relations

$$\langle\rho^0|V_\mu^{(3)}(0)|0\rangle=\frac{i}{(2\Omega E_\rho)^{1/2}}v_\rho\xi_\mu^{(M)*}(\rho)\,, \tag{A7}$$

$$\langle A_1^+|A_\mu^{(+)}(0)|0\rangle=\frac{i}{(2\Omega E_{A_1})^{1/2}}\sqrt{2}a_{A_1}\xi_\mu^{(M)*}(A_1)\,, \tag{A8}$$

and

$$\langle A_1^+;\mathbf{p}_{A_1},\xi^{(M)}(A_1)|V_\mu^{(3)}(0)|\pi^+;\mathbf{p}_\pi\rangle=\frac{1}{(2\Omega E_{A_1})^{1/2}(2\Omega E_\pi)^{1/2}}[\xi_\mu^{(M)*}(A_1)F_V^{(1)}[\pi^+\to A_1^+;(p_{A_1}-p_\pi)^2]$$
$$+\{\xi^{(M)*}(A_1)\cdot p_\pi\}\{(p_{A_1}-p_\pi)_\mu F_V^{(2)}[\pi^+\to A_1^+;(p_{A_1}-p_\pi)^2]+(p_{A_1}+p_\pi)_\mu F_V^{(3)}[\pi^+\to A_1^+;(p_{A_1}-p_\pi)^2]\}]. \tag{A9}$$

The remaining form factors are defined by Eqs. (8a) and (8c), together with the isotopic-spin transformation properties of $A_\mu^{(i)}$.

PHYSICAL REVIEW VOLUME 162, NUMBER 5 25 OCTOBER 1967

Remarks on the Pole Dominance Version of the Hypothesis of Partially Conserved Axial-Vector Current*

C. W. KIM AND MICHAEL RAM†
Department of Physics, The Johns Hopkins University, Baltimore, Maryland

(Received 15 June 1967)

We review and discuss the dispersion-theory version of the PCAC hypothesis in the case of nucleon leptonic weak decays. The discussion is extended to the case of meson weak decays, and the feasibility of a direct test of the Goldberger-Treiman relation for the meson case is considered.

I. INTRODUCTION

THERE have recently been many applications of the PCAC (partially conserved axial-vector current) hypothesis, particularly in conjunction with the derivation of sum rules from current algebra. The PCAC hypothesis is essential in these applications, as it relates weak-interaction form factors (which are in most cases difficult, if not impossible, to measure at present) appearing in the sum rules, to strong-coupling constants which can be determined from decay widths or scattering experiments. In general, to test the PCAC hypothesis directly, one would have to measure independently both the weak form factors and strong-coupling constants. The best and only known case that has been tested directly to date is that involving nucleon $n\to p$ weak form factor (the famous Goldberger-Treiman relation).[1]

In this paper we would like to review the possibility of directly testing the PCAC hypothesis in the case of meson decays. As we shall see, no direct test (as in the nucleon case) is feasible. We shall only discuss the dispersion-theory (pole-dominance) version[2] of the PCAC hypothesis. The more commonly used version due to Gell-Mann and Lévy[3] which relates the divergence of the axial-vector current to the pion field will not be considered.

In Sec. II, we review the application of PCAC to nucleon leptonic weak interactions. In Sec. III, the results of Sec. II are extended to the case of meson leptonic weak interactions. Throughout this paper we use natural units ($\hbar=c=1$).

II. APPLICATION OF PCAC TO NUCLEON LEPTONIC WEAK INTERACTIONS

Let us first review the application of PCAC to nucleon leptonic weak interactions. Consider the matrix element $\langle 0|A_\mu^{(+)}(0)|\bar{p},n;\text{in}\rangle$, where $|\bar{p},n;\text{in}\rangle$ represents an antiproton and neutron "in" state with antiproton and neutron 4-momenta $\bar{p}=(\bar{\mathbf{p}},iE_{\bar{p}})$ and $n=(\mathbf{n},iE_n)$, respectively. $A_\mu^{(+)}(x)$, with $\mu=1,\cdots 4$, is the strangeness-conserving axial-vector weak hadron current operative in β decay and muon capture. This matrix element is easily related to the matrix element $\langle p|A_\mu^{(+)}(0)|n\rangle$ involved in β decay and muon capture,

* Work supported in part by the National Science Foundation.
† Present address: Physics Department, State University of New York at Buffalo, Buffalo, New York.
[1] Even for the nucleon case, the PCAC hypothesis has been tested at only two momentum transfers, corresponding to those occurring in β decay ($q^2\approx 0$) and muon capture ($q^2\approx m_\mu^2$).

