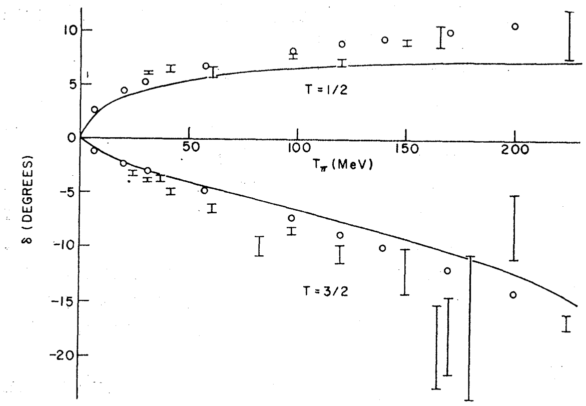

FIG. 3. Comparison of predicted phase shifts with experiment. The points indicated by open circles are taken from Ref. 7. The other points are taken from Ref. 6.

eters are allowed complete freedom. However, as we have seen, when very reasonable constraints are imposed on the scattering lengths then only one type of solution appears to be present.

6. CONCLUSIONS

We have attempted to obtain low-energy s-wave π-N scattering amplitudes which are approximately unitary and crossing symmetric by imposing the requirement that the scattering amplitudes calculated from twice-subtracted dispersion relations are consistent with input unitary trial functions in both the low-energy physical region and part of the nearby crossed u-channel physical region. If the parameters of the trial are allowed to be completely free, than no unique solution is found. However, if very plausible constraints are imposed on the scattering lengths then the resulting solution is both reasonably self-consistent and compatible with experiment. Thus, as in the case of resonant partial waves, the method is a useful tool for the semi-phenomenological analysis of low-energy scattering phenomena.

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Phenomenological Analysis of $K \to \pi\pi(I=2)$ Amplitudes and Violation of CP Invariance*

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A phenomenological analysis of the $K \to \pi\pi(I=2)$ amplitudes, $K^0 \to \pi\pi(I=2)$, $\bar{K}^0 \to \pi\pi(I=2)$, and $K^\pm \to \pi^+\pi^0$, in terms of their irreducible isospin amplitudes corresponding to the $\Delta I=\frac{3}{2}$ and $\Delta I=\frac{5}{2}$ transitions, is given with reference to the observed violation of CP invariance. The unitarity condition for the K^0-$\bar{K}^0$ system is also discussed, and an evaluation of the parameters describing the 2π decay modes made. Terms in the unitarity condition referring to the modes $\pi\pi(I=0)$, $\pi\pi(I=2)$, 3π, and semileptonic are explicitly taken into account.

I. INTRODUCTION

SINCE the discovery by Christenson, Cronin, Fitch, and Turlay[1] that the long-lived component K_L of the K^0-$\bar{K}^0$ system decays into the $\pi^+\pi^-$ mode, much effort has been dedicated to the search for other effects which may confirm the violation of CP invariance in K^0-$\bar{K}^0$ decay. Recently, preliminary results on the measurement of the decay rate of K_L into the $\pi^0\pi^0$ mode have been reported by two groups.[2,3] The observation of $K_L \to \pi^0\pi^0$ decays, together with the earlier observations of $K_L \to \pi^+\pi^-$ decays,[4] and the interference effects[5–9] between K_L and the short-lived component K_S, constitutes the present evidence for the violation of CP invariance.

The experiments mentioned above have resulted, either directly or indirectly, in a knowledge of the

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[2] J. M. Gaillard *et al.*, Phys. Rev. Letters 18, 20 (1967).

[3] J. W. Cronin, P. F. Kunz, W. S. Risk, and P. C. Wheeler, Phys. Rev. Letters 18, 25 (1967).

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[5] V. L. Fitch, R. F. Roth, J. S. Russ, and W. Vernon, Phys. Rev. Letters, 15, 73 (1965).

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[7] M. Bott-Bodenhausen *et al.*, Phys. Letters 20, 212 (1966); 23, 777 (1966).

[8] A. Firestone *et al.*, Phys. Rev. Letters 16, 556 (1966).

[9] R. E. Mischke *et al.*, Phys. Rev. Letters 18, 138 (1967).

parameters

$$\eta_{+-}=\frac{A(K_L\to\pi^+\pi^-)}{A(K_S\to\pi^+\pi^-)}\quad\text{and}\quad|\eta_{00}|=\left|\frac{A(K_L\to\pi^0\pi^0)}{A(K_S\to\pi^0\pi^0)}\right|,$$

where, e.g., $A(K_L\to\pi^0\pi^0)$ denotes the amplitude for K_L decay into the $\pi^0\pi^0$ mode. Their values to date are

$$|\eta_{+-}|=(1.94\pm0.09)\times10^{-3},$$
$$|\eta_{00}|=(4.87\pm0.44)\times10^{-3},$$

and for the phase of η_{+-}, which we call θ_{+-},[10]

$$\theta_{+-}=77°\pm19°.$$

Many suggestions have been made as to the origin of these CP-noninvariant effects,[11] none of which can be said to be conclusive at this time. In fact, some of the theories proposed are already in conflict with the recently reported experiments,[12] which conclusively show that

$$|\eta_{+-}|\neq|\eta_{00}|.$$

As we shall see later, this inequality implies that the violation of CP invariance in $K_L\to\pi\pi$ decays is partially due to the interference between the dominant $K^0\to\pi\pi(I=0)$ amplitude and the $K^0\to\pi\pi(I=2)$ amplitude, where I refers to the total isospin of the final $\pi\pi$ system.

The phenomenological description of the K^0-$\bar{K}^0$ system was first made by Gell-Mann and Pais,[13] and subsequently developed by Treiman and Sachs,[14] and Good.[15] The implications of possible noninvariance under time reversal and charge conjugation were discussed by Lee, Oehme, and Yang,[16] and, within the framework of field theory, by Sachs.[17] After the discovery of Christenson, Cronin, Fitch, and Turlay,[1] various phenomenological analyses of the K^0-$\bar{K}^0$ have been made, firstly by Wu and Yang,[18] and subsequently by Wolfenstein[19] and by Bell and Steinberger,[20] and more recently by Lee and Wu.[21]

In this paper we follow the phenomenological viewpoint of these authors, and analyze the $K\to2\pi(I=2)$ amplitudes: $K^0\to\pi\pi(I=2)$, $\bar{K}^0\to\pi\pi(I=2)$, and $K^\pm\to\pi^\pm\pi^0$, in terms of their irreducible isospin amplitudes corresponding to the $\Delta I=\frac{3}{2}$ and $\Delta I=\frac{5}{2}$ transitions. Although a complete determination of the latter amplitudes is not possible at this time, the present experimental information already allows some interesting conclusions to be drawn. We also present a detailed evaluation of the various parameters describing the K^0-$\bar{K}^0$ system.

The phenomenology of $K\to2\pi$ decays is reviewed in Sec. II, where our notations and phase conventions are stated. A discussion of the unitarity condition of the K^0-$\bar{K}^0$ system is also included in view of its significance for the numerical estimates of the various parameters. The numerical analysis is presented in Sec. III. In Sec. IV we discuss the possible configurations for the $\Delta I=\frac{3}{2}$ and $\Delta I=\frac{5}{2}$ amplitudes which are compatible with the present data, with emphasis on four particular cases which are of special theoretical significance. We conclude in Sec. V with a review of suggested experiments relevant to the preceding discussions.

