resonance at $s=m_\rho^2$; (2) find the C_i so that the resonance widths are correct.

Step (1) is accomplished by first finding the matrix θ that diagonalizes the matrix $M_{ij}=C_i'C_j'$ where the C_i' are from Table I. Define $R_{ij}(s)=R_i(s)\delta_{ij}$. Then $\tilde{M}=\theta^T M\theta$ is a diagonal matrix; in fact,

$$\tilde{M}=\begin{bmatrix} C'^2 & & & & & & \\ & 0 & & & 0 & & \\ & & 0 & & & & \\ & & & 0 & & & \\ & & & & 0 & & \\ & 0 & & & & 0 & \\ & & & & & & 0 \end{bmatrix}$$

where

$$C'^2=\sum_{i=1}^{7} C_i'^2.$$

Let $\tilde{R}(s)=\theta^T R\theta$ and $\tilde{R}'(s_p)=\theta^T R'(s_p)\theta$, where $R'(s_p)=\partial/\partial s[R(s,s_p)]|_{s=m_\rho^2}$.

Then we find s_p by solving

$$\tilde{M}\tilde{R}'(s_p)=1.$$

Step (2) is accomplished by setting $\tilde{N}_{11}=-1/\tilde{R}_{11}$; we recover $B_{ij}=C_iC_j$ by using $B=\theta\tilde{N}\theta^T$. Notice that this prescription implies that the C_i of formula (2) are proportional to the C_i' of Table I; $C_i=k_fC_i'$, where k_f depends on the $R_i(s)$ and s_p.

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Spin-Parity-$\frac{3}{2}^-$ Baryons and Singlet-Octet Mixing*

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Recent experimental data are used to examine the effects of possible $SU(3)$ mixing among the spin-parity-$\frac{3}{2}^-$ baryons. It is found that the mixing model is a quite reasonable one to account for the various decay widths of the members. With a mixing angle $\theta=22.5°$, the following optimum values of the parameters are obtained: $F/D\approx1.1$, the ratio of the coupling of $\{8\}\otimes\{8\}$ to $\{1\}$ and $\{8\}\otimes\{8\}$ to $\{8\}\approx2.5$, and inverse interaction radius $X\approx1000$ MeV. Predicted decay widths of the members are also presented.

THE recently observed $Y_0^*(1700)$[1,2] appears to be a neutral member of a spin-parity-$\frac{3}{2}^-$ baryon octet which is completed by the $N_{1/2}^*(1525)$, $Y_1^*(1660)$, and $\Xi^*(1815)$. It has also been speculated that the $Y_0^*(1518)$ belongs to a unitary singlet state. The experimental situation concerning some members of this multiplet is still unclear according to Rosenfeld *et al.*[3] There are large contradictions among the measured branching ratios of the $Y_1^*(1660)$, and poorly observed branching ratios of the $\Xi^*(1815)$. It also seems that the branching ratio for the $Y_0^*(1518)$ is not as well confirmed as indicated by the earlier experiments.

The $Y_0^*(1518)$ has been thought of as a unitary singlet because the earlier data for the branching ratio[4] $(\bar{K}N)/(\Sigma\pi)$ were compatible with the $SU(3)$ prediction, which is about 0.43.[5–9] The ratio from the recent compilation is 39/51 (=0.76),[3] and it may be even higher if the recently observed value of 52/37 by Dauber *et al.*[10] is taken into account. This suggests that the $Y_0^*(1518)$ can hardly be assigned to a pure unitary singlet state. On the other hand, if we assign the newly observed $Y_0^*(1700)$ as the neutral member of the pure octet state, the mass (1700 MeV) departs by 30 MeV from the computed mass based on the Gell-Mann–Okubo mass formula.[11,12] The presently observed evidence that the $Y_0^*(1700)$ couples strongly to the $\Sigma\pi$ channel (~50%) and weakly to the $\bar{K}N$ (~20%) contradicts the assignment of the $Y_0^*(1700)$ to a pure octet state.[13] These deviations suggest that there might be

* Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-881, COO-881-108.

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TABLE I. Calculated and experimental decay widths.