[2] Y. Nambu, Phys. Rev. Letters 4, 380 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento 17, 757 (1960).
[3] M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).

through crossing symmetry. From parity and Lorentz invariance, we can write[4]

$$\langle 0|A_\mu^{(+)}(0)|\bar{p},n;\text{in}\rangle=\frac{1}{\Omega}(M^2/E_pE_n)^{1/2}\bar{v}(-\bar{\mathbf{p}})\times\left[\gamma_\mu F_A(q^2)+i\frac{2M}{m_\pi^2}q_\mu F_P(q^2)\right]\gamma_5 u(\mathbf{n})\,, \quad (1)$$

where $v(-\bar{\mathbf{p}})$ and $u(\mathbf{n})$ are the antiproton and neutron spinors, respectively, and $q=-(\bar{p}+n)$. We neglect electromagnetic mass splittings and denote the nucleon and pion masses by M and m_π, respectively. In writing relation (1), we have assumed that the weak hadron currents are first-class currents.[5] F_A and F_P are axial-vector and induced pseudoscalar $n\to p$ weak form factors. From Eq. (1) it is easy to show that

$$\langle 0|\partial_\mu A_\mu^{(+)}(0)|\bar{p},n;\text{in}\rangle=\Omega^{-1}(M^2/E_pE_n)^{1/2}2M\bar{v}(-\bar{\mathbf{p}})\gamma_5 u(\mathbf{n})\Phi(q^2)\,, \quad (2)$$

where

$$\Phi(q^2)=F_A(q^2)+(q^2/m_\pi^2)F_P(q^2)\,. \quad (3)$$

If we now assume that

$$\lim_{q^2\to\infty}\langle 0|\partial_\mu A_\mu^{(+)}|\varphi\rangle=0\,, \quad (4)$$

where $|\varphi\rangle$ is an arbitrary state and $q=-p_\varphi$, one can write the following unsubtracted dispersion relation for $\Phi(q^2)$ (Ref. 6):

$$\Phi(q^2)=-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{Im}\Phi(-q'^2)}{q'^2+q^2+i\epsilon}dq'^2\,, \quad (5)$$

where

$$2M\{\bar{v}(-\bar{\mathbf{p}})\gamma_5 u(\mathbf{n})\}(2i)\,\mathrm{Im}\Phi(q^2)=-i\left(\frac{\Omega E_n}{M}\right)^{1/2}\bar{v}(-\bar{\mathbf{p}})(2\pi)^4\sum_i\delta^{(4)}(p+n-p_i)\times\langle 0|\partial_\mu A_\mu^{(+)}|i\rangle\langle i|\eta_p(0)|n\rangle\,, \quad (6)$$

and

$$(\gamma_\mu\partial_\mu+M)\psi_p(x)=\eta_p(x)\,. \quad (7)$$

$\psi_p(x)$ is the proton field. Assumption (4) is one of the versions of the PCAC hypothesis.[2] In this form it is sometimes also referred to as the ACAC (asymptotically conserved axial-vector current) hypothesis.[7] In Table I we have summarized the properties of the intermediate states $|i\rangle$ contributing to Eq. (6). The state β represents the effective 3π, 5π, $\cdots J^P=0^-$ contribution which we approximate by a pole of mass m_β.[7,8]

[4] The Dirac γ matrices (γ_μ and γ_5) are chosen to be Hermitian. Furthermore, we use periodic boundary conditions and normalize wave functions in a box of volume Ω. Also, $\bar{v}=v^\dagger\gamma_4$, where $\dagger$ denotes Hermitian conjugation. We use a metric with imaginary fourth component.

[5] S. Weinberg, Phys. Rev. **112**, 1375 (1958).

[6] M. Goldberger and S. Treiman, Phys. Rev. **111**, 354 (1958).