II. PHENOMENOLOGY OF $K\to2\pi$ DECAYS

Phenomenological analyses of neutral K decays into two pions, without assuming CP invariance, have been discussed by several authors.[18–21] We shall review them briefly, in part of this section, in order to define our notations, phase conventions, and approximations.

A. Notations

Throughout this work we will assume CPT invariance, and we will adopt the convention which relates $|K^0\rangle$ and $|\bar{K}^0\rangle$ in such a way that

$$CP|K^0\rangle=-|\bar{K}^0\rangle.$$

In terms of these states we may define the states $|K_S\rangle$ and $|K_L\rangle$ as the linear combinations

$$|\rangle K_S=p|K^0\rangle-q|\bar{K}^0\rangle,$$
$$|K_L\rangle=p|K^0\rangle+q|\bar{K}^0\rangle, \qquad (1)$$

where p and q are complex parameters with

$$|p|^2+|q|^2=1.$$

We shall denote the various $K\to\pi\pi$ decay ampli-

[10] This value of θ_{+-} is considerably different from the previously accepted value [given, e.g., by V. L. Fitch, in *Proceedings of the Thirteenth International Conference on Elementary Particles, Berkeley, 1967* (University of California Press, Berkeley, California, 1967)]. The change is partly due to a more reliable calculation of the regeneration phase based on data which have become available or K-copper scattering, and partly due to an error in the original calculations of this quantity. See the references in Table I.

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tudes as follows:

$$A^+(+0)\equiv A(K^+\to\pi^+\pi^0),\quad A^0(+-)\equiv A(K^0\to\pi^+\pi^-),$$
$$A^0(00)\equiv A(K^0\to\pi^0\pi^0),$$
$$A^-(-0)\equiv A(K^-\to\pi^-\pi^0),\quad \bar{A}^0(+-)\equiv A(\bar{K}^0\to\pi^+\pi^-),$$
$$\bar{A}^0(00)\equiv A(\bar{K}^0\to\pi^0\pi^0),$$
$$A_S(+-)\equiv A(K_S\to\pi^+\pi^-),\quad A_S(00)\equiv A(K_S\to\pi^0\pi^0),$$
$$A_L(+-)\equiv A(K_L\to\pi^+\pi^-),\quad A_L(00)\equiv A(K_L\to\pi^0\pi^0),$$

and

$$A_S(I=0)\equiv A[K_S\to(\pi\pi)_{I=0}],$$
$$A_S(I=2)\equiv A[K_S\to(\pi\pi)_{I=2}],$$
$$A_L(I=0)\equiv A[K_L\to(\pi\pi)_{I=0}],$$
$$A_L(I=2)\equiv A[K_L\to(\pi\pi)_{I=2}],$$

where I denotes the isospin of the final $\pi\pi$ system.

In terms of the above amplitudes, two useful sets of parameters to describe neutral K decays into two pions may be defined. They are

$$\epsilon=\frac{A_L(I=0)}{A_S(I=0)},\quad \epsilon'=\frac{A_L(I=2)}{A_S(I=0)},\quad \omega=\frac{A_S(I=2)}{A_S(I=0)},\quad (2)$$

and

$$\eta_{+-}=\frac{A_L(+-)}{A_S(+-)},\quad \eta_{00}=\frac{A_L(00)}{A_S(00)},\quad \eta=\frac{A_S(00)}{A_S(+-)}.\quad (3)$$

These two sets of parameters are related as follows:

$$\eta_{+-}=\frac{(2)^{1/2}\epsilon+\epsilon'}{(2)^{1/2}+\omega},\quad (4a)$$

$$\eta_{00}=\frac{\epsilon-(2)^{1/2}\epsilon'}{1-(2)^{1/2}\omega},\quad (4b)$$

$$\eta=\frac{(2)^{1/2}\omega-1}{(2)^{1/2}+\omega}.\quad (4c)$$

Each set of parameters has certain advantages, and both are widely used.

B. Isospin Amplitudes and Sum Rules

Using CPT invariance and taking into account final-state interactions, but neglecting electromagnetic corrections, the transition amplitudes for K^0 and $\bar{K}^0$ into the $(\pi\pi)_{I=0}$ and $(\pi\pi)_{I=2}$ state may be written

$$A[K^0\to(\pi\pi)_{I=0}]=iA_0e^{i\delta_0},$$
$$A[\bar{K}^0\to(\pi\pi)_{I=0}]=-iA_0{}^*e^{i\delta_0},$$
$$A[K^0\to(\pi\pi)_{I=2}]=iA_2e^{i\delta_2},$$
$$A[\bar{K}^0\to(\pi\pi)_{I=2}]=-iA_2{}^*e^{i\delta_2},$$

where δ_0 and δ_2 are the s-wave $I=0$ and $I=2$ $\pi\pi$ phase shifts at the energy corresponding to the kaon mass.

It is conventional[22] to fix the relative phase of $|K\rangle$ and $|\bar{K}\rangle$ in such a way that the amplitude A_0 can be taken real and positive

$$A_0=A_0{}^*>0.$$

We shall also adopt this *convention*. Then the parameters ϵ, ϵ', and ω, expressed in terms of A_0 and A_2, become

$$\epsilon=(p-q)/(p+q),\quad (5a)$$

$$\epsilon'=\frac{pA_2-qA_2{}^*}{pA_0+qA_0}e^{i(\delta_2-\delta_0)},\quad (5b)$$

$$\omega=\frac{pA_2+qA_2{}^*}{pA_0+qA_0}e^{i(\delta_2-\delta_0)}.\quad (5c)$$

In $K^\pm\to\pi^\pm\pi^0$ decays, the final $\pi^\pm\pi^0$ system is in an $I=2$ state. Again, using CPT invariance and taking into account final-state interaction, but neglecting electromagnetic corrections, we may write

$$A^+(+0)=iA_2{}^+e^{i\delta_2},\quad A^-(-0)=-i(A_2{}^+)^*e^{i\delta_2}.$$

These relations lead to the equality of the decay rates for $K^+\to\pi^+\pi^0$ and $K^-\to\pi^-\pi^0$. In the presence of electromagnetic interactions this equality need not be exact. However, parity conservation forbids any electromagnetic (or strong) transition between a $J=0$ 2π state and a $J=0$ 3π state; and, furthermore, the radiative decays of K^+ are known to have very small branching ratios. Consequently, the ratio of the two decay rates for $K^+\to\pi^+\pi^0$ and $K^-\to\pi^-\pi^0$ is expected to be unity to an accuracy $\sim O(10^{-4})$, independent of the magnitude of the CP nonconserving amplitude. Taking this ratio of decay rates unity will be a sufficiently good approximation for our purposes.

