Electromagnetic Simulation of T Violation in Beta Decay*

CURTIS G. CALLAN, JR., AND S. B. TREIMAN
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received 12 June 1967)

β-decay correlation terms of the form $\mathbf{J}\cdot\mathbf{p}_e\times\mathbf{p}_\nu$ can arise, insofar as time-reversal invariance is valid, only from the influence of final-state electromagnetic interactions. For allowed transitions governed by V, A couplings, the effects are recoil-dependent. They are computed here, to lowest relevant recoil and electromagnetic orders, for the special class of spin-$\frac{1}{2}$ mirror transitions, of which the decay process $Ne^{19}\rightarrow F^{19}+e^{+}+\nu$ is an example. On the conserved-vector-current hypothesis, the correlation effect in question is seen to be dominated by the phenomenon of weak magnetism.

I. INTRODUCTION

THE observation of CP violation in neutral K-meson decays has stimulated renewed efforts aimed at testing time-reversal invariance for weak decays generally. One of the classical tests, in nuclear β decay, involves the search for a correlation term in the decay spectrum of the form $(\mathbf{J}/J)\cdot(\mathbf{p}_e\times\mathbf{p}_\nu)$, where $\mathbf{J}/J$ is the polarization of the parent nucleus and $\mathbf{p}_e$ and $\mathbf{p}_\nu$ are, respectively, the electron and neutrino momenta. Insofar as final-state electromagnetic interactions can be ignored, such a correlation term is forbidden by the principle of time-reversal invariance. A small upper limit on the coefficient of this "triple-product" correlation term, as we shall call it, was established some years ago for neutron β decay,[1] and a still smaller limit has been set more recently for the mirror decay $Ne^{19}\rightarrow F^{19}+\beta^{+}+\nu$.[2]

These processes are examples of allowed transitions between spin-$\frac{1}{2}$ systems. On the V, A coupling picture the decay spectrum, summed over final spin polarizations, has the following form in leading approximation[3]:

$$dw=\frac{F(\mp Z,E_e)}{(2\pi)^5}p_eE_e(E_0-E_e)^2dE_ed\Omega_ed\Omega_\nu\xi\left\{1+a\frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_eE_\nu}+\frac{\mathbf{J}}{J}\cdot\left[A\frac{\mathbf{p}_e}{E_e}+B\frac{\mathbf{p}_\nu}{E_\nu}+D\frac{\mathbf{p}_e\times\mathbf{p}_\nu}{E_eE_\nu}\right]\right\},\quad(1)$$

where the coefficients ξ, a, A, B, D depend in a definite way on the vector and axial-vector coupling "constants" (form factors, effectively constant over the spectrum). For the coefficient D of interest here, present experimental results are as follows: for neutron β decay, $D(n)=0.04\pm0.05$; for Ne^{19} decay, $D(Ne^{19})=0.002\pm0.014$.

The difficult measurements which these results summarize are of impressively high accuracy and still more precise measurements appear to be in prospect. With the upper limit on the coefficient D approaching the size of the fine-structure constant, it is timely to undertake a computation of this coefficient for a regime in which time-reversal invariance is assumed to obtain but final-state electromagnetic interactions are taken into account. Superficially, one might expect electromagnetic effects to generate a coefficient D of order $Z\alpha$, where α is the fine structure constant and Ze is the charge of the daughter nucleus. If final-state effects were in fact of this size, their accurate computation would be essential before one could draw conclusions bearing on time reversal invariance from the already existing limits set by experiment on the coefficient D.

However, it has been known[3] for some time that on a V, A coupling model for allowed transitions, final-state scattering, in leading approximation with respect to expansion in inverse powers of nuclear mass, does not generate the triple-product term at all. This suggests that the final-state effects, if any, depend on nuclear recoil and thus give to the coefficient D a size of order $Z\alpha E_e/M$, where M is the nuclear mass. This estimate corresponds to a value $D\approx10^{-5}-10^{-6}$, which is so incredibly small that any practical detection of the triple-product term, at a forseeable level, would immediately signify the breakdown of time-reversal invariance in β decay.

