new experimental decay rates of $Y_0^*(1518)$. The model rather favors a higher decay rate for the $Y_0^*(1518) \rightarrow \bar{K}N$ mode than that given in the compiled data.[3]

(3) For the optimum value of the F/D ratio, the $(\Sigma\pi)$ decay rate of $Y_1^*(1660)$ is two to four times higher than that of the $(\Lambda\pi)$ decay. The currently reported $Y_1^*(1680)$[21] with a large ratio of $\Gamma(\Lambda\pi)/\Gamma(\Sigma\pi)$ might belong to another unitary multiplet if its spin-parity is $\frac{3}{2}^-$.

(4) The fits to $\Xi^*(1815)$ decays are not so good if we take the existing experimental data. Because of the meager experimental situation, we do not consider the disagreement severe. If the $\Xi^*(1815)$ is a real object and if the $\Lambda\bar{K}$ dominance of its decay is true, some other mechanism to suppress the $\Sigma\bar{K}$ decay mode is necessary.[22]

[21] M. Derrick, T. Fields, J. Loken, R. Ammar, R. E. P. Davis, W. Kropac, J. Mott, and F. Schweingruber, Phys. Rev. Letters **18**, 266 (1967).

[22] R. H. Capps (unpublished).

It is surprising that the F/D ratio obtained in this paper is nearly equal to the values derived from different methods and different input data.[22,23]

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Note added in proof. After this paper was written, G. B. Yodh published a paper [Phys. Rev. Letters **18**, 810 (1967)], in which he showed the incompatibility of a pure $SU(3)$ singlet assignment for $Y_0^*(1518)$ and also suggested the difficulties of the singlet-octet mixing scheme. In this paper we predict rather the decay widths which are experimentally uncertain by taking the most reliable data as input. More data for the decays of the $Y_1^*(1660)$ and $\Xi^*(1815)$ should be accumulated before the predictions can be judged.

[23] A. Kernan and W. M. Smart, Phys. Rev. Letters **17**, 832 (1965).

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Drell-Hearn-Gerasimov Sum Rule: Examples and Counterexamples*

G. BARTON
University of Sussex, Brighton, Sussex, England

AND

NORMAN DOMBEY†
Department of Physics, Harvard University, Cambridge, Massachusetts

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We investigate by means of examples whether the Drell-Hearn-Gerasimov sum rule can hold simultaneously for a lightly bound state and for its constituents. Subject to certain assumptions, whose applicability is discussed, we find in particular that if the rule holds for the nucleons, then it holds for the deuteron but fails for He^3 and H^3. If neutron and proton masses were appreciably unequal, then the rule would fail for the deuteron as well.

1. INTRODUCTION

RECENTLY Gerasimov and Drell and Hearn[1] have proposed the following sum rule for the absorption of photons by protons:

$$8\pi^2\alpha\left(\frac{\kappa_p}{2M}\right)^2 = 2\pi^2\alpha\frac{\kappa_p^2}{M^2} = \int_0^\infty \frac{d\omega}{\omega}[\sigma_p{}^P(\omega) - \sigma_p{}^A(\omega)] \equiv J_p{}^P - J_p{}^A \equiv J_p. \quad (1.1)$$

Here, $\alpha = e^2 \approx 1/137$ is the fine structure constant (we use natural units, $\hbar = 1 = c$), $\kappa_p \approx 1.79$ is the anomalous magnetic moment of the proton in units of $e/2M$, M is the nucleon mass, and $\sigma_p{}^P(\omega)$ $(\sigma_p{}^A(\omega))$ is the total absorption cross section for photons of frequency ω with spins parallel (antiparallel) to the initial proton spin. Corresponding rules are implied for any spin-$\frac{1}{2}$ particle; we shall call them DHG rules in the following.

To derive (1.1) (to order α), one needs two results rigorously provable from microcausality and charge conservation (gauge invariance), plus two further independent assumptions. The first of the proved results is the dispersion relation for the forward Compton scattering amplitude[2] $f(\omega)$:

$$f(\omega) = \boldsymbol{\varepsilon}'^* \cdot \boldsymbol{\varepsilon} f_1(\omega) + i\omega\boldsymbol{\sigma} \cdot \boldsymbol{\varepsilon}'^* \times \boldsymbol{\varepsilon} f_2(\omega), \quad (1.2)$$

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† Permanent address: University of Sussex, Brighton, Sussex, England.