[7] H. Primakoff, in *Proceedings of the International School of Physics "Enrico Fermi" Course* XXXII (Academic Press Inc., New York, 1964).

[8] P. Dennery and H. Primakoff, Phys. Rev. Letters **8**, 350 (1962).

TABLE I. Properties of intermediate states contributing to Eqs. (6) and (17). The last column lists the possible known single-particle and resonance (both real and effective) candidates. Q, S, and B refer to the charge (in units of proton charge), strangeness, and baryon number. $J^P=$ (intrinsic spin)$^{\text{parity}}$. $I^G=$ (isotopic spin)$^{G\text{ parity}}$.

	Q	S	B	I^G	J^G	Known candidates
$\|i\rangle$:	-1	0	0	1^-	0^-	π,β
$\|j\rangle$:	-1	0	0	1^-	$0^-,1^+$	π,β,α

From Eqs. (5) and (6) and Table I, one can show that

$$\Phi(q^2)=F_A(q^2)+\frac{q^2}{m_\pi^2}F_P(q^2)\cong\frac{a_\pi m_\pi^2 f_{\pi np}(-m_\pi^2)}{q^2+m_\pi^2}+\frac{a_\beta m_\beta^2 f_{\beta np}(-m_\beta^2)}{q^2+m_\beta^2}\,, \quad (8)$$

where a_π and $f_{\pi np}$ are defined by

$$\langle 0|A_\mu^{(+)}(0)|\pi^-,p_\pi\rangle=\frac{ia_\pi m_\pi}{(2\Omega E_\pi)^{1/2}}(p_\pi)_\mu \quad (9)$$

and

$$\langle\pi^-;p_\pi|\eta_p(0)|n\rangle=\frac{1}{(2\Omega E_\pi)^{1/2}}\left(\frac{M}{\Omega E_n}\right)^{1/2}\frac{2M}{m_\pi}\times f_{\pi np}[(p+n)^2]\gamma_5 u(n)\,. \quad (10)$$

The definitions of a_β and $f_{\beta np}$ are completely analogous. The constant a_π is determined from the observed pion decay rate.

Let us now assume that $|a_\beta f_{\beta np}|\ll|a_\pi f_{\pi np}|$, so that $\Phi(q^2)$ is dominated by the one-pion pole for $|q^2|\lesssim m_\pi^2$. With this assumption, we can write

$$\Phi(q^2)=F_A(q^2)+(q^2/m_\pi^2)F_P(q^2)\cong\frac{a_\pi m_\pi^2 f_{\pi np}(-m_\pi^2)}{q^2+m_\pi^2}\,,\quad |q^2|\lesssim m_\pi^2\,. \quad (11)$$

Equation (11) reduces to the famous Goldberger-Treiman relation

$$F_A(0)\cong a_\pi f_{\pi np}(-m_\pi^2)\,, \quad (12)$$

when we set $q^2=0$. $F_A(0)$ has been determined from nuclear β-decay rates, and the observed value, 1.18, agrees with the one determined from relation (12), 1.35, to within 13%.[9] Had we not neglected the β-state contribution, we would have found that

$$F_A(0)=a_\pi f_{\pi np}+a_\beta f_{\beta np}\,. \quad (13)$$

We therefore see that in this version of the PCAC hypothesis we can attribute the 13% discrepancy in relation (12) as due to neglect of the term $a_\beta f_{\beta np}$. In contrast, in the Gell-Mann–Lévy version of PCAC, the

[9] See, for example, C. W. Kim and H. Primakoff, Phys. Rev. **139**, B1447 (1965).

13% discrepancy is attributed to the approximation $f_{\pi np}(0)\approx f_{\pi np}(-m_\pi^2)$ that has to be made in order to derive Eq. (12) in that version.