Transitions from a K state, which is $I=\frac{1}{2}$, to a $(\pi\pi)_{I=2}$ state may be via $\Delta I=\frac{3}{2}$ and/or $\Delta I=\frac{5}{2}$. Thus the amplitudes A_2 and $A_2{}^+$ are in principle a superposition of irreducible isospin amplitudes $A^{(3/2)}$ and $A^{(5/2)}$:

$$A_2=\frac{1}{\sqrt{2}}(A^{(3/2)}-A^{(5/2)}),\quad (6)$$

$$A_2{}^+=\frac{1}{\sqrt{3}}(A^{(5/2)}+\tfrac{3}{2}A^{(3/2)}).\quad (7)$$

Here, it is useful to introduce a parameter α such that

$$\alpha=(\sqrt{\tfrac{2}{3}})\frac{A_2}{A_2{}^+}=\frac{A^{(3/2)}-A^{(5/2)}}{A^{(5/2)}+\frac{3}{2}A^{(3/2)}}.\quad (8)$$

Then, by eliminating the irreducible isospin amplitude $A^{(1/2)}$ between the physical amplitudes $A^0(00)$ and $A^0(+-)$, we have the following sum rule:

$$2A^0(00)+\sqrt{2}A^0(+-)=3\alpha A^+(+0),\quad (9)$$

[22] See, e.g., T. T. Wu and C. N. Yang, Ref. 18.

and using CPT invariance

$$2\bar{A}^0(00)+\sqrt{2}\bar{A}^0(+-)=3\alpha^*A^-(-0). \qquad (10)$$

We see that if $\Delta I=\frac{5}{2}$ transitions are forbidden then $\alpha=\frac{2}{3}$ and

$$A^0(00)+\sqrt{2}A^0(+-)=2A^+(+0), \quad \text{(Ref. 23)}.$$

If $\Delta I=\frac{3}{2}$ transitions are forbidden, $\alpha=-1$ and

$$A^0(00)+\sqrt{2}A^0(+-)+3A^+(+0)=0.$$

Other values of α, in general complex, correspond to different types of mixing between the $\Delta I=\frac{3}{2}$ and $\Delta I=\frac{5}{2}$ transitions.

Instead of using Eqs. (9) and (10) directly, it is more convenient to express them in terms of the set of parameters introduced in Eqs. (2), or in Eqs. (3). Firstly, using the definitions of $|K_L\rangle$ and $|K_S\rangle$ given in Eq. (1), we may rewrite Eqs. (9) and (10) in the form

$$2A_S(00)+\sqrt{2}A_S(+-)=3[\alpha pA^+(+0)-\alpha^*qA^-(-0)],$$
$$2A_L(00)+\sqrt{2}A_L(+-)=3[\alpha pA^+(+0)+\alpha^*qA^-(-0)].$$

Adding and subtracting, and using the parameters defined in Eqs. (3), we have

$$\sqrt{2}\eta(1+\eta_{00})+(1+\eta_{+-})=3\sqrt{2}\alpha pA^+(+0)/A_S(+-),$$
$$\sqrt{2}\eta(1-\eta_{00})+(1-\eta_{+-})=-3\sqrt{2}\alpha^*qA^-(-0)/A_S(+-),$$

or equivalently, from Eqs. (2)

$$\frac{\omega+\epsilon'}{\sqrt{2}+\omega}=\sqrt{2}\alpha p\frac{A^+(+0)}{A_S(+-)}, \qquad (11)$$

$$\frac{\omega-\epsilon'}{\sqrt{2}+\omega}=-\sqrt{2}\alpha^*q\frac{A^-(-0)}{A_S(+-)}. \qquad (12)$$

Writing α in polar form

$$\alpha=|\alpha|e^{i\theta_\alpha},$$

$|\alpha|$ may be found by adding the squared moduli in Eqs. (11) and (12) giving

$$|\omega|^2+|\epsilon'|^2=|\alpha|^2[\gamma^+(+0)/\gamma_S(+-)] \times[2+|\omega|^2+2\sqrt{2}\,\mathrm{Re}\omega], \qquad (13)$$

where $\gamma^+(+0)$ and $\gamma_S(+-)$ are the decay rates for $K^+\to\pi^+\pi^0$ and $K_S\to\pi^+\pi^-$ divided by their corresponding phase-space factors.[24]

To solve for θ_α, the phase of α, we divide Eq. (12) by Eq. (11)

$$\frac{\epsilon'-\omega}{\epsilon'+\omega}=\frac{q}{p}\frac{A^-(-0)}{A^+(+0)}e^{-2i\theta_\alpha}. \qquad (14)$$

From the definition of α in Eq. (8) it can be readily seen that the phases of α, A_2, and A_2^+ are related. With

$$A_2=|A_2|e^{i\theta_2} \quad \text{and} \quad A_2^+=|A_2^+|e^{i\theta_+},$$

we have

$$\theta_2=\theta_\alpha+\theta_+ \quad \text{up to} \quad \pm 2n\pi. \qquad (15)$$

This completes the analysis of neutral and charged K decays into two pions. Before proceeding to the details of the numerical estimates for the parameters describing $K\to\pi\pi$ decays, we will close this section with a discussion of some aspects of the unitarity condition of the K^0-$\bar{K}^0$ system. This provides an extra piece of information which we shall use later.

C. Unitarity Condition for the $K^0-\bar{K}^0$ System

Let M_S and M_L be the complex masses

$$M_S=m_S-i\Gamma_S/2 \quad \text{and} \quad M_L=m_L-i\Gamma_L/2, \qquad (16)$$

where m_S and m_L are the observed real masses of K_S and K_L, and Γ_S and Γ_L their corresponding total decay rates. The unitarity condition is the statement of the following relations:

$$\Gamma_S=\sum_F|\langle F|H|K_S\rangle|^2; \quad \Gamma_L=\sum_F|\langle F|H|K_L\rangle|^2,$$

and

$$-i(M_L^*-M_S)\langle K_L|K_S\rangle=\sum_F\langle F|H|K_L\rangle^*\langle F|H|K_S\rangle, \qquad (17)$$

where the summations are extended over all possible final states. From Eqs. (1) and (16), the left-hand side of Eq. (17) reads

$$[\tfrac{1}{2}(\Gamma_L+\Gamma_S)-i\Delta m](|p|^2-|q|^2), \quad \text{with} \quad \Delta m\equiv m_L-m_S.$$

To estimate the right-hand side of Eq. (17) we separately take into account contributions from the $(\pi\pi)_{I=0}$ and $(\pi\pi)_{I=2}$ modes, from the semileptonic modes, and from the 3π modes. These should provide a good approximation to the total sum.

(i) Contributions from the π-π Decays

Using the parametrization defined in Eq. (2), we have

$$\sum_{I=0,2}\langle(\pi\pi)_I|H|K_L\rangle^*\langle(\pi\pi)_I|H|K_S\rangle =(\epsilon^*+\epsilon'^*\omega)\Gamma_S(I=0). \qquad (18)$$

(ii) Contributions from the Semileptonic Decays

With neglect of "possible" neutral semileptonic decays, we are limited to final states

$$|\pi^+l^-\bar{\nu}_l\rangle \quad \text{and} \quad |\pi^-l^+\nu_l\rangle,$$

with l=electron and muon. Then, if we define the following set of amplitudes:

$\Delta S=\Delta Q$ amplitudes	$\Delta S=-\Delta Q$ amplitudes
$A(K^0\to\pi^-l^+\nu_l)=f$	$A(\bar{K}^0\to\pi^-l^+\nu_l)=g$
$A(\bar{K}^0\to\pi^+l^-\bar{\nu}_l)=f^*$	$A(K^0\to\pi^+l^-\bar{\nu}_l)=g^*$,

[23] This sum rule has been previously discussed in the literature [see, e.g., E. C. G. Sudarshan, Nuovo Cimento 41, A283 (1966)].