A closer inspection of possible recoil effects modifies this conclusion however. For definiteness, and with the experiment on Ne^{19} β decay in mind, we shall restrict ourselves here to the general class of mirror transitions between spin-$\frac{1}{2}$ nuclei. Two kinds of recoil phenomena are to be distinguished. For one thing, in addition to pure Coulomb scattering in the final state, the recoil phenomenon of electron scattering by the nuclear magnetic moment has to be considered. This recoil effect, as we shall see, conforms to the estimate given above for the size of the D coefficient. Beyond this, however, the structure of the basic β-decay interaction (in the absence of electromagnetism) may involve recoil-dependent terms, which can then be modified by the effects of pure Coulomb scattering to give rise to a triple-product term. If we suppose that the vector and axial-vector currents are of the "first kind," in the sense of Weinberg,[4] then the basic β-decay interaction can

* Work supported by the U. S. Air Force Office of Research, Air Research and Development Command under contract number AF49(638)-1545.

[1] M. T. Burgy, V. E. Krohn, T. B. Novey, G. R. Ringo, and V. L. Telegdi, Phys. Rev. 120, 1829 (1960); see also M. A. Clark and J. M. Robson, Can. J. Phys. 38, 693 (1960).

[2] F. P. Calaprice, E. D. Commins, H. M. Gibbs, and G. L. Wick, Phys. Rev. Letters 18, 918 (1967).

[3] J. D. Jackson, S. B. Treiman, and H. W. Wyld, Jr., Nucl. Phys. 4, 206 (1957); also Phys. Rev. 106, 517 (1960).

[4] S. Weinberg, Phys. Rev. 112, 375 (1958).

contain, in addition to the usual vector and axial-vector terms, only two additional pieces, both of them recoil-dependent: an induced pseudoscalar, negligibly small for our purposes,[5] and a term of the weak magnetism sort. On the conserved-vector-current (CVC) hypothesis,[6] the size of this latter term can be inferred from the magnetic moments of the parent and daughter nuclei. As we shall see, this weak magnetism term, with strength estimated on the basis of the CVC hypothesis, contributes to the D coefficient a quantity of order $(Z\alpha E_e/M)(M/m)(\mu_f-\mu_i)$, where μ_f and μ_i are the magnetic moments of the daughter and parent nuclei, measured in units of the *nucleon* Bohr magneton, and m is the *nucleon* mass. This represents an increase, over our crudest expectation for the size of D, which can be quite substantial. For Ne^{19} β decay we shall find that D is of order 2×10^{-4} for the most energetic positrons. This is of course still incredibly small, but it is amusing that the triple-product term, insofar as time-reversal invariance holds true, becomes such a direct test of the CVC hypothesis. In any case, it is not unthinkable that effects of this size can eventually be measured—for favorably chosen allowed transitions the coefficient D can perhaps become as large as 10^{-3}, though still larger values are not indicated for any cases known to us.

II. DETAILS

We consider the β-decay processes

$$(Z\mp1,A)\rightarrow(Z,A)+\beta^{\mp}+({}_{\nu}{}^{\nu}),$$

where parent and daughter nuclei are members of a common isospin doublet, with ordinary spin $J=\frac{1}{2}$. The four-momenta of parent nucleus, daughter nucleus, electron, and neutrino are denoted, respectively, by n, p, l, q, with $n^2=p^2=-M^2$, $l^2=m_e^2$, $q^2=0$. For definiteness we discuss here the case of negatron decay, indicating at the end the changes that must be made for the case of positron decay. In the absence of electromagnetic corrections, the invariant transition amplitude A_0 has the following structure, up to an over-all constant:

$$A_0=\bar{u}(p)[\gamma_\lambda+g\gamma_\lambda\gamma_5+f_2(\sigma_{\lambda\nu}(n-p)_\nu/2M)]u(n)\times\bar{u}(l)\gamma_\lambda(1+\gamma_5)v(q). \quad (2)$$