[1] S. B. Gerasimov, J. Nucl. Phys. (USSR) **2**, 598 (1965); [English transl.: Soviet J. Nucl. Phys. **2**, 430 (1966)]; S. D. Drell and A. C. Hearn, Phys. Rev. Letters **16**, 908 (1966). See also L. I. Lapidus and Chou Kuang-Chao, Zh. Eksperim. i Teor. Fiz. **41**, 1546 (1961) [English transl.: Soviet Phys.—JETP **14**, 1102 (1962)]; M. Hosoda and K. Yamamoto, Progr. Theoret. Phys. (Kyoto) **30**, 425 (1966).

[2] M. Gell-Mann, M. L. Goldberger, and W. Thirring, Phys. Rev. **95**, 1612 (1954).

where ε and ε' are the polarization vectors of the initial and final photons. In particular one needs the dispersion relation for the spin-dependent part f_2, which we write, provisionally, as

$$f_2(\omega)=f_2(\infty)-\frac{1}{4\pi^2}\int_0^\infty \frac{d\omega'\omega'}{\omega'^2-\omega^2}[\sigma_p{}^P(\omega')-\sigma_p{}^A(\omega')]. \tag{1.3}$$

The second proved result is a theorem[3] on the threshold value of f_2:

$$f_2(0)=-\alpha\kappa_p{}^2/2M^2. \tag{1.4}$$

Beyond (1.3) and (1.4), one must also assume: (i) that the integral in (1.3) converges, and (ii), that the real part of f_2 vanishes at infinity, i.e., that

$$f_2(\infty)=0. \tag{1.5}$$

Then (1.1) follows immediately from (1.3), (1.4), and (1.5). It is known,[2] of course, that the corresponding assumption $f_1(\infty)=0$ is untenable, because it leads to a contradiction[4] between the dispersion relation for f_1 and the Thomson limit $f_1(0)=-\alpha/M$.

The present paper aims to test the plausibility of the assumption (1.5) by investigating whether the DHG rule can apply, simultaneously, both to a bound state and to its constituents. We shall consider only nonrelativistic bound states, whose binding energy is negligible compared to the masses of the constituents. In Sec. 2 we outline the general arguments and the calculation for a simple model. Other more realistic cases are dealt with later by straightforward adaptations of this prototype calculation and the hydrogen atom[5] is simply related to the model. Sections 3 and 4 deal with the deuteron and with the $A=3$ isodoublet He^3, H^3. Section 5 sets out the corresponding results for isoscalar and isovector photons. At the end of Sec. 3, and in Sec. 5, we compare our results with those of Gerasimov,[6] of Pagels,[6a] and of Konisi and Yamamoto.[7] Section 6 contains some final comments and conclusions.

2. CALCULATION FOR A SIMPLE MODEL

We first became doubtful of the unrestricted applicability of the DHG rule by asking how the two sides of Eq. (1.1) would be affected if the particle in question could be bound, lightly, to another. In the case of binding in an $S_{1/2}$ state to a neutral scalar particle, the total magnetic moment would be unaltered, but from the point of view of the bound state it would be partitioned differently between its "Dirac" and "anomalous" parts; and *a priori* it would be surprising if this purely kinematic reapportioning were exactly matched by the changes in the integral J. To investigate this we define the following model.

Suppose (1.1) holds exactly for a certain particle which we call the "proton" p; let the threshold of the integral J_p (analogous to the pion photoproduction threshold) be T. Suppose also that a neutral scalar particle σ exists, of mass m, and that it can be bound lightly to p in an $S_{1/2}$ state, producing a particle p' of mass $M'\approx M+m$. The binding energy is to be negligible compared to M, m, and T. Then the total magnetic moments of p and p' coincide; $\mu=\mu'$. Hence,

$$\frac{\kappa'+1}{2M'}=\frac{\kappa'+1}{2(M+m)}=\frac{\kappa+1}{2M},$$
$$\frac{\kappa'}{2M'}=\left(\frac{\kappa}{2M}+\frac{m}{2MM'}\right); \tag{2.1}$$

therefore the DHG rule for p' reads

$$\frac{2\pi^2\alpha}{M^2}\left(\kappa+\frac{m}{M+m}\right)^2=J_p{}', \tag{2.2}$$

where $J_p{}'$ is defined by analogy with (1.1).