[24] More precisely, the relation between, e.g., $A^+(+0)$ and the corresponding decay rate is

$$\Gamma(K^+\to\pi^+\pi^0)=(1/16\pi M^3)[M^2-(m_++m_0)^2]^{1/2} \times[M^2-(m_+-m_0)^2]^{1/2}|A^+(+0)|^2.$$

using Eq. (2) we have

$$\sum_{F\epsilon(\pi l\nu)} \langle F|H|K_L\rangle^* \langle F|H|K_S\rangle$$
$$=(|p|^2-|q|^2)(|f|^2+|g|^2)+4i\,\mathrm{Im}(pfq^*g^*). \quad (19)$$

(iii) Contributions from 3π Decay Modes

Here, the possible final states are

$$|\pi^+\pi^-\pi^0\rangle \quad \text{and} \quad |\pi^0\pi^0\pi^0\rangle.$$

Too little is still known about the 3π decay modes to enable an analysis in terms of eigenstates of total isospin to be made, similar to that for the 2π decay modes. A convenient procedure to take into account present experimental data is the following: We define the amplitudes

$$a_S(+-0)\equiv A[K_S\to \pi^+\pi^-\pi^0],$$
$$a_L(+-0)\equiv A[K_L\to \pi^+\pi^-\pi^0],$$
$$a_S(000)\equiv A[K_S\to \pi^0\pi^0\pi^0],$$
$$a_L(000)\equiv A[K_L\to \pi^0\pi^0\pi^0],$$

with

$$a_S(+-0)/a_L(+-0)=x(+-0)+iy(+-0);$$
$$a_S(000)/a_L(000)=x(000)+iy(000).$$

Then

$$\sum_{F\epsilon(3\pi)} \langle F|H|K_L\rangle^* \langle F|H|K_S\rangle$$
$$=\{[x(+-0)+iy(+-0)]B_L(+-0)$$
$$+[x(000)+iy(000)]B_L(000)\}\Gamma_L, \quad (20)$$

where $B_L(+-0)$ and $B_L(000)$ are the total branching ratios of K_L into the $\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$ modes.

Taking the imaginary part in both sides of Eq. (17) and using Eqs. (18)–(20), we have

$$-\Delta m(|p|^2-|q|^2)=\mathrm{Im}(\epsilon^*+\epsilon'^*\omega)\Gamma_S+4\mathrm{Im}(pfq^*g^*)$$
$$+[y(+-0)B_L(+-0)+y(000)B_L(000)]\Gamma_L, \quad (21)$$

where we have taken $\Gamma_S(I=0)\simeq\Gamma_S$, and neglected contributions to the summation over final states other than those coming from 2π modes, semileptonic modes, and 3π modes.

To extract information from Eq. (21) we shall rewrite it in terms of quantities which are, in principle, directly measurable. Firstly, we recall [see Eq. (5a)] that

$$q/p=(1-\epsilon)/(1+\epsilon). \quad (22)$$

Then

$$|p|^2-|q|^2=2\,\mathrm{Re}\epsilon/(1+|\epsilon|^2)$$

and

$$2pq^*=\frac{1-|\epsilon|^2}{1+|\epsilon|^2}+i\frac{2\,\mathrm{Im}\epsilon}{1+|\epsilon|^2}.$$

Also, experimental results on semileptonic decays of neutral K mesons are usually expressed in terms of a complex parameter x such that

$$g/f=|x|e^{i\varphi}.$$

Thus we can write

$$4\,\mathrm{Im}(pfq^*g^*)=\frac{2|f|^2|x|}{1+|\epsilon|^2}\{2\,\mathrm{Im}\epsilon\cos\varphi-(1-|\epsilon|^2)\sin\varphi\},$$

where θ_ϵ is the phase of ϵ

$$\epsilon=|\epsilon|e^{i\theta_\epsilon}.$$

In terms of measured quantities $|f|^2$ is given by

$$|f|^2\simeq B_L(\text{leptons})\Gamma_L/2,$$

where B_L(leptons) is the total branching ratio of K_L into *all* semileptonic modes.

With these transformations, Eq. (21) may be written

$$\tan\theta_\epsilon=\frac{2\Delta m}{\Gamma_S}\left[\frac{1}{1+|\epsilon|^2}\right]+\frac{\mathrm{Im}(\epsilon'^*\omega)}{\mathrm{Re}\epsilon}$$
$$+\frac{\Gamma_L}{\Gamma_S}\left\{\frac{|x|}{1+|\epsilon|^2}\frac{B_L(\text{leptons})}{\mathrm{Re}\epsilon}[2\,\mathrm{Im}\epsilon\cos\varphi\right.$$
$$-(1-|\epsilon|^2)\sin\varphi]+\frac{1}{\mathrm{Re}\epsilon}[y(+-0)B_L(+-0)$$
$$\left.+y(000)B_L(000)]\right\}. \quad (23)$$

We shall use this relation in the next section to obtain numerical estimates of θ_ϵ, the phase of ϵ.

III. ESTIMATE OF PARAMETERS DESCRIBING $K\to 2\pi$ DECAYS

The purpose of this section is to discuss how to obtain numerical estimates of the various parameters introduced in the last section from present experimental data. The experimental input data which we shall use have been compiled in Table I. The absence of an entry for $y(000)$ will be explained below.

From the beginning we shall exploit the fact that

TABLE I. Experimental input data. (All other data used are taken from the tables of Rosenfeld *et al.*[a])

Quantity	Experimental value	Reference
$\Delta m\equiv m_L-m_S$	$(5.41\pm0.25)\times10^9$ sec^{-1}	a
$\lvert\eta_{+-}\rvert$	$(1.94\pm0.09)\times10^{-3}$	c
θ_{+-}	$77°\pm19°$	b
$\lvert\eta_{00}\rvert$	$(4.87\pm0.44)\times10^{-3}$	d
φ	$50°\pm26°$	e
X	(0.26 ± 0.10)	e
$y(+-0)$	(0.33 ± 0.61)	f

[a] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).
[b] Mean of C. Rubbia and J. Steinberger, Phys. Letters **24B**, 531 (1967) and M. Bott-Bodenhausen *et al.*, *ibid.* **24B**, 438 (1967).
[c] Compilation by V. L. Fitch. Quoted by J. W. Cronin *et al.*, Phys. Rev. Letters **18**, 25 (1967).
[d] Mean of the experiments of Refs. 2 and 3.
[e] Reference 28.
[f] Mean of Ref. 27 and J. A. Anderson *et al.*, Phys. Rev. Letters **14**, 475 (1965); **16**, 968(E) (1966).

$\epsilon \ll 1$ and write Eqs. (5b) and (5c) in the following form:

$$\epsilon' = i \frac{\mathrm{Im} A_2}{A_0} e^{i(\delta_2 - \delta_0)}, \tag{24a}$$

$$\omega = \frac{\mathrm{Re} A_2}{A_0} e^{i(\delta_2 - \delta_0)}. \tag{24b}$$

Also, to a first approximation, we shall neglect the contribution of ω to η_{+-} and η_{00} [see Eqs. (4a) and (4b)]. Thus

$$\eta_{+-} = \epsilon + \frac{\epsilon'}{\sqrt{2}}, \tag{25a}$$

$$\eta_{00} = \epsilon - \sqrt{2}\epsilon'. \tag{25b}$$

Eventually, we wish to get numerical estimates on θ_{00}, the phase of η_{00}, and on the complex parameters ϵ, ϵ', and ω. Then, from $|\epsilon'|$ and ω, and using Eq. (13), we shall evaluate $|\alpha|$. Also, we are interested in obtaining numerical estimates of the possible solutions for the amplitude A_2.

A. The Wu-Yang Triangle Construction

This method[25] starts with the expressions of η_{+-} and η_{00} as given in Eqs. (25a) and (25b). At present, only η_{+-} and $|\eta_{00}|$ are known from experiment (see Table I). Therefore, no unique solution for ϵ and ϵ' is known *a priori*. However, a "first approximation" to θ_ϵ, the phase of ϵ, may be obtained from the unitarity condition, Eq. (23). With neglect of all contributions to the right-hand side except for the first term, we have

$$\tan\theta_\epsilon \sim 2\Delta m/\Gamma_S,$$

i.e.,

$$\theta_\epsilon \sim 44° \quad \text{or} \quad 224°.$$

To each of these values there corresponds a solution for ϵ, ϵ', and θ_{00}, as can be easily seen by plotting in an Argand diagram the complex numbers related by Eqs. (25a) and (25b).