The parameter g is defined by

$$g=-\frac{g_A}{g_V}\frac{M_{\rm F}M_{\rm GT}}{\sqrt{3}|M_{\rm F}|^2}, \quad (3)$$

where $g_A/g_V\simeq-1.18$ is the ratio of axial-vector and vector coupling constants in elementary nucleon β decay, a real quantity if T invariance holds, and $M_{\rm F}$ and $M_{\rm GT}$ are, respectively, the Fermi and Gamow-Teller nuclear matrix elements. For example, in neutron β decay $g\simeq1.18$; in Ne^{19} β decay $g\simeq-0.99\pm0.05$. The term involving the coefficient f_2 has the weak magnetism structure (we are ignoring, as already mentioned, the induced pseudoscalar term and terms of Weinberg's "second" type). On *any* model of the weak interactions a weak magnetism term is to be qualitatively expected. But, in the case of a mirror transition, the CVC hypothesis relates the coefficient f_2 to the nuclear magnetic moments in a definite way. Namely, for a related quantity $f_1=1+2Mf_2$ we have

$$f_1=1+2Mf_2=(M/m)[\mu(+)-\mu(-)], \quad (4)$$

where $M/m=A$ is the atomic number of the nuclei in question and $\mu(+)$, $\mu(-)$ are the total nuclear magnetic moments corresponding, respectively, to the $I_z=+\frac{1}{2}$ and $I_z=-\frac{1}{2}$ members of the mirror pair, measured in units of the *nucleon* Bohr magneton. Strictly speaking, the parameters g and f_2 are functions of the momentum transfer $(n-p)^2$. But to lowest relevant order in nuclear recoil, we can evaluate these quantities at zero momentum transfer.

Electromagnetism acts to modify the structure of Eq. (2), altering the coefficients that already appear and giving rise to new kinds of terms. We are interested, however, only in those effects which serve to produce a triple-product term in the decay spectrum. If time reversal invariance holds, this correlation arises only as a result of final-state electromagnetic scattering. To lowest order, it is enough to take for the electromagnetically corrected β-decay matrix element the expression

$$A=A_0+i\,{\rm Im}A, \quad (5)$$

where the absorptive part ImA is itself determined, via unitarity, by the product of the zeroth electromagnetic order β-decay amplitude A_0 and the amplitude for electron, daughter nucleus scattering. The amplitude for the latter process, with l' and l, respectively, the initial and final electron momenta, p' and p the initial and final daughter nucleus momenta, is given by

$$T=\frac{Ze^2}{k^2}\bar{u}(p)\left[\gamma_\mu+G_2\frac{\sigma_{\mu\nu}k_\nu}{2M}\right]u(p')\bar{u}(l)\gamma_\mu u(l'), \quad (6)$$

where $k=p'-p=l-l'$ and, to lowest relevant order in nuclear recoil,

$$2MG_2=\left(\frac{\mu_f{}^{(a)}}{Z}\right)\left(\frac{M}{m}\right)=\frac{A}{Z}\mu_f{}^{(a)}, \quad (7)$$

where $\mu_f{}^{(a)}$ is the anamolous magnetic moment, in units of the *nucleon* Bohr magneton, of the daughter nucleus. For practical purposes, Eq. (6) is most conveniently written in the form

$$T=(Ze^2/k^2)\bar{u}(p)G_\mu u(p')\bar{u}(l)\gamma_\mu u(l'), \quad (6')$$

with

$$G_\mu=G_1\gamma_\mu+2iG_2p_\mu, \quad (6'')$$

$$G_1=1+2MG_2, \quad (6''')$$

[5] L. Wolfenstein, Nuovo Cimento 8, 882 (1958); M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958).

[6] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

and Eq. (2) can be written

$$A_0=\bar{u}(p)M_\lambda u(n)\bar{u}(l)\gamma_\lambda(1+\gamma_5)v(q)\,, \qquad (2')$$

with

$$M_\lambda(p,n)=f_1\gamma_\lambda+g\gamma_\lambda\gamma_5+if_2(n+p)_\lambda. \qquad (2'')$$

The unitarity equation for ImA is given by the expression

$$\mathrm{Im}A=Ze^2\frac{(2\pi)^4}{2}\int\frac{d\mathbf{p}'d\mathbf{l}'}{(2\pi)^6}\frac{Mm_e}{p_0'l_0'}\delta(p'+l'-p-l)\frac{1}{(l-l')^2}$$
$$\times\bar{u}(p)G_\mu\Lambda(p')M_\lambda(p',n)u(n)$$
$$\times\bar{u}(l)\gamma_\mu\Lambda(l')\gamma_\lambda(1+\gamma_5)v(q)\,, \qquad (8)$$

where $\Lambda(p')=(-i\gamma\cdot p'+M)/2M$ is the projection operator for the daughter nucleus of momentum p' and $\Lambda(l')$ is the corresponding projection operator for the electron of momentum l'.