Because the threshold T of J_p is taken to be high compared to the binding energy, it seems physically reasonable to decompose $J_p{}'$ into a low-frequency part, $K\equiv K^P-K^A$, coming from photodisintegration of the bound state into p and σ without excitation of the constituents, and into a high-frequency part. It seems equally reasonable as a first approximation to take the high-frequency contribution equal to J_p+J_σ (in our case $J_\sigma=0$, since σ has zero spin). We base this simple additivity on the argument that at high frequencies each particle acts merely as a spectator of photoabsorption by the other, and feel that for sufficiently small binding energy and for a sufficiently extended wave function, corrections are likely to be negligible as regards the total cross sections σ^P and σ^A that are relevant for us. The reader is asked to suspend for the moment his reservations both about the model as such, and about the adequacy of the additivity assumption. The model is designed primarily to introduce the physical examples discussed later on; and we shall see in Sec. 6B that exact additivity is far from necessary for our conclusions, though it is convenient in presenting the argument. At this stage we would point out only that even if our model and approximations are not strictly applicable to the real world, nevertheless they define a dynamical framework to which the DHG rule should be no less relevant than it is to any other case.

Using (1.1) for the high-frequency part, the DHG rule (2.2) can be written in the form

$$\frac{2\pi^2\alpha}{M^2}\left(\kappa+\frac{m}{M+m}\right)^2=K+\frac{2\pi^2\alpha\kappa^2}{M^2}; \tag{2.3}$$

[3] F. E. Low, Phys. Rev. **96**, 1428 (1954); M. Gell-Mann and M. L. Goldberger, *ibid.* **96**, 1433 (1954).
[4] We emphasize that in principle this contradiction exists irrespective of whether $f_1(\infty)$ can be finite or not, i.e., it exists even if the unsubtracted dispersion integral for f_1 [analogous to (1.3)] converges.
[5] S. D. Drell and J. R. Primack (to be published).
[6] S. B. Gerasimov, J. Nucl. Phys. (USSR) **5**, 1263 (1967).
[6a] H. R. Pagels, Phys. Rev. **158**, 1566 (1967), Appendix B.
[7] G. Konisi and K. Yamamoto, Progr. Theoret. Physics (Kyoto) **37**, 538 (1967).

its validity now depends on the value of K, which we can calculate by ordinary nonrelativistic quantum mechanics, once the coupling to the radiation field is given to the right order[8] in M^{-1}. To obtain it, we apply the Foldy-Wouthuysen transformation[9] for a proton with Pauli moment[10] $e\kappa/2M$, and find, to order M^{-3}:

$$H'=\{-(e/M)\mathbf{A}\cdot\mathbf{p}-(e/4M^2)(1+2\kappa)\boldsymbol{\sigma}\cdot\mathbf{E}\times\mathbf{p} -(e/2M)(1+\kappa)\boldsymbol{\sigma}\cdot\mathbf{H}\}. \quad (2.4)$$

The second term is responsible for the spin-orbit splitting when $\mathbf{E}$ is a central field rather than the radiation field; we shall call this the spin-orbit coupling. Its importance was pointed out by Drell and Primack.[11] Note the factor of 2 prefacing κ, which will be important in the following.

It follows by standard methods[12,13] that K^P, say, is given by

$$K^P=4\pi^2\sum_f\frac{1}{\omega^2}\left|\langle f|\left\{\frac{e}{M}\boldsymbol{\varepsilon}\cdot\mathbf{p}+\frac{e}{4M^2}(1+2\kappa)\boldsymbol{\sigma}\cdot i\omega\boldsymbol{\varepsilon}\times\mathbf{p}+\frac{e}{2M}(1+\kappa)\boldsymbol{\sigma}\cdot i\boldsymbol{\varepsilon}\times\mathbf{k}\right\}e^{i\mathbf{k}\cdot\mathbf{r}}|0\rangle\right|^2; \quad (2.5)$$

the sum runs over all excited states, discrete and continuous, of the two-particle system; $\omega\equiv(E_f-E_0)$; $\mathbf{k}=\omega\hat{z}$ is the photon momentum, and if $|0\rangle$ is taken as the ground state with its spin parallel to $\mathbf{k}$ then $\boldsymbol{\varepsilon}=(-)(\hat{x}+i\hat{y})/\sqrt{2}$. Here $\hat{x}$, $\hat{y}$, $\hat{z}$ are the unit coordinate vectors.

