi}_{\epsilon K})_\beta\,. \tag{5}$$

Introducing $\zeta(N^*)$, which is defined in Ref. 9 as

$$\zeta(N^*)=i\gamma_{\mu\nu\lambda}N^{*\prime}{}_\mu N^{*\prime}{}_\nu N^{*\prime}{}_\lambda/m^3\,, \tag{6}$$

the energy projection operator for the spin-$\frac{3}{2}$ particle is

$$\Lambda_\epsilon=\tfrac{1}{2}[1+\epsilon\zeta(N^*)]\,. \tag{7}$$

The polarization projection operator for arbitrary spin s has been introduced by Michel[11] as

$$P_K{}^{(s)}=\frac{(-1)^{s-K}}{(s-K)!(s+K)!}\prod_{K'=-s,K'\neq K}^{s}(T_\mu\eta'_\mu-K')\,, \tag{8}$$

where T_μ is the polarization operator and η'_μ is a four-vector used in Ref. 1. For spin $\frac{3}{2}$ this becomes

$$\begin{aligned}
P_{\frac{3}{2}}{}^{(\frac{3}{2})}&=\tfrac{1}{6}[T_\mu\eta'_\mu-\tfrac{1}{2}][T_\mu\eta'_\mu+\tfrac{1}{2}][T_\mu\eta'_\mu+\tfrac{3}{2}]\,,\\
P_{\frac{1}{2}}{}^{(\frac{3}{2})}&=-\tfrac{1}{2}[T_\mu\eta'_\mu-\tfrac{3}{2}][T_\mu\eta'_\mu+\tfrac{1}{2}][T_\mu\eta'_\mu+\tfrac{3}{2}]\,,\\
P_{-\frac{1}{2}}{}^{(\frac{3}{2})}&=\tfrac{1}{2}[T_\mu\eta'_\mu-\tfrac{3}{2}][T_\mu\eta'_\mu-\tfrac{1}{2}][T_\mu\eta'_\mu+\tfrac{3}{2}]\,,\\
P_{-\frac{3}{2}}{}^{(\frac{3}{2})}&=-\tfrac{1}{6}[T_\mu\eta'_\mu-\tfrac{3}{2}][T_\mu\eta'_\mu-\tfrac{1}{2}][T_\mu\eta'_\mu+\tfrac{1}{2}]\,.
\end{aligned} \tag{9}$$

These can be written as

$$\begin{aligned}
P_K{}^{(\frac{3}{2})}=&[(-1)^{\frac{3}{2}-K}/(\tfrac{3}{2}-K)!(\tfrac{3}{2}+K)!]\\
&\times[(T_\mu\eta'_\mu)^3+a(T_\mu\eta'_\mu)^2+b(T_\mu\eta'_\mu)+c]\,,
\end{aligned} \tag{10}$$

where $a=(\frac{3}{2},\frac{1}{2},-\frac{1}{2},-\frac{3}{2})$, $b=(-1/4,-9/4,-9/4,-1/4)$, and $c=(-3/8,-9/8,9/8,3/8)$ for $K=(\frac{3}{2},\frac{1}{2},-\frac{1}{2},-\frac{3}{2})$, respectively. For spin $\frac{3}{2}$ one also gets

$$\begin{aligned}
T_\mu&=-(3/32m)\zeta(N^*)\epsilon_{\mu\nu\rho\sigma}\gamma_{\alpha\beta\nu}\gamma_{\alpha\beta\rho}N^{*\prime}{}_\sigma\,,\\
T_\mu\eta'_\mu&=-(3/2m^2)i\gamma_5\gamma_{\mu\nu\lambda}N^{*\prime}{}_\mu N^{*\prime}{}_\nu\eta'_\lambda\,,\\
(T_\mu\eta'_\mu)^2&=i(3/2m)\zeta(N^*)\gamma_{\mu\nu\lambda}N^{*\prime}{}_\mu\eta'_\nu\eta'_\lambda+\tfrac{3}{4}\\
(T_\mu\eta'_\mu)^3&=-\tfrac{3}{4}i\gamma_5\gamma_{\mu\nu\lambda}\eta'_\mu\eta'_\nu\eta'_\lambda-(21/8m^2)\\
&\qquad\times i\gamma_5\gamma_{\mu\nu\lambda}N^{*\prime}{}_\mu N^{*\prime}{}_\nu\eta'_\lambda\,.
\end{aligned} \tag{11}$$