Decay mode	Coupling constants	Calculated width (MeV) Case (I) Case (a)	(Case (b))	Calculated width (MeV) Case (II) Case (a)	(Case (b))	Calculated[a] width (MeV) (3-body)	Experimental width (MeV)
$N_{1/2}^*(1525)$							
$\to N\pi$	$D+F$	Input	(Input)	Input	(Input)		68.0
$\to N^*(1238)\pi$	$-2/\sqrt{5}$					(Input)	21.0
$Y_1^*(1660)$							
$\to \bar{K}N$	$-(\sqrt{\frac{2}{3}})D+(\sqrt{\frac{2}{3}})F$	0.2[b]	(0.2)	0.1	(0.1)		Small
$\to \Lambda\pi$	$\frac{2}{3}D$	6.8[b]	(4.8)	5.1	(5.2)		?
$\to \Sigma\pi$	$(2\sqrt{2}/\sqrt{3})F$	14.9[b]	(21.0)	19.7	(19.2)		?
$\to Y_1^*(1385)\pi$	$\sqrt{(2/15)}$					3.0	
$\to Y_0^*(1405)\pi$							Large
$\Xi_{1/2}^*(1815)$							
$\to \Lambda\bar{K}$	$-\frac{1}{3}D+F$	2.1[b]	(4.2)	3.8	(3.7)		~10.4
$\to \Xi\pi$	$-D+F$	0.3[b]	(0.2)	0.1	(0.1)		~1.6
$\to \Sigma\bar{K}$	$D+F$	9.5[b]	(9.3)	9.1	(9.1)		
$\to \Xi^*(1530)\pi$	$1/\sqrt{5}$					4.4	~3.2
$Y_0^*(1700)$							
$\to \bar{K}N$	$(\frac{1}{3}\sqrt{2}D+\sqrt{2}F)\cos\theta - \frac{1}{2}\beta\sin\theta$	Input[b]	(Input)	Input	(Input)		8.0
$\to \Sigma\pi$	$(-2/\sqrt{3})D\cos\theta - (\sqrt{3}/2\sqrt{2})\beta\sin\theta$	46.3[b]	(45.9)	45.6	(45.6)		
$\to Y_1^*(1385)\pi$	$(\sqrt{\frac{3}{5}})\cos\theta$					13.5	
$Y_0^*(1518)$							
$\to \bar{K}N$	$[(\frac{1}{3}\sqrt{2})D+\sqrt{2}F]\sin\theta + \frac{1}{2}\beta\cos\theta$	Input[b]	(8.1)	Input	(7.4)		6.24(7.52)[c]
$\to \Sigma\pi$	$(-2/\sqrt{3})D\sin\theta + (\sqrt{3}/2\sqrt{2})\beta\cos\theta$	5.3[b]	(Input)	7.4	(Input)		8.16(7.2)[c]
		No Mixing					
$Y_0^*(1700)$ (pure octet)							
$\to \bar{K}N$	$\frac{1}{3}\sqrt{2}D+\sqrt{2}F$	40.1[d]	(47.7[d])	46.2[d]	(45.7[d])		8.0
$\to \Sigma\pi$	$-(2/\sqrt{3})D$	15.1[d]	(10.5[d])	11.1[d]	(11.4[d])		
$\to Y_1^*(1385)\pi$	$-\sqrt{(\frac{3}{5})}$					17.6	
$Y_0^*(1518)$ (pure singlet)							
$\to \bar{K}N$	$\frac{1}{2}$	Input[e]	(3.7)[e]	Input[e]	(3.2[e])		6.24(7.52)[c]
$\to \Sigma\pi$	$\sqrt{3}/2\sqrt{2}$	13.9[e]	(Input[e])	16.7[e]	(Input[e])		8.16(7.2)[c]

[a] The three-body decay widths into the decuplet baryon and pseudoscalar meson are calculated by using the formula (4) independently from the calculations of the widths into the octet baryon and pseudoscalar meson.
[b] These values are obtained by using $X=950$ MeV, $F/D=0.89$, and $\beta=2.3$, which are chosen so that $\Gamma(Y_1^*(1660)\to\Sigma\pi)=15$ MeV.
[c] The decay widths inside the parenthesis are from Hardy's data (Ref. 17).
[d] These values are obtained by assuming the pure octet state assignment for $Y_0^*(1700)$ with the same parameter values of a, F/D, and X as in the case of singlet-octet mixing.
[e] These values are obtained by assuming the pure singlet state assignment for $Y_0^*(1518)$ with $X=1000$ MeV. Note that there is no large X dependence in the calculated values of the decay widths in this case.

mixing between the $Y=0$, $I=0$ member of the octet and the singlet states.