Only a very rough picture is obtained by restricting the unitarity condition to this approximation.[26] In fact, for small values of Reϵ, other terms in the right-hand side of Eq. (23) can be dominant. Even if $2\Delta m/\Gamma_S$ is the dominant term, corrections from the other terms may not be negligible. It is clear that a more detailed numerical analysis is necessary.

The calculation procedure which we have used is the following:

(i) We start with the values

$$\theta_\epsilon = 44° \quad \text{and} \quad \theta_\epsilon = 224°$$

obtained by the "first approximation" mentioned above.

(ii) From Eqs. (25a) and (25b) it can be seen that $|\epsilon|$ satisfies the quadratic equation

$$a|\epsilon|^2 + b|\epsilon| + c = 0,$$

with

$$a = 9,$$
$$b = -12|\eta_{+-}|\cos(\theta_\epsilon - \theta_{+-}),$$
$$c = 4|\eta_{+-}|^2 - |\eta_{00}|^2.$$

We solve this equation for each of the values $\theta_\epsilon = 44°$ and $\theta_\epsilon = 224°$. It is clear that for $c \leq 0$ (the experimental situation) and $0 \leq \theta_{+-} \leq \pi/2$ (again the experimental situation) each value of θ_ϵ leads to a unique value for $|\epsilon|$, and hence ϵ.

(iii) Knowing ϵ and η_{+-}, we can determine ϵ' from Eq. (25a), and from Eq. (24a): $\delta_2 - \delta_0$ up to $\pm n\pi$.

(iv) Next we go to Eq. (4c). Taking the squared modulus and solving for Reω we get

$$\mathrm{Re}\omega = \frac{1 - 2|\eta|^2 + 2|\omega|^2 - |\eta|^2|\omega|^2}{2\sqrt{2}(1 + |\eta|^2)}. \tag{26}$$

Knowing the value of $\delta_2 - \delta_0$ obtained in (iii), and using Eq. (24b), we can solve Eq. (26) for $|\omega|$ (and hence Reω) if we assume $|\omega|^2 < 1$.

(v) We may now recalculate θ_ϵ using the full expresrion Eq. (23). The whole iteration scheme is then sepeated until a stable set of parameters results. The errors are likewise calculated by the same iteration procedure.

We have performed this calculation evaluating contributions to the unitarity condition coming from the 2π, 3π, and semileptonic decay modes. The effect of the 3π and semileptonic contributions was estimated by using the preliminary results of two recent experiments for $(\pi\pi)_{I=2}$ term. As can be seen from our calculation below, this approximation is only good in the case where Reϵ is large.

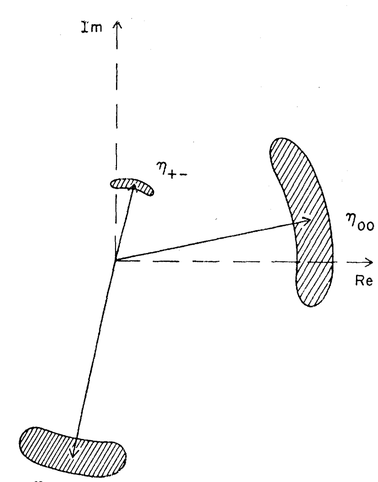

FIG. 1. Argand diagram of η_{+-} and η_{00} for the possible solutions given in Table II (errors included).

[25] See T. T. Wu and C. N. Yang, Ref. 18.

[26] This approximation was followed in a preliminary version of this work [B. R. Martin and E. de Rafael, Brookhaven National Laboratory BNL Report No. 10933 (unpublished)]. See also E. Yen, Phys. Rev. Letters **18**, 513 (1967). This author has also evaluated $\tan\theta_\epsilon$ partially taking into account contributions from the semileptonic terms and the 3π terms. However, he neglects the

these modes.[27–29] (See the corresponding input in Table I.) These experiments [27,28] are not the only ones which have been made, although the results are compatible with those of other experiments.[30] However, a more careful choice of input data for these modes is not warranted at this time since the value of $y(000)$ is not known from experiment. Because of this lack of knowledge we have set $y(000)$ to be zero within an error which we have assumed to be equal to the error on $y(+-0)$. The final set of parameters shown in Table II is compatible, within the errors, with the results obtained by neglecting all but the 2π modes. For solution a, the term $2\Delta m/\Gamma_S$ is dominant in Eq. (23), the other terms giving corrections of the order of a few percent. However, for solution b, the term $\mathrm{Im}(\epsilon'^*\omega)/\mathrm{Re}\epsilon$, and the contribution from the semileptonic terms, are both significantly larger than the contribution from the first term. In Fig. 1 we have plotted the vectors η_{+-}, (known experimentally), and η_{00} for each solution with their errors.

TABLE II. Values of parameters describing $K\to\pi\pi$ decays. Contributions to the unitarity condition from all the terms in Eq. (23) have been considered. The phase-shift difference $\delta_2-\delta_0$ is determined only to $\pm n\pi$.

Parameter	Solution a	Solution b
$\mathrm{Re}\epsilon$	$(1.90\pm0.51)\times10^{-3}$	$-(0.48\pm1.23)\times10^{-4}$
θ_ϵ	$36°\pm9°$	$-(98°\pm21°)$
$\mathrm{Re}\omega$	$(3.1\ \pm1.3)\times10^{-2}$	$(2.8\ \pm1.3)\times10^{-2}$
$\delta_2-\delta_0$	$71°+16°$	$-(12°\pm16°)$
θ_{00}	$12°\pm25°$	$-(102°\pm16°)$
$\lvert\epsilon'\rvert$	$(2.2\ \pm2.4)\times10^{-3}$	$(3.2\ \pm0.4)\times10^{-3}$

B. Estimates of $|\alpha|$ and A_2

From the values obtained for $|\epsilon'|$, $|\omega|$, and $\mathrm{Re}\omega$, in Table II, we can solve Eq. (13) for $|\alpha|$. The values corresponding to solutions a and b are given in Table III. The value for solution b excludes the possibility of a pure $\Delta I=\frac{5}{2}$ contribution to the $K\to\pi\pi$ $(I=2)$ amplitudes A_2 and A_2^+, in which case $\alpha=-1$; however, the case of a pure $\Delta I=\frac{3}{2}$ contribution, for which $\alpha=\frac{2}{3}$, is only one standard deviation from the estimated value. For solution a, both extreme possibilities, pure $\Delta I=\frac{3}{2}$ or pure $\Delta I=\frac{5}{2}$ contribution to the A_2 and A_2^+ amplitudes, lie within the estimated errors.

The modulus of A_2 can be now directly estimated from the values of $|\alpha|$ given in Table III. By definition [see Eq. (8)]

$$|A_2|=(\sqrt{\tfrac{3}{2}})|\alpha|[\gamma_+(+0)]^{1/2},$$

which leads to the values given in Table III.

[27] Y. Cho *et al.*, Brookhaven National Laboratory Report No. BNL 10609, 1967 (unpublished).

[28] D. G. Hill *et al.*, Phys. Rev. Letters **19**, 668, 1967.

[29] We wish to thank Dr. Sakitt for an interesting discussion concerning the work of Refs. 27 and 28.