The first equation in Eq. (11) was given in Ref. 9 and the other relations can be obtained using the properties of the matrix S given in the Appendix. From Eqs. (1), (5), (7), and (10) it follows for the positive-energy state that

$$\begin{aligned}
\chi_{\dot\alpha\dot\beta\dot\kappa}\chi_{\mu\nu\lambda}=&(384E_{N^*}|K|)^{-1}\textstyle\sum_p\sum_{p'}[N^{*\prime}+K_1m\eta']_{\dot\alpha\mu}\\
&\times[N^{*\prime}+K_2m\eta']_{\dot\beta\nu}[N^{*\prime}+K_3m\eta']_{\dot\kappa\lambda}\,,\\
\varphi^{\mu\nu\lambda}\varphi^{\dot\alpha\dot\beta\dot\kappa}=&(384E_{N^*}|K|)^{-1}\textstyle\sum_p\sum_{p'}[N^{*\prime}-K_1m\eta']^{\mu\dot\alpha}\\
&\times[N^{*\prime}-K_2m\eta']^{\nu\dot\beta}[N^{*\prime}-K_3m\eta']^{\lambda\dot\kappa}\,,
\end{aligned} \tag{12}$$

where $\eta'^{\mu\dot\alpha}$ is the spinor corresponding to the four-vector η'_μ and $K_1=K_2=K_3=+1$ for $K=\frac{3}{2}$, $K_1=K_2=+1$, and $K_3=-1$ for $K=\frac{1}{2}$, $K_1=+1$, and $K_2=K_3=-1$ for $K=-\frac{1}{2}$, and $K_1=K_2=K_3=-1$ for $K=-\frac{3}{2}$. Here $\sum_p\sum_{p'}$ means the sum over all permutations of $(\dot\alpha\dot\beta\dot\kappa)$ and $(\mu\nu\lambda)$. When one sums over the polarizations of the spin-$\frac{3}{2}$ particle, Eq. (12) becomes

$$\begin{aligned}
\textstyle\sum\chi_{\dot\alpha\dot\beta\dot\kappa}\chi_{\mu\nu\lambda}&=(12E_{N^*})^{-1}\textstyle\sum_p N^{*\prime}{}_{\dot\alpha\mu}N^{*\prime}{}_{\dot\beta\nu}N^{*\prime}{}_{\dot\kappa\lambda}\,,\\
\textstyle\sum\varphi^{\mu\nu\lambda}\varphi^{\dot\alpha\dot\beta\dot\kappa}&=(12E_{N^*})^{-1}\textstyle\sum_p N^{*\prime\mu\dot\alpha}N^{*\prime\nu\dot\beta}N^{*\prime\lambda\dot\kappa}\,,
\end{aligned} \tag{13}$$

where $\sum_p$ means the sum over all permutations of either $(\dot\alpha\dot\beta\dot\kappa)$ or $(\mu\nu\lambda)$. Here the physical energy-momentum operator $N^{*\prime}{}_\mu$ is related to the gradient operator $N^*{}_\mu=-i\partial/\partial x_\mu$ by $N^{*\prime}{}_\mu=(H/E)N^*{}_\mu$. Since for the reaction presently considered, $\nu_l+N\rightarrow N^*+l$, only particles are involved, $N^{*\prime}{}_\mu$ and $N^*{}_\mu$ are identical and they will not be distinguished hereafter.