For our purposes we can neglect retardation, i.e., set $\exp(i\mathbf{k}\cdot\mathbf{r})=1$; then the matrix elements of the last operator in (2.5) vanish because of the orthogonality of the spatial wave functions in $|0\rangle$ and $|f\rangle$. Thus there is no magnetic dipole ($M1$) contribution. The electric dipole ($E1$) term $e\boldsymbol{\varepsilon}\cdot\mathbf{p}/M$ is spin-independent; hence only cross terms between it and the spin-orbit coupling need be retained to our order[14] in M^{-1}:

$$K^P-K^0=\frac{\pi^2\alpha}{M^3}(1+2\kappa)\times\sum_f\{\langle 0|\boldsymbol{\varepsilon}^*\cdot\mathbf{p}|f\rangle\langle f|i\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\times\mathbf{p}|0\rangle-\langle 0|i\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}^*\times\mathbf{p}|f\rangle\langle f|\boldsymbol{\varepsilon}\cdot\mathbf{p}|0\rangle\}\frac{1}{\omega}, \quad (2.6)$$

where K^0 does not involve the spin.

[8] Actually we reckon in powers of M^{-1} and m^{-1} interchangeably, so that $m/(M+m)$ for instance is of order unity.

[9] L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950).

[10] H. Neuer and P. Urban, Acta Phys. Austriaca, **15**, 380 (1962).

[11] S. D. Drell and J. Primack (private communication).

[12] J. S. Levinger, *Nuclear Photodisintegration* (Oxford University Press, New York, 1960).

[13] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer-Verlag, Stuttgart, Germany, 1957), paragraphs 61 and 69–73.

[14] At first sight one might suspect that contributions of the same order arise from cross terms between the $E1$ and the once-retarded $M1$ terms, where in the latter one replaces $\exp(i\mathbf{k}\cdot\mathbf{r})$ by $(i\mathbf{k}\cdot\mathbf{r})$. But by following the method outlined below one finds readily that these contributions vanish.

In order to carry out the summation, the first step is to rewrite the $E1$ operator $\boldsymbol{\varepsilon}\cdot\mathbf{p}$ by using the standard equivalence $\mathbf{p}=M_R\dot{\mathbf{r}}$, where $\mathbf{r}$ is the relative coordinate of the two particles and M_R is the reduced mass:

$$M_R=Mm/(M+m); \quad (2.7)$$

$$\langle 0|\boldsymbol{\varepsilon}^*\cdot\mathbf{p}|f\rangle=-i\omega M_R\langle 0|\boldsymbol{\varepsilon}^*\cdot\mathbf{r}|f\rangle,\quad \langle f|\boldsymbol{\varepsilon}\cdot\mathbf{p}|0\rangle=+i\omega M_R\langle f|\boldsymbol{\varepsilon}\cdot\mathbf{r}|0\rangle. \quad (2.8)$$

The presence of the reduced mass will prove crucial in the sequel. Indeed Siegert's theorem[12,15,16] tells us that the expressions on the right of (2.8) are more generally valid than those on the left, for instance in the presence of exchange forces. In more complicated systems such as we shall meet later, the operator $\boldsymbol{\varepsilon}\cdot\mathbf{p}/M$ in (2.5) is therefore replaced by $i\omega\boldsymbol{\varepsilon}\cdot\mathbf{D}$, where $\mathbf{D}$ is the standard electric dipole operator for the system relative to its center of mass. (Recall that only relative coordinate operators, but not the coordinate of the mass center itself, can lead to photon absorption.) In these more complicated situations, the spin-orbit operator in (2.5) is simply summed over all the particles, each taken with the appropriate value of charge, mass, and anomalous moment.