[30] J. A. Anderson *et al.*, Phys. Rev. Letters **14**, 475 (1965); **16**, 986(E) (1966); M. Baldo-Ceolin *et al.*, Nuovo Cimento **38**, 684 (1965); B. Aubert, Phys. Letters **17**, 59 (1965); P. Franzini, L. Kirsch, P. Schmidt, J. Steinberger, and R. J. Plano, Phys. Rev. **140**, B127 (1965).

TABLE III. Estimated values of $|\alpha|$, A_2, and θ_2 from the experimental input listed in Table I and the parameters listed in Table II. The angle θ_2 is determined only to $\pm n\pi$.

Parameter	Solution a	Solution b
$\lvert\alpha\rvert$	$1.4\ \pm1.3$	0.44 ± 0.20
$\lvert A_2\rvert$	$(4.81\pm4.47)\times10^{16}\ \mathrm{sec}^{-1}$	$(1.51\pm0.69)\times10^{16}\ \mathrm{sec}^{-1}$
$\theta_2(=\arg A_2)$	$-(1.3°\pm1.9°)$	$(6.3°\pm2.9°)$

The phase of A_2, θ_2, can be obtained, up to $\pm n\pi$, from the approximate expression of ϵ' and ω given in Eqs. (24a) and (24b). We have

$$\cos\theta_2=\frac{|\omega|}{(|\omega|^2+|\epsilon'|^2)^{1/2}}\quad\text{and}\quad\sin\theta_2=\frac{\mp|\epsilon'|}{(|\omega|^2+|\epsilon'|^2)^{1/2}},$$

where the minus sign has to be taken in the case corresponding to the solution a of the Wu-Yang triangle construction. The values we obtain are given in the third line of Table III. This determination of θ_2 can be improved, if necessary, using Eqs. (14) and (15).

IV. POSSIBLE CONFIGURATIONS OF $A^{(3/2)}$ AND $A^{(5/2)}$ AMPLITUDES LEADING TO THE OBSERVED VIOLATION OF *CP* INVARIANCE

As can be seen from Eqs. (6) and (7), the determination of the irreducible isospin amplitudes $A^{(3/2)}$ and $A^{(5/2)}$ requires a knowledge of θ_+, the phase of A_2^+. Unfortunately, the determination of this phase is not directly accessible to experiment, and it will probably be some time before an indirect measurement is available. (We shall return to this point in the next section.) With θ_+ unknown, the possible configurations of $A^{(3/2)}$ and $A^{(5/2)}$ amplitudes can be seen using a geometrical construction. In fact, as we concluded in the last section, the amplitude A_2 has two solutions (see Table III), which means we need to draw two diagrams. They are shown in Figs. 2(a) and 2(b), corresponding to the values of A_2 for solution a and solution b, respectively. In these diagrams, the vector OA represents the amplitude A_2, and $OC=(\frac{2}{5})A_2$. With center at C we have drawn a circle of radius $\frac{1}{5}\sqrt{6}|A_2^+|$. Then, it can be easily seen that for a given value of θ_+ there corresponds a point B in the circle such that

$$OB=\frac{1}{\sqrt{2}}A^{(3/2)}\quad\text{and}\quad AB=\frac{1}{\sqrt{2}}A^{(5/2)}.$$

It is also convenient to construct a diagram which, for a fixed value of $|\alpha|$, gives the ratio $|A^{(5/2)}/A^{(3/2)}|$ as a function of θ_α, which is the relative phase of A_2 with respect to A_2^+: $\theta_\alpha=\theta_2-\theta_+$. With

$$R\equiv A^{(5/2)}/A^{(3/2)}$$

and using Eq. (8), the expression for $|R|$ as a function of $|\alpha|$ and θ_α can be readily obtained. In Figs. 3(a) and 3(b) we have plotted, in polar coordinates, $|R|$ and $|\alpha|$ as a function of θ_α. The radii of the circles drawn

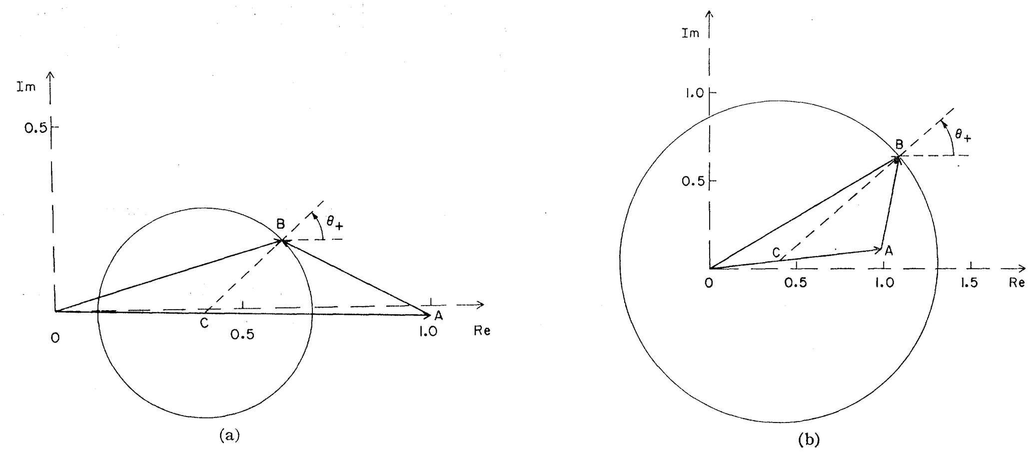

FIG. 2. (a) Geometrical construction of the amplitudes $A^{(3/2)}$ and $A^{(5/2)}$ for a given value of θ_+, the phase of A_2^+. The vector OA is fixed and represents the value of A_2 for solution a in Table III. The circle drawn has its center at C ($OC=\frac{2}{5}A_2$) and radius equal to $\frac{1}{5}(\sqrt{6})|A_2^+|$. Then $OB=A^{(3/2)}/\sqrt{2}$ and $AB=A^{(5/2)}/\sqrt{2}$. The vector OA is taken as unity. (b) Geometrical construction of the amplitudes $A^{(3/2)}$ and $A^{(5/2)}$ for a given value of θ_+, the phase of A_2^+. The vector OA is fixed and represents the values of A_2 for solution b in Table III. The circle drawn has its center at C ($OC=\frac{2}{5}A_2$) and radius equal to $\frac{1}{5}(\sqrt{6})|A_2^+|$. Then $OB=A^{(3/2)}/\sqrt{2}$ and $AB=A^{(5/2)}/\sqrt{2}$. The vector OA is taken as unity.

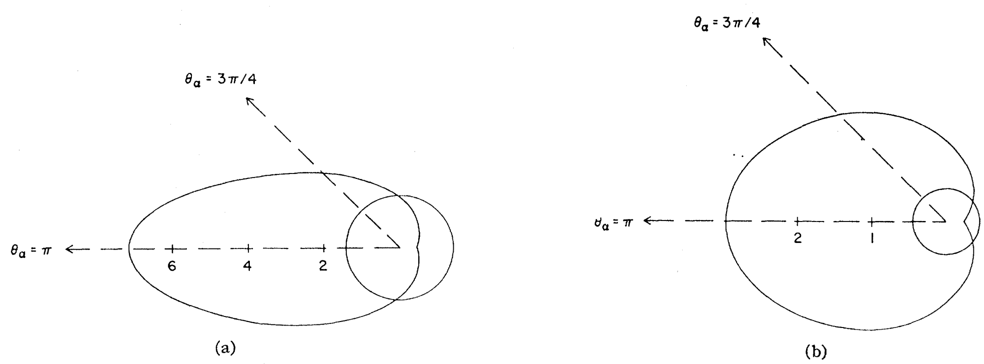

FIG. 3. (a) Polar diagram of $|R|$ and $|\alpha|$ as a function of θ_α, corresponding to solution a. The radius of the circle drawn corresponds to the value $|\alpha|=1.4$, which fixes the scale of the figure. (b) Polar diagram of $|R|$ and $|\alpha|$ as a function of θ_α, corresponding to solution b. The radius of the circle drawn corresponds to the value $|\alpha|=0.44$, which fixes the scale of the figure.

correspond to the values $|\alpha|=1.4$ in Fig. 3(a) and $|\alpha|=0.44$ in Fig. 3(b). These are the mean values of $|\alpha|$ obtained in the preceding section for solutions a and b in Table II.