III. PRODUCTION CROSS SECTION AND DENSITY MATRIX

In Ref. 1 the interaction Hamiltonian for the process $\nu_l+N\rightarrow N^*+l$ has been expressed in terms of 14 form factors in general. If it is assumed that the interaction Hamiltonian is the current-current form and γ_5 symmetry holds for the lepton current, only eight form factors need to be considered. Several authors[2-5] obtained explicit values for these form factors by using different methods. Here only those derived from N^* photoproduction and $SU(6)$ theory, which were considered by Berman and Beltman[2] and by Albright and Liu,[3] will be discussed. Then the interaction Hamiltonian given in Ref. 1 is

$$\begin{aligned}
\langle f|H|i\rangle=&(G/m_{N^*}V)[F_9\chi_{\alpha\beta\gamma}N^{*\alpha\dot\kappa}\varphi_{l\dot\kappa}\varphi_N{}^\beta\varphi_\nu{}^\gamma\\
&+F_{10}\chi_{\alpha\beta\gamma}N^{*\alpha\dot\kappa}N_{\dot\kappa}{}^\beta\chi_N{}^{\dot\sigma}\varphi_{l\dot\sigma}\varphi_\nu{}^\gamma+F_{13}\varphi^{\dot\sigma\dot\kappa\dot\rho}N^*{}_{\dot\rho\gamma}\chi_{N\dot\sigma}\varphi_{l\dot\kappa}\varphi_\nu{}^\gamma\\
&+F_{14}\varphi^{\dot\sigma\dot\kappa\dot\rho}N^*{}_{\dot\rho\gamma}N^\gamma{}_{\dot\kappa}\varphi_{N\delta}\varphi_\nu{}^\delta\varphi_{l\dot\sigma}]\,,
\end{aligned} \tag{14}$$

where the subscripts N, ν, N^*, and l denote the nucleon, neutrino, N^* resonance, and lepton, respectively. The explicit results given in Ref. 3 are

$$|F_1{}^A(0)|=-0.87\,,\quad |F_1{}^V(0)|=-|F_2{}^V(0)|=5.6\,, \tag{15}$$

when N^* photoproduction and conserved vector current (CVC) are considered and

$$|F_1{}^A(0)|=-0.83\,,\quad |F_1{}^V(0)|=-|F_2{}^V(0)|=3.75\,, \tag{16}$$

when $SU(6)$ theory and CVC are considered. The relation between the form factors in Eq. (14) and those of Albright and Liu, $F_i{}^V$ and $F_i{}^A$, are given in Ref. 1 as

$$\begin{aligned}
F_9&=-i\sqrt{2}(F_1{}^A+F_1{}^V)\,,\quad F_{10}=-(i\sqrt{2}/\Delta)F_2{}^V\,,\\
F_{13}&=-i\sqrt{2}(F_1{}^A-F_1{}^V)\,,\quad F_{14}=(i\sqrt{2}/\Delta)F_2{}^V\,,
\end{aligned} \tag{17}$$

where Δ is $m_{N^*}+m_N$.

Consider the absolute square of the matrix element of the interaction Hamiltonian. Averaging it over the polarizations of the initial particles and summing over the polarizations of the outgoing lepton give, in the

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rest system of the N^* resonance,

$$\tfrac{1}{2}|\langle f|H|i\rangle|^2=(G^2m_{N^*}^2/96|K|E_NE_lE_\nu)\times\{[3+K_1K_2+K_1K_3+K_2K_3]A\times[5(K_1+K_2+K_3)+3K_1K_2K_3]B_is_i+[K_1K_2+K_1K_3+K_2K_3]C_{ij}s_{ij}+K_1K_2K_3D_{ijk}s_{ijk}\}\,, \quad (18)$$

where the s_{ij} and s_{ijk} are defined as

$$s_{ij}=3s_is_j-\delta_{ij}\,,$$
$$s_{ijk}=(9/10)(5s_is_js_k-\delta_{ij}s_k-\delta_{jk}s_i-\delta_{ki}s_j) \quad (19)$$

and the A, B_i, C_{ij}, and D_{ijk} are as follows:

$$\begin{aligned}
A=&|F^A|^2(E_N+m_N)[3E_lE_\nu-(\mathbf{l}\cdot\mathbf{v})]+2\Delta^{-1}|F^AF^V|(E_N-m_{N^*})[E_\nu(\mathbf{N}\cdot\mathbf{l})-E_l(\mathbf{N}\cdot\mathbf{v})]\\
&+|F^V|^2\{(E_N-m_N)[3E_lE_\nu-(\mathbf{l}\cdot\mathbf{v})]-2\Delta^{-1}(E_N-m_N)[E_\nu(\mathbf{N}\cdot\mathbf{l})+E_l(\mathbf{N}\cdot\mathbf{v})]\\
&+2\Delta^{-1}[(\mathbf{N}\cdot\mathbf{v})(\mathbf{N}\cdot\mathbf{l})-N^2(\mathbf{v}\cdot\mathbf{l})+2N^2E_\nu E_l]-2\Delta^{-2}N^2[E_\nu(Nl)+E_l(N\nu)+m_N(l\nu)]\}\,,\\
B_i=&-|F^A|^2(E_N+m_N)[E_l\nu_i-E_\nu l_i]+\tfrac{1}{5}|F^AF^V|\{\Delta^{-1}(E_N+m_N)[2(\mathbf{v}\cdot\mathbf{l})N_i-3(\mathbf{N}\cdot\mathbf{v})l_i-3(\mathbf{N}\cdot\mathbf{l})\nu_i]\\
&+2[(\mathbf{N}\cdot\mathbf{v})l_i+(\mathbf{N}\cdot\mathbf{l})\nu_i+(\mathbf{l}\cdot\mathbf{v})N_i-5E_\nu E_lN_i]+3\Delta^{-1}[E_l(\mathbf{N}\cdot\mathbf{v})+E_\nu(\mathbf{N}\cdot\mathbf{l})]N_i-\Delta^{-1}N^2[E_\nu l_i+E_l\nu_i]\}\\
&+|F^V|^2\{(E_N-m_N)(E_\nu l_i-E_l\nu_i)-\Delta^{-1}(E_N-m_N)[(\mathbf{N}\cdot\mathbf{v})l_i-(\mathbf{N}\cdot\mathbf{l})\nu_i]-\tfrac{2}{5}\Delta^{-2}(E_N+m_N)\\
&\times[(E_\nu(\mathbf{N}\cdot\mathbf{l})-E_l(\mathbf{N}\cdot\mathbf{v}))N_i-2N^2(E_\nu l_i-E_l\nu_i)]-\tfrac{4}{5}\Delta^{-2}N^2[(\mathbf{N}\cdot\mathbf{v})l_i-(\mathbf{N}\cdot\mathbf{l})\nu_i]\\
&+\tfrac{1}{5}\Delta^{-1}[9N^2(E_\nu l_i-E_l\nu_i)-7(E_\nu(\mathbf{N}\cdot\mathbf{l})-E_l(\mathbf{N}\cdot\mathbf{v}))N_i]\}\,, \quad (20)\\
\tfrac{1}{2}C_{ij}=&-|F^A|^2(E_N+m_N)l_i\nu_j-\Delta^{-1}|F^AF^V|\{(E_N+3m_N+2m_{N^*})(E_\nu l_i-E_l\nu_i)N_j+[(\mathbf{N}\cdot\mathbf{l})\nu_i-(\mathbf{N}\cdot\mathbf{v})l_i]N_j\}\\
&+|F^V|^2\{-(E_N-m_N)l_i\nu_j+\Delta^{-1}(E_N-m_N)(E_\nu l_i+E_l\nu_i)N_j+\Delta^{-2}[E_\nu(Nl)+E_l(N\nu)+m_N(l\nu)]N_iN_j\\
&+\Delta^{-1}[(\mathbf{N}\cdot\mathbf{l})\nu_iN_j+(\mathbf{N}\cdot\mathbf{v})l_iN_j-2N^2l_i\nu_j-2E_\nu E_lN_iN_j]\}\,,\\
\tfrac{1}{4}D_{ijk}=&\Delta^{-1}|F^AF^V|[2(E_N+2m_N+m_{N^*})l_i\nu_jN_k-(E_\nu l_i+E_l\nu_i)N_jN_k]+\Delta^{-2}|F^V|^2\\
&\times\{(E_N+2m_N+m_{N^*})(E_l\nu_i-E_\nu l_i)N_jN_k+[(\mathbf{N}\cdot\mathbf{v})l_i-(\mathbf{N}\cdot\mathbf{l})\nu_i]N_jN_k\}\,,
\end{aligned}$$

where F^A is $F_1{}^A$ and F^V is $F_1{}^V=-F_2{}^V$. Note that $K=\frac{1}{4}(3+K_1K_2+K_2K_3+K_3K_1)$. Therefore, one has, for the unpolarized cross section,