In this paper, we pursue the puzzling status of the $\frac{3}{2}^-$ baryon members on the basis of the proposed mixing scheme between the octet and singlet. First we write the wave functions for the physical $Y_0^*(1700)$ and $Y_0^*(1518)$ as follows:

$$\begin{aligned} |Y_0^*(1700)\rangle &= \cos\theta|Y_0^*(8)\rangle - \sin\theta|Y_0^*(1)\rangle, \\ |Y_0^*(1518)\rangle &= \sin\theta|Y_0^*(8)\rangle + \cos\theta|Y_0^*(1)\rangle, \end{aligned} \tag{1}$$

where $|Y_0^*(8)\rangle$ and $|Y_0^*(1)\rangle$ are the pure octet isosinglet and unitary singlet states, respectively, and θ is the mixing angle. To determine the mixing parameters, we write the Hamiltonian in the form

$$H=H_0+V=\begin{pmatrix} m_0(8)+V_{88} & V_{18} \\ V_{81} & m_0(1)+V_{11} \end{pmatrix}, \tag{2}$$

where $V_{18}=V_{81}$, the diagonal element $m_0(8)+V_{88}$ satisfies the Gell-Mann–Okubo mass formula, and $m_0(1)+V_{11}$ is the unknown mass of the pure singlet state $Y_0^*(1)$. Using the facts that the $|Y_0^*(1700)\rangle$ and $|Y_0^*(1518)\rangle$ are the eigenstates of H with known eigenvalues, and that $m_0(8)+V_{88}$ is known from the mass formula, we find that

$$\theta=22.5°,\quad V_{18}=-64.2\text{ MeV},\quad Y_0^*(1)=1545\text{ MeV}. \tag{3}$$

To calculate the decay width of a resonance, we use the formula given by Glashow and Rosenfeld,[14]

$$\Gamma=a(\text{C.G.})\ [p^2/(p^2+X^2)]^l p/M, \tag{4}$$

where p is the momentum of decay products of a resonance of mass M, and l is the orbital angular momentum. The inverse interaction radius is X; C. G. (a Clebsch-Gordan coefficient) represents the coupling constant with singlet-octet mixing,[15] and a is the over-all

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reduced factor. In Table I the coupling constants of the resonances are given in terms of the two types of the coupling constants D and F of $\{8\}\otimes\{8\}$ to $\{8\}$, singlet-octet mixing angle θ, and the coupling constant β of $\{8\}\otimes\{8\}$ to $\{1\}$. Here $D+F$ is normalized to unity.

In determining four unknown parameters a, F (or D), β, and X, we treat X as a free parameter. We have the option of choosing any three of the 11 experimentally known decay widths as input. We prefer the data obtained mainly from formation experiments, without having large contradictions in each experiment. Along this line, we take for input $\Gamma(N^*\to N\pi)$, $\Gamma(Y_0^*(1700)\to\bar{K}N)$ together with $\Gamma(Y_0^*(1518)\to\bar{K}N)$ as case (a), and $\Gamma(Y_0^*(1518)\to\Sigma\pi)$ as case (b). The decay width $\Gamma(Y_0^*(1700)\to\bar{K}N)$ would also be acceptable as an input, in spite of the few observations, since the existing data[1,2] are in good agreement and there seems no neighboring $I=0$ baryon resonance which decays strongly into $\bar{K}N$.[16]

Taking into account the possible variation in the data for the partial decay widths of the $Y_0^*(1518)$, we examine two sets of data. One is the compiled data[3] (case I), and the other is Hardy's[17] (case II), which gives a higher ratio of $(\bar{K}N)/(\Sigma\pi)$. In Figs. 1 and 2 the X dependence of the various quantities are shown for cases I and II, respectively. The uncertainties associated with the individual errors of the input data are indicated by bars and bands.

It is evident from Figs. 1 and 2 that both the F/D ratio and the decay widths are rather sensitive to