For a given value of $|R|$, the phase of R, θ_R, which is the relative phase of $A^{(5/2)}$ with respect to $A^{(3/2)}$, is determined, in general, up to a sign.

Of all the possible configurations for $A^{(3/2)}$ and $A^{(5/2)}$ which can be drawn at present, there are some of special interest for which we give numerical estimates in Tables IV and V. (Table IV refers to solution a, Table V to solution b.) They correspond to one of the following assumptions:

(i) The amplitude A_2^+ is real;
(ii) Maximal violation of CP invariance in $K^\pm\to\pi^\pm\pi^0$ decays;
(iii) $A^{(3/2)}$ complex, $A^{(5/2)}$ real;
(iv) $A^{(5/2)}$ complex, $A^{(3/2)}$ real.

If CP is conserved in $K^\pm\to\pi^\pm\pi^0$ decays, then (i) becomes a necessary consequence.[31] Assumption (ii) corresponds to the case where the real parts of the amplitudes $A^{(3/2)}$ and $A^{(5/2)}$, when combined to form the amplitude A_2^+, cancel. Then A_2^+ is pure imaginary. It is in this sense that we mean maximal violation of CP invariance in $K^\pm\to\pi^\pm\pi^0$ decays.

If the violation of CP invariance, observed as an interference between A_2 and A_0, is due to the $A^{(3/2)}$ amplitude alone, then we have (iii); if it is due to the $A^{(5/2)}$ amplitude alone, then we have (iv).

If the origin of the violation of CP invariance in $K\to\pi\pi$ decays is due to C-nonconserving electromagnetic interactions, as suggested by Bernstein,

[31] Since the discovery that the A_2 amplitude has an imaginary part we know that either $A^{(3/2)}$ and/or $A^{(5/2)}$ must also have an imaginary component. Therefore, unless there is an accidental cancellation of the imaginary parts of $A^{(3/2)}$ and $A^{(5/2)}$ in Eq. (7), we can expect that, in general, A_2^+ has also an imaginary part. It is interesting, nevertheless, to see which values of $A^{(3/2)}$ and $A^{(5/2)}$ correspond to the case where A_2^+ is real.

TABLE IV. Estimated values of the amplitudes $A^{(3/2)}/|A_2|$ and $A^{(5/2)}/|A_2|$, in the case of solution a, for the particular configurations discussed in Sec. IV. As can be seen from the geometrical construction in Fig. 2(a), there are two possible solutions corresponding to each assumption.

Assumption	$\|A^{(3/2)}\|/\|A_2\|$	Phase of $A^{(3/2)}$	$\|A^{(5/2)}\|/\|A_2\|$	Phase of $A^{(5/2)}$
A_2^+ real	0.96	−1°	0.46	179°
	0.17	−1°	1.2	179°
A_2^+ pure imaginary	0.69	−36°	0.94	−156°
	0.69	33°	0.94	154°
$\mathrm{Im}A^{(5/2)}=0$	0.96	−2°	0.46	18°
	0.18	−10°	1.2	180°
$\mathrm{Im}A^{(3/2)}=0$	0.17	0°	1.2	178°
	0.96	0°	0.46	176°

TABLE V. Estimated values of the amplitudes $A^{(3/2)}/|A_2|$ and $A^{(5/2)}/|A_2|$ in the case of solution b, for the particular configurations discussed in Sec. IV. As can be seen from the geometrical construction in Fig. 2(b) there are two possible solutions corresponding to each assumption.

Assumption	$\|A^{(3/2)}\|/\|A_2\|$	Phase of $A^{(3/2)}$	$\|A^{(5/2)}\|/\|A_2\|$	Phase of $A^{(5/2)}$
A_2^+ real	1.8	2°	0.48	−11
	0.73	176°	2.1	−177°
A_2^+ pure imaginary	1.5	68°	1.5	125°
	1.3	−65°	1.6	−149°
$\mathrm{Im}A^{(5/2)}=0$	1.8	5°	0.47	180°
	0.73	169°	2.1	180°
$\mathrm{Im}A^{(3/2)}=0$	1.8	0°	0.48	160°
	0.73	180°	2.1	−175°

Feinberg, and Lee,[32] and Barshay,[33] then, in general, we can expect that both amplitudes $A^{(3/2)}$ and $A^{(5/2)}$ are present and they are complex. Assuming that the nonleptonic Hamiltonian is of the current×current type, with currents belonging to octets of $SU(3)$, we can expect contributions to $\mathrm{Re}A^{(3/2)}$ due to the $\Delta I=\frac{3}{2}$ part of the Hamiltonian and/or to weak $(\Delta I=\frac{1}{2})$ ×electromagnetic (*CP* even) interactions; contributions to $\mathrm{Re}A^{(5/2)}$ due to weak×electromagnetic (*CP* even) interactions; and contributions to $\mathrm{Im}A^{(3/2)}$ and $\mathrm{Im}A^{(5/2)}$ due to weak×electromagnetic (*CP* odd) interactions. It is clear that in this case, accidental cancellation of the imaginary parts of $A^{(3/2)}$ and $A^{(5/2)}$ could lead to case (i); while accidental cancellation of their real parts could lead to case (ii).

The violation of *CP* invariance could also be an intrinsic property of the weak interactions.[34] Let us denote by H_W the total weak nonleptonic Hamiltonian,

$$H_W=H_W(CP\text{ even})+H_W(CP\text{ odd}).$$

If H_W (*CP* even) and H_W (*CP* odd) are current×current Hamiltonians, with the currents belonging to $SU(3)$ octets, then *only* $A^{(3/2)}$ can be complex. (This is due to the fact that no $\Delta I=\frac{5}{2}$ component can be accommodated in the product of two octets.) In this case, contributions to $\mathrm{Re}A^{(5/2)}$ would be due to H_W $(CP\text{ even})\times H_\gamma$, where H_γ is the electromagnetic Hamiltonian. Contributions to $\mathrm{Im}A^{(5/2)}$ would be due to H_W $(CP\text{ odd})\times H_\gamma$ and therefore we would have $|A^{(5/2)}|\ll|A^{(3/2)}|$. This could lead to some of the situations corresponding to (iii) above.

If H_W (*CP* odd) induces pure $\Delta I=\frac{5}{2}$ transitions, then $A^{(5/2)}$ can be complex; contributions to $\mathrm{Im}A^{(3/2)}$ being now due to H_W $(CP\text{ odd})\times H_\gamma$ and therefore $|\mathrm{Im}A^{(3/2)}|\ll|\mathrm{Im}A^{(5/2)}|$. This could lead to the situation mentioned in (iv) above.

V. OUTLOOK

We will conclude with a brief review of suggested further experiments on K decays and their implications for the "degeneracy" of present solutions.