$$\left(\frac{d\sigma}{dt}\right)_{\text{unp}}=\frac{V^2E_NE_lE_\nu m_{N^*}}{2\pi(s-m_N{}^2)^2}|\langle f|H|i\rangle|^2{}_{\text{unp}} \quad (21)$$
$$=\frac{G^2m_{N^*}}{6\pi(s-m_N{}^2)^2}A\,,$$

where $s=-(N+\nu)^2$ and $t=-(l-\nu)^2$. One also has from Eqs. (18) and (21)

$$\frac{d\sigma_K}{dt}=\frac{1}{4}\left(\frac{d\sigma}{dt}\right)_{\text{unp}}\left[1+\frac{5(K_1+K_2+K_3)+3K_1K_2K_3}{4|K|A}B_is_i+\frac{K_1K_2+K_2K_3+K_3K_1}{4|K|A}C_{ij}s_{ij}+\frac{K_1K_2K_3}{4|K|A}D_{ijk}s_{ijk}\right], \quad (22)$$

or

$$(3d\sigma_{\frac{3}{2}}+d\sigma_{\frac{1}{2}}-d\sigma_{-\frac{1}{2}}-3d\sigma_{-\frac{3}{2}})/\textstyle\sum_K d\sigma_K=5B_is_i/A$$
$$(d\sigma_{\frac{3}{2}}-d\sigma_{\frac{1}{2}}+d\sigma_{-\frac{1}{2}}-d\sigma_{-\frac{3}{2}})/\textstyle\sum_K d\sigma_K=C_{ij}s_{ij}/2A \quad (23)$$
$$(d\sigma_{\frac{3}{2}}-3d\sigma_{\frac{1}{2}}+3d\sigma_{-\frac{1}{2}}-d\sigma_{-\frac{3}{2}})/\textstyle\sum_K d\sigma_K=5D_{ijk}s_{ijk}/6A\,.$$

Experimentally B_i, C_{ij}, and D_{ijk} can be obtained by considering the angular distribution of the decay particles of the N^* resonance. In order to get the density matrix of the N^* resonance in its rest system before it decays into a nucleon and a pion, one replaces the K_js_i's by the $(\sigma_i)_{\alpha\beta}$'s for any K_j and the 1's by the $(1)_{\alpha\beta}$'s in Eq. (18) so that each term of the equation can be expressed in terms of three pairs of subscripts. One also multiplies it by $\sum_p\sum_{p'}$ from the left in order to symmetrize the subscripts and by the matrix T from the right and left where the matrix T is defined in the Appendix. Then using the properties of the matrix S given in the Appendix, one get the density matrix

$$\rho=\tfrac{1}{4}[1+2(B_i\mathbf{S}_i/A)+C_{ij}\mathbf{S}_{ij}/2A+(D_{ijk}\mathbf{S}_{ijk}/6A)]\,, \quad (24)$$

where the $\mathbf{S}_i$ are the usual spin-$\frac{3}{2}$ matrices and the $\mathbf{S}_{ij}$, $\mathbf{S}_{ijk}$ are defined in Ref. 9 as

$$\mathbf{S}_{ij}=\mathbf{S}_i\mathbf{S}_j+\mathbf{S}_j\mathbf{S}_i-\tfrac{5}{2}\delta_{ij}$$
$$\mathbf{S}_{ijk}=\mathbf{S}_i\mathbf{S}_j\mathbf{S}_k+\mathbf{S}_i\mathbf{S}_k\mathbf{S}_j+\mathbf{S}_j\mathbf{S}_k\mathbf{S}_i+\mathbf{S}_j\mathbf{S}_i\mathbf{S}_k+\mathbf{S}_k\mathbf{S}_i\mathbf{S}_j+\mathbf{S}_k\mathbf{S}_j\mathbf{S}_i-(41/10)(\delta_{ij}\mathbf{S}_k+\delta_{jk}\mathbf{S}_i+\delta_{ki}\mathbf{S}_j)\,. \quad (25)$$