Returning to our model, we substitute (2.8) into (2.6); the ω's cancel, and the completeness of the states $|f\rangle$ allows us to write

$$\begin{aligned}K^P-K^0&=(\alpha\pi^2M_R/M^3)(1+2\kappa)\langle 0|[(\boldsymbol{\varepsilon}^*\cdot\mathbf{r})\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\times\mathbf{p}+\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}^*\times\mathbf{p}(\boldsymbol{\varepsilon}\cdot\mathbf{r})]|0\rangle,\\&=(\alpha\pi^2M_R/M^3)(1+2\kappa)\langle 0|\sigma_0\{L_0-i[x,p_x]\}|0\rangle,\\&=(\alpha\pi^2M_R/M^3)(1+2\kappa)\langle 0|\sigma_0(L_0+1)|0\rangle,\\K^P-K^0&=(\alpha\pi^2M_R/M^3)(1+2\kappa).\end{aligned} \quad (2.9)$$

Here, L_0 is the z component of the orbital angular momentum, whose expectation value vanishes since $|0\rangle$ is an S state. Evidently we also find $(K^A-K^0)=-(K^P-K^0)$, whence

$$K^P-K^A=K=2\pi^2\alpha M_R(1+2\kappa)/M^3. \quad (2.10)$$

Thus (2.3), (2.10), and (2.7) show that the DHG rules for p and p' are compatible only if the following condition is satisfied:

$$\left(\kappa+\frac{m}{M+m}\right)^2=\kappa^2+(1+2\kappa)\frac{m}{M+m}. \quad (2.11)$$

The terms involving κ^2 and κ do indeed cancel in (2.11), but the remaining "Dirac" contributions lead to the condition

$$m/(M+m)=1, \quad (2.12)$$

which is satisfied only in the special case $M/m\to 0$.

With some changes, our model reduces to the hydrogen atom, neglecting the nuclear spin. Our "p" then

[15] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Reading, Massachusetts, 1953).

[16] M. Danos, University of Maryland Technical Report No. 221, 1961 (unpublished).

corresponds to the electron, and σ to the nucleus; σ is now charged oppositely to "p". Hence both σ and "p" contribute to the $E1$ operator $\mathbf{D}$, which becomes just $e\mathbf{r}$, while the atom, p', becomes neutral. We can even allow a hypothetical Pauli moment (i.e., one independent of α); then (2.1) is replaced by $\kappa'/2M'=(\kappa+1)/2M$, (2.10) by $K=2\pi^2\alpha(1+2\kappa)/M^2$, and the consistency condition (2.11) by $(\kappa+1)^2=\{\kappa^2+(1+2\kappa)\}$, which is satisfied identically. Thus hydrogen obeys the DHG rule; for infinite proton mass this has been pointed out already by Drell and Primack.[5] However, for nuclear charge $Z\neq1$, the rule would fail again.

3. THE DEUTERON

In order to discuss the deuteron we need the analog of (1.1) for particles of spin 1. The dispersion relations for Compton scattering and the threshold theorem have been generalized to particles of any spin by Lapidus and Chou Kuang-Chao.[17] From their results, supplemented by convergence assumptions like those discussed in Sec. 1, the DHG rule for the deuteron follows on exactly the same footing as (1.1):

$$4\pi^2[\mu_d-e/M_d]^2=\int\frac{d\omega}{\omega}[\sigma_d{}^P(\omega)-\sigma_d{}^A(\omega)]\equiv J_d. \quad (3.1)$$

For the moment we allow for the hypothetical possibility of significantly different neutron and proton masses; and for simplicity confine ourselves to a "model" without tensor forces, D-state admixtures, and exchange currents. Then the deuteron is a $^3S_1$ state, and its total magnetic moment μ_d is given by

$$\mu_d=e(1+\kappa_p)/2M_p+e\kappa_n/2M_n. \quad (3.2)$$

As before, we split J_d into a high-frequency part equal to (J_p+J_n), and a low-frequency part K due to photodistintegration without excitation of the individual nucleons. Assuming the DHG rule for the nucleons, we get

$$4\pi^2\alpha\left(\frac{1+\kappa_p}{2M_p}+\frac{\kappa_n}{2M_n}-\frac{1}{M_p+M_n}\right)^2 =2\pi^2\alpha\left(\frac{\kappa_p{}^2}{M_p{}^2}+\frac{\kappa_n{}^2}{M_n{}^2}\right)+K. \quad (3.3)$$