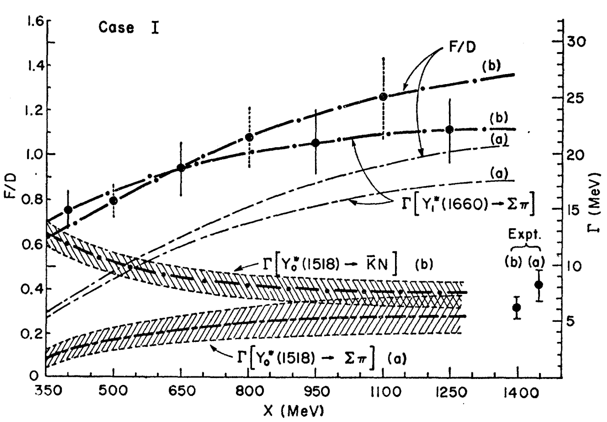

FIG. 1. Plots of F/D ratio and various decay widths as a function of inverse interaction radius X, for case I. The dashed-dotted and heavy dashed-dotted curves correspond to the central values of the input taken for cases (a) and (b), respectively. The length of vertical bars along the curves are associated with a 26% error of the input $\Gamma(Y_0^*(1700)\to\bar{K}N)$. The hatched bands indicate the uncertainties corresponding to the individual errors of an alternative partial decay width of $Y_0^*(1518)$ taken as input.

[16] A neighboring resonance is the $Y_0^*(1670)$. However, the effective cross section for $Y_0^*(1670)\to\bar{K}N$ is about 1 mb, which is $\frac{1}{10}$ as large as that of $Y_0^*(1700)\to\bar{K}N$. See D. Berley, P. L. Connolly, E. L. Hart, D. C. Rahm, D. L. Stonehill, B. Thevenet, W. J. Willis, and S. S. Yamamoto, Phys. Rev. Letters **15**, 641 (1965).

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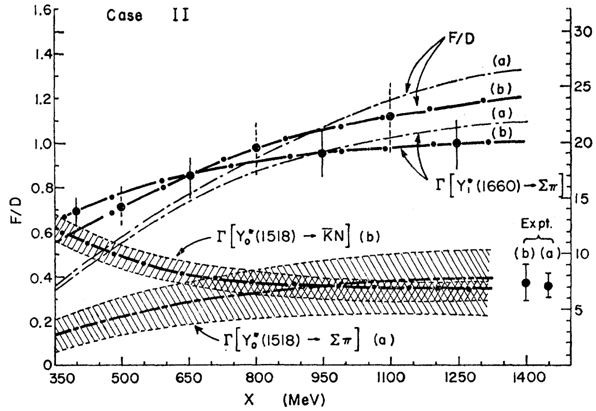

FIG. 2. Same as Fig. 1, for case II.

changes in X, unlike the case of no mixing.[18] In order to estimate the optimum value of X, we insist on the following restrictions: (1) The calculated $\Gamma(Y_0^*(1518)\to\bar{K}N)$ from the input of $\Gamma(Y_0^*(1518)\to\Sigma\pi)$ must lie in the allowed region and vice versa. (2) The calculated $\Gamma(Y_1^*(1660)\to\Sigma\pi)$ must be lower than about 21 MeV, which is a possible upper limit.[19] The latter requirement seems to be necessary because the $(\Sigma\pi)$ rate rises with increasing X and all data reported so far are consistent as far as the upper limit is concerned. The following best values are obtained:

Case I

Case (a): $F/D=0.9$–1.1, $X=950$–∞, $\beta=2.1$–2.6;
Case (b): $F/D=\sim1.2$, $X=\sim950$, $\beta=\sim2.6$.

Case II

Case (a): $F/D=\sim1.1$, $X=\sim1000$, $\beta=\sim2.5$;
Case (b): $F/D=\sim1.1$, $X=\sim1000$, $\beta=\sim2.5$.

The decay widths calculated from the above values are listed in Table I. In addition, the width of the three-body decay of the resonance into the decouplet and pseudoscalar meson [computed from Eq. (4)] is also given in order to see the effects of mixing on the decay width of $Y_0^*(1700)\to Y_1^*(1385)+\pi$.

We can see that the model is quite a reasonable one. As a result, we may discuss the following:

(1) The small fraction of $(\bar{K}N)$ decay mode and the large fraction of $(\Sigma\pi)$ decay mode for $Y_0^*(1700)$ are treated consistently in this model, in contrast to the case of no singlet-octet mixing. This mixing drastically changes the ratio of $(\bar{K}N)/(\Sigma\pi)$.[20] Further accurate measurements of this ratio would be quite interesting.

(2) This model also reproduces very closely the

[18] M. Goldberg, J. Leitner, R. Musto, and L. O'Raifeartaigh, Nuovo Cimento **45**, 169 (1966).

[19] The value taken here is about $1\frac{1}{2}$ standard deviations higher than the average of the existing data.