These matrices are traceless and symmetric with respect to the interchange of any two indices, and they give zero when any two indices are contracted. Moreover, they are orthogonal in the sense that product of any two of 1, $\mathbf{S}_i$, $\mathbf{S}_{ij}$, and $\mathbf{S}_{ijk}$ is traceless. Equation (23) can also be written as

$$(\tfrac{3}{2}d\sigma_{3/2}+\tfrac{1}{2}d\sigma_{1/2}-\tfrac{1}{2}d\sigma_{-1/2}-\tfrac{3}{2}d\sigma_{-3/2})/\textstyle\sum_K d\sigma_K=s_i\,\mathrm{Tr}(\rho\mathbf{S}_i)$$
$$(d\sigma_{3/2}-d\sigma_{1/2}+d\sigma_{-1/2}-d\sigma_{-3/2})/\textstyle\sum_K d\sigma_K=\tfrac{1}{6}s_{ij}\,\mathrm{Tr}(\rho\mathbf{S}_{ij})$$
$$(d\sigma_{3/2}-3d\sigma_{1/2}+3d\sigma_{-1/2}-d\sigma_{-3/2})/\textstyle\sum_K d\sigma_K=(10/81)s_{ijk}\,\mathrm{Tr}(\rho\mathbf{S}_{ijk})\,. \quad (23')$$

Gottfried and Jackson[10] gave the angular distribution of the decay particle in terms of the density matrix of the particle with arbitrary spin before its decay. Therefore, the measurement of the angular distribution of the decay particle will give some information concerning the density matrix, i.e., the production process. Parity is not conserved in this case of the N^* production.

Several authors[2–5] expressed the form factors in terms of those of zero momentum transfer as

$$F_i^{V,A}(t)=F_i^{V,A}(0)/[1-(t/b)]^n, \qquad (26)$$

where n is an integer and b a cutoff parameter. If Eq. (26) is used, the density matrix of Eq. (24) can be expressed in terms of the $F_i(0)$, i.e., only the form factors of zero momentum transfer need to be considered in the density matrix. The measurement of the angular distribution of the decay particle might also determine which of the several sets of form factors is the suitable one in the present cases and when more form factors are used.

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APPENDIX

The matrices $S_{\mu\nu\lambda}$ for spin $\frac{3}{2}$ can be expressed by products of the σ matrices. This is very convenient for calculations. Every expression given here can be extended immediately to the arbitrary spin.

Let the matrix component of $S_{\mu\nu\lambda}$ be $(S_{\mu\nu\lambda})_{\alpha\alpha'}$ where α and α' run from 1 to 4. α and α' can be expressed by (ijk) and $(i'j'k')$, i.e., $\alpha=1$, 2, 3, and 4 correspond to (111), (112), (122), and (222), respectively. Introducing a diagonal matrix

$$T_{(ijk)(i'j'k')}=\begin{bmatrix}1&0&0&0\\0&\sqrt{3}&0&0\\0&0&\sqrt{3}&0\\0&0&0&1\end{bmatrix}, \qquad (A1)$$

$S_{\mu\nu\lambda}$ can be expressed as

$$(S_{\mu\nu\lambda})_{(ijk)(i'j'k')}=(-i/36)T_{(ijk)(abc)} \times[\textstyle\sum_p\sum_{p'}(\sigma_\mu)_{aa'}(\sigma_\nu)_{bb'}(\sigma_\lambda)_{cc'}]T_{(a'b'c')(i'j'k')}, \qquad (A2)$$

where $\sum_p\sum_{p'}$ means the sum over all permutations of (abc) and $(a'b'c')$ and summation convention is used. Then the product of two S matrices is

$$(S_{\mu\nu\lambda}S_{\xi\eta\zeta})_{(ijk)(i'j'k')}=(-i/36)^2T_{(ijk)(abc)} \times[\textstyle\sum_p\sum_{p'}(\sigma_\mu)_{aa'}(\sigma_\nu)_{bb'}(\sigma_\lambda)_{cc'}] \times[\sum_{p'}\sum_{p''}(\sigma_\xi)_{a'a''}(\sigma_\eta)_{b'b''}(\sigma_\zeta)_{c'c''}] \times T_{(a''b''c'')(i'j'k')}. \qquad (A3)$$