In evaluating the integral of Eq. (8), and in all other aspects of the present calculation, we are focussing only on the triple product coefficient D, evaluated, moreover only to lowest order in the recoil parameter E_e/M, where E_e is the electron total energy in the rest frame of the parent nucleus. To this order the evaluation is straightforward, although already sufficiently tedious. The triple-product term of interest arises from the interference of A_0 and ImA in the net squared matrix element $|A|^2=|A_0+\mathrm{Im}A|^2$; the tedium is somewhat relieved by dropping at the outset terms in ImA which are proportional to A_0 itself, since such terms contribute nothing more than a common phase change to all the pieces of A_0 and cannot therefore produce the correlation effect of interest.

We shall not display all of the algebra here but shall content ourselves, rather, with summarizing the final results for the D coefficient. The following remark is, however, appropriate here. Our discussion so far has centered on the example of β^- decay. For β^+ decay one can of course start anew, making the obvious changes. But it is clear that in the D coefficient, which relates to a parity conserving correlation, the β^- and β^+ cases can differ only in the sign of vector–axial-vector interference terms, since the vector and axial-vector lepton currents behave oppositely under charge conjugation. In effect, the expressions for D can differ as between β^- and β^+ decay only in the sign of the parameter g, wherever it occurs. This is of course apart from an overall sign change related to the reversal of electric charge as between β^+ and β^-.

We may now summarize. In terms of the parameters g, f, and G_1 introduced previously, we have for $\beta^\mp$ decays the expression

$$D(\beta^\mp)=\pm\frac{Z}{A}\frac{\alpha}{4(1+3g^2)}\bigg\{(1\pm3g)[(f_1\mp g)-3G_1(1\mp g)]$$
$$+\frac{m_e^2}{E_e^2}[(3\pm g)(f_1\mp g)+3G_1(1\pm3g)(1\mp g)]\bigg\}\left(\frac{E_e}{p_e}\right)\left(\frac{E_e}{m}\right), \qquad (9)$$

where $\alpha=1/137$ is the fine structure constant, p_e the electron three-momentum, E_e the total electron energy, m_e the electron mass, and m the *nucleon* mass. It is useful to rewrite here the formulas relating f_1 and G_1 to the *total* magnetic moments μ_f and μ_i of daughter and parent nuclei, expressed in *nucleon* magnetons. From Eqs. (7) and (6‴) it is evident that

$$G_1=(A/Z)\mu_f\,, \qquad (9')$$

and from Eq. (4) we have

$$f_1=A[\mu(+)-\mu(-)]. \qquad (9'')$$

The quantity f_1, whose origin lies in weak magnetism and CVC, can evidently become quite large compared to unity in favorable cases and thereby dominate the other terms in Eq. (9).

In the rather timely case of $Ne^{19}\to F^{19}+\beta^++\nu$, the relevant numbers are

$$g\simeq-0.99\,,\quad \mu_f=\mu(-)=2.63\,,$$
$$\mu_i=\mu(+)=-1.89\,,\quad (p_e)_{\max}=2.75\ \mathrm{MeV}/c\,,$$

and the D coefficient turns out to be (neglecting the positron mass)

$$D=(2.6\times10^{-4})p_e/(p_e)_{\max}.$$

This is a factor 50 smaller than the present experimental accuracy, leaving considerable room for experimental improvement before one has to worry about confusing true T violation with final-state scattering effects.

On the other hand, the final-state scattering contribution to the triple-product correlation can be dominated, as it is in the example of Ne^{19} decay, by the effects of weak magnetism—insofar as the CVC hypothesis is valid. Thus one deals here with a distinctive kind of test of the CVC hypothesis. For favorable prospects one wants a mirror transition with large energy release, large atomic number A, and large magnetic-moment difference $\mu(+)-\mu(-)$. The magnetic-moment parameters are not available for all candidate transitions, but it seems unlikely that one can find a situation in which the coefficient D is larger than about 10^{-3}, so that Ne^{19} decay is not far from being the best practical candidate. Nonmirror transitions, both allowed and forbidden are of course another matter, requiring independent analysis.