It remains to estimate K. Because of the spin dependence of the binding forces, the orbital wave functions of the continuum $^1S_0$ states are not now orthogonal to the bound (triplet) state; hence in the sum corresponding to (2.5) there is a contribution $K(M)$ from $M1$ disintegrations into these $^1S_0$ states.[15] An elementary calculation, relying on the completeness of their orbital wave functions, yields a closed form for $K(M)$, which after spin averaging would reduce to the sum rule of Rustgi and Levinger[18]:

$$K(M)=-\pi^2\alpha\left(\frac{1+\kappa_p}{M_p}-\frac{\kappa_n}{M_n}\right)^2. \quad (3.4)$$

The negative sign arises because conservation of angular momentum along the incident direction allows photons to be absorbed in these $M1$ transitions only if their spins are antiparallel to the deuteron spin.

In addition to $K(M)$, there is also a contribution $K(E)$ from $E1$ transitions. This is readily calculated along the lines of Sec. 2, recalling that the reduced mass is given by

$$M_R=M_pM_n/(M_p+M_n), \quad (3.5)$$

and that in the spin-orbit coupling as written in (2.4) the factor $(1+2\kappa)\boldsymbol{\sigma}/4M^2$ must now be replaced by

$$[(1+2\kappa_p)/4M_p{}^2]\boldsymbol{\sigma}_p+[2\kappa_n/4M_n{}^2]\boldsymbol{\sigma}_n.$$

The calculation is made easy by observing that only triplet states are relevant; it yields

$$K(E)=2\pi^2\alpha\frac{M_n}{M_p+M_n}\left(\frac{1+2\kappa_p}{M_p{}^2}-\frac{2\kappa_n}{M_n{}^2}\right). \quad (3.6)$$

Recall finally that because of their different parities, there is no interference between $E1$ and $M1$ contributions, so that

$$K=K(M)+K(E). \quad (3.7)$$

Then it follows from (3.3), (3.4), (3.6), and (3.7) that the DHG rules for the nucleons and for the deuteron are compatible only subject to the following condition:

$$\left(\frac{1+\kappa_p}{M_p}+\frac{\kappa_n}{M_n}-\frac{2}{M_p+M_n}\right)^2=2\left(\frac{\kappa_p{}^2}{M_p{}^2}+\frac{\kappa_n{}^2}{M_n{}^2}\right) -\left(\frac{1+\kappa_p}{M_p}-\frac{\kappa_n}{M_n}\right)^2+\frac{2M_n}{M_p+M_n}\left(\frac{2\kappa_p+1}{M_p{}^2}-\frac{2\kappa_n}{M_n{}^2}\right). \quad (3.8)$$

Most remarkably, all terms involving κ_p and κ_n do cancel in (3.8), which reduces to

$$\left[\frac{M_n-M_p}{M_p(M_n+M_p)}\right]^2=\frac{M_n-M_p}{M_p{}^2(M_p+M_n)}. \quad (3.9)$$

For the physical deuteron, $M_p=M_n$ (to order α), and (3.9) is indeed satisfied. It would also be satisfied in the limit, reminiscent of (2.12) above, where $M_p/M_n\to0$. However, it appears that for "general" values of M_p/M_n there is an inconsistency; for $M_p>M_n$ even the signs fail to agree.

The DHG rule for the deuteron has already been considered by Gerasimov[6] and by Pagels.[6a] However,

[17] L. I. Lapidus and Chou Kuang-Chao, Zh. Eksperim. i Teor. Fiz. **39**, 1286 (1966) [English transl.: Soviet Phys.—JETP **12**, 898 (1961)].

[18] M. L. Rustgi and J. S. Levinger, Progr. Theoret. Phys. (Kyoto) **18**, 100 (1957).

they omit the contribution $K(E)$ (as we also did in the original version of the present paper), and conclude therefore that the rule breaks down even in the physical, equal-mass case. We are informed that conclusions identical to ours have also been reached independently by Primack (unpublished), both as regards the deuteron and as regards the hydrogen atom.