(1) To distinguish between the two solutions of the Wu-Yang construction *at least one* of the quantities

$$\mathrm{Re}\epsilon,\quad \delta_2-\delta_0,\quad \theta_{00}$$

must be measured. Preliminary experiments to measure θ_{00} using regeneration techniques are in progress. An experiment to measure $\mathrm{Re}\epsilon$ was suggested by Kaplan,[35] and independently by Byers, McDowell, and Yang.[18] It consists of determining the charge asymmetry in three-body semileptonic K_L decay

$$A=\Gamma(K_L\to\pi^+l^-\bar{\nu}_l)/\Gamma(K_L\to\pi^-l^+\nu_l).$$

In our notations

$$A=\left|\frac{pg^*+qf^*}{pf+qg}\right|^2\simeq 1-4\,\mathrm{Re}\epsilon\left(\frac{1-|x|^2}{1-2|x|\cos\varphi+|x|^2}\right). \quad (27)$$

A sufficiently accurate measurement of A could distinguish between the two values of $\mathrm{Re}\epsilon$ available at present (see Table II). The phase-shift difference $\delta_2-\delta_0$ is not directly accessible to experiment. However, there is evidence from peripheral-model calculations[36] that δ_2 is small and negative below 500 MeV, and that δ_0 is *positive*, but nonresonant, in this region. This latter conclusion is supported by evidence from many other sources[37] and so may be taken as some indication in favor of solution b, even allowing for the $n\pi$ ambiguity in the determination of this phase.

(2) It would be of *great interest* to have more accurate measurements of $|\eta_{00}|$ and $|\eta|$, for then the errors on most of the parameters listed in Table II could be

[32] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965).

[33] S. Barshay, Phys. Letters **17**, 78 (1965).

[34] The possibility of *CP* nonconservation from a $\Delta I\geq\frac{3}{2}$ amplitude alone has been suggested by Truong [T. N. Truong, Phys. Rev. Letters **13**, 358A (1965)]. See also Wu and Yang, Ref. 18.

[35] J. M. Kaplan, Phys. Rev. **139**, B1065 (1965).

[36] See, e.g., E. West, J. H. Boyd, A. R. Erwin, and W. D. Walker, Phys. Rev. **149**, 1089 (1966); L. W. Jones *et al.*, Phys. Letters **21**, 590 (1966); W. D. Walker, J. Carroll, A. Garfinkel, and B. Y. Oh, Phys. Rev. Letters **18**, 630 (1967).

[37] See, e.g., P. Singer, Lectures presented at the Finnish Summer School in Theoretical Physics, Helsinki, 1966 (unpublished); W. D. Walker, Rev. Mod. Phys. **39**, 695 (1967).

reduced considerably. We recall that both $|\eta_{00}|$ and $|\eta|$ are directly accessible to experiment:

$$|\eta_{00}|^2=\frac{\Gamma(K_L\to\pi^0\pi^0)}{\Gamma(K_S\to\pi^0\pi^0)}$$

and

$$|\eta|^2=0.986\frac{\Gamma(K_S\to\pi^0\pi^0)}{\Gamma(K_S\to\pi^+\pi^-)},$$

where the numerical factor on the right-hand side is the phase-space correction.

(3) The determination of θ_+, the phase of the amplitude $A_2{}^+=(A^{(3/2)}-A^{(5/2)})/\sqrt{2}$ is a much more difficult task. Recently, it has been suggested by Cline,[38] that θ_+ could, in principle, be obtained from the measurement of the π^+-energy spectrum in $K^+\to\pi^+\pi^0\gamma$ decays. The method, however, needs the *a priori* knowledge of the direct matrix elements, and therefore becomes very much model-dependent. Experiments on $K^\pm\to\pi^\pm\pi^0\gamma$ decays (measurement of decay rates, spectra, and polarizations) are nevertheless very important in the sense that they can lead to the detection of sizeable *CP*-noninvariant effects.[39] A better understanding of these decays could lead, eventually, to the determination of the phase of the $A_2{}^+$ amplitude.

Note added in proof. Recently two values have been obtained for Reϵ by measuring the asymmetry in three-body semileptonic K_L decay [see Eq. (27)]. Ignoring the small $\Delta S=-\Delta Q$ correction term, Dorfan *et al.*[40] find (from $K_L\to\pi\mu\nu$) Re$\epsilon=(2.0\pm0.7)\times10^{-3}$, and Bennett *et al.*[41] find (from $K_L\to\pi e\nu$) Re$\epsilon=(1.11\pm0.18)\times10^{-3}$, both experiments showing that solution *a* is the physical one. However, this solution (see Table II) predicts a value for $(\delta_2-\delta_0)$ which is inconsistent with the evidence from a considerable body of other experiments[37] (even allowing for the fact that $(\delta_2-\delta_0)$ is determined only up to $\pm n\pi$). If one accepts the latter estimates[37] of $(\delta_2-\delta_0)$, then it is likely that the value of at least one of the input parameters in the $K^0-\bar{K}^0$ system is in error. A possible candidate is θ_{+-} whose value has fluctuated considerably in the past. The latest "world-average" value for θ_{+-} is[42] $60°\pm12°$, but the spread on the individual experiments is still considerable.

Using the techniques of Sec. III and the data of Table I[43] we find that[37] $-90°\lesssim(\delta_2-\delta_0)\lesssim0°$ implies $20°\lesssim\theta_{+-}\lesssim55°$, and for this range of θ_{+-}, $1.90\lesssim$ Re$\epsilon\lesssim2.05$. We note that for a value of $\theta_{+-}\sim45°$ a consistent picture emerges for the parameters of $K\to2\pi$ decay provided that Re$\epsilon\simeq2\times10^{-3}$. It is clear that an accurate measurements of θ_{+-} is needed.

ACKNOWLEDGMENT

We wish to thank the members of the Theory Group at Brookhaven for discussions.

[38] D. Cline, Nuovo Cim. **48A**, 566 (1967).

[39] Some of the possible effects have been recently discussed by G. Costa and P. K. Kabir, Phys. Rev. Letters **18**, 429 (1967); S. Barshay, *ibid.* **18**, 515 (1967); and N. Christ, Phys. Rev. **159**, 1292 (1967).

[40] D. Dorfan *et al.*, in Proceedings of the Stanford Conference on Electron and Photon Interactions at High Energies, 1967 (unpublished).

[41] B. Bennett *et al.*, in Proceedings of the Stanford Conference on Electron and Phonon Interactions at High Energies, 1967 (unpublished).

[42] V. L. Fitch, Lectures at the Second Hawaii Topical Conference, 1967 (unpublished).

[43] For $|\eta_{00}|$ we use the most recent value of $|\eta_{00}|=(4.17\pm0.30)\times10^{-3}$ as given by J. W. Cronin [in Proceedings of the Rochester Conference on Elementary Particles and Fields, 1967 (unpublished)].

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Model for Electron Excitation of the Nucleon*

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A very crude model of oscillations of the meson field in the nucleon is made which gives an excitation spectrum similar to that of the nucleon and which allows us to calculate the transition form factors for the allowed normal-parity Coulomb transitions $\frac{1}{2}^+\to\frac{3}{2}^-, \frac{5}{2}^+, \frac{7}{2}^-$, etc. The results are compared with the recent Cambridge Electron Accelerator (CEA) data, and predictions, applicable to the planned Stanford Linear Accelerator (SLAC) experiments, are made for the other relevant isobars and other momentum transfers.

1. INTRODUCTION

THE advent of very high-energy electron accelerators makes electron excitation a practical means of studying the details of the excited states of the nucleon. The well-known $J^\pi=\frac{3}{2}^+$, $T=\frac{3}{2}$ (1236 MeV) resonance has already been studied extensively with existing machines.[1–4] However, the nucleon is now

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