Muon-capture experiments also seem to indicate that relation (11) is reasonably well satisfied for $q^2\approx m_\mu^2$, where m_μ is the muon mass.[10]

Assuming that

$$\lim_{q^2\to\infty} F_A(q^2)=0 \quad\text{and}\quad \lim_{q^2\to\infty} F_P(q^2)=0\,, \tag{14}$$

one can write down the following unsubtracted dispersion relations:

$$F_A(q^2)=-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{Im}F_A(-q'^2)}{q^2+q'^2+i\epsilon}dq'^2\,, \tag{15}$$

$$F_P(q^2)=-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{Im}F_P(-q'^2)}{q^2+q'^2+i\epsilon}dq'^2\,, \tag{16}$$

where

$$\begin{aligned}\{\bar v(-\bar{\mathbf{p}})\gamma_\mu\gamma_5 u(\mathbf{n})\}\ \mathrm{Im}F_A(q^2)&\\ +i\frac{2M}{m_\pi^2}\{\bar v(-\bar{\mathbf{p}})q_\mu\gamma_5 u(\mathbf{n})\}\ \mathrm{Im}F_P(q^2)&\\ =-\tfrac{1}{2}(\Omega E_n/M)^{1/2}\bar v(-\bar{\mathbf{p}})(2\pi)^4\textstyle\sum_j\delta^{(4)}(p+n-p_j)&\\ \times\langle 0|A_\mu^{(+)}(0)|j\rangle\langle j|\eta_p(0)|n\rangle\,.&\end{aligned} \tag{17}$$

In Table I we have summarized the properties of the intermediate states $|j\rangle$ contributing to Eq. (17). The state α represents the effective 3π, 5π, $\cdots J^P=1^+$ contribution which we approximate by a pole of mass m_α.[7,8] It is interesting to note that the $A_1(1080)$ enhancement[11] that has been observed in high-energy experiments seems to have exactly the same quantum numbers as the α state. It may therefore be possible to identify most of the α contributions as coming from the A_1 enhancement, particularly if the latter turns out to be a bonafide resonance.[12]

Equations (15), (16), and (17) together with Table I give

$$F_A(q^2)\cong\sqrt{2}a_\alpha m_\alpha^2 f_{\alpha np}/(q^2+m_\alpha^2)\,, \tag{18}$$

and

$$F_P(q^2)\cong-\frac{m_\pi^2 a_\pi f_{\pi np}}{q^2+m_\pi^2}-\frac{m_\pi^2 a_\beta f_{\beta np}}{q^2+m_\beta^2}+\frac{\sqrt{2}m_\pi^2 a_\alpha f_{\alpha np}}{q^2+m_\alpha^2}, \tag{19}$$

where[13] a_α and $f_{\alpha np}$ are defined by

$$\langle 0|A_\mu^{(+)}|\alpha^-;p_\alpha,\xi^{(M)}(\alpha)\rangle=\frac{-i}{(2\Omega E_\alpha)^{1/2}}\sqrt{2}a_\alpha\xi_\mu^{(M)}(\alpha)\,, \tag{20}$$

and

$$\langle\alpha^-;p_\alpha,\xi^{(M)}(\alpha)|\eta_p(0)|n\rangle=\frac{1}{\Omega}\left(\frac{M}{2E_\alpha E_n}\right)^{1/2}\times im_\alpha^2 f_{\alpha np}\xi_\mu^{(M)\star}(\alpha)\gamma_\mu\gamma_5 u(\mathbf{n})\,. \tag{21}$$

It is interesting to note that the induced pseudoscalar form factor $F_P(q^2)$ also gets a contribution from the $J^P=1^+$ intermediate state. In fact, such a contribution is implied by Eqs. (11) and (18) and is necessary if one is to obtain the "modified" Goldberger-Treiman relation (13) from Eqs. (2), (3), (4), (18), and (19). This can easily be seen as follows: Eqs. (2), (3), (4), and (18) imply

$$\lim_{q^2\to\infty} q^2F_P(q^2)=0\,. \tag{22}$$

Substituting Eq. (19) into (22) we find that

$$\sqrt{2}a_\alpha f_{\alpha np}=a_\pi f_{\pi np}+a_\beta f_{\beta np}\,. \tag{23}$$

This, together with Eq. (18), leads to the "modified" Goldberger-Treiman relation, Eq. (13). Had we neglected the $J^P=1^+$ contribution to $F_P(q^2)$ we would have obtained

$$a_\pi f_{\pi np}+a_\beta f_{\beta np}=0\,, \tag{24}$$

which, with Eq. (13) implies $F_A(0)=0$, in contradiction with the experimental value of 1.18. Furthermore, as we have already indicated, the fact that the Goldberger-Treiman relations is in agreement with experiment to within 13% seems to indicate that

$$|a_\pi f_{\pi np}|\gg|a_\beta f_{\beta np}|\,, \tag{25}$$

and not Eq. (24).[14]