The traces of Eqs. (A2) and (A3) are

$$\mathrm{Tr}(S_{\mu\nu\lambda})=(-i/36)[\textstyle\sum_p\sum_{p'}(\sigma_\mu)_{aa'}(\sigma_\nu)_{bb'} \times(\sigma_\lambda)_{cc'}]_{a=a',b=b',c=c'} \qquad (A4)$$

$$\mathrm{Tr}(S_{\mu\nu\lambda}S_{\xi\eta\zeta})=(-i/36)^2[\textstyle\sum_p\sum_{p'}(\sigma_\mu)_{aa'}(\sigma_\nu)_{bb'}(\sigma_\lambda)_{cc'}] \times[\sum_{p'}\sum_p(\sigma_\xi)_{a'a}(\sigma_\nu)_{b'b}(\sigma_\lambda)_{c'c}]. \qquad (A5)$$

From Eq. (A2) one gets $S_{444}=-1$ and $S_{44i}=\frac{2}{3}i\mathbf{S}_i$ and

$$[\mathbf{S}_i,S_{\mu\nu\lambda}]_+=i\delta_{\lambda4}S_{\mu\nu i}+i\delta_{\nu4}S_{\mu i\lambda}+i\delta_{\mu4}S_{i\nu\lambda}-i\delta_{\lambda i}S_{\mu\nu4} -i\delta_{\nu i}S_{\mu4\lambda}-i\delta_{\mu i}S_{4\nu\lambda}. \qquad (A6)$$

From this equation one can get Eq. (15) of Ref. 9. Also Eq. (15) of Ref. 9 can be expressed as

$$\begin{aligned}&1=-S_{444},\\&\mathbf{S}_i=-\tfrac{3}{2}iS_{44i},\\&\mathbf{S}_{ij}=3S_{ij4}+\delta_{ij}S_{444},\\&\mathbf{S}_{ijk}=(9i/10)[5S_{ijk}+\delta_{ij}S_{44k}+\delta_{jk}S_{44i}+\delta_{ki}S_{44j}].\end{aligned} \qquad (A7)$$

These are used to get the density matrix from the square of the interaction Hamiltonian. These matrices are traceless and symmetric with respect to the interchange of any two indices, and they give zero when any two indices are contracted. Also the product of any two of the matrices in Eq. (A7) is traceless and for the product of the same matrices, one gets

$$\begin{aligned}&\mathrm{Tr}(\mathbf{S}_i\mathbf{S}_j)=5\delta_{ij},\\&\mathrm{Tr}(\mathbf{S}_{ij}\mathbf{S}_{kl})=12(\delta_{il}\delta_{jk}+\delta_{ik}\delta_{jl})-8\delta_{ij}\delta_{kl},\\&\mathrm{Tr}(\mathbf{S}_{ijk}\mathbf{S}_{pqr})=(27/5)\textstyle\sum_p(5\delta_{ip}\delta_{jq}\delta_{kr}-\delta_{ij}\delta_{kp}\delta_{qr}\\&\qquad-\delta_{jk}\delta_{ip}\delta_{qr}-\delta_{ki}\delta_{jp}\delta_{qr}),\end{aligned} \qquad (A8)$$

where $\sum_p$ means the sum of permutations of either (ijk) or (pqr). These relations were used to get Eq. (23′).

Equation (11) can be obtained by using Eqs. (5), (A2), and (A3). Other useful formulas are

$$\gamma_{\mu\nu\lambda}N^*{}_\mu N^*{}_\nu N^*{}_\lambda\gamma_{\tau\xi\eta}N^*{}_\xi N^*{}_\eta=\tfrac{1}{8}m_{N^*}{}^4N^*{}_\lambda\gamma_{\lambda\alpha\beta}\gamma_{\tau\alpha\beta}, \qquad (A9)$$

$$\gamma_{\mu\nu\lambda}N^*{}_\mu N^*{}_\nu N^*{}_\lambda\gamma_{\rho\sigma\tau}N^*{}_\tau=\tfrac{3}{8}m_{N^*}{}^2N^*{}_\lambda N^*{}_\tau\gamma_{\lambda\tau\alpha}\gamma_{\rho\sigma\alpha}. \qquad (A10)$$