4. THE $A=3$ ISODOUBLET

He^3 and H^3 have exactly the same quantum numbers as the nucleons. For simplicity we treat their orbital wave functions as totally symmetric in the three nucleon coordinates, and again we neglect exchange currents; these are reasonably good approximations. Then one has

$$\mu(\mathrm{He}^3)=\mu_n,\quad \mu(\mathrm{H}^3)=\mu_p. \tag{4.1}$$

By the same procedure as before, we write the DHG rule for He^3, say, as follows (recalling that $Z=2$):

$$8\pi^2\alpha(\kappa_n/2M-2/6M)^2=J(\mathrm{He}^3)=J_n+K(\mathrm{He}^3). \tag{4.2}$$

But by a coincidence, the bound-state wave function is an eigenfunction of the $M1$ operator, so that there is no $M1$ contribution[19] to K. The $E1$ contribution follows straightforwardly along the lines of Sec. 2:

$$K(\mathrm{He}^3)=-8\alpha\pi^2\kappa_n/3M^2. \tag{4.3}$$

For H^3 one finds

$$K(\mathrm{H}^3)=+4\alpha\pi^2(1+2\kappa_p)/3M^2.$$

This can be interpreted very simply by comparing it to (2.10), and noting that, as seen by the $E1$ operator, H^3 consists of two neutrons forming a subsystem of spin 0, plus an odd proton, and that the reduced mass is $\frac{1}{3}2M$.

Now, (4.2) and (4.3) show that the DHG rules for nucleons and for He^3 are compatible only subject to the following condition:

$$(\kappa_n-\tfrac{2}{3})^2=\kappa_n^2-\tfrac{4}{3}\kappa_n. \tag{4.4}$$

The corresponding condition for H^3 reads

$$(\kappa_p+\tfrac{2}{3})^2=\kappa_p+\tfrac{2}{3}(1+2\kappa_p). \tag{4.5}$$

The terms involving the κ's cancel in (4.4) and (4.5), but in both cases the remainder is absurd.

5. ISOTOPIC SUM RULES

By starting from the $SU(2)$ isotopic-spin current algebra, separate DHG rules have been derived for the absorption of isoscalar and isovector photons.[20,21] In the present section we test these rules by the methods already used above.

The model defined in Sec. 2 can be adapted trivially by introducing a neutral particle n which forms an isodoublet together with p, and assigning i spin zero to σ; then p' and n' also form an isodoublet. Because the contradiction in (2.11) affects only the Dirac part, the DHG rules for n and n' are compatible; hence the incompatibility between p and p' entails a simultaneous breakdown in both isoscalar and isovector rules.

The physical deuteron is isoscalar; so, of course, is its magnetic moment. Hence the isoscalar analog of (3.1) reads

$$4\pi^2[\mu_d-e/M_d]^2=J_d^s, \tag{5.1}$$

where J_d^s is defined as in (3.1), except that it refers to the absorption only of isoscalar photons. But both the $M1$ and the $E1$ disintegrations are isovector; therefore J_d^s has only a high-frequency part, which equals $2J_N^s$. Thus (5.1) becomes

$$(\alpha\pi^2/M^2)(\kappa_p+\kappa_n)^2=2(2\pi^2\alpha/M^2)\kappa_s^2 =\frac{4\pi^2\alpha}{M^2}\left(\frac{\kappa_p+\kappa_n}{2}\right)^2; \tag{5.2}$$

in other words it is satisfied identically. Similarly the isovector moment is zero, so that the isovector rule reads

$$\begin{aligned}0&=K(M)+K(E)+2J_N^v,\\ &=-(\pi^2\alpha/M^2)(1+\kappa_p-\kappa_n)^2+(\pi^2\alpha/M^2)(1+2\kappa_p-2\kappa_n)\\ &\quad+2\frac{2\pi^2\alpha}{M^2}\left(\frac{\kappa_p-\kappa_n}{2}\right)^2,\end{aligned} \tag{5.3}$$

which is also true. [In (5.3), we have used (3.4) and (3.6) with $M_p=M_n=M$.]

For He^3 and H^3 the disintegration contributions, like all $E1$ transitions,[12,15] are again pure isovector. Therefore the isoscalar DHG rule reads

$$8\pi^2\tfrac{1}{4}[(\mu(\mathrm{He}^3)-2e/6M)+(\mu(\mathrm{H}^3)-e/6M)]^2=J_N^s =\frac{2\pi^2\alpha}{M^2}\left(\frac{\kappa_p+\kappa_n}{2}\right)^2, \tag{5.4}$$

which is satisfied by virtue of (4.1). By contrast, it is trivial to verify that the isovector rule breaks down.