III. APPLICATION OF PCAC TO MESON LEPTONIC WEAK INTERACTIONS

In this section, we wish to consider the application of the PCAC hypothesis to the matrix element

$$\langle 0|A_\mu^{(+)}(0)|\pi^0,k;\rho^-,p,\xi^{(M)}(\rho);\mathrm{in}\rangle\,,$$

where $|\pi^0,k;\rho^-,p,\xi^{(M)}(\rho);\mathrm{in}\rangle$ is a π^0 and ρ^- "in" state with pion and ρ-meson 4-momenta $k=(\mathbf{k},i\omega)$ and $p=(\mathbf{p},iE)$, respectively. $\xi^{(M)}(\rho)$ is the ρ-meson polarization vector. The reason we have chosen to discuss this specific matrix element will be clarified later. Applying parity and Lorentz invariance, we can write

$$\begin{aligned}&\langle 0|A_\mu^{(+)}(0)|\pi^0,k;\rho^-,p,\xi^{(M)}(\rho);\mathrm{in}\rangle\\ &=\frac{i}{(2\Omega)(\omega E)^{1/2}}[\xi_\mu^{(M)}F^{(1)}(q^2)+(\xi^{(M)}\cdot q)\\ &\qquad\times\{q_\mu F^{(2)}(q^2)+Q_\mu F^{(3)}(q^2)\}]\,,\end{aligned} \tag{26}$$

where

$$q=-(k+p)\,,\quad Q=-(k-p)\,. \tag{27}$$

[10] See, for example, C. W. Kim and H. Primakoff, Phys. Rev. **140**, B566 (1965).

[11] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

[12] A recent calculation by S. Weinberg [Phys. Rev. Letters **18**, 507 (1967)] based on the algebra of currents seems to indicate that $m_\alpha/m_\rho\approx\sqrt{2}$, in agreement with the experimental value of m_{A_1}/m_ρ.

[13] $p_\alpha=(\mathbf{p}_\alpha,iE_\alpha)$ is the α-state 4-momentum and $\xi^{(M)}(\alpha)$ $(M=1,2,3)$ its polarization. $\xi_\mu^{(M)\star}=\xi_\mu^{(M)*}$, if $\mu=1,2,3$ (* denotes complex conjugation), and $\xi_4^{(M)\star}=-\xi_4^{(M)*}$.

[14] In Ref. 7, the $J^P=1^+$ contribution to $F_P(q^2)$ was in fact neglected, and as a result, Eq. (24) followed.

From Eq. (26), one readily obtains

$$\langle 0|\partial_\mu A_\mu^{(+)}(0)|\pi^0,k;\rho^-,p,\xi^{(M)}(\rho);\text{in}\rangle = \frac{1}{2\Omega(\omega E)^{1/2}}\{\xi^{(M)}(\rho)\cdot q\}\Psi(q^2), \quad (28)$$

where

$$\Psi(q^2)=F^{(1)}(q^2)+q^2F^{(2)}(q^2)+(m_\rho^2-m_\pi^2)F^{(3)}(q^2)\,; \quad (29)$$

m_ρ is the ρ-meson mass. As in the nucleon case, application of the PCAC hypothesis, Eq. (4), gives

$$\Psi(q^2)=F^{(1)}(q^2)+q^2F^{(2)}(q^2)+(m_\rho^2-m_\pi^2)F^{(3)}(q^2) = \frac{a_\pi m_\pi^3 g_{\rho\pi\pi}(-m_\pi^2)}{q^2+m_\pi^2}+\frac{a_\beta m_\beta^3 g_{\rho\beta\pi}(-m_\beta^2)}{q^2+m_\beta^2}, \quad (30)$$

where $g_{\rho\pi\pi}$ is defined by

$$\langle\pi^-;k|j_\pi^{(0)}(0)|\rho^-;p,\xi^{(M)}(\rho)\rangle = \frac{-1}{2\Omega(\omega E)^{1/2}}(\xi^{(M)}\cdot k)g_{\rho\pi\pi}[(k-p)^2], \quad (31)$$