[20] Although the associated errors are not shown in Table I, if one uses a possible upper limit of $\Gamma(Y_0^*(1700)\to\bar{K}N)$ as input in case I(a), one can obtain the branching ratio of $\Gamma(Y_0^*(1700)\to\bar{K}N)/\Gamma(Y_0^*(1700\to\Sigma\pi)\sim0.3$. A crude estimate of the ratio with presently available data is about 0.4.

new experimental decay rates of $Y_0^*(1518)$. The model rather favors a higher decay rate for the $Y_0^*(1518) \to \bar{K}N$ mode than that given in the compiled data.[3]

(3) For the optimum value of the F/D ratio, the $(\Sigma\pi)$ decay rate of $Y_1^*(1660)$ is two to four times higher than that of the $(\Lambda\pi)$ decay. The currently reported $Y_1^*(1680)$[21] with a large ratio of $\Gamma(\Lambda\pi)/\Gamma(\Sigma\pi)$ might belong to another unitary multiplet if its spin-parity is $\frac{3}{2}^-$.

(4) The fits to $\Xi^*(1815)$ decays are not so good if we take the existing experimental data. Because of the meager experimental situation, we do not consider the disagreement severe. If the $\Xi^*(1815)$ is a real object and if the $\Lambda\bar{K}$ dominance of its decay is true, some other mechanism to suppress the $\Sigma\bar{K}$ decay mode is necessary.[22]

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It is surprising that the F/D ratio obtained in this paper is nearly equal to the values derived from different methods and different input data.[22,23]

One of us (S.M.) wishes to thank Professor M. L. Good for his encouragement and Professor R. W. Hartung for his support. We are very grateful to Dr. A. R. Swift and Dr. M. A. Thompson for reading the manuscript.

Note added in proof. After this paper was written, G. B. Yodh published a paper [Phys. Rev. Letters **18**, 810 (1967)], in which he showed the incompatibility of a pure $SU(3)$ singlet assignment for $Y_0^*(1518)$ and also suggested the difficulties of the singlet-octet mixing scheme. In this paper we predict rather the decay widths which are experimentally uncertain by taking the most reliable data as input. More data for the decays of the $Y_1^*(1660)$ and $\Xi^*(1815)$ should be accumulated before the predictions can be judged.

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Drell-Hearn-Gerasimov Sum Rule: Examples and Counterexamples*

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(Received 20 June 1967)

We investigate by means of examples whether the Drell-Hearn-Gerasimov sum rule can hold simultaneously for a lightly bound state and for its constituents. Subject to certain assumptions, whose applicability is discussed, we find in particular that if the rule holds for the nucleons, then it holds for the deuteron but fails for He^3 and H^3. If neutron and proton masses were appreciably unequal, then the rule would fail for the deuteron as well.

1. INTRODUCTION

Recently Gerasimov and Drell and Hearn[1] have proposed the following sum rule for the absorption of photons by protons:

$$8\pi^2\alpha\left(\frac{\kappa_p}{2M}\right)^2=2\pi^2\alpha\frac{\kappa_p^2}{M^2}=\int_0^\infty\frac{d\omega}{\omega}[\sigma_p{}^P(\omega)-\sigma_p{}^A(\omega)]$$
$$\equiv J_p{}^P-J_p{}^A\equiv J_p. \quad (1.1)$$

Here, $\alpha=e^2\approx 1/137$ is the fine structure constant (we use natural units, $\hbar=1=c$), $\kappa_p\approx 1.79$ is the anomalous magnetic moment of the proton in units of $e/2M$, M is the nucleon mass, and $\sigma_p{}^P(\omega)$ $(\sigma_p{}^A(\omega))$ is the total absorption cross section for photons of frequency ω with spins parallel (antiparallel) to the initial proton spin. Corresponding rules are implied for any spin-$\frac{1}{2}$ particle; we shall call them DHG rules in the following.

To derive (1.1) (to order α), one needs two results rigorously provable from microcausality and charge conservation (gauge invariance), plus two further independent assumptions. The first of the proved results is the dispersion relation for the forward Compton scattering amplitude[2] $f(\omega)$:

$$f(\omega)=\boldsymbol{\varepsilon}'^*\cdot\boldsymbol{\varepsilon}f_1(\omega)+i\omega\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}'^*\times\boldsymbol{\varepsilon}f_2(\omega), \quad (1.2)$$

* Supported in part by the Air Force Office of Scientific Research Contract No. AF 49(638)-1380.
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