The isoscalar rule for He^3 and H^3 has been considered already by Konisi and Yamamoto.[7] They do not introduce the disintegration cross sections at all; as we have seen, these are indeed absent from (5.4), so that the correct conclusion is reached, namely that the isoscalar rule holds. These authors also discuss the isoscalar rule for the deuteron in the same way.

6. COMMENTS AND CONCLUSIONS

In this section we attempt to forestall some possible objections which we believe are invalid or irrelevant, and list some doubts that we do entertain.

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[19] For references, and for experimental data on He^3, see V. N. Fetisov, A. N. Gorbunov, and A. T. Varfolomeev, Nucl. Phys. **71**, 305 (1965).

[20] M. A. B. Bég, Phys. Rev. Letters **17**, 333 (1966).

[21] K. Kawarabayashi and W. W. Wada, Phys. Rev. **152**, 1286 (1966).

A. One might be tempted to hope that even though the integrals J converge for nucleons, they will diverge for bound states, and thus necessitate a subtraction which would automatically prevent one from trying to apply the DHG rule. But our arguments suggest that the integrals in question either all diverge or all converge, unless the additivity assumption fails even for photons of arbitrarily high frequency, or unless there are additional and asymptotically dangerous contributions to the distintegration integrals, such as might arise for instance from derivative couplings in the strong (binding) interaction.

B. The additivity assumption as regards the J's is certainly not fulfilled exactly. But we see that it would have to fail very badly in order to restore all the DHG rules; moreover it would have to fail in a way precisely dictated by the mass ratios and the anomalous moments involved [witness for instance the left-hand side of (3.8)], but which is completely independent of the bound-state wave function. This possibility of escape would appear to be rather remote.[22]

C. There may be a subtle ambiguity associated with the use of the Foldy-Wouthuysen transformation leading to (2.4). This transformation, if carried further, introduces couplings which contain higher derivatives of the radiation field, compensated dimensionally by coefficients of higher order in M^{-1}. Such terms contribute to the summand in (2.5) with extra powers of ω/M, and it is conceivable that eventually the sums cease to converge. We underline that hitherto no such ambiguities are known in applications of the transformation; we point out the possibility not because we expect it to be relevant, but because we cannot rule it out rigorously. Even if present, it would presumably imply only that the series resulting from the Foldy-Wouthuysen transformation is an asymptotic one; to affect our conclusions one would need to show that already the next-to-leading term, with which we are concerned, is misleading.

D. The ambiguity discussed in paragraph C could be avoided if we could make a fully covariant calculation of the disintegration integrals. Then the treatment would become independent of the assumption, implicit in the foregoing, that closure (over positive energy states) and the Foldy-Wouthuysen transformation commute. Unfortunately, because of the need to account for center-of-mass and reduced-mass effects in our two-body and three-body systems, we know of no way to do this. Of course, this is just the reason why we have restricted ourselves to discussing nonrelativistic examples after first carrying out the Foldy-Wouthuysen transformation.

E. Subject to these provisos, we conclude from our explicit examples that in general the assumption (1.5), ($f_2(\infty)=0$), underlying the DHG rule is violated either for lightly bound states, or for their constituents, or for both. Since an additive constant $f_1(\infty)$ must certainly enter the spin-independent part of the forward Compton amplitude (irrespective of any possible need for subtractions to secure convergence), the presence of a similar constant $f_2(\infty)$ in the spin-dependent part as well is not in itself surprising. The need for it is unlikely to depend critically on the precise magnitude of the binding energy; hence there would seem to remain no clear-cut criterion for deciding, *a priori*, whether the DHG rule should be expected to apply to any given particle.

Two complementary conclusions follow. First, empirical verifications of the rule, if they are only rough, become less significant, because rough agreement might well be accidental. Second, close verification becomes even more remarkable than before, because, far from confirming a tautology, it would establish a property which some particles (e.g., either H^3, or the proton, or both) certainly lack. In particular, it becomes very interesting to check just how closely the proton does satisfy the sum rule.[23]

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[22] For instance, if additivity is seriously upset, it would be expected that the κ-dependent terms will no longer cancel, barring yet another fantastic coincidence.

[23] Y. C. Chau, Norman Dombey, and R. G. Moorhouse (to be published).