and an analogous expression for $g_{\rho\beta\pi}[(k-p)^2]$. In Eq. (31)

$$\left[\left(\nabla^2-\frac{\partial^2}{\partial t^2}\right)-m_\pi^2\right]\varphi_\pi^{(0)}(x)=j_\pi^{(0)}(x), \quad (32)$$

where $\varphi_\pi^{(0)}(x)$ is the π^0 field. Assuming here that

$$|m_\pi a_\pi g_{\rho\pi\pi}(-m_\pi^2)|\gg|m_\beta a_\beta g_{\rho\beta\pi}(-m_\beta^2)|, \quad (33)$$

i.e., that $\Psi(q^2)$ is dominated by the pion pole for $|q^2|\lesssim m_\pi^2$, one can write

$$\Psi(q^2)=F^{(1)}(q^2)+q^2F^{(2)}(q^2)+(m_\rho^2-m_\pi^2)F^{(3)}(q^2) \cong \frac{a_\pi m_\pi^3 g_{\rho\pi\pi}(-m_\pi^2)}{q^2+m_\pi^2}, \quad |q^2|\lesssim m_\pi^2. \quad (34)$$

Setting $q^2=0$ in the above, we obtain

$$F^{(1)}(0)+(m_\rho^2-m_\pi^2)F^{(3)}(0)\cong a_\pi m_\pi g_{\rho\pi\pi}(-m_\pi^2). \quad (35)$$

This is the analog of the Goldberger-Treiman relation, Eq. (12), in the meson case. To check relation (35) directly, one would have to make independent measurements of $F^{(1)}(0)$, $F^{(3)}(0)$, and $g_{\rho\pi\pi}(-m_\pi^2)$. The last constant, $g_{\rho\pi\pi}(-m_\pi^2)$, is easily determined from the observed decay width of the reaction $\rho\to 2\pi$. On the other hand, to determine $F^{(1)}(0)$ and $F^{(3)}(0)$ directly, one would have to observe, e.g., the weak decay process $\rho\to\pi+l+\nu$ ($l\equiv$lepton). This is practically impossible owing to the fact that the ρ decays considerably faster into two pions via the strong interaction. A direct comparison of relation (35) with experiment does not therefore seem feasible. One is forced to search for less direct ways of testing Eq. (35).

If we assume

$$\lim_{q^2\to\infty} F^{(i)}(q^2)=0, \quad i=1, 2, 3, \quad (36)$$

then, in analogy to Eqs. (18) and (19), one can show that

$$F^{(1)}(q^2)=\frac{\sqrt{2}a_\alpha g_{\alpha\rho\pi}^{(1)}(-m_\pi^2)}{q^2+m_\alpha^2}, \quad (37)$$

$$F^{(2)}(q^2)=-\frac{a_\pi m_\pi g_{\rho\pi\pi}(-m_\pi^2)}{q^2+m_\pi^2}-\frac{a_\beta m_\beta g_{\rho\beta\pi}(-m_\beta^2)}{q^2+m_\beta^2}+\frac{\sqrt{2}a_\alpha m_\alpha f_{\alpha\rho\pi}(-m_\pi^2)}{q^2+m_\alpha^2}, \quad (38)$$

$$F^{(3)}(q^2)=\frac{\sqrt{2}a_\alpha[-\frac{1}{2}g_{\alpha\rho\pi}^{(2)}(-m_\pi^2)]}{q^2+m_\alpha^2}, \quad (39)$$

where $g_{\alpha\rho\pi}^{(1)}$, $g_{\alpha\rho\pi}^{(2)}$, and $f_{\alpha\rho\pi}$ are defined by

$$\langle\alpha^-;p_\alpha,\xi^{(M')}(\alpha)|j_\pi^{(0)}(0)|\rho^-;p,\xi^{(M)}(\rho)\rangle = -\frac{1}{2\Omega(EE_\alpha)^{1/2}}\{[\xi^{(M')\star}(\alpha)\cdot\xi^{(M)}(\rho)] \times g_{\alpha\rho\pi}^{(1)}[(p_\alpha-p)^2]+[\xi^{(M)}(\rho)\cdot p_\alpha] \times[\xi^{(M')\star}(\alpha)\cdot p]g_{\alpha\rho\pi}^{(2)}[(p_\alpha-p)^2]\}, \quad (40)$$

and

$$f_{\alpha\rho\pi}(-m_\pi^2)=\frac{1}{m_\alpha^3}[g_{\alpha\rho\pi}^{(1)}(-m_\pi^2) +\tfrac{1}{2}(m_\pi^2-m_\rho^2)g_{\alpha\rho\pi}^{(2)}(-m_\pi^2)]. \quad (41)$$

Substituting Eqs. (37) and (39) into Eq. (35), we find that

$$\sqrt{2}a_\alpha m_\alpha f_{\alpha\rho\pi}(-m_\pi^2)\cong a_\pi m_\pi g_{\rho\pi\pi}(-m_\pi^2). \quad (42)$$

All quantities on the right-hand side of Eq. (42) are well known. If, as mentioned in Sec. II, we identify most of the α contribution as coming from the $A_1(1080)$ "enhancement,"[11] we can write

$$\sqrt{2}a_{A_1}m_{A_1}f_{A_1\rho\pi}(-m_\pi^2)\cong a_\pi m_\pi g_{\rho\pi\pi}(-m_\pi^2). \quad (43)$$

In principle, one can determine $f_{A_1\rho\pi}(-m_\pi^2)$ from measurements on the strong decay $A_1\to\rho+\pi$. This is the reason why we chose to consider the specific matrix element $\langle 0|A_\mu^{(+)}(0)|\pi^0;\rho^-\rangle$, as the A_1 decays predominantly into $\rho\pi$. On the other hand, measurement of a_{A_1} is not feasible as it is related to the rate of the weak decay $A_1\to l+\nu$ which is many orders of magnitude smaller than the rate of the dominant strong-decay mode $A_1\to\rho+\pi$. There is, therefore, really no hope of testing relation (43) directly. The most one can hope to obtain from relation (43) is a determination of the constant a_{A_1}. As we will now indicate, such an estimate of a_{A_1} can be useful when correlated with other theoretical calculations.

From Eqs. (41) and (43) we find that

$$a_{A_1}=m_{A_1}^2\left[\frac{a_\pi m_\pi g_{\rho\pi\pi}(-m_\pi^2)}{\sqrt{2}g_{A_1\rho\pi}(-m_\pi^2)\{1-[(m_\rho^2-m_\pi^2)(m_{A_1}^2+m_\rho^2)/(m_{A_1}^2-m_\rho^2)^2]g_{A_1\rho\pi}'(-m_\pi^2)\}}\right], \tag{44}$$

where

$$g_{A_1\rho\pi}=g_{A_1\rho\pi}^{(1)} \quad\text{and}\quad g_{A_1\rho\pi}'=\frac{(m_{A_1}^2-m_\rho^2)^2}{2(m_{A_1}^2+m_\rho^2)}\frac{g_{A_1\rho\pi}^{(2)}}{g_{A_1\rho\pi}^{(1)}}. \tag{45}$$

Knowledge of the decay rate $\Gamma(A_1\rightarrow\rho+\pi)$ is not sufficient to determine $g_{A_1\rho\pi}(-m_\pi^2)$ and $g_{A_1\rho\pi}'(-m_\pi^2)$ uniquely, but just gives a relation between these two constants. To determine both parameters uniquely, a knowledge of the angular distribution or polarization of the decay products is also required. In the absence of such detailed information on the decay $A_1\rightarrow\rho+\pi$, we shall rely upon a recent[15] theoretical estimate of $g_{A_1\rho\pi}(-m_\pi^2)$ and $g_{A_1}{}'_{\rho\pi}(-m_\pi^2)$ based on *charge-charge* commutators and PCAC. This estimate gives the following two possible solutions[16]:

$$|g_{A_1\rho\pi}^{(\mathrm{I})}(-m_\pi^2)|\cong 4.05\times10^3\text{ MeV},$$
$$g_{A_1\rho\pi}'^{(\mathrm{I})}(-m_\pi^2)\cong 2.04, \tag{46I}$$

and

$$|g_{A_1\rho\pi}^{(\mathrm{II})}(-m_\pi^2)|\cong 4.05\times10^3\text{ MeV},$$
$$g_{A_1\rho\pi}'^{(\mathrm{II})}(-m_\pi^2)\cong -0.04. \tag{46II}$$

Substituting (46I) and (46II) into Eq. (44), together with the values

$$a_\pi=0.95,\quad g_{\rho\pi\pi}(-m_\pi^2)=11.4, \tag{47}$$

we find that

$$a_{A_1}^{(\mathrm{I})}=(m_{A_1}^2/\sqrt{2})\times0.081 \tag{48I}$$

and

$$a_{A_1}^{(\mathrm{II})}=(m_{A_1}^2/\sqrt{2})\times0.336. \tag{48II}$$

As shown by Sakurai,[17] ρ dominance together with the CVC (conserved vector current) hypothesis implies

$$v_\rho g_{\rho\pi\pi}=2m_\rho^2, \tag{49}$$

where v_ρ is defined by

$$\langle\rho^0;p,\xi^{(M)}(\rho)|V_\mu^{(3)}|0\rangle=\frac{i}{(2\Omega E)^{1/2}}v_\rho\xi_\mu^{(M)\star}(\rho). \tag{50}$$

The state $|\rho^0;p,\xi^{(M)}(\rho)\rangle$ is a ρ^0 vector-meson state of 4-momentum $p=(\mathbf{p},iE)$ and polarization $\xi^{(M)}(\rho)$, and $V_\mu^{(i)}(x)$ $(i=1,2,3;\mu=1\cdots4)$ is the isotopic-spin current. Combining relations (47), (48I), (48II), and (49), we find that

$$(v_\rho/a_{A_1})_{\mathrm{I}}=(m_\rho^2/m_{A_1}^2)3.08\cong1.52, \tag{51I}$$

$$(v_\rho/a_{A_1})_{\mathrm{II}}=(m_\rho^2/m_{A_1}^2)0.74\cong0.36. \tag{51II}$$

Neither of these values is in good agreement with recent[18] theoretical estimates based on *charge-current* commutators and PCAC which seem to indicate

$$(v_\rho/a_{A_1})=1. \tag{52}$$

It has been pointed out[19] that result (52) implies that the vector meson is coupled to the vacuum by the vector current $V_\mu^{(i)}$ with the same constant as the axial-vector meson is coupled to the vacuum by the axial-vector current $A_\mu^{(i)}$. Though such a conclusion may be valid for bare coupling constants [just like $(G_A/G_V)_{\text{bare}}=1$],[20] it need not hold for the renormalized constants appearing in our equations [just like $(G_A/G_V)_{\text{renormalized}}=1.18$].

In conclusion, we have shown that: (1) the induced pseudoscalar form factors $F_P(q^2)$ and $F^{(2)}(q^2)$ get contributions from the $J^P=1^+$ intermediate state, and that such a contribution is essential if one is to obtain the correct Goldberger-Treiman relation; (2) a direct test of the PCAC hypothesis in the case of meson matrix elements is not feasible (at least not in the near future).

[15] C. W. Kim and Michael Ram, preceding paper, Phys. Rev. **162**, 1576 (1967).

[16] Since in Ref. 15 we used the Gell-Mann–Lévy version of PCAC, the constants determined there were actually $g_{A_1\rho\pi}(0)$ and $g_{A_1\rho\pi}'(0)$. We are therefore making the assumption that $g_{A_1\rho\pi}(-m_\pi^2)\cong g_{A_1\rho\pi}(0)$ and $g_{A_1\rho\pi}'(-m_\pi^2)\cong g_{A_1\rho\pi}'(0)$, which, as mentioned in Sec. II, is an assumption commonly adopted when using the Gell-Mann–Lévy version of PCAC. As shown in Ref. 15, both solutions (46I) and (46II) are not inconsistent with the observed decay width $\Gamma(A_1\rightarrow\rho\pi)$, within the limits of the approximations used.

[17] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960); M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).

[18] D. A. Geffen, Ann. Phys. (N. Y.) 42, 1 (1967); C. W. Kim and Michael Ram (Ref. 15).

[19] D. A. Geffen, Ref. 18.

[20] G_A and G_V are, respectively, the renormalized axial-vector and vector weak-coupling constants in neutron β decay.